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Robust Incentive Stackelberg Strategy for Markov Jump Delay Stochastic Systems via Static Output Feedback ⋆

Hiroaki Mukaidani, * **Saravanakumar Ramasamy,** * **Hua Xu,** ** **Weihua Zhuang** ***

* *Hiroshima University, 1-7-1 Kagamiyama, Higashi-Hiroshima 739-8521, Japan* (e-mail: mukaida@hiroshima-u.ac.jp, saravanamaths30@gmail.com).
** *The University of Tsukuba, 3-29-1, Otsuka, Bunkyo-ku, Tokyo, 112-0012 Japan* (e-mail: xu@mbaib.gsbs.tsukuba.ac.jp).
*** *University of Waterloo, 200 University Avenue West, Waterloo, ON N2L3G1, Canada* (e-mail: wzhuang@uwaterloo.ca).

Abstract: A static output feedback (SOF) incentive Stackelberg game (ISG) for a continuous-time Markov jump delay stochastic system (MJDSS) is discussed. The existence conditions on the SOF incentive Stackelberg strategy set are established in terms of the solvability of a set of higher-order cross-coupled stochastic algebraic Lyapunov-type equations (CCSALTEs). A classical Lagrange-multiplier technique is used to derive the CCSALTEs, thereby avoiding having to solve the bilinear matrix inequalities (BMIs), a well-known NP-hard problem in designing the SOF strategy. A heuristic algorithm is proposed to solve CCSALTEs such that convergence is attained by applying the Krasnoselskii-Mann (KM) iterative algorithm. A simple numerical example demonstrates the efficiency of the SOF incentive Stackelberg strategy.

Keywords: Stackelberg games, H_∞ control, stochastic systems, numerical algorithms.

1. INTRODUCTION

Over the past few decades, there has been a considerable amount of research on various control problems for MJDSSs to overcome stochastic switching. MJDSS control problems and related applications have attracted research attention because many physical systems involve rapid failure processes and sudden changes in operating points (Dragan et al. (2016); Mariton (1990)). Moreover, research on the ISG as the hierarchy strategy for MJDSSs has advanced rapidly in recent years to obtain the induced strategy set (see, e.g. Mukaidani et al. (2017a); Mukaidani (2020)).

A well-known drawback in the practical implementation of the state-feedback strategy set is that the required full state information of the overall system is not always available because of limited observations. Moreover, for complicated and/or distributed large-scale systems, such state information is difficult to observe. To overcome these drawbacks, the static output feedback (SOF) strategy is a powerful approach. Therefore, many researchers have focused on designing SOF control solutions, and there are many useful results for MJDSSs (Vargas et al. (2015); Dolgov and Hanebeck (2017)). In particular, SOF robust dynamic games for MJDSSs have been investigated (Mukaidani et al. (2018b)). Subsequently, the incentive Stackelberg problem has been studied using of the SOF strategy for MJDSSs (Mukaidani et al. (2018c)). However, the manner of developing SOF strategies for ISGs of MJDSSs remains an open problem. It is important to develop an incentive Stackelberg strategy for such systems because the delay appears in practical hierarchical systems such as network systems.

To address the aforementioned challenges, in this study, we investigate the ISG using the SOF strategy for a class of MJDSSs. Compared with recent results (Mukaidani et al. (2019b)), a distinct difference is that SOF incentive Stackelberg strategies for MJDSSs are developed for the first time. The existence conditions of the SOF incentive Stackelberg strategy are provided in terms of the solvability of a set of cross-coupled stochastic algebraic Lyapunov type equations (CCSALTEs). In particular, because a classical Lagrange-multiplier technique is used to solve the CCSALTEs, the bilinear matrix inequalities (BMIs) constraint is not considered here and the required strategy set can be obtained directly. As another important feature of this paper, a heuristic algorithm is proposed to solve the CCSALTEs. By applying the Krasnoselskii-Mann (KM) iterative algorithm (Yao et al. (2009)), it is also shown that convergence is attained. Finally, to demonstrate the effectiveness of the SOF incentive Stackelberg strategy for MJDSSs, a simple numerical example is discussed.

Notation: The notations used in this paper are fairly standard: **block diag** denotes the block diagonal matrix; I_n denotes the $n \times n$ identity matrix; **vec** denotes the column vector of a matrix; $\|\cdot\|$ denotes the Euclidean norm of a matrix; $\mathbb{E}[\,\cdot \mid r_t = k]$ stands

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for the conditional expectation operator with respect to the event $\{r_t = k\}$; $\mathbf{M}^s_{n,m}$ denotes space of all $\boldsymbol{S} = (S(1), \ldots, S(s))$ with $S(k)$ being $n \times m$ matrix, $k \in \mathscr{D}$, $\mathscr{D} = \{1, \ldots, s\}$. Moreover, the component of $\boldsymbol{S} + \boldsymbol{T}\boldsymbol{U}$ is defined as $\boldsymbol{S} + \boldsymbol{T}\boldsymbol{U} = (S(1) + T(1)U(1), \ldots, S(s) + T(s)U(s))$; $L^2_F([0,\ \infty),\ \mathbb{R}^n)$ denotes the space of all measurable functions, which is F_t-measurable for every $t \geq 0$, and $\mathbb{E}[\int_0^\infty \|u(t)\|^2 dt | r_t = k] < \infty$, $k \in \mathscr{D}$.

2. PRELIMINARY RESULTS

Let $w(t)$ be the one-dimensional Wiener process that is defined on a given filtered probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t \geq 0}, \mathscr{P})$, and r_t, $t \geq 0$, be a right continuous homogeneous Markov process taking values in a finite state space, $\mathscr{D} = \{1,\ 2, \ldots, s\}$. Without loss of generality, it is assumed that $\{w(t)\}_{t \geq 0}$ and $\{r_t\}_{t \geq 0}$ are independent stochastic processes. Furthermore, the transition probabilities are given by

$$\mathbf{P}\{r_{t+\delta t} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\delta t + o(\delta t), & \text{if } i \neq j \\ 1 + \pi_{ii}\delta t + o(\delta t), & \text{else} \end{cases}, \tag{1}$$

where $\delta t > 0$, $\pi_{k\ell} \geq 0$, $k \neq \ell$, $\pi_{kk} = -\sum_{\ell=1,\ \ell \neq k}^{s} \pi_{k\ell}$, $\lim_{\delta t \to 0} o(\delta t)/\delta t = 0$.

