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Stabilization and Adaptive Output Tracking for MIMO Systems with Distinct Input Delays*

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Abstract: In the paper the twofold problem of stabilization and output adaptive tracking is addressed for the class of multi-input multi-output (MIMO) linear time-invariant (LTI) unstable plants with known parameters, unmeasurable state, and known distinct input delays. The reference is represented by the vector of multi-harmonic time functions and is generated by an autonomous linear dynamic model (exosystem) with known order but unknown parameters. The amplitudes, phase shifts, and frequencies of the harmonics of these functions are unknown. The solution proposed is based on a robust predictor-feedback stabilizing control law, suitable parameterization of the tracking error, special implementation of the augmented error scheme, and direct adaptation algorithm providing asymptotic tracking without identification of the exosystem parameters. The stabilizing part of control palliates negative influence of distinct input delays. Regardless of the values of input delays, the adaptive control law designed ensures boundedness of all signals in the closed-loop system and drives the tracking error to zero.

Keywords: Distinct input delays, predictor-feedback stabilizing control, adaptive output tracking, multi-input multi-output system.

1. INTRODUCTION

In this paper the problem of output feedback adaptive tracking is resolved for a class of linear time invariant (LTI) multi-input multi-output (MIMO) systems with distinct input delays. The reference to be tracked is modeled as the vector output of linear exosystem with unknown parameters but known order. This paper develops results of the authors' results recently reported in Gerasimov et al. (2019a) and considering stable MIMO plants only. Therefore, the present paper extends the results of Gerasimov et al. (2019a) to the class of unstable MIMO plants with distinct delays. In other words, the control law to be designed should not only ensure tracking objective, but also stabilize the plant with distinct delays.

Theoretical interest in delayed systems is motivated by numerous practical applications. It is revealed from practice that many processes include aftereffect in their dynamics. Examples of such processes can be observed in technical systems and engineering, communication, networked controlled systems, chemistry, biology and are presented in monographs of Kolmanovskii and Myshkis (1999), Niculescu (2001), surveys Richard (2003), Gu and Niculescu (2003), and the references therein.

It is well-known that stabilization of even a linear systems with input delay is a nontrivial problem. In early works of Lewis (1979), Manitius and Olbrot (1979), Kwon and Pearson (1980), Watanabe and Ito (1981), Artstein (1982) stabilization techniques were developed for linear systems with single input delay, or with multiple delays of the same control signal. Recently multi-input systems with, potentially different, delays in each individual input channel were considered in Bekiaris-Liberis and Krstić (2017), Tsubakino et al. (2016), Kharitonov (2017), Zhu et al. (2018), Cai et al. (2019). In this case it is said about *distinct* input delays. In paper Kharitonov (2017) the case of both input and state delays in linear system was considered, while in papers Bekiaris-Liberis and Krstić (2017), Tsubakino et al. (2016), Zhu et al. (2018), Cai et al. (2019) backstepping design procedure was developed to design predictor-based state feedback controller for linear and nonlinear systems. In all these recent papers the design procedure has iterative character and is presented by a chain of expressions. In this paper we introduce relatively simple expressions with better computational robustness and demonstrate applicability of the approach to a special type of control laws with feedforward loops.

The control problem becomes much more difficult in the case when not just stabilization of delayed system is required, but also reference tracking and/or disturbance compensation. One important example of a such complex problem is presented by adaptive implementation of the *internal model principle* for systems with input delays. This problem has been attracting serious attention of the control community during the last years. Among the solutions proposed there are identification-based ones (see Pyrkin and Bobtsov (2016), Pyrkin and Bobtsov (2012), Wang et al. (2015)) as well as solutions based on implementation of backstepping procedure (see Bresch-Pietri and Krstić (2009), Basturk and Krstić (2014), Basturk and Krstić (2015)). As usual, the mentioned solutions require some *a priori* knowledge about boundaries of external signal frequencies and satisfaction of the persistent excitation (PE) condition.

