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A Generalized Scattering Framework for Teleoperation with Communication Delays ⋆

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Abstract: The scattering (wave) based teleoperation is currently one of the most popular approaches to bilateral teleoperation with communication delays. Limitations of the conventional scattering-based teleoperation are mostly related to the underlying passivity requirements imposed on the master-human and slave-environment subsystems. In this paper, a generalized scattering framework for bilateral teleoperation with communication delays is outlined, and basic stability results for generalized scattering-based teleoperator systems with delays are established. The proposed framework removes many limitations of the existing scattering-based teleoperation, and allows for much higher flexibility in the control design for the master and slave subsystems.

Keywords: Telerobotics, teleoperation, scattering transformations, wave variables, stability, communication delays.

1. INTRODUCTION

The scattering (wave) based approach to teleoperation was introduced in (Anderson and Spong (1989); Niemeyer and Slotine (1991)) and has become one of the most popular and thoroughly developed approaches to bilateral teleoperation with communication delays (Niemeyer and Slotine (2004); Nuño et al. (2011); Sun et al. (2014)) with a wide range of applications from space to telesurgery. In spite of all the success of this approach, a number of shortcomings is generally associated with the conventional scattering-based teleoperation. Many of these shortcomings are related, either directly or indirectly, to the underlying passivity assumption. In the conventional scattering-based teleoperator system, the human operator, the master device, the slave device, and the environment must be passive with respect to power variables $\dot{\mathbf{x}}$ (velocity) and F (force). In some cases, such a passivity assumption may be overly restrictive. There exist environments which are not passive (Atashzar et al., 2012; Li et al., 2016), and behavior of the human operator may also fail to satisfy the passivity requirements (Dyck et al., 2013), see also discussion in (Jazayeri and Tavakoli, 2015, Section 1.4). Moreover, passivity requirement imposed on the slave device appears to be in contradiction with the trajectory tracking performance. This is related to the fact that passivity with respect to power variables $(F, \dot{\mathbf{x}})$ does not directly allow for inclusion of position information into the signals transmitted between the master and the slave, which in particular results in position drift (although some partial solutions exist, see for example (Niemeyer and Slotine, 2004, Section 6)). Moreover, the existing control laws for robot manipulators that guarantee tracking of a time-varying trajectory (Chung et al., 2008) inevitably result in nonpassivity of the closed-loop dynamics with respect to the input-output pair $(F, \dot{\mathbf{x}})$. Consequently, the existing results related to trajectory tracking in scattering-based teleoperators are relatively weak and at best guarantee boundedness of trajectories and position regulation, the latter is under the assumption that the human operator forces are totally equal to zero (Nuño et al., 2011, Section 4). Finally, actuators and sensors dynamics can contribute to passivity violations (Tanner and Niemeyer, 2004).

On the other hand, assumption of passivity of the subsystems (the human operator, the master device, the slave device, and the environment) may be overly crude in many cases. Even if every subsystem involved satisfies the passivity assumption, their actual dynamic behavior may form a small subset of all possible passive behaviors. In these cases, design based solely on passivity can be overly conservative and, in particular, may lead to unnecessary performance limitations. Consequently, of interest is a more general approach to scattering-based design of teleoperator systems which is based on a more precise description of the subsystems' dynamics and, on the other hand, not only allows for nonpassivity of the components but also removes other limitations of passivity-based design such as the requirement that the subsystems must have equal number of inputs and outputs.

Extensions of the scattering-based design for teleoperator systems with not necessarily passive components were addressed previously (Hirche and Buss, 2012). Recently, generalized scattering-based methods for stabilization of interconnections of so-called non-planar conic systems,

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with as well as without communication delays, were developed in (Usova et al., 2018). The notion of non-planar conicity, which is a multi-dimensional extension of the traditional conicity notion (originally introduced in (Zames, 1966)), can also be viewed as a special parametrization of quadratic supply rates. Based on this connection, a method for scattering-based stabilization of networks of dissipative systems with quadratic supply rates was subsequently developed in (Usova et al., 2019). In this paper, based on the approach developed in (Usova et al., 2019), a framework for generalized scattering-based bilateral teleoperation with communication delays is outlined. The framework presented in this paper fundamentally relaxes the underlying assumption of passivity of the master-human and slave-environment subsystems and potentially removes many other limitations of the existing scattering-based approaches to teleoperation.

The remaining part of the paper has the following structure. In Section 2, notation is established and some necessary definitions are given. Conventional passivity approach to scattering-based teleoperation with communication delays is discussed in Section 3. In Section 4, the proposed generalized scattering framework for bilateral teleoperation with communication delays is described, and basic stability results are established. Section 5 concludes the paper with a summary of the contribution and discussion of future research directions. Appendices A and B contain proofs of the stability results (Theorems 2 and 5, respectively).