Consider the following MJDSS

$$dx(t) = [A(r_t)x(t) + A_h(r_t)x(t-h) + D(r_t)v(t)]dt + A_p(r_t)x(t)dw(t),\ x(t) = \phi(t),\ t \in [-h,\ 0], \tag{2a}$$

$$z(t) = H(r_t)x(t), \tag{2b}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector, $v(t) \in \mathbb{R}^{m_v}$ the external disturbance, $z(t) \in \mathbb{R}^{n_z}$ the controlled output, $w(t) \in \mathbb{R}$ a one-dimensional standard Wiener process defined in the filtered probability space, $h > 0$ the time-delay of the MJDSSs, and $\phi(t)$ a real-valued initial function. Without loss of generality, it is assumed that, for all $\delta \in [-h,\ 0]$, there exists scalar $\sigma > 0$ such that $\|x(t+\delta)\| \leq \sigma\|x(t)\|$ (Wang et al. (2002)).

In coefficients $\boldsymbol{A}$, $\boldsymbol{A}_h$, $\boldsymbol{A}_p \in \mathbb{M}^s_{n,n}$ and $\boldsymbol{B}_v \in \mathbb{M}^s_{n,m_v}$, $A(k)$, $A_h(k)$, $A_p(k)$ and $D(k)$, $k \in \mathscr{D}$, are constant matrices.

First, the related definition and lemmas are introduced.

Definition 1. (Wang et al. (2002); Cao and Lam (2000)) The MJDSS is said to be stochastically stable if, when $v(t) \equiv 0$, for all finite $\phi(t) \in \mathbb{R}^n$ defined on $[-h,\ 0]$ and initial mode $r_0 = k \in \mathscr{D}$, there exists $\tilde{M} > 0$ satisfying

$$\lim_{t_f \to \infty} \mathbb{E}\left[\int_0^{t_f} x^T(t,\phi,r_0)x(t,\phi,r_0)dt \middle| \phi, r_0 = k\right] \leq x^T(0)\tilde{M}x(0). \tag{3}$$

The following result can be proved by using the previous result in (Mukaidani (2020)) as a special case.

Lemma 2. Let γ denote the required disturbance attenuation level. Consider a set of symmetric positive semidefinite matrices $\boldsymbol{W} \geq 0$ and $U > 0$, such that the following CCSMIs holds for every $k \in \mathscr{D}$:

$$\Lambda(\boldsymbol{W}, U, k) < 0, \tag{4}$$

where $k = 1, \ldots, s$,

$$\Lambda(\boldsymbol{W}, U, k) := \begin{bmatrix} \Phi^{11}(k) & W(k)A_h(k) & W(k)D(k) \\ A_h^T(k)W(k) & -U & 0 \\ D^T(k)W(k) & 0 & -\gamma^2 I_{m_v} \end{bmatrix},$$

$\Phi^{11}(k) := W(k)A(k) + A^T(k)W(k) + H^T(k)H(k) + U + \sum_{\ell=1}^{s} \pi_{k\ell}W(\ell) + A_p^T(k)W(k)A_p(k)$.

Then, we have the following results:

i) The MJDSS in (2) is stochastically stable internally with $v(t) \equiv 0$;

ii) The following inequality holds:

$$\|z\|_2^2 < \gamma^2\|v\|_2^2 + \mathscr{F}_W(W(k), U), \tag{5}$$

where $\|z\|_2^2 := \mathbb{E}\left[\int_0^\infty \|z(t)\|^2 dt \middle| r_0 = k\right]$,
$\|v\|_2^2 := \mathbb{E}\left[\int_0^\infty \|v(t)\|^2 dt \middle| r_0 = k\right]$,
$\mathscr{F}_W(W(k), U) := x^T(0)W(k)x(0) + \int_{-h}^{0} \phi^T(s)U\phi(s)ds$;

iii) The worst-case disturbance is given by

$$v^*(t) = F^*_\gamma(r_t)x(t) = \gamma^{-2}D^T(r_t)W(r_t)x(t). \tag{6}$$

The following corollary can be established by tracing the proof of Lemma 2 with some change.

Corollary 1. Define the corresponding cost function for MJDSS (2) with $v(t) \equiv 0$ as follows:

$$J := \mathbb{E}\left[\int_0^\infty x^T(t,\phi,r_0)Q(r_t)x(t,\phi,r_0)dt \middle| \phi, r_0 = k\right], \tag{7}$$

where $Q(r_t) = Q^T(r_t) > 0$. Consider a set of symmetric positive semidefinite matrices $\boldsymbol{P} \geq 0$, $V > 0$ and positive scalars $\varepsilon(k)$ and $\nu(k)$, such that the following CCSMIs holds:

$$\Gamma(\boldsymbol{P}, V, k) < 0, \tag{8}$$

where $k = 1, \ldots, s$,

$$\Gamma(\boldsymbol{P}, V, k) := \begin{bmatrix} \Psi^{11}(k) & P(k)A_h(k) \\ A_h^T(k)P(k) & -V \end{bmatrix}.$$

$\Psi^{11}(k) := P(k)A(k) + A^T(k)P(k) + Q(k) + V + \sum_{\ell=1}^{s} \pi_{k\ell}P(\ell) + A_p^T(k)P(k)A_p(k)$.

