Robust Adaptive Control of a Hydraulic Press

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Abstract: The aim of this paper is to illustrate the design and the simulative validation of an adaptive controller developed for an hydraulic press. With respect to controllers actually implemented, the proposed solution assures similar performances in nominal conditions, but it is simpler to be tuned and it is able to estimate the leakage inside the hydraulic piston. Such an estimate is used to maintain the performances acceptable in a wide set of operative conditions and for predictive maintenance. By using singular perturbation arguments, we show robustness to slowly increasing leakage gains, which is a typical situation in a real-world application. The control scheme is validated on a simulative Simscape model.

Keywords: hydraulic systems, adaptive control, backstepping control, nonlinear control, exponential stability, singular perturbations

1. INTRODUCTION

This paper deals with the robust control of hydraulic presses used in many industrial sectors. Such system is characterised by a nonlinear behaviour and by the presence of several undesired elastic modes and delays, which makes its control definitely complicated, especially because of the challenging tracking error requirements. Furthermore, the system is highly time-varying due to the wear-out of the hydraulic components, and it is typically subject to huge external forces that can be treated as disturbances. Such forces have an order of magnitude of dozens of kilo-newtons (kN), and are responsible for tracking and steady-state errors if not properly compensated. Another major problem is that the position error is increasing in time because of the wear-out effect in the components. This fact is responsible not only for the degraded performances in the production cycle, but very often for the interruption of the machine to re-calibrate the controller. In addition, no information about the health of the hydraulic components is available e.g. for (predictive) maintenance.

The requirements that the controlled system has to meet are a maximum tracking error of $0.1 \div 0.5$ mm during the transient phase, and a maximum steady state error of one order of magnitude lower. Moreover, the maximum overshoot has to be lower than the 1%. Such severe requirements, in conjunction with the system non-idealities mentioned before, forces the adoption of nonlinear adaptive control systems that assure performances in nominal working conditions, as well as in uncertain conditions, when external forces and wear-out of the components are present. In this work we are mainly driven by the adaptive robust control framework, Yao (1997); Yao and Tomikuza (1997), and by the use of classical backstepping techniques, Krstic et al. (1995); Sepulchre et al. (1997). Such techniques have been already successfully employed to control hydraulic systems, see e.g. Bu and Yao (2000); Yao et al. (2000); Bu and Yao (2001); Yao et al. (2001), machine tools Yao et al. (1997), and other engineering systems.

An inner / outer loop control strategy is adopted. The outer loop is responsible for controlling the motion of the press having the force applied to the piston as input, while the inner one for generating such pressure by acting on a servo-valve. On top of this scheme, two adaptive laws have been designed. The first one is associated with the outer loop and responsible for compensating not only the external forces that are generated during the pressing phase, but also all the unmodelled effects that act on the mechanical sub-system, such as friction, non-idealities in the pressure supply system, and fluid compressibility. The second adaptive law is devoted to reduce the performance degradation due to the wear-out of the components, whose major effect is the generation of a leakage flux in the servo-valve. Such phenomenon is time-varying, but characterised by a dynamic that is largely much slower than the one associated with the working cycle of the press. Since the closed-loop system resulting from the inner / outer loop structure, together with the two adaptive laws, is exponentially stable when the unknown parameters are constant, we show that the control solution guarantees a limited performance degradation when the leakage slowly increases. The result is proved via singular perturbations, see Khalil (1996).

The paper is organised as follows. In Section 2, the main characteristics are presented, and the mathematical model used for control design is introduced in Section 3. Section 4 presents the control design. The performances of the obtained control law are then evaluated on a "virtual"

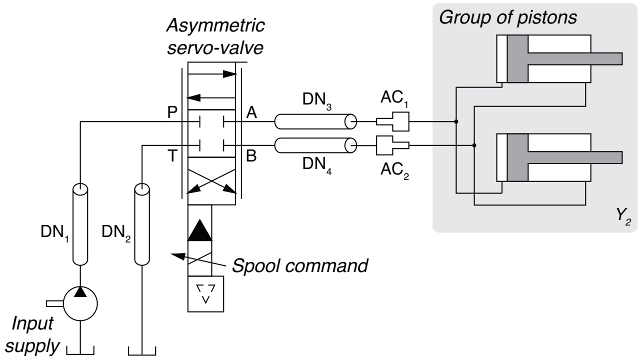

Fig. 1. Diagram of the Y_2-axis of the hydraulic press.

plant, i.e. a Simscape simulative model, in Section 5. Finally, conclusions are discussed in Section 6.

