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Collision-avoiding decentralized control for vehicle platoons: a mechanical perspective

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Abstract: A new bidirectional decentralized control algorithm for vehicle platoons is proposed, which guarantees absence of collisions between the vehicles. The algorithm exploits an elegant parallel between vehicles platoon and chains of interconnected mass-spring-damper systems and the idea of barrier certificates. Stability and robustness properties of the algorithm are examined. The results are illustrated by numerical examples, simulating different driving scenarios.

1. INTRODUCTION

Autonomous vehicles and automated driving are recognized as key technologies for future mobility and infrastructures of smart cities (Medina-Tapia and Robusté, 2018). One of the classical "benchmark problems" in automated driving is the design of a cooperative adaptive cruise control (CACC) algorithm for longitudinal motion of a vehicle platoon, see, e.g., Xiao and Gao (2010).

Linear CACC algorithms are the most widely studied in the literature. They range from the simplest predecessor-follower models of vehicle strings (Herman et al., 1958; Bender and Fenton, 1970) to more recent algorithms based on the ideas of decentralized and multi-agent control applicable to platoons with more general communication topologies, see, e.g., (Middleton and Braslavsky, 2010; Ploeg et al., 2014; Zheng et al., 2016; Sabău et al., 2017; Firooznia et al., 2017). These works focus on establishing conditions for stability and string stability, that is, attenuation of disturbances propagating through the platoon. While these algorithms ensure *asymptotic* convergence of position errors (entailing collision avoidance), it is difficult to examine the transient dynamics and, in particular, to prove safety at any time. The same problem exists in analysis of more advanced algorithms able to cope with parametric uncertainties (Guo et al., 2016).

CACC algorithms, for which safety at any time can be proved rigorously, can be divided into three large classes. The first class is constituted by MPC-based algorithms (Dunbar and Caveney, 2012; Kianfar et al., 2015; Zheng et al., 2017) that include safety constraints directly into the optimization problem. The price to be paid is the necessity to implement complicated optimization solvers and transmit large amount of data between the vehicles. Moreover, restructuring the platoon (e.g., adding a new vehicle), requires re-initialization of the optimization procedure. The second approach to collision-avoiding CACC design is based on analysis of reachability sets (Lygeros et al., 1998; Kianfar et al., 2013; Alam et al., 2014; Nilsson et al., 2016; Ligthart et al., 2018), aiming to estimating the set of initial conditions for which safety can be guaranteed at all subsequent time instants. The resulting algorithms are simpler than optimization-based methods, however, in practice the estimates of safe sets appear to be rather conservative (Ligthart et al., 2018). The third class of CACC controllers with guaranteed safety exploits the idea of barrier functions, which are widely used in multi-robot coordination algorithms (Tanner et al., 2007; Wang et al., 2017; Ames et al., 2017; Chen et al., 2018; Ligthart et al., 2018). Often the barrier function grows unbounded as the distance between some two robots vanishes (Tanner et al., 2007); every algorithm providing its boundedness then automatically ensures absence of collisions.

The contribution of this paper is twofold. First, we point out an important analogy between linear bidirectional algorithms for control of vehicle platoons (Barooah et al., 2009) and the Lagrangian equations for a chain of interconnected heterogeneous spring-damper systems. Second, we examine a nonlinear algorithm that is obtained by modifying the Lagrangian equations by augmenting the potential energy with a barrier function that ensures the absence of collisions. A similar modification of the Lagrangian has been used, in particular, for continuous approximations of hybrid mechanical systems (Menini and Tornambe, 2000; Menini and Tornambe, 2003; Menini et al., 2018). It should be noticed that the algorithm uses only distances between adjacent vehicles and a relative velocity of a vehicle with respect to its predecessor and follower (which can be measured, e.g., by two radars/lidars, installed in the front and in the rear of a vehicle). Convergence and robustness properties of the algorithm are examined; the results are illustrated by numerical simulations corresponding to different driving scenarios (Mullakkal-Babu et al., 2016).