Then,we have the following inequality

$$J < x^T(0)P(k)x(0) + \int_{-h}^{0} \phi^T(s)V\phi(s)ds := \mathscr{F}_P(P(k), V). \tag{9}$$

3. PROBLEM FORMULATION

Consider the following MJDSS with one leader and one follower:

$$dx(t) = \Big[A(r_t)x(t) + A_h(r_t)x(t-h) + B_0(r_t)u_0(t) + B_1(r_t)u_1(t) + D(r_t)v(t)\Big]dt + A_p(r_t)x(t)dw(t), \tag{10a}$$

$$x(t) = \phi(t),\ t \in [-h,\ 0], \tag{10b}$$

$$z(t) = \begin{bmatrix} E(r_t)x(t) \\ G_c(k)u_c(t) \end{bmatrix}, \tag{10c}$$

$$y_c(t) = C_c(r_t)x(t), \tag{10d}$$

with $u_c(t) = \begin{bmatrix} u_0(t) \\ u_1(t) \end{bmatrix}$, $y_c(t) = \begin{bmatrix} y_0(t) \\ y_1(t) \end{bmatrix}$, $C_c(r_t) = \begin{bmatrix} C_0(r_t) \\ C_1(r_t) \end{bmatrix}$, $G_c(k) = \textbf{block diag}(\ G_0(r_t)\ \ G_1(r_t)\ )$, $G_i^T(r_t)G_i(r_t) = I_{m_i}$, where $u_0(t) \in \mathbb{R}^{m_0}$ represents the leader's control input, $u_1(t) \in \mathbb{R}^{m_1}$ represents the follower's control input, $v(t) \in \mathbb{R}^{n_v}$ represents the external disturbance, $y_0(t) \in \mathbb{R}^{p_0}$ represents the leader's output measurement vector, and $y_1(t) \in \mathbb{R}^{p_1}$ represents the follower's output measurement vector. Other variables are the same as (2). The coefficients $\boldsymbol{A}$, $\boldsymbol{B}_0$, $\boldsymbol{B}_1$, $\boldsymbol{E}$, $\boldsymbol{A}_p$, $\boldsymbol{C}_0$, $\boldsymbol{C}_1$ are constant matrices of compatible dimensions.

Cost functionals of the leader and the follower are defined as follows:

$$J_0\left(u_0,u_1,v;x^0,k\right)=\mathbb{E}\left[\int_0^\infty\left\{x^T(t)Q_0(r_t)x(t)+u_0^T(t)R_{00}(r_t)u_0(t)+u_1^T(t)R_{01}(r_t)u_1(t)\right\}dt\Big|r_0=k\right], \quad (11a)$$

$$J_1\left(u_0,u_1,v;x^0,k\right)=\mathbb{E}\left[\int_0^\infty\left\{x^T(t)Q_1(r_t)x(t)+u_0^T(t)R_{10}(r_t)u_0(t)+u_1^T(t)R_{11}(r_t)u_1(t)\right\}dt\Big|r_0=k\right], \quad (11b)$$

where $k\in\mathscr{D}$, $Q_i(k)=Q_0^T(k)\geq 0$, $R_{ii}(k)=R_{ii}^T(k)>0$, $i=0,\ 1$, $R_{ij}(k)=R_{ij}^T(k)\geq 0$, $ij=10,\ 01$.
For the incentive Stackelberg game, the leaders announce the following incentive strategy to the follower ahead of time:

$$\begin{aligned}u_0^\dagger(t)&=F_0(r_t)C_0(r_t)x(t)+\Xi(r_t)\left[u_1(t)-F_1(r_t)C_1(r_t)x(t)\right]\\&=\Theta(r_t)x(t)+\Xi(r_t)u_1(t),\end{aligned} \quad (12)$$

where $\Theta(k)=F_0(k)C_0(k)-\Xi(k)F_1(k)C_1(k)$.
The parameters $\Theta(k)$ and $\Xi(k)$ are determined in accordance with the follower.
Finally, a robust SOF incentive Stackelberg game for MJLSSs can be formulated as follows.

Problem: For a given disturbance attenuation level $\gamma>0$, find, if possible, the SOF strategies

$$u_0^*(t)=F_0^*(r_t)y_0(t)=F_0^*(r_t)C_0(r_t)x(t), \quad (13a)$$
$$u_1^*(t)=F_1^*(r_t)y_1(t)=F_1^*(r_t)C_1(r_t)x(t), \quad (13b)$$

such that the following hold:

(i) The trajectory of MJDLSS in (10) satisfies the following inequalities in the sense that H_2/H_∞ control concept (Mukaidani et al. (2018b)) holds:

$$J_0(u_c^*,v^*;x^0,k)\leq J_0(u_c,v^*;x^0,k), \quad (14a)$$
$$0\leq J_\gamma(u_c^*,v^*;x^0,k)\leq J_\gamma(u_c^*,v;x^0,k), \quad (14b)$$

where $J_\gamma(u_c,v;x^0,\ k)=\mathbb{E}\left[\int_0^\infty\left\{\gamma^2\|v(t)\|^2-\|z(t)\|^2\right\}dt\Big|r_0=k\right]$,
$\|z(t)\|^2=x^T(t)E^T(r_t)E(r_t)x(t)+u_c^T(t)u_c(t)$.
On the other hand, consider the leader's incentive strategy in (12) and the worst-case disturbance $v^*(t)\in\mathscr{L}^2_{\mathscr{F}}(\mathbb{R}_+,\ \mathbb{R}^{n_v})$. The follower's decision $u_1^*(t)\in\mathscr{L}^2_{\mathscr{F}}(\mathbb{R}_+,\ \mathbb{R}^{n_1})$ can be selected as follows.
(ii) The bound $\mathscr{F}_{P_1}(P_1(k),V_1)$ of objective function (11b) should be minimized such that the following inequalities are satisfied:

$$\begin{aligned}J_1(u_0^\dagger,u_1,v^*;x^0,k)&=\mathbb{E}\left[\int_0^\infty\left\{x^T(t)Q_1(r_t)x(t)\right.\right.\\&+[\Theta(r_t)x(t)+\Xi(r_t)u_1(t)]^TR_{10}(r_t)[\Theta(r_t)x(t)+\Xi(r_t)u_1(t)]\\&\left.\left.+u_1^T(t)R_{11}(r_t)u_1(t)\right\}dt\Big|r_0=k\right],\\&<x^T(0)P_1(k)x(0)+\mathbf{Tr}[LL^TV_1]:=\mathscr{F}_{P_1}(P_1(k),V_1),\end{aligned} \quad (15a)$$

$$\Gamma_1(\boldsymbol{P}_1,V_1,\Xi(k),F_1(k),F_\gamma(k),k)<0, \quad (15b)$$

where $k=1,\ldots,s$,

$$\Gamma_1(\boldsymbol{P}_1,V_1,\Xi(k),F_1(k),F_\gamma(k),k):=\begin{bmatrix}\Psi_1^{11}(k) & P_1(k)A_h(k)\\ A_h^T(k)P_1(k) & -V_1\end{bmatrix},$$