2. SYSTEM DESCRIPTION

In the press, 8 hydraulic axes moving in a synchronised manner are present. Even if different in size, maximum velocity and generated pressure, all the axes share a similar structure, schematically reported in the simplified diagram of Fig. 1. In this work, the focus is on a specific axis, referred to as Y_2, whose main components are:

- The pressure supply, responsible for providing a nominal pressure of 280 bar, even if in the real application the working pressure is generated by a hydraulic sub-system able to minimise the pressure oscillations caused by the load.
- The flexible pipes with circular section, denoted by DN_i, $i = 1, \ldots, 4$.
- Resistive elements, denoted by AC_j, $j = 1, 2$, taking into account the interconnection between the piston chambers and the pipes.
- A group of 2 pistons in which compressibility effect of the fluid are taken into account.
- An asymmetric 4-ways servo-valve, to move the piston back and forth.

One of the main requirements is to identify and compensate for the leakage in the hydraulic circuit. Leakage is one of the most critical problems in hydraulic systems, and the components usually responsible for it are pistons and valves. Generally speaking, a leakage flow is an undesired flow between two paths. Leakage flow depends on several environmental conditions and on their time-evolution, and thus it is hard to be modelled. In our model, it is thought of small and time-varying orifices that suddenly appear in the system. In the hydraulic system under study, leakage effects appear either in cylinders and in valves. In pistons, leakage phenomena can be divided into two main types, namely the internal ones, which are associated with a hydraulic flow between the two chambers, and the external ones, associated with a fluid flow from one chamber to the environment. In this work, for the sake of simplicity, only internal leakage has been taken into account.

Similar considerations can be drawn for the leakage flows in the valve. Usually, they are caused by small openings among the lines of the different chambers, mainly because of the wear-out of the spool seas, especially for zero overlapped valves. However, such effect has not been taken into account in this paper, since the control algorithm discussed in Section 4 is robust enough for compensating such disturbance and able to guarantee the desired performances.

3. MODEL FOR CONTROL DESIGN

Starting from the general description provided in Section 2, a mathematical representation of the Y_2 axis of the hydraulic machine is now obtained for control design purposes. It has been already pointed out that the plant consists of 2 cylinders operating in parallel, and a single valve. To simplify the model, only one piston is described, but with double areas with respect to the ones used in the real device. This means that velocities, flow rates and pressures appearing in the model correspond to the "real" ones. The state space model of the plant is

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{1}{m}\big[A_1 x_3(t) - A_2 x_4(t) - C_v x_2(t) - \\ \qquad\qquad - F_f(x_2(t))\big] + d(t) - g \\ \dot{x}_3(t) = \dfrac{\beta_e}{V_1(x_1(t))}\big[Q_1(x_3(t), u(t)) - \\ \qquad\qquad - A_1 x_2(t) - q_{leak}(t)\big] \\ \dot{x}_4(t) = \dfrac{\beta_e}{V_2(x_1(t))}\big[- Q_2(x_4(t), u(t)) + \\ \qquad\qquad + A_2 x_2(t) + q_{leak}(t)\big] \end{cases} \tag{1}$$

where

- $x_1(t)$ and $x_2(t)$ are the cylinder position and velocity,
- m is the piston mass,
- C_v is the viscous friction coefficient,
- $F_f(x_2)$ takes into account unmodelled friction effects, with F_f unknown but Lipschitz-continuous,
- $d(t)$ are the unknown external disturbances / forces acting on the piston,
- g is the gravity acceleration,
- A_1 and A_2 are the areas of the piston chambers,
- $x_3(t)$ and $x_4(t)$ are the pressures in the lower and upper chambers, respectively,
- β_e is the bulk modulus of the fluid,
- $V_1(x_1)$ and $V_2(x_1)$ are known functions that give the actual volumes of the lower and upper chambers, respectively,
- $u(t)$ is the spool displacement of the valve, i.e. the control input,
- $q_{leak}(t)$ is the unknown internal leakage flow,
- $Q_1(x_3, u)$ and $Q_2(x_4, u)$ are the input and output flow rates in the lower and upper chambers, respectively.

In (1), the control input is the spool displacement. Since the valve has an asymmetric behaviour, the flow rates Q_1 and Q_2 are clearly asymmetric. More precisely, as far as Q_1 is concerned, it turns out that

$$Q_1(x_3, u) = C_{01} u \sqrt{|\Delta P_1|}\,\mathrm{sign}(\Delta P_1) \tag{2}$$

where

$$\Delta P_1 = \begin{cases} P_{source} - x_3 & \text{if } u \geq 0 \\ x_3 - P_r & \text{if } u < 0 \end{cases} \tag{3}$$

where $P_{source} = 280$ bar is the supplied pressure, $P_r = 0$ bar is the reference (environment) pressure, and and C_{01} is the flow rate gain of the valve. Similarly, for the upper chamber it turns out that

$$Q_2(x_4, u) = C_{02} u \sqrt{|\Delta P_2|}\, \mathrm{sign}(\Delta P_2) \tag{4}$$

where

$$\Delta P_2 = \begin{cases} x_4 - P_r & \text{if } u \geq 0 \\ P_{source} - x_4 & \text{if } u < 0 \end{cases} \tag{5}$$

and C_{02} is the flow rate gain of the valve.