2. NOTATION AND DEFINITIONS

Throughout the paper, $\mathbb{R} := (-\infty, +\infty)$, $\mathbb{R}_+ := [0, +\infty)$, $\mathbb{R}^p$ denotes a set of p-dimensional vectors with real coefficients, and $\mathbb{R}^{p\times q}$ a set of real $p \times q$-matrices. For a square matrix $A \in \mathbb{R}^{p\times p}$, notation $A \succ 0$ ($A \prec 0$) means A is positive (negative) definite, and $A \succeq 0$ ($A \preceq 0$) means A is positive (negative) semi-definite.

Let $\mathbb{B} := \{0, 1\}$ denote a binary set, and $\mathbb{B}^{p\times q}$ denote a set of $p \times q$ binary matrices, *i.e.*, matrices which comprise ones and zeros. A square matrix $\mathrm{P} \in \mathbb{B}^{p\times p}$ is called *permutation matrix* if exactly one entry in each row and each column is equal to one, and all other entries are zeros. Right multiplication of a matrix A by a permutation matrix P, *i.e.*, $A\mathrm{P}$, permutes the columns of A, while left multiplication $\mathrm{P}A$ permutes the rows of A. For a diagonal matrix Λ, the transformation $\Lambda \to \mathrm{P}^T\Lambda\mathrm{P}$ permutes the elements on the main diagonal (see for example (Horn and Johnson, 2013, p. 32)).

A signal $x \colon \mathbb{R} \to \mathbb{R}^n$ is said to belong to $\mathcal{L}_2^{(a,b)}$ ($x \in \mathcal{L}_2^{(a,b)}$), where $-\infty \le a \le b \le +\infty$ if

$$\int_a^b |x(t)|^2 dt < \infty.$$

Now consider a nonlinear dynamical system of the form

$$\Sigma : \begin{cases} \dot{x} = f(x, \eta), \\ y = h(x, \eta), \end{cases} \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ is the state, $\eta \in \mathbb{R}^q$ the input, and $y \in \mathbb{R}^p$ the output of system (1), respectively. Functions $f(\cdot,\cdot)$, $h(\cdot,\cdot)$ are assumed to be locally Lipschitz continuous in their arguments. A system (1) is called *dissipative* with a storage function $\mathcal{V} \colon X \to R_+$ and a supply rate $w \colon U \times V \to R$ if the dissipation inequality

$$\mathcal{V}(x(t)) - \mathcal{V}(x(t_0)) \le \int_{t_0}^t w(\eta(\tau), y(\tau))\, d\tau$$

holds along trajectories of the system (1). A special case of dissipativity which is of interest for our purposes is where the supply rate is a quadratic function of the form

$$w := \begin{bmatrix} \eta \\ y \end{bmatrix}^T W \begin{bmatrix} \eta \\ y \end{bmatrix}, \quad W \in \mathbb{R}^{(q+p)\times(q+p)}. \tag{2}$$

A system (1) is called *finite $\mathcal{L}_2$-gain stable* with a gain less than or equal to $\gamma > 0$ if it is dissipative with quadratic supply rate (2) where the matrix W has the form

$$W := \begin{bmatrix} \frac{\gamma^2}{2}\,\mathbb{I}_q & \mathbb{O} \\ \mathbb{O} & -\frac{1}{2}\,\mathbb{I}_p \end{bmatrix}. \tag{3}$$

A system (1) with $q = p$ is called *passive* if it is dissipative with quadratic supply rate (2) where matrix W is of the form

$$W := \begin{bmatrix} \mathbb{O} & \frac{1}{2}\mathbb{I}_p \\ \frac{1}{2}\mathbb{I}_p & \mathbb{O} \end{bmatrix} \in \mathbb{R}^{2p\times 2p}. \tag{4}$$

3. CONVENTIONAL SCATTERING-BASED TELEOPERATION

The block diagram of a conventional scattering-based teleoperator system with delays is shown in Figure 1. The underlying idea of the scattering (wave) based teleoperation is that the master and the slave subsystems communicate wave variables $\mathbf{u}$, $\mathbf{v}$ rather than conventional power variables $\dot{\mathbf{x}}$ (velocity) and F (force), as shown in Figure 1. The wave variables are related to the power variables according to the formulas

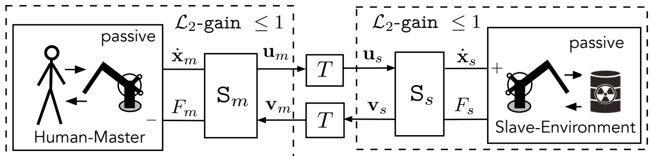

Fig. 1. Scattering (wave) teleoperator system with delays

$$\begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{b}{2}} & \sqrt{\frac{1}{2b}} \\ \sqrt{\frac{b}{2}} & -\sqrt{\frac{1}{2b}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ F \end{bmatrix} \Leftrightarrow \begin{bmatrix} \dot{\mathbf{x}} \\ F \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{1}{2b}} & \sqrt{\frac{1}{2b}} \\ \sqrt{\frac{b}{2}} & -\sqrt{\frac{b}{2}} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \end{bmatrix}, \tag{5}$$

where $b > 0$ is a parameter called the characteristic wave impedance. Exchanging the wave variables instead of the power variables over communication channels with constant delays guarantees stability of the teleoperator system as long as the master-human and the slave-environment terminations are passive.