$LL^T:=\int_{-h}^0\phi(s)\phi^T(s)ds$, $\Psi_1^{11}(k):=P_1(k)\tilde{A}(k)+\tilde{A}^T(k)P_1(k)+\tilde{Q}_1(k)+V_1+\sum_{\ell=1}^s\pi_{k\ell}P_1(\ell)+A_p^T(k)P_1(k)A_p(k)$, $\tilde{A}(k):=A_\gamma(k)+B_0(k)\Theta(k)+[B_1(k)+B_0(k)\Xi(k)]F_1(k)C_1(k)$, $A_\gamma(r_t):=A(r_t)+D(r_t)F_\gamma(r_t)$, $\tilde{Q}_1(k):=Q_1(k)+\Theta^T(k)R_{10}(k)\Theta(k)+\Theta(k)R_{10}(k)\Xi(k)F_1(k)C_1(k)+C_1^T(k)F_1^T(k)\Xi^T(k)R_{10}(k)\Theta^T(k)+C_1^T(k)F_1^T(k)[R_{11}(k)+\Xi^T(k)R_{10}(k)\Xi(k)]F_1(k)C_1(k)$.

4. MAIN RESULTS

In this section, the leader and follower's strategy set under disturbance attenuation condition is derived.

4.1 Leader's Strategy

The leader's team strategy set $(u_c^*(t),v^*(t))$ is investigated in terms of how they attenuate the disturbance under an H_∞ constraint. For this purpose, let us configure the MJDSS as the centralized system with any $v(t)=F_\gamma(r_t)x(t)$:

$$\begin{aligned}dx(t)&=\left[A_\gamma(r_t)x(t)+A_h(r_t)x(t-h)+B_c(r_t)u_c(t)\right]dt\\&+A_p(r_t)x(t)dw(t),\end{aligned} \quad (16a)$$

$$z(t)=\begin{bmatrix}E(r_t)x(t)\\ u_c(t)\end{bmatrix}, \quad (16b)$$

$$u_c(t)=F_c(r_t)y_c(t)=F_c(r_t)C_c(r_t)x(t), \quad (16c)$$

where $B_c(k)=[\,B_0(k)\ B_1(k)\,]$,
$F_c(k)=\textbf{block diag}\,(\,F_0(k)\ F_1(k)\,)$, $k\in\mathscr{D}$.
Furthermore, the cost functional in (11a) can be changed as follows:

$$J_0(u_0,u_1,v;x^0,k)=\mathbb{E}\left[\int_0^\infty\left\{x^T(t)Q_0(r_t)x(t)+u_c^T(t)R_c(r_t)u_c(t)\right\}dt\Big|r_0=k\right], \quad (17)$$

where $R_c(k)=\textbf{block diag}\Big(\,R_{00}(k)\ R_{01}(k)\,\Big)$.
Using the result of Corollary 1, $J_0(u_0,u_1,v;x^0,k)$ has the following cost bound:

$$J_0(u_0,u_1,v;x^0,k)<\mathscr{F}_{P_0}(P_0(k),V_0), \quad (18)$$

when

$$\Gamma_0(\boldsymbol{P}_0,V_0,F_c(k),F_\gamma(k),k):=\begin{bmatrix}\Psi_0^{11}(k) & P_0(k)A_h(k)\\ A_h^T(k)P_0(k) & -V_0\end{bmatrix}<0,$$

$\Psi_0^{11}(k):=P_0(k)\tilde{A}_\gamma+\tilde{A}_\gamma^TP_0(k)+C_c^T(k)F_c^T(k)R_c(k)F_c(k)C_c(k)+Q_0(k)+V_0+\sum_{\ell=1}^s\pi_{k\ell}P_0(\ell)+A_p^T(k)P_0(k)A_p(k)$,
$\tilde{A}_\gamma(k):=A(k)+B_c(k)F_c(k)C_c(k)+D(k)F_\gamma(k)$.
In order to obtain the leader's centralized strategy set, $F_c^*(k)$, the Karush-Kuhn-Tucker (KKT) conditions are derived. Define the following Lagrangian:

$$\mathscr{L}_0(k)=\mathbf{Tr}\,[P_0(k)]+\mathbf{Tr}[LL^TV_0]+\sum_{k=1}^s\mathbf{Tr}\,[S_0(k)\Delta_0(k)], \quad (19)$$

where $S_0(k)$ is the symmetric matrix of the Lagrange multiplier, and we set $r_0=k$. Furthermore, we have

$$\begin{aligned}\Delta_0(k)&:=\Delta_0(\boldsymbol{P}_0,V_0,F_c(k),F_\gamma,k)\\&=\Psi_0^{11}(k)+P_0(k)A_h(k)V_0^{-1}A_h^T(k)P_0(k).\end{aligned} \quad (20)$$