4. CONTROL DESIGN

The aim of this section is to show how to design a robust and adaptive control system for (1) that is able to dynamically react against high magnitude external disturbances $d(t)$ generated during the pressing process, and also able to compensate the offset caused by the slowly varying internal leakage effect $q_{leak}(t)$. A reliable estimate of the latter quantity is also necessary for diagnostic and maintenance purposes. From (1) and due to (2) and (4), the plant is described by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m}\big[A_1 x_3 - A_2 x_4 - C_v x_2 - \\ \qquad\qquad - F_f(x_2)\big] + d - g \\ \dot{x}_3 = \dfrac{\beta_e}{V_1(x_1)}\big[C_{01} u \sqrt{|\Delta P_1|}\, \mathrm{sign}\, \Delta P_1 - \\ \qquad\qquad - A_1 x_2 - q_{leak}\big] \\ \dot{x}_4 = \dfrac{\beta_e}{V_2(x_1)}\big[- C_{02} u \sqrt{|\Delta P_2|}\, \mathrm{sign}\, \Delta P_2 + \\ \qquad\qquad + A_2 x_2 + q_{leak}\big] \end{cases} \tag{6}$$

where $V_1(x_1) = V_{01} + A_1 x_1$, $V_2(x_1) = V_{02} - A_2 x_1$, and ΔP_1 and ΔP_2 are defined in (3) and (5), respectively. Let $x_1^\star(t)$ be a desired (periodic) reference trajectory for the piston defining the nominal working cycle for the press, and let $x_2^\star(t) := \dot{x}_1^\star(t)$ be the associated velocity profile. The following assumptions are then made:

- The parameters C_v, C_{01}, C_{02}, m, β_e, A_1, A_2, V_{01} and V_{02} are known, and constant;
- The function F_f is globally Lipschitz, i.e. there exist $\bar{F} > 0$ such that for all x_a and x_b we have that
$$|F_f(x_b) - F_f(x_a)| \leq \bar{F}|x_b - x_a| \tag{7}$$
and for any reference profile $x_2^\star(t)$ we have that
$$F_f(x_2^\star(t)) = \sum_{i=1}^{N_f} \phi_i \Phi_i(x_2^\star(t)) := F_\phi(x_2^\star(t)) \tag{8}$$
where each $\Phi_i(x_2^\star)$ is a known and bounded function, while the ϕ_i are real and unknown constants, with $i = 1, \dots, N_f$.
- For any reference trajectory $x_1^\star(t)$, there exist a set of known and bounded functions $\Gamma_i(t)$, a set of unknown parameters γ_i, $i = 1, \dots, N_d$, and a known and bounded function $F_{ext}(x_1)$ such that
$$d(t) = F_{ext}(x_1(t)) + d_\gamma(t) \tag{9}$$
being
$$d_\gamma(t) := \sum_{i=1}^{N_d} \gamma_i \Gamma_i(t) \tag{10}$$
- As for the internal leakage, we assume that
$$q_{leak}(t) = C_{0l} x_l(t) \sqrt{|x_3(t) - x_4(t)|} \cdot \\ \cdot\, \mathrm{sign}\,(x_3(t) - x_4(t)) \tag{11}$$
where C_{0l} is the discharge gain of an undesired orifice responsible for the leakage flow, and $x_l(t)$ the associated opening. In this respect, both C_{0l} and $x_l(t)$ are unknown, so that (11) can be more compactly rewritten as
$$q_{leak}(t) = g_l(t) G_l(x_3(t), x_4(t)) \tag{12}$$
where $G_l(x_3, x_4) = \sqrt{|x_3 - x_4|}\, \mathrm{sign}\,(x_3 - x_4)$ and $g_l(t)$, which is a sort of time-varying "leakage gain," bounded and unknown. Moreover, since the dynamic associated to the change in the leakage flow is much slower than the hydraulic system dynamic, we have that $0 < \dot{g}_l(t)$ and bounded, and such that $\dot{g}_l(t) \simeq 0$.
- The state vector $x = (x_1, x_2, x_3, x_4)$ is measurable.