One possible way to interpret the stabilizing effect of the scattering (wave) transformation is to consider the equations of energy balance for the master-human and the slave-environment subsystems. Assuming both subsystems are passive, and taking into account the sign convention as shown in Figure 1 as well as formulas (5), the equations for energy balance are

$$E_m(t) - E_m(t_0) \leq \int_{t_0}^{t} -F_m^T \dot{\mathbf{x}}_m \, \mathrm{d}\tau = \frac{1}{2} \int_{t_0}^{t} \left(|\mathbf{v}_m|^2 - |\mathbf{u}_m|^2 \right) d\tau$$

$$E_s(t) - E_s(t_0) \leq \int_{t_0}^{t} F_s^T \dot{\mathbf{x}}_s \, \mathrm{d}\tau = \frac{1}{2} \int_{t_0}^{t} \left(|\mathbf{u}_s|^2 - |\mathbf{v}_s|^2 \right) d\tau,$$

where E_m, E_s are the energy stored in the master-human and the slave-environment subsystems, respectively. Since the energy stored is always nonnegative, the above formulas show that both the master-human subsystem (with input $\mathbf{v}_m$ and output $\mathbf{u}_m$) and the slave-environment subsystem (with input $\mathbf{u}_s$ and output $\mathbf{v}_s$) are finite $\mathcal{L}_2$-gain input-output stable with $\mathcal{L}_2$-gains less than or equal to one. As a result, the wave-based teleoperator system can be seen as a feedback interconnection of two subsystems with $\mathcal{L}_2$-gains ≤ 1 and, using small gain arguments, one can conclude that for any constant communication delays the interconnection is marginally stable; it is stable under some mild assumptions such as existence of nonzero damping in at least one of the subsystems.

4. GENERALIZED SCATTERING-BASED TELEOPERATION

In this section, a more general approach to scattering (wave) based teleoperation with communication delays is described which is based on recent results on stabilization of networks of dissipative systems with delays using generalized scattering transformations (Usova et al., 2019). Consider a teleoperator system with communication delays schematically shown in Figure 2. In this figure, $\eta_m \in \mathbb{R}^q$,

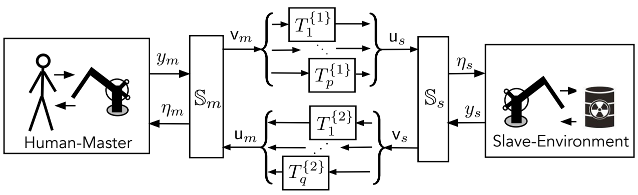

Fig. 2. Generalized scattering-based teleoperator system with delays

$y_m \in \mathbb{R}^p$ are the input and the output of the master-human subsystem, while $\eta_s \in \mathbb{R}^p$, $y_s \in \mathbb{R}^q$ are the input and the output of the slave-environment subsystem, respectively. Note that, in contrast with the conventional scattering-based approach described above, the number of inputs of each subsystem is not required to be equal to the number of its outputs, *i.e.*, in general $q \neq p$. The input-output pairs (η_m, y_m), (η_s, y_s) are related to the corresponding pairs of generalized scattering (wave) variables $(\mathsf{u}_m, \mathsf{v}_m)$, $(\mathsf{u}_s, \mathsf{v}_s)$, respectively, through generalized scattering transformations of the form

$$\begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix} := \mathbb{S}_m \begin{bmatrix} \eta_m \\ y_m \end{bmatrix}, \quad \begin{bmatrix} \mathsf{u}_s \\ \mathsf{v}_s \end{bmatrix} := \mathbb{S}_s \begin{bmatrix} \eta_s \\ y_s \end{bmatrix}, \tag{6}$$

where $\mathbb{S}_m, \mathbb{S}_s \in \mathbb{R}^{(p+q)\times(p+q)}$ are matrices of the generalized scattering transformations. The master and the slave scattering variables are exchanged over delayed communication channels according to the formulas

$$\mathsf{u}_s := \mathsf{v}_m^{\mathbf{d}}, \quad \mathsf{u}_m := \mathsf{v}_s^{\mathbf{d}}, \tag{7}$$

where

$$\mathsf{v}_m^{\mathbf{d}}(t) := \begin{bmatrix} \mathsf{v}_{m1}\left(t - T_1^{\{1\}}\right) \\ \vdots \\ \mathsf{v}_{mp}\left(t - T_p^{\{1\}}\right) \end{bmatrix}, \ \mathsf{v}_s^{\mathbf{d}}(t) := \begin{bmatrix} \mathsf{v}_{s1}\left(t - T_1^{\{2\}}\right) \\ \vdots \\ \mathsf{v}_{sq}\left(t - T_q^{\{2\}}\right) \end{bmatrix},$$

and $T_1^{\{1\}}, \dots, T_p^{\{1\}}, T_1^{\{2\}}, \dots T_q^{\{2\}} \geq 0$ are constant communication delays.