In this case, we have the following cross coupled stochastic matrix equations (CCSMEs):

$$\frac{\partial \mathscr{L}_0(k)}{\partial P_0(k)} = \Delta_0^1(k) = \Delta_0^1(\boldsymbol{S}_0, P_0(k), V_0, F_c(k), F_\gamma(k), k) = 0, \quad (21a)$$

$$\frac{\partial \mathscr{L}_0(k)}{\partial S_0(k)} = \Delta_0(k) = 0, \quad (21b)$$

$$\frac{1}{2}\cdot\frac{\partial \mathscr{L}_0(k)}{\partial F_i(k)} = \Delta_0^2(k) = \Delta_0^2(\boldsymbol{S}_0, P_0(k), F_c(k), k) = 0, \quad (21c)$$

where $\Delta_0^1(k) = I_n + S_0(k)[A_\gamma(k) + B_c(k)F_c(k)C_c(k)]^T + [A_\gamma(k) + B_c(k)F_c(k)C_c(k)]S_0(k) + \sum_{\ell=1}^{s} \pi_{\ell k} S_0(\ell) + A_p(k)S_0(k)A_p^T(k) + \big[S_0(k)P_0(k)A_h(k)V_0^{-1}A_h^T(k) + A_h(k)V_0^{-1}A_h^T(k)P_0(k)S_0(k)\big]$,
$\Delta_0^2(k) = [R_c(k)F_c(k)C_c(k) + B_c^T(k)P_0(k)]S_0(k)C_c^T(k)$.
From $\Delta_0^1(k) = 0$, we have $S_0(k) > 0$. Therefore, from $\Delta_0^2(k) = 0$, the following strategy set can be obtained:

$$u_c(t) = F_c^*(r_t)C_c(r_t)x(t), \quad (22)$$

where $F_c^*(k) = \textbf{block diag}\,(\,F_0^*(k)\ \ F_1^*(k)\,)$, $F_i^*(k) = -[R_{0i}(k)]^{-1}B_i^T(k)P_0(k)S_0(k)C_i^T(k)[C_i(k)S_0(k)C_i^T(k)]^{-1}$.

4.2 Followers' Strategy

Second, the follower's strategy is established. Let us consider the minimization problem for the cost bound, $\mathscr{F}_{P_1}(P_1(k), V_1)$, of (15a) such that LMI in (15b) is satisfied. In order to solve this optimization problem, consider the following Lagrangian:

$$\mathscr{L}_1(k) = \textbf{Tr}\ [P_1(k)] + \textbf{Tr}[LL^T V_1] + \sum_{k=1}^{s} \textbf{Tr}\ [S_1(k)\Delta_1(k)], \quad (23)$$

where $S_1(\ell) = S_1^T(\ell)$ is the Lagrange multipliers,

$$\Delta_1(k) := \Delta_1(\boldsymbol{P}_1, V_1, F_1(k), F_\gamma(k), k) = \Psi_1^{11}(k) + P_1(k)A_h(k)V_1^{-1}A_h^T(k)P_1(k). \quad (24)$$

As a necessary condition, the following equations can be derived by using the KKT condition:

$$\frac{\partial \mathscr{L}_1(k)}{\partial P_1(k)} = \Delta_1^1(k) = \Delta_1^1(\boldsymbol{S}_1, P_1(k), V_1, F_1(k), F_\gamma(k), k) = 0, \quad (25a)$$

$$\frac{\partial \mathscr{L}_1(k)}{\partial S_1(k)} = \Delta_1(k) = 0, \quad (25b)$$

$$\frac{1}{2}\cdot\frac{\partial \mathscr{L}_1(k)}{\partial F_1(k)} = \Delta_1^2(k) = \Delta_1^2(\boldsymbol{S}_1, P_1(k), F_1(k), k) = 0, \quad (25c)$$

where $\Delta_1^1(k) = I_n + S_1(k)\tilde{A}^T(k) + \tilde{A}(k)S_1(k) + \sum_{\ell=1}^{s} \pi_{\ell k} S_1(\ell) + A_p(k)S_1(k)A_p^T(k) + \big[S_1(k)P_1(k)A_h(k)V_1^{-1}A_h^T(k) + A_h(k)V_1^{-1}A_h^T(k)P_1(k)S_1(k)\big]$, $\Delta_1^2(k) = \big[R_{11}(k) + \Xi^T(k)R_{10}(k)\Xi(k)\big]F_1(k)C_1(k)S_1(k)C_1^T(k) + \big[\big(B_1(k) + B_0(k)\Xi(k)\big)^T P_1(k) + \Xi^T(k)R_{10}(k)\Theta(k)\big]S_1(k)C_1^T(k)$.
Therefore, if $C_1(k)S_1(k)C_1^T(k)$ is nonsingular, the gain of the leader's strategy, $F_1(k)$, can be computed as follows:

$$F_1^\dagger(k) = -\big[R_{11}(k) + \Xi^T(k)R_{10}(k)\Xi(k)\big]^{-1} \times\big[\big(B_1(k)+B_0(k)\Xi^T(k)\big)^T P_1(k) + \Xi^T(k)R_{10}(k)\Theta(k)\big] \times S_1(k)C_1^T(k)\big[C_1(k)S_1(k)C_1^T(k)\big]^{-1}. \quad (26)$$

In this case, since $F_1^\dagger(k) = F_1^*(k)$, the incentive of $\Xi(k)$ can be computed by

$$\Xi^T(k)\big[B_0^T(k)P_1(k) + R_{10}(k)F_0^*(k)C_0(k)\big] + R_{11}(k)F_1^*(k)C_1(k) + B_1^T(k)P_1(k) = 0. \quad (27)$$

4.3 Disturbance Attenuation Condition

Finally, the disturbance attenuation condition is derived. Consider the closed-loop MJDSS and the cost functions. For arbitrary $u_i(t) = F_i(r_t)y_i(t) = F_i(r_t)C_i(r_t)x(t)$, $i = 1, 2$, the closed-loop MJDSS is established as

$$dx(t) = \Big[\bar{A}(r_t)x(t) + A_h(r_t)x(t-h) + D(r_t)v(t)\Big]dt + A_p(r_t, t)x(t)dw(t), \quad (28a)$$

$$z(t) = \begin{bmatrix} E(r_t)x(t) \\ G_0(r_t)F_0^*(r_t)C_0(r_t) \\ G_1(r_t)F_1^*(r_t)C_1(r_t) \end{bmatrix} x(t), \quad (28b)$$

where $\bar{A}(r_t) := A(r_t) + B_c(r_t)F_c^*(r_t)C_c(r_t)$.
Thus, we have the following CCSMIs, using Lemma 2:

$$\tilde{\Lambda}(\boldsymbol{W}, U, k) < 0, \quad (29)$$

where $k = 1, \ldots, s$,
$\tilde{\Lambda}(\boldsymbol{W}, U, k) := \begin{bmatrix} \tilde{\Phi}^{11}(k) & W(k)A_h(k) & W(k)D(k) \\ A_h^T(k)W(k) & -U & 0 \\ D^T(k)W(k) & 0 & -\gamma^2 I_{m_v} \end{bmatrix}$, $\tilde{\Phi}^{11}(k) := W(k)\bar{A}(k) + \bar{A}^T(k)W(k) + E^T(k)E(k) + U + \sum_{\ell=1}^{s} \pi_{k\ell}W(\ell) + C_c^T(k)F_c^{*T}(k)F_c^*(k)C(k) + A_p^T(k)W(k)A_p(k)$.
Furthermore, the worst-case disturbance is given by

$$v^*(t) = \gamma^{-2}D^T(r_t)W(r_t)x(t). \quad (30)$$

Theorem 3. Consider the MJDSS in (12) with one leader $u_0(t)$, one follower $u_1(t)$ and deterministic disturbance $v(t)$. Assume that there exist the solution sets of (21), (25), (27) and (29). In this case, the incentive strategy (12) is worked such that the follower's strategy can be induced to the leader's strategy.

It should be noted that Markov jump processes without a state delay is a special case of this paper, e.g., set $A_h(r_t) \equiv 0$ in Mukaidani (2020).

5. HEURISTIC ALGORITHM

In order to compute the robust incentive SOF Stackelberg strategy set for the MJDSS, optimization problems (15) and (18) should be solved. However, it is difficult to obtain the solution set. Hence, we propose the following heuristic algorithm based on the KM iterations (Yao et al. (2009)):

Step 1. *Set the initial values: choose $F_i^{(0)}(k)$, $i = 0, 1$, $k = 1, \ldots, s$, such that closed-loop* MJDSS *in* (16a) *is stochastically stable; choose an appropriate κ value for $W^{(0)}(k) = \kappa I_n$ and compute $F_\gamma^{(0)}(k) = \gamma^{-2}D^T(k)W^{(0)}(k)$;*
Step 2-1. *Solve the following optimization problem for $P_0^{(n+1)}(k)$ and $V_0^{(n+1)}$ for variable α_0:*

$$\min_{\boldsymbol{\alpha}_0} \textbf{Tr}\left[\sum_{k=1}^{s} P_0^{(n+1)}(k) + LL^T V_0^{(n+1)}\right], \quad (31a)$$

s.t. $\boldsymbol{\alpha}_0 := (\boldsymbol{P}_0^{(n+1)}, V_0^{(n+1)})$ satisfies (31b),
$\Gamma_0(\boldsymbol{P}_0^{(n+1)}, V_0^{(n+1)}, F_c^{(n)}(k), F_\gamma^{(n)}(k), k)$

$$:= \begin{bmatrix} \boldsymbol{\Psi}_0^{11(n)}(k) & P_0^{(n+1)}(k)A_h(k) \\ A_h^T(k)P_0^{(n+1)}(k) & -V_0^{(n+1)} \end{bmatrix} < 0, \tag{31b}$$

where $k = 1,\dots,s$, $\boldsymbol{\Psi}_0^{11(n)}(k) := P_0^{(n+1)}(k)\tilde{A}_\gamma^{(n)}(k) + \tilde{A}_\gamma^{(n)T}(k)P_0^{(n+1)}(k) + Q_0(k) + V_0^{(n+1)} + \sum_{\ell=1}^s \pi_{k\ell}P_0^{(n+1)}(\ell) + C_c^T(k)F_c^{(n)T}(k)R_c(k)F_c^{(n)}(k)C_c(k) + A_p^T(k)P_0^{(n+1)}(k)A_p(k)$, $\tilde{A}_\gamma^{(n)}(k) := A(k) + B_c(k)F_c^{(n)}(k)C_c(k) + D(k)F_\gamma^{(n)}(k)$;

Step 2-2. *Solve the following* CCSMEs *for* $S_0^{(n+1)}(k)$:

$$\Delta_0^1(\boldsymbol{S}_0^{(n+1)}, P_0^{(n+1)}(k), V_0^{(n+1)},\ F_c^{(n)}(k), F_\gamma^{(n)}(k), k) = 0; \tag{32}$$

Step 2-3. *Compute* $F_i^{(n+1)}(k)$, $i = 0, 1$:

$$F_i^{(n+1)}(k) = -[R_{0i}(k)]^{-1}B_i^T(k)P_0^{(n+1)}(k)S_0^{(n+1)}(k)C_i^T(k) \times [C_i(k)S_0^{(n+1)}(k)C_i^T(k)]^{-1}; \tag{33}$$

Step 2-4. *Solve the following optimization problem for* $W^{(n+1)}(k)$ *for variables* $\boldsymbol{\beta}$:

$$\min_{\boldsymbol{\beta}} \sum_{k=1}^s \mathbf{Tr}[W^{(n+1)}(k) + LL^TU^{(n+1)}], \tag{34a}$$

s.t. $\boldsymbol{\beta} := (\boldsymbol{W}^{(n+1)}, U^{(n+1)})$ satisfies (34a),

$$\tilde{\Lambda}(\boldsymbol{W}^{(n+1)}, U^{(n+1)}, k) := \begin{bmatrix} \tilde{\Phi}^{11(n)}(k) & W^{(n+1)}(k)A_h(k) & W^{(n+1)}(k)D(k) \\ A_h^T(k)W^{(n+1)}(k) & -U^{(n+1)} & 0 \\ D^T(k)W^{(n+1)}(k) & 0 & -\gamma^2 I_{m_v} \end{bmatrix} < 0, \tag{34b}$$

where $k = 1,\dots,s$, $\tilde{\Phi}^{11(n)}(k) := W^{(n+1)}(k)\bar{A}^{(n)}(k) + \bar{A}^{(n)T}(k)W^{(n+1)}(k) + E^T(k)E(k) + C_c^T(k)F_c^{(n)}(k)F_c^{(n)}(k)C_c + U^{(n+1)} + \sum_{\ell=1}^s \pi_{k\ell}W^{(n+1)}(\ell) + A_p^T(k)W^{(n+1)}(k)A_p(k)$;

Step 2-5. *Set*

$$\boldsymbol{Z}_0^{(n+1)} \leftarrow \theta_0^{(n)}\boldsymbol{Z}_0^{(n+1)} + (1-\theta_0^{(n)})\boldsymbol{Z}_0^{(n)} \tag{35}$$

where $\boldsymbol{Z}_0^{(n)} := \begin{bmatrix} \boldsymbol{P}_0^{(n)} & \boldsymbol{S}_0^{(n)} & \boldsymbol{W}^{(n)} & V_0^{(n)} & U^{(n)} \end{bmatrix}$.