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2. PROBLEM STATEMENT

We consider a platoon of N vehicles indexed 1 through N in the upstream direction so that N is the number of the platoon leader and 1 is the last vehicle of the platoon, see Fig. 1. In CACC algorithm design, a vehicle is typically modeled as a third- or second-order linear control system based on feedback linearization (Swaroop et al., 1994). The third-order model takes into account the engine internal dynamics, which causes a discrepancy between the commanded and actual accelerations (Ploeg et al., 2014; Zheng et al., 2016). Following Swaroop et al. (1994); Dunbar and Caveney (2012); Barooah et al. (2009); Ames et al. (2017), we confine ourselves to a simpler point-mass (double integrator) dynamics that neglects the engine dynamics:

$$a_i = \dot{v}_i = \ddot{x}_i = \frac{1}{m_i} u_i,$$

where x_i, v_i, a_i, u_i denotes the position, velocity, acceleration and the control input (force) on the ith vehicle.

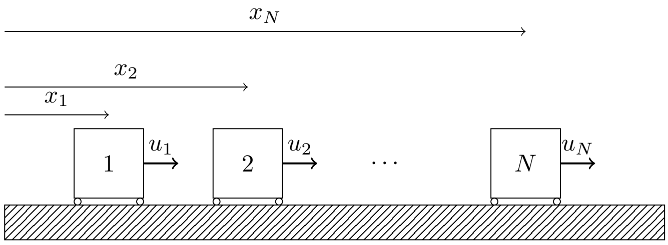

Fig. 1. Platoon of N vehicles.

We introduce two sets of parameters: for each pair of adjacent vehicles $(i, i+1)$ let $\ell_{i,i+1} > 0$ stand for the *safe* distance which should be kept at any time and $r_{i,i+1} \geq \ell_{i,i+1}$ denote the *desired* distance [1].

The algorithm proposed below assumes that

- each vehicle i (except for the leader $i = N$) knows the distance to and relative velocity of the predecessor;
- each vehicle i (except for the last one $i = 1$) knows the distance to and relative velocity of the follower;
- the leader knows the desired platoon's speed $v_d \geq 0$.

Notice that if vehicles are equipped with front and rear radars or a 360-degree vision radar, the proposed algorithm can be implemented without communication among the vehicles which, however, can be useful for detecting sensor faults and providing sensor redundancy. If a car is equipped only by a front radar, an undirected communication follower-predecessor is necessary.

The purpose of the algorithm is to provide safety, distance policy and the predefined speed of the platoon

$$x_{i+1}(t) - x_i(t) > \ell_{i,i+1}, \ \forall i = 1, \ldots, N-1, t \geq 0, \quad (1)$$

$$\lim_{t\to\infty} (x_{i+1} - x_i(t)) = r_{i,i+1}, \ \forall i = 1, \ldots, N-1, \quad (2)$$

$$\lim_{t\to\infty} v_i(t) = v_d, \qquad \forall i = 1, \ldots, N. \quad (3)$$

[1] For simplicity, we consider here a constant spacing policy: the desired distance between two adjacent vehicles is independent of their velocities. The algorithm examined below can be modified to a more general situation where $r_{i,i+1} = r^0_{i,i+1} + h_i v_i$, where $r^0_{i,i+1}$ is the standstill distance and $h_i > 0$ is the time headway constant.

The design of the algorithm (Section 4) is inspired by an analogy between a vehicle platoon and a mechanical mass-spring-damper system, described in Section 3.

3. A MASS-SPRING-DAMPER SYSTEM AND A LINEAR PLATOONING ALGORITHM

Consider a system (Fig. 2) constituted by N masses $m_1, m_2, \ldots, m_N$ that are pairwise connected by springs and dampers whose stiffness and damping coefficients are $k_{i,i+1} > 0$ and $d_{i,i+1} > 0$ and the lengths at rest are $r_{i,i+1} > 0$. Mass m_N is influenced by external force $u(t)$.