An alternative approach consists in direct adaptation that does not require identification of the external signal parameters, and the upper bound of the number of harmonics in external signal is only used as prior knowledge (see Nikiforov (1997), Nikiforov (1998), Nikiforov (2001), Nikiforov (2003)). In Gerasimov et al. (2019b) the problem of adaptive tracking for single-input single-output (SISO) LTI system with measurable state and single input delay was considered. In Gerasimov et al. (2019a) the results were extended to the case of LTI MIMO system with unmeasurable state and distinct delays. However, the plant was assumed to be stable and its stabilization was not required. In this paper we consider much more general case of LTI unstable MIMO system with distinct delays.

The main contributions of the paper are the following:
1) simple expressions with better computational robustness for design of predictor-feedback control stabilizing LTI MIMO system with distinct delays;
2) special scheme of *augmented error* allowing one to completely compensate for the influence of distinct input delays on stability of the adaptation algorithm;
3) scheme of adaptive tracking *a priori* uncertain reference signal for MIMO systems with distinct input delays and unmeasurable state.

The paper is organized as follows. In Section II the problem statement with assumptions accepted is formulated. Section III presents preliminary results concerning stabilization of LTI MIMO systems with distinct delays, while Section IV presents the main results — adaptive control law design procedure. Simulation results are presented in Section V.

2. PROBLEM STATEMENT

Consider LTI MIMO plant of the form

$$\dot{x} = A_0 x + \sum_{i=1}^{q} b_i u_i(t-\tau_i), \quad y = C^\top x, \tag{1}$$

where $x \in \mathbb{R}^n$ is the unmeasurable state, each control signal $u_i \in \mathbb{R}^q$ is delayed by $\tau_i \geq 0$, $i = \overline{1,q}$, $y \in \mathbb{R}^q$ is the vector of output variables, $n \geq q$, A_0, $B = [b_1, \dots, b_q]$, C are the known matrices of appropriate dimensions, $x(t) = \phi(t), t \in [-\tau_{max}, 0]$ is the functional initial condition, $\tau_{max} = \max_i \{\tau_i\}$. Without loss of generality we assume that $\tau_1 \leq \tau_2 \ldots \leq \tau_q$.

The problem considered is to design an output-feedback control providing boundedness of all the closed-loop signals and ensuring the control objective

$$\lim_{t\to\infty} \|g(t) - y(t)\| = 0, \tag{2}$$

where $g \in \mathbb{R}^q$ is the vector of reference signals [1] .

The following assumptions are accepted.

Assumption 1. Triple (A_0, B, C) is controllable and observable, transfer matrix $W(s) = C^\top (sI - A_0)^{-1} B$ is minimum-phase and invertable, $s = d/dt$ is the differential operator.

Assumption 2. The reference g can be modeled as the output of the exosystem:

$$\dot{z} = \Gamma z, \qquad g = H^\top z, \tag{3}$$

where $z \in \mathbb{R}^m$ is unmeasurable state vector with unknown initial condition, Γ and H are the constant matrices of appropriate dimensions, and Γ has simple eigenvalues on the imaginary axis. Without loss of generality the pair (Γ, H) is assumed to be observable.

Assumption 3. The parameters of matrices Γ and H are unknown, while dimension m is known. The reference g is measurable.

[1] In this paper all the signals (control, reference, disturbance etc.) are vectors. Therefore the term "vector" will be omitted.

Remark 1. In contrast to Gerasimov et al. (2019a), we do not use restrictive assumption that the plant matrix A_0 is Hurwitz. Thus, the adaptive controller designed should stabilize the plant with distinct input delays. We will show, that based on approach introduced in Kharitonov (2017), Cai et al. (2019), this nontrivial problem can be resolved: a) with the use of compact iterative expressions; b) independently from design of feedforward loop responsible for asymptotic tracking.

3. PRELIMINARY RESULTS: ROBUST PREDICTION

We start with preliminary results concerning the problem of state-feedback stabilization of the plant (1).