By changing coordinate as $x_3 \mapsto \bar{x}_3 := A_1 x_3 - A_2 x_4$, it turns out that the first three dynamics in (6) read as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{1}{m}\big[\bar{x}_3 - C_v x_2 - F_f(x_2)\big] + d - g \\ \dot{\bar{x}}_3 = f_1(x) u - f_2(x_1) x_2 - f_3(x_1) q_{leak} \end{cases} \tag{13}$$

where

$$f_1(x) := \frac{\beta_e A_1}{V_1(x_1)} C_{01} \sqrt{|\Delta P_1|}\, \mathrm{sign}\, \Delta P_1 + \\ + \frac{\beta_e A_2}{V_2(x_1)} C_{02} \sqrt{|\Delta P_2|}\, \mathrm{sign}\, \Delta P_2$$

$$f_2(x_1) := \beta_e \left[\frac{A_1^2}{V_1(x_1)} + \frac{A_2^2}{V_2(x_1)}\right]$$

$$f_3(x_1) := \beta_e \left[\frac{A_1}{V_1(x_1)} + \frac{A_2}{V_2(x_1)}\right].$$

This system is in semi-strict-feedback form, Polycarpou and Ioannou (1993).

We face the problem of letting the state (x_1, x_2) of (13) to track the reference signal $(x_1^\star, x_2^\star)$. To achieve this, we start by considering the first two dynamics regarded as a second order system with state (x_1, x_2) and virtual control input $\bar{x}_3$. By defining the error coordinates $z_1(t) = x_1(t) - x_1^\star(t)$ and $z_2(t) = x_2(t) - x_2^\star(t)$, the system in question reads as

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = \dfrac{1}{m}\big[\bar{u} - C_v(z_2 + x_2^\star) - F_f(z_2 + x_2^\star)\big] + \\ \qquad\qquad + F_{ext}(z_1 + x_1^\star) + d_\gamma - g \end{cases} \tag{14}$$

in which we denoted by $\bar{u}$ the state variable $\bar{x}_3$ to emphasise the fact the latter is regarded as virtual input for (14). For this system, a feedback linearising control action $\bar{u}(t)$ that makes the origin of (14) globally exponentially stable when $F_f(x_2) = 0$ and $d_\gamma(t) = 0$ can be simply obtained. Such control action is given by

$$\bar{u} = -m\big[K_1 z_1 + K_2 z_2 + F_{ext}(z_1 + x_1^\star) - g\big] + \\ + C_v(z_2 + x_2^\star) \tag{15}$$

with K_1, $K_2 > 0$. The next proposition builds on (15) to present an adaptive control law that makes the origin of (14) globally exponentially stable when $F_f(x_2)$ and $d_\gamma(t)$ are present. Before presenting the result, we make the following assumption.

Hypothesis 1. Consider the functions $F_f(x_2^\star(t))$ and $d_\gamma(t)$ introduced in (8) and in (10), respectively, and let

$$\Theta(t) := \left(-\frac{1}{m}\Phi_1(t), \ldots, -\frac{1}{m}\Phi_{N_f}(t), \Gamma_1(t), \ldots, \Gamma_{N_d}(t)\right)^{\mathrm{T}}. \tag{16}$$

The entries of $\Theta : \mathbb{R}_+ \to \mathbb{R}^{N_f+N_d}$ are C^1, and there exists positive constants D_0 and D_1 such that $\max_{t\in\mathbb{R}_+} \|\Theta(t)\| \leq D_0$ and $\max_{t\in\mathbb{R}_+} \|\dot{\Theta}(t)\| \leq D_1$, where $\|\cdot\|$ denotes the Euclidean norm. Moreover, there exists positive constants μ and T such that $\int_{t-T}^{t} \Theta(\tau)\Theta^{\mathrm{T}}(\tau)\,\mathrm{d}\tau \geq \mu I$, for all $t \geq T$. This last requirement is the classical persistency of excitation condition, Ioannou and Sun (1985).

Proposition 2. Consider system (14) in which $F_f(x_2)$ satisfies conditions (7) and (8), $d_\gamma(t)$ is defined as in (9), and $\Theta(t)$ as in (16). Moreover, suppose that Hypothesis 1 holds true. Let $\theta := (\phi_1, \ldots, \phi_{N_f}, \gamma_1, \ldots, \gamma_{N_d})^{\mathrm{T}}$. Then, the trajectories of the closed-loop system resulting from (14), with the control action

$$\begin{aligned} \bar{u} = -m\big[K_1 z_1 + K_2 z_2 + F_{ext}(z_1 + x_1^\star) - g\big] + \\ + C_v(z_2 + x_2^\star) + m\xi_\theta^{\mathrm{T}}\Theta \end{aligned} \tag{17}$$

and the adaptive law

$$\dot{\xi}_\theta(t) = -\big[\varepsilon_1 z_1(t) + z_2(t)\big]K_\theta\Theta(t) \tag{18}$$