Using the generalized scattering approach developed in (Usova et al., 2019), stability of the teleoperator system shown in Figure 2 with interconnections described by (6), (7) can be guaranteed in the case where the master-human and the slave-environment terminations belong to a class which is fundamentally wider than that of passive systems. Specifically, our basic assumption is that both the master-human and the slave-environment subsystems are dissipative with quadratic supply rates and, additionally, satisfy the so-called "liveness" condition (Willems and Trentelman, 2002). The liveness condition requires that the number of nonnegative eigenvalues of the matrix of the quadratic supply rate is equal to the number of system's inputs. Formally, the assumption is formulated as follows.

Assumption 1. The human-master and the slave - environment interconnections are dissipative with quadratic supply rates of the form

$$w_m := \begin{bmatrix} \eta_m \\ y_m \end{bmatrix}^T W_m \begin{bmatrix} \eta_m \\ y_m \end{bmatrix}, \quad w_s := \begin{bmatrix} \eta_s \\ y_s \end{bmatrix}^T W_s \begin{bmatrix} \eta_s \\ y_s \end{bmatrix}, \tag{8}$$

respectively, where $W_m = W_m^T \in \mathbb{R}^{(p+q)\times(p+q)}$ has exactly q nonnegative eigenvalues, and $W_s = W_s^T \in \mathbb{R}^{(p+q)\times(p+q)}$ has exactly p nonnegative eigenvalues.

The liveness condition is not restrictive; in fact, it is shown in (Willems and Trentelman, 2002, Proposition 2), (Usova et al., 2019, Lemma 1) that, under mild technical assumptions, the number of nonnegative eigenvalues of the matrix of supply rate is always greater than or equal to the number of system's inputs. It is worth to mention that in the case of passivity (*i.e.*, where $p = q$ and the supply rate is of the form (2), (4))), the liveness condition is obviously satisfied as the matrix (4) has p eigenvalues equal to $1/2$ and the other p eigenvalues equal to $-1/2$.

Under Assumption 1, the scattering transformations (6) that guarantee stability of the teleoperator system with delays can be constructed using the following procedure. Let the eigenvalues $\lambda_i^{\{m\}}$, $i = 1, \dots, p+q$, of $W_m = W_m^T$ be indexed in descending order, *i.e.*,

$$\lambda_1^{\{m\}} \geq \ldots \geq \lambda_q^{\{m\}} \geq 0 > \lambda_{q+1}^{\{m\}} \geq \ldots \geq \lambda_{q+p}^{\{m\}},$$

and let $g_1^{\{m\}}, \dots, g_{q+p}^{\{m\}} \in \mathbb{R}^{p+q}$ be the corresponding set of orthonormal eigenvectors such that

$$W_m g_i^{\{m\}} = \lambda_i^{\{m\}} \cdot g_i^{\{m\}}, \quad i = 1, \dots, p+q.$$

Similarly, let

$$\lambda_1^{\{s\}} \geq \ldots \geq \lambda_p^{\{s\}} \geq 0 > \lambda_{p+1}^{\{s\}} \geq \ldots \geq \lambda_{p+q}^{\{s\}}$$

be the eigenvalues of $W_s = W_s^T$, and $g_1^{\{s\}}, \dots, g_{p+q}^{\{s\}} \in \mathbb{R}^{p+q}$ be the corresponding orthonormal set of eigenvectors. Denote

$$G_m := \left[g_1^{\{m\}} \dots g_{q+p}^{\{m\}}\right] \in \mathbb{R}^{(p+q)\times(p+q)},$$
$$G_s := \left[g_1^{\{s\}} \dots g_{p+q}^{\{s\}}\right] \in \mathbb{R}^{(p+q)\times(p+q)}.$$

The matrices of scattering transformations (6) have the form

$$\mathbb{S}_m := \begin{bmatrix}\Gamma_m^+ & \mathbb{O}\\ \mathbb{O} & \Gamma_m^-\end{bmatrix} G_m^T, \quad \mathbb{S}_s := \begin{bmatrix}\Gamma_s^+ & \mathbb{O}\\ \mathbb{O} & \Gamma_s^-\end{bmatrix} G_s^T, \tag{9}$$

where $\Gamma_m^+ \in \mathbb{R}^{q\times q}$, $\Gamma_m^- \in \mathbb{R}^{p\times p}$, $\Gamma_s^+ \in \mathbb{R}^{p\times p}$, $\Gamma_s^- \in \mathbb{R}^{q\times q}$ are positive definite diagonal gain matrices.