3.1 Case study: $q = 2$

To present the main idea consider the plant

$$\dot{x} = A_0 x + b_1 u_1(t-\tau_1) + b_2 u_2(t-\tau_2), \tag{4}$$

where $\tau_1 < \tau_2$ and x is measurable. Following Kharitonov (2017), Cai et al. (2019) at first step we derive the control law for u_1:

$$u_1 = -k_1^\top x_{\tau 1}, \tag{5}$$

where $x_{\tau 1}$ is calculated as

$$\begin{aligned} x_{\tau 1}(t) &= \exp(A_0 \tau_1) x(t) + \\ &\int_{t-\tau_1}^{t} \exp(A_0(t-\mu))(b_1 u_1(\mu) + b_2 u_2(\mu - D_{2,1}))d\mu, \end{aligned} \tag{6}$$

k_1 is the constant vector defined later, and $D_{2,1} = \tau_2 - \tau_1$. It can be shown (see Krstić (2009)) that for $t \geq 0$ we have $x_{\tau 1}(t) = x(t + \tau_1)$. Therefore, substituting (5) into (4) we obtain:

$$\dot{x} = A_1 x + b_2 u_2(t - \tau_2), \ \ \forall t \geq \tau_1, \tag{7}$$

where $A_1 = A_0 - b_1 k_1^\top$. Now, we use model (7) to derive the control law for the second component u_2:

$$u_2 = -k_2^\top x_{\tau 2}, \tag{8}$$

where

$$x_{\tau 2}(t) = \exp(A_1 \tau_2) x(t) + \int_{t-\tau_2}^{t} \exp(A_1(t-\mu)) b_2 u_2(\mu) d\mu.$$

Taking into account that $x_{\tau 2} = x(t + \tau_2)$ for $t \geq \tau_1$, and replacing (8) in (7) we finally obtain $\dot{x} = A_2 x$ for $t \geq \tau_1 + \tau_2$, where $A_2 = A_0 - b_1 k_1^\top - b_2 k_2^\top$. Therefore, control (5), (8) provides stabilization of the plant (4) if the vectors k_1 and k_2 are chosen so that matrix A_2 is Hurwitz.

3.2 General case

Consider plant (1) with measurable x. Then generalizing procedure introduced above we define the control law by the following equations (for $i = \overline{1,q}$):

$$u_i = -k_i^\top x_{\tau i}, \tag{9}$$

where

$$\begin{aligned} x_{\tau i}(t) &= \exp(A_{i-1}\tau_i) x(t) + \\ &\int_{t-\tau_i}^{t} \exp(A_{i-1}(t-\mu)) \sum_{j=i}^{q} b_j u_j(\mu - D_{j,i}) d\mu, \end{aligned} \tag{10}$$

$D_{j,i} = \tau_j - \tau_i$ and

$$A_i = A_0 - \sum_{j=1}^{i} b_j k_j^\top, \tag{11}$$

while the vectors k_i are chosen so that the *closed-loop system matrix* A_q is Hurwitz. It is worth noting that $x_{\tau i}(t) = x(t + \tau_i)$

for $t \geq \sum_{j=0}^{i-1} \tau_j$, ($\tau_0 = 0$). Therefore, replacing all the control laws u_j in (1) we obtain $\dot{x} = A_q x$ for $t \geq \sum_{i=1}^{q} \tau_i$.

Remark 2. In Cai et al. (2019) the following relations were derived to calculate predictions:

$$x_{\tau i}(t) = \exp(A_{i-1}D_{i,i-1})x_{\tau(i-1)}(t) + \int_{t-D_{i,i-1}}^{t} \exp(A_{i-1}(t-\mu)) \sum_{j=i}^{q} b_j u_j(\mu - D_{j,i}) d\mu. \tag{12}$$

It can be shown that in this case $x_{\tau_i}(t) = x(t+\tau_i)$ for $t \geq 0$, and the closed-loop model takes the form $\dot{x} = A_q x$ for $t \geq \tau_q$. In comparison with (10), expressions (12) provide earlier prediction of $x(t+\tau_i)$ (right at the moment $t = \tau_i$) and better transient performance. However, in this case each prediction $x_{\tau i}$ involves previous one $x_{\tau(i-1)}$, while in expression (10) actual value of the state $x(t)$ is used. Therefore, we can conclude that in the case of unstable matrices A_i expressions (12) can demonstrate property of accumulating calculation errors, while predictions (10) are more robust.