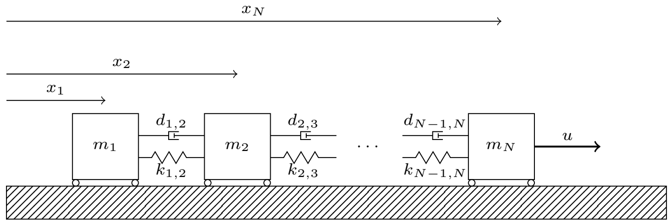

Fig. 2. A mass-spring-damper system.

Introducing the total kinetic energy T, total potential energy U and the Rayleigh dissipation function of the system D (Goldstein et al., 2002)

$$T(v_1, \ldots, v_N) = \tfrac{1}{2} \sum_{i=1}^{N} m_i v_i^2,$$

$$U(x_1, \ldots, x_N) = \tfrac{1}{2} \sum_{i=1}^{N-1} k_{i,i+1}(x_{i+1} - x_i - r_{i,i+1})^2,$$

$$D = \tfrac{1}{2} \sum_{i=1}^{N-1} d_{i,i+1}(v_{i+1} - v_i)^2,$$

the standard Euler-Lagrange equations of motion are

$$\dot{v}_1 = \tfrac{k_{1,2}}{m_1}(x_2 - x_1 - r_{1,2}) + \tfrac{d_{1,2}}{m_1}(v_2 - v_1), \quad (4a)$$

$$\dot{v}_i = \tfrac{k_{i,i+1}}{m_i}(x_{i+1} - x_i - r_{i,i+1}) + \tfrac{d_{i,i+1}}{m_i}(v_{i+1} - v_i) + \tfrac{k_{i-1,i}}{m_i}(x_{i-1} + r_{i-1,i} - x_i) + \tfrac{d_{i-1,i}}{m_i}(v_{i-1} - v_i), \quad (4b)$$
$$(i = 2, \ldots, N-1)$$

$$\dot{v}_N = \tfrac{k_{N-1,N}}{m_N}(x_{N-1} + r_{N-1,N} - x_N) + \tfrac{d_{N-1,N}}{m_N}(v_{N-1} - v_N) + \tfrac{1}{m_N} u, \quad (4c)$$

The parallel with mechanics allows to deduct the natural fact: in the absence of external force, the springs return to their rest positions and the oscillation of masses decays due to the dissipation of energy. In view of the system's linearity, the properties (6) will hold also for any decaying force ($\lim_{t\to+\infty} u(t) = 0$). In particular, these properties are preserved by a simple proportional controller maintaining the desired speed of the "leading" mass

$$u(t) = \sigma(v_d - v_N(t)). \quad (5)$$

Mathematically, these properties are formulated as follows.

Theorem 1. Let $u = 0$. Then, the trajectories of system (4) are such that, for $i = 1, \ldots, N-1$,

$$\lim_{t\to\infty} v_{i+1}(t) - v_i(t) = 0, \quad (6a)$$

$$\lim_{t\to\infty} x_{i+1}(t) - x_i(t) = r_{i,i+1}. \quad (6b)$$

Under the speed control policy (5), conditions (6) hold and, additionally, $\lim_{t\to\infty} v_i(t) = v_d$, $i = 1, \dots, N$.

Theorem 1 inspires the following linear CACC algorithm for a platoon of vehicles, providing the conditions (2), (3)

$$\begin{aligned} u_1 &= k_{1,2}\left(x_2 - x_1 - r_{1,2}\right) + d_{1,2}\left(v_2 - v_1\right), \\ u_i &= k_{i,i+1}\left(x_{i+1} - x_i - d_{i,i+1}\right) + d_{i,i+1}\left(v_{i+1} - v_i\right) \\ &\quad + k_{i-1,i}\left(x_{i-1} + r_{i-1,i} - x_i\right) + d_{i-1,i}\left(v_{i-1} - v_i\right), \\ u_N &= k_{N-1,N}\left(x_{N-1} + r_{N-1,N} - x_N\right) \\ &\quad + d_{N-1,N}\left(v_{N-1} - v_N\right) + \sigma(v_d - v_N) \end{aligned} \tag{7}$$

where $d_{i,i+1} > 0, k_{i,i+1} > 0$, $i = 1, \dots, N-1$, and $\sigma > 0$.