In order to formulate stability result, let us denote

$$\Lambda_m^+ := \mathrm{diag}\{\lambda_1^{\{m\}}, \dots, \lambda_q^{\{m\}}\} \in \mathbb{R}^{q\times q},$$
$$\Lambda_m^- := -\mathrm{diag}\{\lambda_{q+1}^{\{m\}}, \dots, \lambda_{q+p}^{\{m\}}\} \in \mathbb{R}^{p\times p},$$
$$\Lambda_s^+ := \mathrm{diag}\{\lambda_1^{\{s\}}, \dots, \lambda_p^{\{s\}}\} \in \mathbb{R}^{p\times p},$$
$$\Lambda_s^- := -\mathrm{diag}\{\lambda_{p+1}^{\{s\}}, \dots, \lambda_{p+q}^{\{s\}}\} \in \mathbb{R}^{q\times q}.$$

By construction, $\Lambda_m^+, \Lambda_s^+ \succeq 0$, and $\Lambda_m^-, \Lambda_s^- \succ 0$. Also, denote

$$\mathbf{T}^{\{1\}} := \max_{i\in\{1,\dots,p\}} T_i^{\{1\}}, \quad \mathbf{T}^{\{2\}} := \max_{j\in\{1,\dots,q\}} T_j^{\{2\}}.$$

The following theorem presents conditions for stability of the generalized scattering-based teleoperator system with delays.

Theorem 2. Consider a teleoperator system shown in Figure 2, where the "human-master" and the "slave - environment" subsystems satisfy Assumption 1. Suppose the interconnection between the "human-master" and the "slave - environment" subsystems is described by (6), (7), (9). If there exists $\alpha > 0$ such that

$$\mathcal{E}_1 := \Lambda_m^- \left(\Gamma_m^-\right)^{-2} - \alpha \cdot \Lambda_s^+ \left(\Gamma_s^+\right)^{-2} \succ 0, \tag{10}$$
$$\mathcal{E}_2 := \alpha \cdot \Lambda_s^- \left(\Gamma_s^-\right)^{-2} - \Lambda_m^+ \left(\Gamma_m^+\right)^{-2} \succ 0, \tag{11}$$

then bounded initial conditions (specifically, $\eta_m, y_m \in \mathcal{L}_2^{(t_0-\mathbf{T}^{\{1\}},t_0)}$, $\eta_s, y_s \in \mathcal{L}_2^{(t_0-\mathbf{T}^{\{2\}},t_0)}$) imply u_m, v_m, η_m, y_m, u_s, v_s, η_s, $y_s \in \mathcal{L}_2^{(t_0,+\infty)}$.

Proof. The proof of Theorem 2 is given in Appendix A.

Remark 3. In Theorem 2, inequalities (10), (11) describe conditions for stability of the generalized scattering teleoperator system with communication delays. In these inequalities, diagonal matrices $\Lambda_m^+ \succeq 0$, $\Lambda_m^- \succ 0$, and $\Lambda_s^+ \succeq 0$, $\Lambda_s^- \succ 0$ are determined by the dynamics of the "human-master" and the "slave - environment" subsystems, respectively, the diagonal matrices Γ_m^+, Γ_m^-, Γ_s^+, $\Gamma_s^- \succ 0$ are design parameters, while $\alpha > 0$ is an auxiliary analytic parameter not used in the design. It is clear that inequalities (10), (11) allow for substantial design freedom. In particular, for arbitrary "human-master" and "slave - environment" dynamics satisfying Assumption 1, stability of the teleoperator system can always be guaranteed by a wide range of choices of the design parameters Γ_m^+, Γ_m^-, Γ_s^+, $\Gamma_s^- \succ 0$ and the auxiliary parameter $\alpha > 0$. This, in particular, allows for substantial freedom in design of the local control algorithms for the master and slave manipulators which can be utilized, for example, for tracking performance improvement as well as optimizing other performance measures.

Remark 4. Some of the features of the proposed generalized scattering framework which make it different from the existing scattering/wave based teleoperator design methods is that it does not require the "human-master" and the "slave - environment" subsystems to have equal number of inputs and outputs ($q \neq p$ in general), nor it imposes constraints on which signals comprise these inputs and outputs. In contrast, in classical scattering/wave teleoperation which relies on passivity properties of the subsystems, the requirement $q = p$ is unavoidable as a part of definition of passivity, and the choice of input-output signals is largely limited to power variables, *i.e.*, velocity and force. The latter leads to well-documented problems with trajectory tracking in wave-based teleoperation. One possible way of using flexibility of the proposed approach is to choose

$$y_m = \begin{bmatrix}\mathbf{x}_m\\ \dot{\mathbf{x}}_m\end{bmatrix}, \quad y_s = F_e,$$

where $\mathbf{x}_m$, $\dot{\mathbf{x}}_m$ are position and velocity of the master manipulator and F_e is the forces which are imposed on the slave due to interaction with the environment. In this case, η_m becomes a force reflection signal, while η_s can be interpreted as the desired position and velocity of the slave, from which a desired acceleration can be restored (for example, using an asymptotic differentiator), and conventional tracking or impedance control algorithms subsequently implemented on the slave side. The latter is not possible in the conventional scattering-based teleoperation, where implementation of tracking and/or impedance control algorithms would result in violation of passivity with respect to the force-velocity pair.