Remark 3. Both expressions (10) and (12) generate predictions $x(t+\tau_i)$ for each distinct delay τ_i. However, if these predictions are really unnecessary, we can use the following approach. Instead of (9) introduce control signals with aligned delays

$$u_i = H_i(D_{q,i})[U_i], \tag{13}$$

where $H_i(D_{q,i})$ is the delay operator (i.e. $H_i(D_{q,i})[U_i(t)] = U_i(t - D_{q,i})$), $U_i = k_i^T \hat{x}$, and prediction $\hat{x}$ will be defined later. Substituting (13) into (1) we obtain

$$\dot{x} = A_0 x + BU(t - \tau_q),$$

where $U = [U_1, U_2, \ldots, U_q]^T$. Then, we can calculate single prediction $\hat{x}(t) = x(t+\tau_q)$ as (see Krstić (2009)):

$$\hat{x} = \exp(A_0\tau_q)x(t) + \int_{t-\tau_q}^{t} \exp(A_0(t-\mu))BU(\mu)d\mu.$$

3.3 Control with feedforward loops

Stabilizing control law (9), (10) contains feedback loops only. Such a specific control law structure allows one to calculate iteratively predicted values of the state vector and stabilize the plant. However, in the problems of state or output regulation a control law can contain feedforward loop that fails to predict the states. Now, we demonstrate that for specific error models invoked in problems of adaptive disturbance compensation or reference tracking we can apply this stabilization approach even if the control law contains additional feedforward loops.

Consider error model of the form (see Gerasimov et al. (2018), Paramonov et al. (2018), Gerasimov et al. (2019a), Gerasimov et al. (2019b))

$$\dot{e} = A_0 e + \sum_{i=1}^{q} b_i(\varphi_i(\theta_i, t-\tau_i) + u_i(t-\tau_i)), \tag{14}$$

where $e \in \mathbb{R}^n$ is the state error (e.g., tracking error), $\varphi_i(\theta_i, t-\tau_i)$ are the known bounded functions dependent on unknown constant parameter θ_i.

Lemma 1. Consider control laws

$$u_i = U_i - \varphi_i(\hat{\theta}_i, t), \tag{15}$$

where

$$U_i = -k_i^\top e_{\tau i}, \tag{16}$$

$$e_{\tau i} = \exp(A_{i-1}\tau_i)e(t) + \int_{t-\tau_i}^{t} \exp(A_{i-1}(t-\mu)) \sum_{j=i}^{q} b_j U_j(\mu - D_{j,i}) d\mu, \tag{17}$$

and bounded adjustable parameters $\hat{\theta}_i$ are tuned so that

$$\tilde{\varphi}_i(t-\tau_i) = \left(\varphi_i(\theta_i, t-\tau_i) - \varphi_i(\hat{\theta}_i, t-\tau_i)\right) \to 0 \; as \; t \to \infty.$$

Then the system closed by control laws (15) applied to model (14) can be represented as

$$\dot{e} = A_q e + \sum_{i=1}^{q} b_i(\tilde{\varphi}_i(t-\tau_i) + k_i^\top \Delta_i(t-\tau_i)), \;\; \forall t \geq \tau_q, \tag{18}$$

where $e \to 0$, $\tilde{\varphi}_i \to 0$, and $\Delta_i \to 0$, while $t \to \infty$.

Proof. Replacing (15) in (14) we obtain

$$\dot{e} = A_0 e + \sum_{i=1}^{q} b_i(\tilde{\varphi}_i(t-\tau_i) + U_i(t-\tau_i)). \tag{19}$$

In view of (19) we can conclude that for $e_{\tau i}$ defined by (17) we have:

$$e_{\tau i}(t) = e(t+\tau_i) - \Delta_i(t), \tag{20}$$

where

$$\Delta_i(t) = \int_{t-\tau_i}^{t} \exp(A_{i-1}(t-\mu)) \sum_{j=i}^{q} b_j \tilde{\varphi}_j(\mu - D_{j,i}) d\mu.$$

Since $\tilde{\varphi}_i(t)$ is bounded and tends to zero, we have $\Delta_i(t) \to 0$ as $t \to \infty$ for $i = \overline{1,q}$. Substituting (16) into (19) in view of (20) we obtain expression for the closed-loop error model (18). Proof is complete.