are bounded and $(z_1(t), z_2(t), \xi_\theta(t)))$ converges to $(0, 0, -\theta)$ globally, uniformly and exponentially, (Khalil, 1996, Definition 3.5). In (18), we have that $\xi_\theta \in \mathbb{R}^{N_f+N_d}$ and K_θ is a $(N_f + N_d) \times (N_f + N_d)$ symmetric and positive definite real matrix. In (17), instead, we have that K_1 and K_2 are positive gains such that

$$\begin{aligned} K_1 &> \bar{\Gamma}_1 + \frac{\varepsilon_2\delta_3 K_1^2}{2\varepsilon_1} \\ K_2 &> \bar{\Gamma}_2 + \frac{\varepsilon_2\delta_3 K_2^2}{2} \end{aligned} \tag{19}$$

where

$$\begin{aligned} \bar{\Gamma}_1 &:= \frac{\delta_1\bar{F}}{2m} + 2\varepsilon_1\varepsilon_2\|K_\theta\|D_0^2 + \frac{\varepsilon_2 D_1}{2\delta_2} \\ \bar{\Gamma}_2 &:= \frac{\bar{F}}{m} + \varepsilon_1\left(1 + \frac{\bar{F}}{2\delta_1 m}\right) + 2\varepsilon_2\|K_\theta\|D_0^2 + \frac{\varepsilon_2\bar{\Gamma}_2'}{2\delta_4} \end{aligned} \tag{20}$$

being $\bar{\Gamma}_2' := T\|K_\theta\|D_0^3 + D_1 + \left(\varepsilon_1 + \frac{\bar{F}}{m}\right)D_0$, $\bar{F} > 0$ a sufficiently large constant so that (7) holds, $\varepsilon_1, \varepsilon_2$ small and positive constants, and δ_i, $i = 1, \ldots, 4$, positive constants. The constants ε_1 and δ_i, $i = 1, \ldots, 4$ have to be selected so that $\frac{\mu}{T} > \bar{\Gamma}_\theta$, where

$$\bar{\Gamma}_\theta := \frac{1}{2}\left(\varepsilon_1\delta_2 D_1 + \delta_4\bar{\Gamma}_2'\right) + \frac{D_0^2}{\delta_3}, \tag{21}$$

while ε_2 small enough so that (19) holds.

The previous result provides the expression (17) for the "virtual" control input $\bar{x}_3(t)$ that assures that the mechanical subsystem in (13) asymptotically tracks a desired reference trajectory $x_1^\star(t)$. By following the standard backstepping procedure, we compute now the true input u. By letting $z_3(t) := \bar{x}_3(t) - \bar{u}(t)$, we obtain

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = \dfrac{1}{m}\big[z_3 + \bar{u} - C_v(z_2 + x_2^\star) - F_f(z_2 + x_2^\star)\big] + \\ \qquad + F_{ext}(z_1 + x_1^\star) + d_\gamma - g \\ \dot{z}_3 = f_1'(z_e, t)u - f_2'(z_1, t)z_2 - f_3'(z_1, t)q_{leak} - \\ \qquad - f_2'(z_1, t)x_2^\star - \dot{\bar{u}} \end{cases} \tag{22}$$

where (14) has been taken into account, and $f_1'(z_e, t)$, $f_2'(z_1, t)$ and $f_3'(z_1, t)$ are the functions $f_1(x)$, $f_2(x_1)$ and $f_3(x_1)$ in (13) in the new error coordinates, with $z_e := (z_1, z_2, z_3, x_4)$. The next proposition presents an adaptive control law by assuming that the internal leakage takes the form (12), and that the "leakage gain" $g_l(t)$ is constant. In this case, global exponential stability is proved. Similarly to Proposition 2, we make the following assumption.

Hypothesis 3. Consider $q_{leak}(t)$ given by (12), in which $G_l(x_3, x_4)$ is a known function and $g_l(t) \equiv \bar{g}_l \geq 0$, but unknown, and the function $f_3'(z_1, t)$ introduced in (22). Define $\Theta_l(t) := G_l'(z_e(t), t)f_3'(z_1(t), t)$, where $G_l'(z_e, t)$ is the function $G_l(x_3, x_4)$ in the new error coordinates, with $z_e = (z_1, z_2, z_3, x_4)$. Similarly to Hypothesis 1, suppose that Θ_l is of class C^1 and that there exists positive constants L_0 and L_1 such that $\max_{t\in\mathbb{R}_+} |\Theta_l(t)| \leq L_0$ and $\max_{t\in\mathbb{R}_+} |\dot{\Theta}_l(t)| \leq L_1$. Moreover, suppose that there exists positive constants μ_l and T_l such that $\int_{t-T_l}^{t} \Theta_l^2(\tau)\,\mathrm{d}\tau \geq \mu_l$, for all $t \geq T_l$.