Theorem 2 can be generalized in a way that allows for additional freedom in design of scattering transformations $\mathbb{S}_m$, $\mathbb{S}_s$ and results in even less conservative stability conditions as compared to (10), (11). The generalization is based on the observation that the proof of Theorem 2 remains valid if the columns of G_m and G_s (which are the orthonormal eigenvectors of W_m and W_s, respectively) are permuted arbitrarily as long as any column that corresponds to a nonnegative eigenvalue is to the left from any column that correspond to a negative eigenvalue. Let $\mathrm{P}_m^{\{q\}}, \mathrm{P}_s^{\{q\}} \in \mathbb{B}^{q\times q}$ and $\mathrm{P}_m^{\{p\}}, \mathrm{P}_s^{\{p\}} \in \mathbb{B}^{p\times p}$ be arbitrary permutation matrices, and denote

$$\mathbb{P}_m := \begin{bmatrix}\mathrm{P}_m^{\{q\}} & \mathbb{O}\\ \mathbb{O} & \mathrm{P}_m^{\{p\}}\end{bmatrix}, \quad \mathbb{P}_s := \begin{bmatrix}\mathrm{P}_s^{\{p\}} & \mathbb{O}\\ \mathbb{O} & \mathrm{P}_s^{\{q\}}\end{bmatrix}. \tag{12}$$

It is clear that, for any $\mathbb{P}_m$ of the form (12), the columns of matrix $G_m\mathbb{P}_m$ are orthonormal eigenvectors of W_m such that the first q columns correspond to nonnegative eigenvalues, while the last p columns to negative eigenvalues of W_m. Similarly, for any $\mathbb{P}_s$ of the form (12), the first p columns of $G_s\mathbb{P}_s$ are eigenvectors of W_s corresponding to nonnegative eigenvalues, while the last q columns to negative eigenvalues of W_s. Denote

$$\hat{G}_m := G_m\mathbb{P}_m, \quad \hat{G}_s := G_s\mathbb{P}_s, \tag{13}$$

where $\mathbb{P}_m$, $\mathbb{P}_s$ are arbitrary permutation matrices of the form (12), and consider scattering transformations of the form

$$\mathbb{S}_m := \begin{bmatrix}\Gamma_m^+ & \mathbb{O}\\ \mathbb{O} & \Gamma_m^-\end{bmatrix} \hat{G}_m^T, \quad \mathbb{S}_s := \begin{bmatrix}\Gamma_s^+ & \mathbb{O}\\ \mathbb{O} & \Gamma_s^-\end{bmatrix} \hat{G}_s^T, \tag{14}$$

where, as before, Γ_m^+, Γ_m^-, Γ_s^+, $\Gamma_s^- \succ 0$ are diagonal design matrices. Denote

$$\hat{\Lambda}_m^+ := (\mathrm{P}_m^{\{q\}})^T \Lambda_m^+ \mathrm{P}_m^{\{q\}},$$
$$\hat{\Lambda}_m^- := (\mathrm{P}_m^{\{p\}})^T \Lambda_m^- \mathrm{P}_m^{\{p\}},$$
$$\hat{\Lambda}_s^+ := (\mathrm{P}_s^{\{p\}})^T \Lambda_s^+ \mathrm{P}_s^{\{p\}},$$
$$\hat{\Lambda}_s^- := (\mathrm{P}_s^{\{q\}})^T \Lambda_s^- \mathrm{P}_s^{\{q\}}.$$

Diagonal matrices $\hat{\Lambda}_m^+$, $\hat{\Lambda}_m^-$, $\hat{\Lambda}_s^+$, $\hat{\Lambda}_s^-$ have the same elements on their main diagonals as Λ_m^+, Λ_m^-, Λ_s^+, Λ_s^-, respectively, however the order of these elements can be prescribed arbitrarily by an appropriate choice of permutation matrices $\mathbb{P}_m$, $\mathbb{P}_s$ in (12). The following generalization of Theorem 2 is valid.

Theorem 5. Consider a teleoperator system shown in Figure 2, where the "human-master" and the "slave - environment" subsystems satisfy Assumption 1. Suppose the interconnection between the "human-master" and the "slave - environment" subsystems is described by (6), (7), (14). If there exists $\alpha > 0$ such that

$$\hat{\mathcal{E}}_1 := \hat{\Lambda}_m^- \left(\Gamma_m^-\right)^{-2} - \alpha \cdot \hat{\Lambda}_s^+ \left(\Gamma_s^+\right)^{-2} \succ 0, \tag{15}$$
$$\hat{\mathcal{E}}_2 := \alpha \cdot \hat{\Lambda}_s^- \left(\Gamma_s^-\right)^{-2} - \hat{\Lambda}_m^+ \left(\Gamma_m^+\right)^{-2} \succ 0, \tag{16}$$

then bounded initial conditions (specifically, $\eta_m, y_m \in \mathcal{L}_2^{(t_0-\mathbf{T}^{\{1\}},t_0)}$, $\eta_s, y_s \in \mathcal{L}_2^{(t_0-\mathbf{T}^{\{2\}},t_0)}$) imply u_m, v_m, η_m, y_m, u_s, v_s, η_s, $y_s \in \mathcal{L}_2^{(t_0,+\infty)}$.