4. ADAPTIVE CONTROL LAW DESIGN

Moving toward our main result we, first, parameterize the reference signal and then construct the structure of adjustable tracking control together with the error model. Later is used for synthesis of adaptation algorithm. Then we design the independent stabilizing component.

4.1 Reference Signal Parameterization and Prediction

Using results reported in Nikiforov (1997), Nikiforov (2001), we present the reference signal in the form of linear regression:

$$g = \Theta^\top \xi + \varsigma, \tag{21}$$

where $\Theta \in \mathbb{R}^{m \times q}$ is the unknown constant matrix, $\xi \in \mathbb{R}^m$ is the state vector of the *reference filter*

$$\dot{\xi} = G\xi + Lg \tag{22}$$

with arbitrary $m \times m$ Hurwitz matrix G, and constant $m \times q$ matrix L chosen such that the pair (G, L) is controllable. The signal ς exponentially decays [2]. It is worth noting that the filter (22) is physically implementable, since it involves measurable signal g.

Substituting (21) into (22) we obtain *canonical form* of the exosystems

$$\dot{\xi} = (G + L\Theta^\top)\xi, \tag{23}$$

which can be used for parameterization of the predicted values of ξ necessary for design of adaptive control. It follows from the fundamental solution of (23) that

$$\xi(t+\tau_i) = R_i^\top \xi_i(t), \tag{24}$$

[2] Signal ς do not influence on stability of the closed-loop system and will be omitted.

where $R_i^\top = \exp((G+L\Theta^\top)\tau_i) \in \mathbb{R}^{q\times m}$ are the unknown matrices.

We will use expression (24) for adaptive controller design.

4.2 Error Model and Control Law Structure

Following standard design procedure (see Davison (1976)) we introduce state and output tracking errors

$$e = M\xi - x, \qquad \varepsilon = g - y, \tag{25}$$

where $n\times m$ matrix M will be defined later. Evaluation of the time derivative of e in view of (23) and the state equation of (1) after some simple transformations gives:

$$\dot{e} = A_0 e + \Big(M(G+L\Theta^\top) - A_0 M\Big)\xi - \sum_{i=1}^{q} b_i u_i(t-\tau_i). \tag{26}$$

Now, take into account that under Assumption 1 there exist matrices M and $\bar{\Theta}$ so that the following equations hold (see Davison (1976), Marino and Tomei (2003)) :

$$M(G+L\Theta^\top) - A_0 M = B\bar{\Theta}^\top, \quad C^\top M = \Theta^\top.$$

In view of (24) we can write:

$$\begin{cases} \dot{e} = A_0 e + \sum_{i=1}^{q} b_i(\psi_i^\top \xi(t-\tau_i) - u_i(t-\tau_i)), \\ \varepsilon = C^\top e, \end{cases} \tag{27}$$

where $\psi_i^\top = \bar{\theta}_i^\top R_i$ is the m-dimensional unknown vector, $\bar{\theta}_i^\top$ is the i-th row of matrix $\bar{\Theta}^\top$.

Analysis of the last equations motivates the following structure of the control law for the i-th channel:

$$u_i = \hat{\psi}_i^\top \xi + U_i, \tag{28}$$

where $\hat{\psi}_i$ is the vector of adjustable parameters, while U_i is a stabilizing component.

4.3 Adaptation algorithm design

Replacing (28) in (27) we obtain the closed-loop error model

$$\begin{cases} \dot{e} = A_0 e + \sum_{i=1}^{q} b_i(\tilde{\psi}_i^\top(t-\tau_1)\xi(t-\tau_i) - U_i(t-\tau_i)), \\ \varepsilon = C^\top e, \end{cases} \tag{29}$$

where $\tilde{\psi}_i = \psi_i - \hat{\psi}_i$ is the i-th vector of parametric errors.