Proposition 4. Consider (22) with $F_f(x_2)$ satisfying the conditions (7) and (8), and $d_\gamma(t)$ defined as in (9). Moreover, suppose that Hypotheses 1 and 3 hold true. Then, the trajectories of the closed-loop system resulting from (22), where the control action is given by

$$\begin{aligned} u = \frac{1}{f_1'(z_e, t)}\Big[-\frac{z_2}{m} - K_3 z_3 + \dot{\bar{u}} + \\ + f_2'(z_1, t)(z_2 + x_2^\star) + \xi_l G_l'(z_e, t)f_3'(z_1, t)\Big] \end{aligned} \tag{23}$$

with $\bar{u}$ given by (17), and with the adaptive laws (18) and

$$\dot{\xi}_l = -K_l G_l'(z_e, t)f_3'(z_1, t)z_3 \tag{24}$$

are bounded and $(z_1(t), z_2(t), z_3(t), \xi_\theta(t), \xi_l(t))$ converges to $(0, 0, 0, -\theta, \bar{g}_l)$ globally, uniformly and exponentially, (Khalil, 1996, Definition 3.5). In (24), we have that $\xi_l \in \mathbb{R}$, $G_l'(z_e, t)$ is the function $G_l(x_3, x_4)$ introduced in (12) in the new error coordinates, with $z_e = (z_1, z_2, z_3, x_4)$, and K_l is a positive gain. In (23), instead, K_1, K_2 and K_3 are positive gains, with K_1 and K_2 appearing in the definition of $\bar{u}$ given in (17), such that

$$\begin{aligned} K_1 &> \bar{\Gamma}_1 + \frac{\delta_5}{2m} + \frac{\varepsilon_2\delta_3 K_1^2}{2\varepsilon_1} \\ K_2 &> \bar{\Gamma}_2 + \frac{\varepsilon_3 L_0}{2m\delta_7} + \frac{\varepsilon_2\delta_3 K_2^2}{2} \\ K_3 &> \frac{\varepsilon_1}{2\delta_5 m} + \frac{\varepsilon_2 D_0}{2\delta_6 m} + \varepsilon_3\left(K_l L_0^2 + \frac{\bar{\Gamma}_3}{2\delta_8}\right) \\ &\quad + \frac{\varepsilon_3\delta_9 K_3^2}{2} \end{aligned} \tag{25}$$

where $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$ are defined in (20), while $\bar{\Gamma}_3 := T_l K_l L_0^3 + L_1$. In addition, $\bar{F} > 0$ is a sufficiently large constant so that (7) holds, ε_i, $i = 1, \ldots, 3$ are small and positive constants, and δ_j, $j = 1, \ldots, 8$ are positive constants. The constants ε_1 and the δ_j have to be selected so that $\frac{\mu}{T} > \bar{\Gamma}_\theta + \frac{\delta_6 D_0}{2m}$, and $\frac{\mu_l}{T_l} > \frac{1}{2}\left(\frac{L_0\delta_7}{m} + \bar{\Gamma}_3\delta_8 + \frac{1}{\delta_9}\right)$, with $\bar{\Gamma}_\theta$ defined in (21), while ε_2 and ε_3 are sufficiently small so that (25) holds.

In the previous proposition, it is proved that when $q_{leak}(t)$ is given by (12) with $g_l(t) = \bar{g}_l \geq 0$, then the equilibrium $(0, 0, 0, -\theta, \bar{g}_l, \bar{g}_l)$ of the closed-loop system

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -K_1 z_1 - K_2 z_2 + \dfrac{z_3}{m} + (\xi_\theta + \theta)^{\mathrm{T}}\Theta - \\ \qquad\qquad - \dfrac{1}{m}\left[F_f(z_2 + x_2^\star) - F_f(x_2^\star)\right] \\ \dot{z}_3 = -\dfrac{z_2}{m} - K_3 z_3 + (\xi_l - g_l) G_l'(z_e, t) f_3'(z_1, t) \\ \dot{\xi}_\theta = -\left[\varepsilon_1 z_1 + z_2\right] K_\theta \Theta \\ \dot{\xi}_l = -K_l G_l'(z_e, t) f_3'(z_1, t) z_3 \\ \dot{g}_l = 0 \end{cases} \tag{26}$$

is exponentially stable. However, in a real-world scenario, the "leakage gain" $g_l(t)$ is not constant, but it increases in time. This variation is largely slower than the dynamics associated with the single production cycle, so the overall system presents a multi-time-scale behaviour.