Proof. The proof of Theorem 5 follows similar lines of reasoning to that of Theorem 2. For details, see Appendix B.

Remark 6. The use of permutation matrices (12) in the scattering transformations (14) allows for an arbitrary change of order of diagonal elements in diagonal matrices $\hat{\Lambda}_m^+$, $\hat{\Lambda}_m^-$, $\hat{\Lambda}_s^+$, $\hat{\Lambda}_s^-$; as a result, the conditions (15), (16) are generally less conservative as compared to (10), (11).

5. CONCLUSION

In this paper, a generalized scattering framework for bilateral teleoperation with communication delays is outlined. The proposed framework is based on the recent results on generalized scattering stabilization of complex interconnections of dissipative systems with delays (Usova et al., 2019), and imposes fundamentally weaker restrictions on the dynamics of "master-human" and "slave-environment" subsystems as compared to the conventional scattering/wave based approaches, which in particular allows for much higher flexibility in the control design for master and slave subsystems. Specific design methods for teleoperator systems based on the proposed framework is a topic of future research.

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Appendix A. PROOF OF THEOREM 2

Substituting the formulas for scattering transformations (6), (9) into the expressions for supply rates of the "human-master" and the "slave - environment" subsystems (8), one obtains

$$\begin{aligned} w_m &= \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}^T \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} G_m^T W_m G_m \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix} \\ &= \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}^T \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} \begin{bmatrix} \Lambda_m^+ & \mathbb{O} \\ \mathbb{O} & -\Lambda_m^- \end{bmatrix} \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix} \\ &= \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}^T \begin{bmatrix} \Lambda_m^+ \left(\Gamma_m^+\right)^{-2} & \mathbb{O} \\ \mathbb{O} & -\Lambda_m^- \left(\Gamma_m^-\right)^{-2} \end{bmatrix} \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}, \end{aligned}$$

and similarly

$$w_s = \begin{bmatrix} \mathsf{u}_s \\ \mathsf{v}_s \end{bmatrix}^T \begin{bmatrix} \Lambda_s^+ \left(\Gamma_s^+\right)^{-2} & \mathbb{O} \\ \mathbb{O} & -\Lambda_s^- \left(\Gamma_s^-\right)^{-2} \end{bmatrix} \begin{bmatrix} \mathsf{u}_s \\ \mathsf{v}_s \end{bmatrix}.$$

Let $\mathcal{V}_m$ and $\mathcal{V}_s$ be storage functions of the "human-master" and the "slave - environment" subsystems, respectively. Consider a storage function candidate $\mathcal{V} := \mathcal{V}_m + \alpha \cdot \mathcal{V}_s$, $\alpha > 0$. Taking into account the interconnection (7), the dissipation inequality becomes

$$\begin{aligned} &\mathcal{V}(t) - \mathcal{V}(t_0) \\ &\leq \int_{t_0}^{t} \left[\left(\mathsf{v}_s^{\mathbf{d}}\right)^T \Lambda_m^+ \left(\Gamma_m^+\right)^{-2} \mathsf{v}_s^{\mathbf{d}} + \left(\mathsf{v}_m^{\mathbf{d}}\right)^T \alpha \Lambda_s^+ \left(\Gamma_s^+\right)^{-2} \mathsf{v}_m^{\mathbf{d}} \right] d\tau \\ &\quad - \int_{t_0}^{t} \left[\mathsf{v}_m^T \Lambda_m^- \left(\Gamma_m^-\right)^{-2} \mathsf{v}_m + \mathsf{v}_s^T \alpha \Lambda_s^- \left(\Gamma_s^-\right)^{-2} \mathsf{v}_s \right] d\tau \end{aligned}$$

The fact that $\Lambda_m^+ \left(\Gamma_m^+\right)^{-2} \succeq 0$, $\Lambda_s^+ \left(\Gamma_s^+\right)^{-2} \succeq 0$ and diagonal implies that

$$\int_{t_0}^{t} \left(\mathsf{v}_s^{\mathbf{d}}\right)^T \Lambda_m^+ \left(\Gamma_m^+\right)^{-2} \mathsf{v}_s^{\mathbf{d}} d\tau \leq \int_{t_0 - \mathbf{T}^{\{2\}}}^{t} \mathsf{v}_s^T \Lambda_m^+ \left(\Gamma_m^+\right)^{-2} \mathsf{v}_s d\tau,$$

$$\int_{t_0}^{t} \left(\mathsf{v}_m^{\mathbf{d}}\right)^T \Lambda_s^+ \left(\Gamma_s^+\right)^{-2} \mathsf{v}_m^{\mathbf{d}} d\tau \leq \int_{t_0 - \mathbf{T}^{\{1\}}}^{t} \mathsf{v}_m^T \Lambda_s^+ \left(\Gamma_s^+\right)^{-2} \mathsf{v}_m d\tau,$$