Furthermore, $\theta_0^{(n)} \in (0,1]$ *is chosen at each iteration to ensure that* $\mathscr{J}_0^{(n)} > \mathscr{J}_0^{(n+1)}$ *with*

$$\mathscr{J}_0^{(n)} = \sum_{k=1}^s \mathbf{Tr}[P_0^{(n)}(k) + S_0^{(n)}(k) + W^{(n)}(k)] + \mathbf{Tr}[V_0^{(n)} + U^{(n)}]; \tag{36}$$

Step 2-6. *If the iterative algorithm consisting of Steps 2-1 to 2-5 converges, we have obtained the iterative solutions as* $F_i^{(\infty)}(k) = F_i^*(k)$, $i = 0, 1$, $k = 1,\dots,s$, $F_\gamma^{(\infty)}(k) = F_\gamma^*(k)$; *otherwise, if the number of iterations reaches a preset threshold, declare that there is no strategy set. Stop.*

Step 3-1. *Solve the following optimization problem for* $P_1^{(m+1)}(k)$ *and* $V_1^{(m+1)}$ *for variable* α_1:

$$\min_{\boldsymbol{\alpha}_1} \mathbf{Tr}\left[\sum_{k=1}^s P_1^{(m+1)}(k) + LL^TV_1^{(m+1)}\right], \tag{37a}$$

$\boldsymbol{\alpha}_1 := (\boldsymbol{P}_1^{(m+1)}, V_1^{(m+1)})$

s.t. $\boldsymbol{\alpha}_1$ satisfies (37b),

$$\Gamma_1(\boldsymbol{P}_1^{(m+1)}, V_1^{(m+1)}, \Xi^{(m)}(k), F_1^*(k), F_\gamma^*(k), k) := \begin{bmatrix} \boldsymbol{\Psi}_1^{11(m)}(k) & P_1^{(m+1)}(k)A_h(k) \\ A_h^T(k)P_1^{(m+1)}(k) & -V_1^{(m+1)} \end{bmatrix} < 0, \tag{37b}$$

where $k = 1,\dots,s$, $\Psi_1^{11(m)}(k) := P_1^{(m+1)}(k)\tilde{A}^{(m)}(k) + \tilde{A}^{(m)T}(k)P_1^{(m+1)}(k) + \tilde{Q}_1^{(m)}(k) + V_1^{(m+1)} + \sum_{\ell=1}^s \pi_{k\ell}P_1^{(m+1)}(\ell) + A_p^T(k)P_1^{(m+1)}(k)A_p(k)$, $\tilde{A}^{(m)}(k) := A(k) + D(k)F_\gamma^*(k) + B_0(k)\Theta^{(m)}(k) + [B_1(k) + B_0(k)\Xi^{(m)}(k)]F_1^*(k)C_1(k)$, $\tilde{Q}_1^{(m)}(k) := Q_1(k) + \Theta^{(m)T}(k)R_{10}(k)\Theta^{(m)}(k) + \Theta^{(m)}(k)R_{10}(k)\Xi^{(m)}(k)F_1^*(k)C_1(k) + C_1^T(k)F_1^{*T}(k)\Xi^{(m)T}(k)R_{10}(k)\Theta^{(m)T}(k) + C_1^T(k)F_1^{*T}(k)[R_{11}(k) + \Xi^{(m)T}(k)R_{10}(k)\Xi^{(m)}(k)]F_1^*(k)C_1(k)$, $\Theta^{(m)}(k) = F_0^*(k)C_0(k) - \Xi^{(m)}(k)F_1^*(k)C_1(k)$;

Step 3-2. *Solve the following* CCSMEs *for* $\boldsymbol{S}_1^{(m+1)}(k)$:

$$\Delta_1^1(\boldsymbol{S}_1^{(m+1)}, P_1^{(m+1)}(k), V_1^{)m+1)}, F_1^*(k), F_\gamma^*(k), k) = 0; \tag{38}$$

Step 3-3. *Compute* $\Xi^{(m+1)}(k)$, $k = 1,\dots,N$:

$$\Xi^{(m+1)T}(k)\big(B_0^T(k)P_1^{(m+1)}(k) + R_{10}(k)F_0^*(k)C_0(k)\big) + R_{11}(k)F_1^*(k)C_1(k) + B_1^T(k)P_1^{(m+1)}(k) = 0. \tag{39}$$

Step 3-4. *Set*

$$\boldsymbol{Z}_1^{(m+1)} \leftarrow \theta_1^{(m)}\boldsymbol{Z}_1^{(m+1)} + (1-\theta_1^{(m)})\boldsymbol{Z}_1^{(m)} \tag{40}$$

where $\boldsymbol{Z}_1^{(m)} := \begin{bmatrix} \boldsymbol{P}_1^{(m)} & \boldsymbol{S}_1^{(m)} & V_1^{(m)} & \boldsymbol{\Xi}^{(m)} \end{bmatrix}$.

Furthermore, $\theta_1^{(m)} \in (0,1]$ *is chosen at each iteration to ensure that* $\mathscr{J}_1^{(m)} > \mathscr{J}_1^{(m+1)}$ *with*

$$\mathscr{J}_1^{(m)} = \sum_{k=1}^s \mathbf{Tr}\big[P_1^{(m)}(k) + S_1^{(m)}(k) + \Xi^{(m)}(k)\big] + \mathbf{Tr}\big[V_1^{(m)}\big]; \tag{41}$$

Step 3-5. *If the iterative algorithm consisting of Steps 3-1 to 3-4 converges, we have obtained the iterative solutions as* $\Xi^{(\infty)}(k) = \Xi(k)$, $k = 1,\dots,s$; *otherwise, if the number of iterations reaches a preset threshold, declare that there is no strategy set. Stop.*

Finally, the convergence property can be stated.