Obviously, the controller cannot guarantee safety, since the equations do not involve safe distances $\ell_{i,i+1}$.

4. PLATOONING ALGORITHM WITH GUARANTEED SAFETY

A natural method allowing to guarantee the absence of collisions is to augment the potential energy with the *barrier function* that grows without bound when the solution leaves the safety set (some distance $x_{i+1} - x_i$, or the length of the ith spring, tends to the minimal value $\ell_{i,i+1}$); see, e.g., Menini and Tornambe (2000).

Assume that an additional nonlinear spring is added between the bodies having mass m_i and m_{i+1}, $i = 1, \dots, N-1$, so that the total potential energy of the mechanical system is *augmented* as follows:

$$U_e(x) = U(x) + \frac{1}{2}\sum_{i=1}^{N-1} \frac{\kappa_{i,i+1}}{(x_{i+1} - x_i - \ell_{i,i+1})^2}, \tag{8}$$

where $\kappa_{i,i+1}$ are strictly positive constants.

Remark 1. Note that the function $\frac{1}{2}\sum_{i=1}^{N-1} \frac{\kappa_{i,i+1}}{(x_{i+1}-x_i-\ell_{i,i+1})^2}$ that is used in (8) to augment the potential energy of the mechanical system can be in principle substituted by any barrier function $\Psi(x)$ that is such that $\Psi(x) \geq 0$ for all $x \in \mathbb{R}^n$ such that $x_{i+1} - x_i > \ell_{i,i+1}\, \forall i = 1, \dots, N-1$ and $\Psi(x) \to +\infty$ if $x_{i+1} - x_i \to \ell_{i,i+1}$ for some i. The choice of barrier function (8) simplifies analysis of equilibria points.

Replacing $U(x)$ with the augmented potential energy $U_e(x)$, the Lagrange equations become as follows

$$\begin{aligned} \dot v_1 &= \tfrac{k_{1,2}}{m_1}\left(x_2 - x_1 - r_{1,2}\right) + \tfrac{d_{1,2}}{m_1}\left(v_2 - v_1\right) \\ &\quad - \tfrac{\kappa_{1,2}}{m_1}\tfrac{1}{(x_2-x_1-\ell_{i,i+1})^3}, \\ \dot v_i &= \tfrac{k_{i,i+1}}{m_i}\left(x_{i+1} - x_i - r_{i,i+1}\right) + \tfrac{d_{i,i+1}}{m_i}\left(v_{i+1} - v_i\right) \\ &\quad + \tfrac{k_{i-1,i}}{m_i}\left(x_{i-1} + r_{i-1,i} - x_i\right) + \tfrac{d_{i-1,i}}{m_i}\left(v_{i-1} - v_i\right) \\ &\quad + \tfrac{\kappa_{i-1,i}}{m_i}\tfrac{1}{(x_i-x_{i-1}-\ell_{i,i-1})^3} - \tfrac{\kappa_{i,i+1}}{m_i}\tfrac{1}{(x_{i+1}-x_i-\ell_{i,i+1})^3}, \\ \dot v_N &= \tfrac{k_{N-1,N}}{m_N}\left(x_{N-1} + r_{N-1,N} - x_N\right) \\ &\quad + \tfrac{d_{N-1,N}}{m_N}\left(v_{N-1} - v_N\right) \\ &\quad + \tfrac{\kappa_{N-1,N}}{m_N}\tfrac{1}{(x_N-x_{N-1}-\ell_{N,N-1})^3} + \tfrac{1}{m_N}u. \end{aligned} \tag{9}$$