To study the amount of performance degradation the (slow) leakage dynamic causes in the controlled system, we rely on the singular perturbations theory, see (Khalil, 1996, Chapter 9). A simple way to take into account the variation of the "leakage gain" is to change the last equation in (26) into $\dot{g}_l(t) = \varepsilon_l^2$. The motivation behind this choice is that ε_l^{-1} is associated to the time constant of the controlled hydraulic system. As a consequence, g_l is practically constant for a large time interval, i.e. the rate of variations of g_l is of several order of magnitude lower than the time derivatives of the signals associated with the plant (controlled) dynamic. The question is how to characterise the evolution of the "slow" dynamic in (26), i.e. of $(z_1, z_2, \xi_\theta, \xi_l)$, when $\dot{g}_l(t) = \varepsilon_l^2$, being $\varepsilon_l > 0$ and small, with respect to the quasi-steady-state trajectory that corresponds to the case in which $\varepsilon_l = 0$. This problem is investigated in the next proposition.

Proposition 5. Assume that the conditions of Proposition 4 holds true, except for the fact that $q_{leak}(t)$ defined in (12) is such that $\dot{g}_l(t) = \varepsilon_l^2$, with $g_l(0) = \bar{g}_l$ and $\varepsilon_l > 0$. For the closed-loop system (26), for any $t_0, t_1 > 0$ but finite and such that $t_0 < t_1$, there exists positive constants η and $\varepsilon_l^\star$ such that for all $\|(z_1, z_2, z_3, \xi_\theta, \xi_l)(0) - (0, 0, 0, -\theta, \bar{g}_l)\| < \eta$, and $0 < \varepsilon_l < \varepsilon_l^\star$, we have that

$$\|(z_1, z_2, z_3, \xi_\theta, \xi_l)(t) - (0, 0, 0, -\theta, \bar{g}_l)\| \le O(\varepsilon_l) \tag{27}$$

uniformly for $t \in [t_0, t_1]$, whenever $\varepsilon_l < \varepsilon_l^\star$.

The previous result follows from (Khalil, 1996, Theorem 9.1), and states that, on a finite interval, the effect of the slow dynamic associated to the leakage is not leading to instability. The inner / outer loops structure, together with the pair of adaptive laws, is able to compensate such phenomenon, and the key property that assures this is that for constant leakage gain the closed-loop system (26) has a globally and exponentially stable equilibrium. In a real-world scenario, since $\varepsilon_l \simeq 0$, relation (27) assures that the performance degradation is in fact neglectable. It is worth noticing that such result is valid on a finite interval due to the fact the the leakage dynamic is assumed to be simply stable, i.e. it is the one of an integrator. On the other hand, if some convergence properties are assumed, the result can be extended on an infinite time interval. A rigorous analysis is beyond the scopes of this paper.

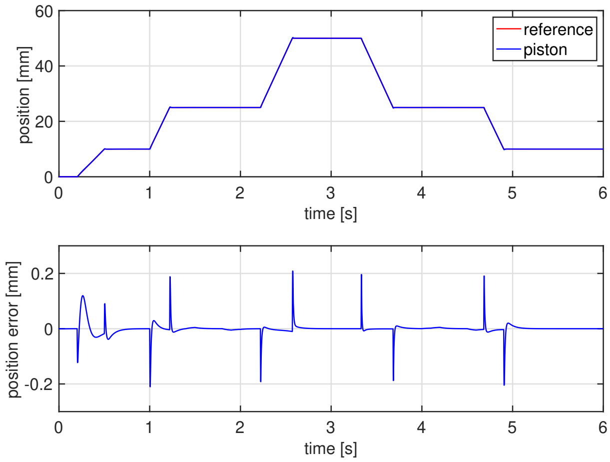

Fig. 2. Single working cycle with external force and no leakage: position tracking achieved with the adaptive controller of Proposition 4 with $K_l = 0$.

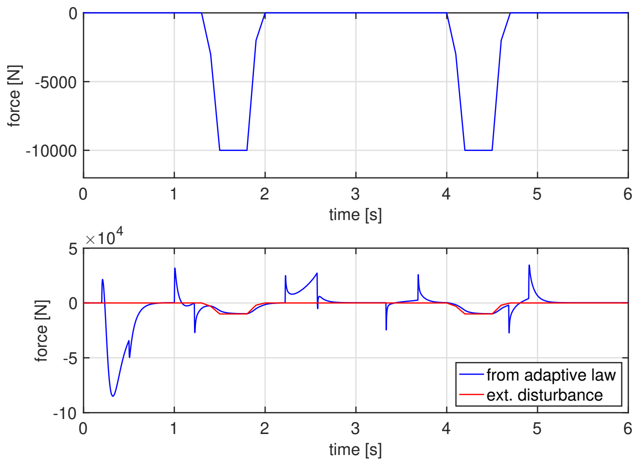

Fig. 3. Single working cycle with external force (top) and no leakage: contribution of the adaptive loop on the force (bottom) introduced in Proposition 2.