However, we cannot use directly model (29) for design of an adaptation algorithm, since the matrix A_0 may be unstable, and the model contains delayed values of the adjustable parameters $\hat{\psi}_i(t-\tau_i)$. In order to overcome these obstacles we use special form of *augmented error* (see Gerasimov et al. (2018), Gerasimov et al. (2019b)) modified here for the considered problem and defined by the following lemma.

Lemma 2. Consider the filter

$$\begin{cases} \dot{\hat{e}} = A_0\hat{e} + L_e(\varepsilon + C^\top\hat{e}) + \\ \quad + \sum_{i=1}^{q} b_i(\hat{\psi}_i^\top(t-\tau_i)\xi(t-\tau_i) + U_i(t-\tau_i)), \\ \hat{\varepsilon} = \varepsilon + C^\top\hat{e} - \Xi^\top\hat{\psi}, \end{cases} \tag{30}$$

where $L_e \in \mathbb{R}^{n\times q}$ is such that the matrix $A_e = A_0 + L_e C^\top$ is Hurwitz, $\hat{\psi} = col(\hat{\psi}_1^\top, \ldots, \hat{\psi}_q^\top) \in \mathbb{R}^{qm}$ is the vector of adjustable parameters, $\Xi \in \mathbb{R}^{qm\times q}$ is the measurable matrix regressor given by

$$\Xi^\top = \big[W_{e1}(s)\,[\xi(t-\tau_1)]\ldots W_{eq}(s)\,[\xi(t-\tau_q)]\big],$$

$W_{ei}(s) = C^\top(sI - A_e)^{-1}b_i$ are the q-dimensional vectors. Then for the *augmented error* $\hat{\varepsilon}$ we have:

$$\hat{\varepsilon} = \Xi^\top\tilde{\psi}, \tag{31}$$

where $\tilde{\psi} = \psi - \hat{\psi}$ is the vector of parametric errors.

Proof. Taking into account (29) and (30) and applying the properties of linear systems it can be shown that

$$\hat{\varepsilon} = \sum_{i=1}^{q} W_{ei}(s)\left[\psi_i^\top\xi(t-\tau_i)\right] - \Xi^\top\hat{\psi} =$$
$$\big[W_{e1}(s)\,[\xi(t-\tau_1)]\ldots W_{eq}(s)\,[\xi(t-\tau_q)]\big]\psi - \Xi^\top\hat{\psi} =$$
$$\Xi^\top\tilde{\psi}.$$

It is known from adaptive control theory (see Narendra and Annaswamy (1989), Ioannou and Sun (1996)) that the error model (31) can be used for design of different adaptation algorithms including the following gradient-based one:

$$\dot{\hat{\psi}} = \gamma\Xi\hat{\varepsilon}, \tag{32}$$

where $\gamma > 0$ is the adaptation gain.

Remark 4. Due to Assumption 2 and Lemma 2, it is known *a priori* that the matrix regressor Ξ is bounded. Therefore, in spite of the fact that adaptation algorithm (32) involves the augmented error $\hat{\varepsilon}$, its normalization is not required.

Algorithm of adaptation (32) has the properties defined by the following lemma.

Lemma 3. Under Assumptions 1–3 algorithm of adaptation (32) together with filter (22), control law (28), and scheme of augmentation (30) being applied to plant (1) provides:

3.1) boundedness of $\|\hat{\varepsilon}\|, \|\hat{\psi}_i\|$;

3.2) asymptotic convergence $\|\tilde{\psi}_i^\top(t)\xi(t)\| \to 0$ and $\|\tilde{\psi}_i^\top(t-\tau_i)\xi(t-\tau_i)\| \to 0$ for $t\to\infty$.

Proof. Consider Lyapunov function candidate

$$V = \frac{1}{2\gamma}\tilde{\psi}^T\tilde{\psi} \tag{33}$$

with the time derivative $\dot{V} = -\hat{\varepsilon}^\top\hat{\varepsilon} \le 0$ evaluated in view of (32). The latter inequality immediately proves Property (3.1), asymptotic convergence $\|\hat{\varepsilon}\| \to 0$ for $t\to\infty$ (see Narendra and Annaswamy (1989), Ioannou and Sun (1996)) and, as a result, $\|\Xi^\top\tilde{\psi}(t)\| \to 0$ for $t\to\infty$.