Combining them with the speed controller (5), one arrives at the following platooning algorithm

$$\begin{aligned} u_1 &= k_{1,2}\left(x_2 - x_1 - r_{1,2}\right) + d_{1,2}\left(v_2 - v_1\right) \\ &\quad - \tfrac{\kappa_{1,2}}{(x_2-x_1-\ell_{i,i+1})^3}, \\ u_i &= k_{i,i+1}\left(x_{i+1} - x_i - r_{i,i+1}\right) + d_{i,i+1}\left(v_{i+1} - v_i\right) \\ &\quad + k_{i-1,i}\left(x_{i-1} + r_{i-1,i} - x_i\right) + d_{i-1,i}\left(v_{i-1} - v_i\right) \\ &\quad + \tfrac{\kappa_{i-1,i}}{(x_i-x_{i-1}-\ell_{i,i-1})^3} - \tfrac{\kappa_{i,i+1}}{(x_{i+1}-x_i-\ell_{i,i+1})^3}, \ (1 < i < N), \\ u_N &= k_{N-1,N}\left(x_{N-1} + r_{N-1,N} - x_N\right) + \\ &\quad + d_{N-1,N}\left(v_{N-1} - v_N\right) + \tfrac{\kappa_{N-1,N}}{(x_N-x_{N-1}-\ell_{N,N-1})^3} + \\ &\quad + \sigma(v_d - v_N), \end{aligned} \tag{10}$$

As a consequence of the potential function's modification, the minima of the potential energy for each spring are shifted and is no longer achieved at the point of rest ($\xi_i = x_{i+1} - x_i - r_{i,i+1} = 0$), but corresponds to the unique root $\xi = \xi_i^0 > \ell_{i,i+1} - r_{i,i+1}$ of the algebraic equation

$$\zeta_i(\xi) := \kappa_{i,i+1} + \xi k_{i,i+1}\left(-r_{i,i+1} + \ell_{i,i+1} - \xi_i\right)^3 = 0. \tag{11}$$

To prove the existence and uniqueness of the root ξ_i^0, it suffices to introduce the new variable $\theta = \xi + r_{i,i+1} - \ell_{i,i+1}$ and note that (11) can be rewritten as

$$-k_{i,i+1}\theta^4 + k_{i,i+1}(r_{i,i+1} - \ell_{i,i+1})\theta^3 + \kappa_{i,i+1} = 0 \tag{12}$$

Due to the Descartes' rule of sign (Sturmfels, 2002), (12) has exactly one positive root $\theta > 0$ and exactly one negative root $\theta < 0$. The minimum point of the potential function thus corresponds to the unique real root of (11) such that $\xi > \ell_{i,i+1} - r_{i,i+1}$ (or, equivalently, $x_{i+1} - x_i > \ell_{i,i+1}$). Notice that for $\kappa_{i,i+1} = 0$ this root is, obviously, $\xi = 0$ (the situation of Theorem 1). Using the implicit function theorem, it can be easily shown that the root ξ_i^0 continuously depends on $\kappa_{i,i+1} \geq 0$ and, in particular,

$$\xi_i^0 \xrightarrow[\kappa_{i,i+1}\to 0]{} 0.$$

Remark 2. Similar to Menini et al. (2018); Menini and Tornambe (2003), it can be shown that the system arising as a limit $\kappa_{i,i+1} \to 0$ corresponds to the hybrid (continuous-discrete) dynamics with *elastic* (energy- and momentum-preserving) collisions between the masses. In the absence of collisions the evolution of the system coincides with (4).

We are now ready to formulate our main result.

Theorem 2. Assume that the initial distances between the vehicles are safe $x_{i+1}(0) - x_i(0) > \ell_{i,i+1}$. The algorithm (10) guarantees safety (1), maintains the desired speed (3) and provides a "relaxed" distance policy

$$x_{i+1}(t) - x_i(t) - r_{i,i+1} \xrightarrow[t\to\infty]{} \xi_i^0,$$

Here ξ_i^0 is the root of (11) such that $\xi_i^0 > \ell_{i,i+1} - r_{i,i+1}$.

Remark 3. As the distance between two vehicles i and $i+1$ become unsafe, the barrier function grows, which inevitably leads to large values of the corresponding force. In practice, actuator limits have to be taken into account $|u_i(t)| \leq u_{i,max}$. Obviously, with limited actuators it is impossible to provide safety for all initial configurations. Nonetheless, the inequality

$$W_e(\xi(t), v(t)) \leq W_e(\xi(0), v(0))$$

can be used to provide an explicit (yet conservative) estimate of the initial data region, starting in which no actuator constraint is saturated.