5. SIMULATIONS

In this section, some simulation results, in which a Simscape model of the plant has been employed, are reported. In particular, the performances of the complete control scheme of Proposition 4, not only in terms of the external force compensation, but also in case of leakage in the hydraulic system are presented. The piston undergoes several working cycles, i.e. different repetitions of the trajectory presented in the top graph in Fig. 2. At the same time, an unknown (to the controller) external force as the one in the top graph in Fig 3 is applied to the piston, and an unknown offset is added to the spool command in order to simulate the leakage in the valve. Such command generates the leakage opening profile reported in red in Fig. 5.

The controller assures quite good performances, since both the force and leakage adaptive loops are able to compensate for such external disturbances. In fact, as reported in Fig. 4, position and velocity tracking errors remain of the same magnitude as in case no leakage is

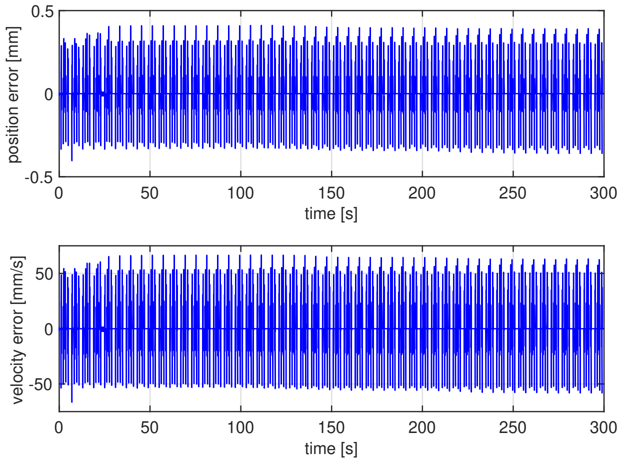

Fig. 4. Multiple working cycles with external force and leakage: position and velocity errors achieved with the adaptive controller of Proposition 4.

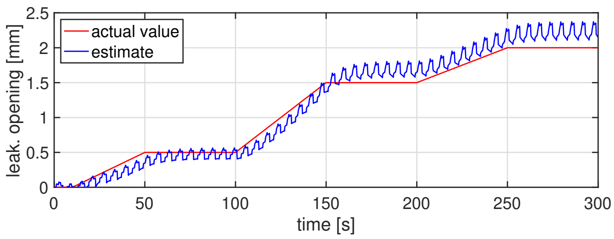

Fig. 5. Multiple working cycles with external force and leakage: leakage opening and its estimate obtained with the adaptive controller of Proposition 4.

present. More precisely, the maximum position tracking error is now equal to 0.41 mm, while the maximum velocity tracking error to 66.71 mm/s. On the other hand, the estimate of the leakage opening is characterised by the response of Fig. 5. It is important to point out that in the real world scenario the leakage dynamic is largely slower than the one simulated here. Such levels of internal leakage are reached after hundred thousands of cycles, and usually the valve is replaced when the intensity of the leakage flow is comparable to the one associated to a (simulated) opening of $1 \div 1.5$ mm, which is in fact the quantity $x_l(t)$ that appears in (11). In any case, the adaptive loop is able to correctly estimate the leakage, and such information is successfully employed in the control loop to let the system to have acceptable performances even in these extreme situations. The ripple in the estimate response is due to the fact that the adaptive loop for the leakage is influenced also by the force dynamic, which is generated at each working cycle. In other words, to follow the leakage opening profile of Fig. 5, the gain of the adaptive loop responsible for its estimate and compensation is larger than the value that would be necessary in reality.

6. CONCLUSIONS

In this paper, the adaptive robust control of a hydraulic press has been presented. The design is based on classical backstepping arguments, and the resulting control law is characterised by an inner / outer loop structure. In particular, the outer loop is responsible for the motion of the press having the hydraulic pressure acting on the piston as input, while the inner loop is in charge of generating such pressure by acting on a servo-valve. Beside, two adaptive laws are implemented. The first one is associated with the outer loop and is capable to compensate for the external forces generated during the pressing phase or due to some unmodelled effects, such as friction, fluid compressibility and non-idealities in the supply system. Instead, the second adaptive law is paired with the inner loop and is designed to compensate for the leakage flow in the servo valve, a major source of performance degradation in the system. The validity and the closed-loop performances that the control scheme provides are illustrated with the help of some simulations in which a detailed Simscape physical model of the plant has been employed.

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