therefore

$$\begin{aligned} \mathcal{V}(t) - \mathcal{V}(t_0) \leq& \int_{t_0 - \mathbf{T}^{\{1\}}}^{t_0} \mathsf{v}_m^T \alpha \Lambda_s^+ \left(\Gamma_s^+\right)^{-2} \mathsf{v}_m d\tau \\ &+ \int_{t_0 - \mathbf{T}^{\{2\}}}^{t_0} \mathsf{v}_s^T \Lambda_m^+ \left(\Gamma_m^+\right)^{-2} \mathsf{v}_s d\tau - \int_{t_0}^{t} \mathsf{v}_m^T \mathcal{E}_1 \mathsf{v}_m d\tau \\ &- \int_{t_0}^{t} \mathsf{v}_s^T \mathcal{E}_2 \mathsf{v}_s d\tau, \end{aligned}$$

where $\mathcal{E}_1, \mathcal{E}_2 \succ 0$ are defined by (10), (11). Now, taking into account $\mathcal{V}(t) \geq 0$, one see that the inequality

$$\begin{aligned} \int_{t_0}^{t} \left[\mathsf{v}_m^T \mathcal{E}_1 \mathsf{v}_m + \mathsf{v}_s^T \mathcal{E}_2 \mathsf{v}_s \right] d\tau \leq& \int_{t_0 - \mathbf{T}^{\{1\}}}^{t_0} \mathsf{v}_m^T \alpha \Lambda_s^+ \left(\Gamma_s^+\right)^{-2} \mathsf{v}_m d\tau \\ &+ \int_{t_0 - \mathbf{T}^{\{2\}}}^{t_0} \mathsf{v}_s^T \Lambda_m^+ \left(\Gamma_m^+\right)^{-2} \mathsf{v}_s d\tau + \mathcal{V}(t_0) \end{aligned}$$

holds for all $t \geq t_0$, and v_m, $\mathsf{v}_s \in \mathcal{L}_2^{(t_0,+\infty)}$ follows due to positive definiteness of $\mathcal{E}_1, \mathcal{E}_2$. From here, u_m, $\mathsf{u}_s \in \mathcal{L}_2^{(t_0,+\infty)}$ follows from (7), while η_m, y_m, η_s, $y_s \in \mathcal{L}_2^{(t_0,+\infty)}$ follows from (6) and the fact that $\mathbb{S}_m$, $\mathbb{S}_s$ have full rank due to (9). The proof is complete.

Appendix B. PROOF OF THEOREM 5

The proof of Theorem 5 is very similar to that of Theorem 2; the differences between the two are outlined below. Substituting (6), (13), and (14) into (8), the following expression for supply rate w_m can be obtained

$$w_m = \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}^T \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} \hat{G}_m^T W_m \hat{G}_m \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}.$$

However,

$$\begin{aligned} \hat{G}_m^T W_m \hat{G}_m &= (\mathbb{P}_m)^T G_m^T W_m G_m \mathbb{P}_m \\ &= (\mathbb{P}_m)^T \begin{bmatrix} \Lambda_m^+ & \mathbb{O} \\ \mathbb{O} & -\Lambda_m^- \end{bmatrix} \mathbb{P}_m = \begin{bmatrix} \hat{\Lambda}_m^+ & \mathbb{O} \\ \mathbb{O} & -\hat{\Lambda}_m^- \end{bmatrix}, \end{aligned}$$

therefore

$$\begin{aligned} w_m &= \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}^T \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} \begin{bmatrix} \hat{\Lambda}_m^+ & \mathbb{O} \\ \mathbb{O} & -\hat{\Lambda}_m^- \end{bmatrix} \begin{bmatrix} \Gamma_m^+ & \mathbb{O} \\ \mathbb{O} & \Gamma_m^- \end{bmatrix}^{-1} \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix} \\ &= \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}^T \begin{bmatrix} \hat{\Lambda}_m^+ \left(\Gamma_m^+\right)^{-2} & \mathbb{O} \\ \mathbb{O} & -\hat{\Lambda}_m^- \left(\Gamma_m^-\right)^{-2} \end{bmatrix} \begin{bmatrix} \mathsf{u}_m \\ \mathsf{v}_m \end{bmatrix}. \end{aligned}$$

Similarly,

$$w_s = \begin{bmatrix} \mathsf{u}_s \\ \mathsf{v}_s \end{bmatrix}^T \begin{bmatrix} \hat{\Lambda}_s^+ \left(\Gamma_s^+\right)^{-2} & \mathbb{O} \\ \mathbb{O} & -\hat{\Lambda}_s^- \left(\Gamma_s^-\right)^{-2} \end{bmatrix} \begin{bmatrix} \mathsf{u}_s \\ \mathsf{v}_s \end{bmatrix}.$$

The remaining part of the proof is exactly the same as that of Theorem (2).