5. EFFECTS OF DISTURBANCES

The main objective of this section is to characterize the effect of disturbances on the behavior of the closed-loop system with the control law given in (10). Toward this end, consider the following lemma.

Lemma 1. Let the force of the ith vehicle be affected by an additive bounded disturbance δ_i, i.e., the actual force pulling the ith vehicle is $\bar{u}_i = u_i + \delta_i$, with $|\delta_i(t)| \leq \Delta_i$ for some $\Delta_i > 0$, $i = 1, \dots, N$. Then the algorithm (10) preserves safety (1) for any "safe" initial condition.

An important question traditionally addressed in the engineering literature on vehicle platooning is the effect of *string instability*, which can roughly be characterized as amplification of the disturbances as they propagate through the platoon in upstream and downstream directions. For nonlinear platoons, the string stability is usually defined in time domain, e.g. as the input-to-state stability in L_p norm (Dolk et al., 2017) and its analysis requires special Lyapunov functions. At the same time, if the disturbance bounds Δ_i are sufficiently small, as well as the parameters $\kappa_{i,i+1}$, the platoon dynamics can be approximated by the following linear system

$$\begin{aligned}
\dot{v}_1 = \ddot{x}_1 &= \frac{k_{1,2}}{m_1}(x_2 - x_1 - r_{1,2}) + \frac{d_{1,2}}{m_1}(v_2 - v_1) + \tilde{\delta}_1, \\
\dot{v}_i = \ddot{x}_i &= \frac{k_{i,i+1}}{m_i}(x_{i+1} - x_i - r_{i,i+1}) + \frac{d_{i,i+1}}{m_i}(v_{i+1} - v_i) + \\
&+ \frac{k_{i-1,i}}{m_i}(x_{i-1} - x_i + r_{i-1,i}) + \frac{d_{i-1,i}}{m_i}(v_{i-1} - v_i) + \tilde{\delta}_i, \\
\dot{v}_N = \ddot{x}_N &= \frac{k_{N-1,N}}{m_N}(x_{N-1} - x_N + r_{N-1,N}) + \\
&+ \frac{d_{N-1,N}}{m_N}(v_{N-1} - v_N) + \tilde{\delta}_N.
\end{aligned} \tag{13}$$

Here the new "disturbance" $\tilde{\delta}_i$ absorbs the actual disturbance δ_i and the nonlinear terms, caused by the potential function. For the linear system (13), string stability can be examined in the *frequency domain*. The minimal frequency-domain string stability means the uniform boundedness (in H_∞-norms) of the transfer function from $\tilde{\delta}_i$ to the position errors of all vehicles ξ_j, $j = 1, \dots, N-1$ (notice that disturbances propagate in both upstream and downstream directions). Based on extensive simulations, we have a conjecture that the string stability can be proved for *homogeneous* platoons where $m_1 = \dots = m_N = m$ and $k_{i,i+1} = k$, $d_{i,i+1} = d$. Notice that often the control input of the vehicle is acceleration rather than force (Hao and Barooah, 2013) and then one can assume that $m_i = 1$, so that the homogeneity can always be provided.

Conjecture. For a homogeneous platoon, the transfer functions from $\tilde{\delta}_i$ to the position errors $\xi_j = x_{j+1} - x_j - r_{j,j+1}$ and the velocity error $\dot{\xi}_j$ are uniformly (in N and $j = 1, \dots, N-1$) bounded in the H_∞-norm. In other words, the maximal amplification gain for each disturbance does not grow with the size of the platoon N.

To illustrate the conjecture, we show the Bode diagram of transfer functions from $\tilde{\delta}_N$ to ξ_1, computed for the parameters $k = d = m$ and $N = 2, \dots, 40$ (Fig. 3).