Since regressor Ξ is bounded, it follows from (32) that $\|\dot{\hat{\psi}}\| \to 0$ for $t\to\infty$. Therefore, taking into account the *swapping lemma* (see Ioannou and Sun (1996)) we have $\sum_{i=1}^{q} W_{ei}(s)[\tilde{\psi}_i^\top\xi(t-\tau_i)] \to \Xi^\top\tilde{\psi}$ for $t\to\infty$. However, $\|\Xi^\top\tilde{\psi}(t)\| \to 0$, therefore $\|\sum_{i=1}^{q} W_{ei}(s)[\tilde{\psi}_i^\top\xi(t-\tau_i)]\| \to 0$ and $\|\sum_{i=1}^{q} W_{ei}(s)[\tilde{\psi}_i^\top(t-\tau_i)\xi(t-\tau_i)]\| \to 0$. Proof is complete.

4.4 Stabilizing component design

To utilize the results of Lemma 1 and then to design the stabilizing components U_i we compare the error model (29) with the form (19) given by Lemma 1 and reduce the problem of tracking to the stabilization one.

Toward this end we, noting that the tracking error e is not measurable (matrix M is unknown, x is not measurable), design the observer in the form

$$\dot{\bar{e}} = A_0\bar{e} + L_s(\varepsilon - C^\top\bar{e}) - \sum_{i=1}^{q} b_i U_i(t-\tau_i), \tag{34}$$

where $\bar{e}$ is the estimate of the vector e, and $L_s \in \mathbb{R}^{n\times q}$ is the matrix selected so that the matrix $A_s = A_0 - L_s C^\top$ is Hurwitz. By introducing the estimation error $\tilde{e} = e - \bar{e}$ and calculating its time derivative in view of (29) and (34) we obtain:

$$\dot{\tilde{e}} = A_s\tilde{e} + \sum_{i=1}^{q} b_i \tilde{\psi}_i^\top(t-\tau_i)\xi(t-\tau_i). \tag{35}$$

It is worth noting that in view of Property 3.2 of Lemma 3, we get: $\|\tilde{e}(t)\| \to 0$ for $t \to \infty$.

Thus, utilizing the results of Lemma 1 we design stabilizing components in the form $(i = \overline{1,q})$

$$U_i = -k_i^\top \bar{e}_{\tau i}, \tag{36}$$

where $\bar{e}_{\tau i}$ are calculated as

$$\bar{e}_{\tau i} = \exp(A_{i-1}\tau_i)\bar{e}(t) - \int_{t-\tau_i}^{t} \exp(A_{i-1}(t-\mu)) \sum_{j=i}^{q} b_j u_{sj}(\mu - D_{j,i})d\mu, \tag{37}$$

and the matrices A_i are defined by (11), while the vectors k_i are chosen so that A_q is Hurwitz.

Then the properties of the closed-loop system are defined by the following theorem.

Theorem 1. Under Assumptions 1–3 control law (28) together with algorithm of adaptation (32), reference filter (22), scheme of augmentation (30), observer of tracking error (34), and stabilizing component (36)-(37) being applied to plant (1) provides the following properties of the closed-loop system:
1) all the closed-loop signals are bounded;
2) control objective (2) is achieved.

Proof. It is worth noting that

$$\bar{e}_{\tau i}(t) = e(t+\tau_i) - \tilde{e}(t+\tau_i) - \Delta_{ei}(t), \tag{38}$$

where $\bar{e}_{\tau i}$ is calculated by (37) and

$$\Delta_{ei}(t) = \int_{t-\tau_i}^{t} \exp(A_{i-1}(t-\mu))L_s C^\top \tilde{e}(\mu+\tau_i)d\mu.$$

Then replacing (36) in (29) in view of (38) we obtain the closed-loop tracking model of the form

$$\dot{e} = A_q e + \sum_{i=1}^{q} b_i \left(\tilde{\psi}_i^\top(t-\tau_i)\xi(t-\tau_i) - k_i^\top(\tilde{e}(t+\tau_i) + \Delta_{ei}(t-\tau_i)\right),$$

where all the terms under the sum operator are bounded and tend to zero, while time tends to infinity. In view of boundedness of e and ξ we have boundedness of x. In view of boundedness of u_{si} from (30) we have boundedness of $\hat{e}$, and from (34) we have boundedness of $\bar{e}$. This completes the proof.