Theorem 4. The proposed heuristic algorithm achieves the convergence if there exists $\theta_0^{(n)} \in (0,1]$ such that for all $n \in \mathbb{N}$, $\mathscr{J}_0^{(n)} > \mathscr{J}_0^{(n+1)}$ in Steps 2. Furthermore, if there exists $\theta_1^{(n)} \in (0,1]$ such that for all $n \in \mathbb{N}$, $\mathscr{J}_1^{(m)} > \mathscr{J}_1^{(m+1)}$ in Steps 3, another algorithm based on the KM iterations also converges.

6. A SIMPLE EXAMPLE

To demonstrate the effectiveness and usefulness of the theoretical results presented in the previous sections, a simple computer simulation example is provided in the following. Consider the set of following parameters in the simulations:

$$s = 2,\ \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} -0.3 & 0.3 \\ 0.7 & -0.7 \end{bmatrix},$$

$$A(1) = \begin{bmatrix} -5 & 0 \\ 0 & -1 \end{bmatrix},\ A_p(1) = 0.2A(1),\ A_h(1) = 0.1A(1)$$
$$B_0(1) = \begin{bmatrix} -0.5 & -1 \\ 0 & 0 \end{bmatrix},\ B_1(1) = \begin{bmatrix} -0.5 & -2.5 \\ 1 & 1 \end{bmatrix}$$
$$H(1) = [\, 1 \ 1 \,],\ C_0(1) = C_1(1) = [\, 1 \ 0 \,]$$
$$A(2) = \begin{bmatrix} -15 & 0 \\ 0 & -1 \end{bmatrix},\ A_p(2) = 0.2A(2),\ A_h(2) = 0.1A(2)$$
$$B_0(2) = \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix},\ B_1(2) = \begin{bmatrix} -1 & -2.5 \\ 1 & 1 \end{bmatrix}$$
$$H(2) = [\, 2 \ 1 \,],\ C_0(2) = C_1(2) = [\, 1 \ 0 \,]$$
$$LL^T := \int_{-h}^{0} \phi(s)\phi^T(s)ds = \text{block}\,(\, 1 \ 0 \,),\ h = 1,$$
$$Q_0(k) = Q_1(k) = I_2$$
$$R_{00}(k) = I_2,\ R_{01}(k) = 2I_2,\ R_{10}(k) = 1.5I_2,\ R_{11}(k) = 0.5I_3.$$

Next, we select $\gamma = 7$. Using the proposed KM iterations, the leader's strategy set $u_c(t) = F_c(r_t)C_c(r_t)x(t)$ and the worst case disturbance are obtained as follows:

$$F_0(1) = \begin{bmatrix} 5.3440\text{e-}2 \\ 1.0688\text{e-}1 \end{bmatrix},\ F_1(1) = \begin{bmatrix} 1.4568\text{e-}2 \\ 1.2145\text{e-}1 \end{bmatrix},$$
$$F_0(2) = \begin{bmatrix} 4.9123\text{e-}2 \\ 4.9123\text{e-}2 \end{bmatrix},\ F_1(2) = \begin{bmatrix} 2.1468\text{e-}2 \\ 5.8310\text{e-}2 \end{bmatrix},$$
$$F_\gamma(1) = -[\, -1.1059\text{e-}3 \ \ -1.7165\text{e-}3 \,],$$
$$F_\gamma(2) = -[\, -5.0705\text{e-}4 \ \ -7.2285\text{e-}4 \,].$$

Second, the related incentives $\Xi(r_t)$ are given by

$$\Xi(1) = -\begin{bmatrix} 1.5319\text{e-}1 & 1.2970 \\ 3.0638\text{e-}1 & 2.5941 \end{bmatrix},$$
$$\Xi(2) = -\begin{bmatrix} 6.4609\text{e-}1 & 1.7558 \\ 6.4609\text{e-}1 & 1.7558 \end{bmatrix}.$$

Finally, it can be observed that the strategy of the follower based on the incentive $\Xi(r_t)$ in (16) is induced to the leader's strategy. In particular, the relation $F_1^*(r_t) = F_1^\dagger(r_t)$ is satisfied.

We employ the proposed KM iterative algorithm to obtain the converged solutions and the strategies. In particular, Step 2 only is demonstrated. The initial gains are set as $F_0^{(0)}(k) = F_1^{(0)}(k) = [\, \kappa \ \kappa \,]^T$, $\kappa = 5.0$ for $k = 1,\ 2$. The initial condition was selected by the trial and error method, such that the closed loop system is stable. The algorithms converge after 33 iterations, with an accuracy of 10^{-7}.

In Steps 2 and 3 of the heuristic algorithms, the value of $\theta_i^{(\tau)}$, $i = 0,\ 1$ are set to 0.5. As a result, it is easy to show that the proposed algorithm generates a non-increasing sequence for the cost.

7. CONCLUSION

In this paper, the robust SOF incentive Stackelberg games in the two-level decision hierarchy for a MJDSS has been studied. Compared to previous studies, this paper differs distinctly in that the robust SOF incentive Stackelberg strategies for the state delay are developed for the first time. The existence conditions are provided in terms of the solvability of a set of CCSALTEs. A classical Lagrange-multiplier technique is used to solve the CCSALTEs, thereby avoiding having to solve the BMIs, which is a well-known NP-hard problem in designing SOF strategies. Furthermore, a novel heuristic algorithm based on the KM iteration is developed to guarantee convergence analytically. A simple numerical example demonstrates the existence of the SOF incentive Stackelberg strategies and the effectiveness of the proposed algorithm.

The robust incentive Stackelberg game is an important recent research area. However, unsolved problems remain. To the best of our knowledge, incentive Stackelberg game for stochastic linear parameter-varying (LPV) system with time-delay has not been investigated. This problem can be addressed in future studies.

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