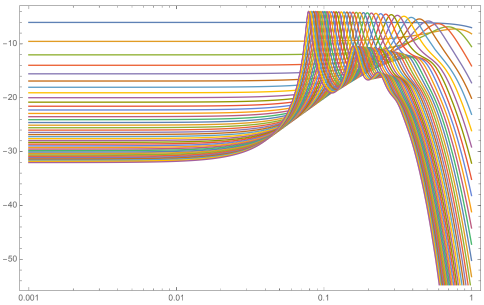

Fig. 3. Transfer functions from $\tilde{\delta}_N$ to ξ_1 (frequency response) for $N = 2, \dots, 40$.

Our conjecture is consonant with the result from Hao and Barooah (2013) stating that in a homogeneous platoon (13) the transfer function from $\tilde{\delta}_N$ to the sum of position errors

$$\sum_{i=1}^{N-1} \xi_i = x_N - x_1 - (r_{1,2} + r_{2,3} + \dots + r_{N-1,N})$$

has asymptotics $O(N)$ as $N \to \infty$.

6. NUMERICAL SIMULATIONS

In this section, we simulate the behavior of a homogeneous platoon (namely, the dynamics of position errors, velocities and accelerations of the vehicles) of $N = 6$ vehicles under the control algorithm (10) with the following parameters: 1) the desired distances are $r_{i,i+1} = 10$ m; 2) the safe distances are $\ell_{i,i+1} = 3$ m; 3) the initial configuration of the platoon is such that $x_{i+1}(0) - x_i(0) = 20$ m (the positions are safe but the distance has to be increased) and $v_i(0) = 20$ m/s for all vehicles; 4) the controller's parameters are chosen so that

$$k_{i,i+1}/m = 1, d_{i,i+1}/m = 1, \kappa_{i,i+1}/m = 10^{-3}, \sigma = 2.9$$

where m is the mass of each vehicle.

Numerical simulations have been carried for three standard driving scenarios: normal highway driving, stop-and-go driving, and emergency braking (Mullakkal-Babu et al., 2016). The three scenarios differ by the profiles of the desired speed $v_d(t)$ (the controller, as usual, is designed to keep a constant predefined speed, however, can be used for the speed that is slowly changing, the variations in the speed are interpreted as the disturbances).

In all experiments, we display the position errors (the red dashed line corresponds to the minimal safe distance $\xi = l_{i,i+1} - r_{i,i+1} = -7$ m), the vehicle speed profiles and the desired speed and the vehicle accelerations. In all three scenarios, the controller exhibits string stability in its engineering meaning: the position errors are ordered

$$|\xi_1(t)| \leq |\xi_2(t)| \leq \dots \leq |\xi_N(t)|.$$

Normal driving (Fig. 4). The first scenario corresponds to a gentle increase and decrease of the desired speed, corresponding to normal highway driving. The speed is varying slowly between 30 m/s and 15 m/s. In this experiment, the nonlinear terms are very small, since all pairwise distances are not less than 7 m. After a short transient period, accelerations of all vehicles are within ± 0.5 m/s^2.

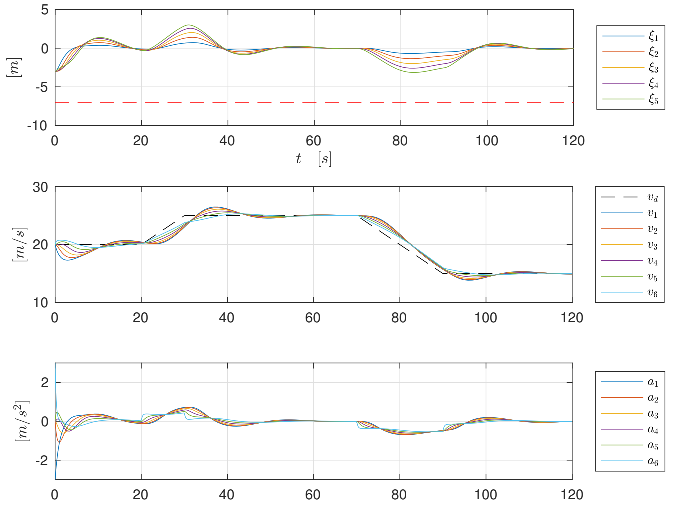

Fig. 4. The normal highway driving scenario.