Remark 5. When the matrices A_i are not Hurwitz control law (36), (37) is not robust and not internally stable due to possible growth of integral in (37). To overcome this undesirable phenomenon, practical implementation of (37) with additional dynamic filters was proposed in Mondié and Michiels (2003) (see also Kharitonov (2017)).

5. SIMULATION RESULTS

Consider unstable plant (1) of the second order with

$$A_0 = \begin{bmatrix} -1.85 & -0.95 \\ 1.9 & 1 \end{bmatrix}, \; b_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \; b_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \; C^\top = \begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix},$$

initial conditions $x(0) = col(1,0)$, input delays $\tau_1 = 2[s]$, $\tau_2 = 3[s]$, and reference signal $g = col(\cos(5t), 4\sin(7t))$ with *a priori* unknown amplitudes, frequencies, and phases. It is worth noting that the pair (A_0, B) is controllable, while neither (A_0, b_1) nor (A_0, b_2) are controllable.

The vector ξ is generated by filter (22) with matrices

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -36 & -60 & -37 & -10 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Augmented error model (30), state error observer (34), and stabilizing component (36) are taken with matrices

$$L_e = \begin{bmatrix} 0.4 & 0.4 \\ -0.7 & -0.8 \end{bmatrix}, \; L_s = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \; k_1 = \begin{bmatrix} 1.05 \\ 1.05 \end{bmatrix}, \; k_2 = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}.$$

Gain of adaptation algorithm (32) is given by $\gamma = 100$.

Simulation results for the control system closed by adaptive control algorithm designed are presented in Fig. 1 and demonstrate achievement of the control objective as well as boundedness of the adjustable parameters despite the influence of delays.

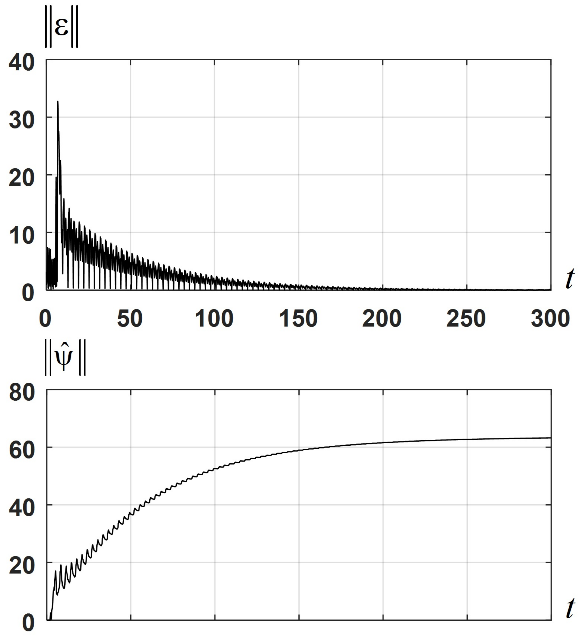

Fig. 1. Transients in the adaptive tracking closed-loop system with stabilization.

6. CONCLUSIONS

The solution of adaptive tracking problem is proposed for the class of MIMO LTI unstable plants with distinct input delays and unmeasurable state. System stabilization is provided by predictor-feedback control given by relatively simple and compact expression. The adaptive tracking control law with augmented error does not require identification of unknown reference model parameters. The output-feedback controller, involving adaptive tracking and stabilizing components, ensures boundedness of all the closed-loop signals and asymptotic convergence of the tracking error to zero for arbitrary input delays.

Our further research is focused on extensions of adaptive servocontroller problem for the plants with unknown input delays and parametric uncertainties.

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