Stop-and-go (Fig. 5). This scenario corresponds to a dense traffic, where the platoon has to decrease its velocity to a full stop and then accelerate again. During braking, the distance between the leader and its predecessor (vehicle 5) comes to the critical value 3m. The accelerations, after a short starting period, stay between -2 and 2.5 $\mathrm{m/s^2}$. Notice that our controller does not use the information about the predecessor's acceleration, so in the case changing of velocities one observes overshoots in the velocity and position tracking.

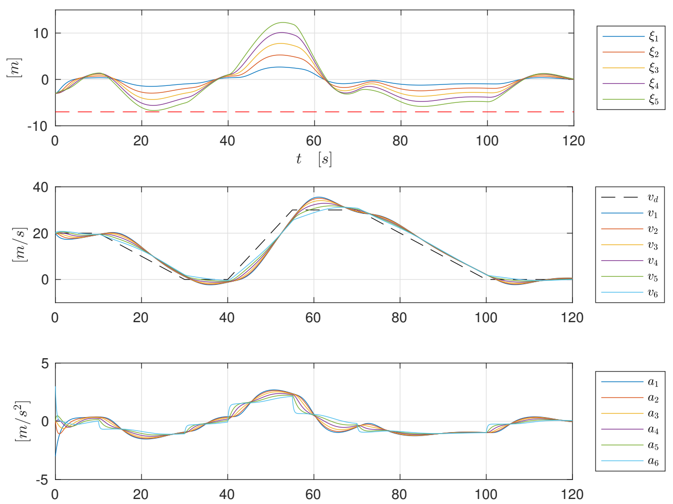

Fig. 5. The stop-and-go scenario.

Emergency braking (Fig. 6). In this scenario the platoon brakes heavily from the initial speed to full stop. In this experiment, two distances (between the leader and vehicle 5, between vehicle 5 and vehicle 4) come to the critical value. Due to the influence of barrier function, all distances are kept safe, however, the accelerations needed for this are quite large in absolute value (e.g. vehicle 5 deceleration reaches (-4.5) $\mathrm{m/s^2}$, although the leader accelerates in order to keep the distance safe).

CONCLUSION

In this paper, we examine a nonlinear algorithm for vehicle platooning, which is inspired by an analogy between a platoon and a mass-spring-damping system and is obtained from the usual Lagrangian equations via augmenting the potential energy with a barrier function. By allowing for bidirectional communication, such an algorithm guarantees convergence to a desired formation of the agents while avoiding collisions during the transient behavior. Robustness properties of such an algorithm have been studied by means of an electrical equivalent system.

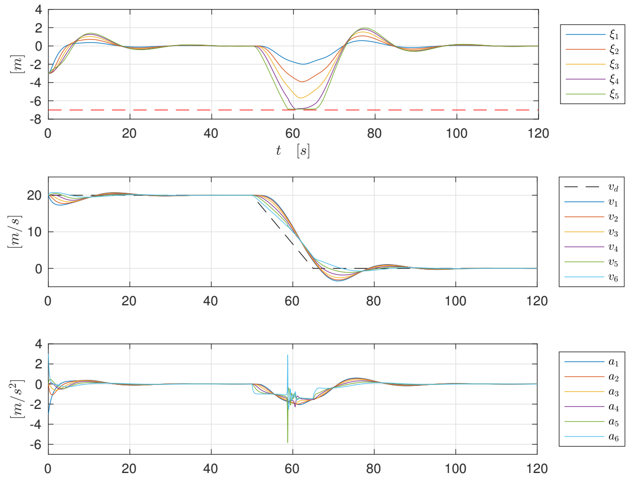

Fig. 6. The emergency braking scenario.

One of the key advantage of the proposed control technique over state-of-the-art CACC algorithms is its simplicity. In fact, in order to implement the control law (10) the ith vehicle just need to measure the relative distance and velocity of its predecessor and of its follower. Then, the applied control input is a simple rational function of these quantities. However, one of the disadvantages of the proposed technique is that it does not account for the saturation naturally occurring when dealing with real vehicles. Overcoming such a limitation will be one of objectives of our future work.

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