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Spatio-Temporal Loop Shaping for Distributed Control of PDE Systems

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Abstract: The systems of interest in this paper are described by a possibly large number of interconnected subsystems, where the spatial discretization into subsystems is induced by applying an array of collocated actuator/sensor pairs. When considering these types of systems in a classical centralized fashion, they have a high number of input/output signals on one hand, and usually a sparse structure on the other. While the method of loop-shaping in classical control is well-established, the notion of spatially distributed systems makes it possible to extend frequency domain loop shaping to spatial frequencies and to include weighting filters with spatio-temporal dynamics. By using a Fourier transform, controller synthesis can be done for each spatial frequency at a time. This method is applied to the control of the temperature distribution of a thin metal rod.

Keywords: Spatially-interconnected systems, distributed systems, spatio-temporal loopshaping, large-scale systems, partial differential equations

1. INTRODUCTION

Distributed parameter systems where the underlying system dynamics involve signals that not only depend on time but also on a - possibly multi-dimensional- spatial variable are subject of a wide field of research for many years now. Especially the application to systems, where the dynamics are described by partial differential equations (PDEs) are of interest in this paper. Possible application involve for instance the heat equation or vibration control of flexible structures. By applying an array of actuator/sensor pairs, a spatial discretization is induced such that the global system can be described as an interconnection of single-input/single-output (SISO) subsystems. If the number of input/output signals is very large, classical control approaches which consider the problem from a global multi-input/multi-output (MIMO) point of view quickly turn impractical due to the high computational complexity of the controller synthesis and complex implementation. The centralized controller needs to receive and process every sensor output and has to have control authority over each actuator. At the same time, these systems are usually sparse structured and there are several research result which aim at exploiting the sparse structure of these systems in order to obtain low-complexity analysis and synthesis results.

In Bamieh et al. (2002) it was shown that the dynamics of spatially invariant systems of infinite extent can be decomposed into a family of finite dimensional systems involving spatial frequencies. A closely related approach is described in D'Andrea et al. (2003), where a shift operator description of the spatially-interconnected system is used to obtain sufficient analysis and synthesis results of the size of a single subsystem. The framework is extended in Langbort et al. (2005) to include systems with bounded spatial domains if they satisfy a condition called spatial reversibility. Another approach which includes possibly spatially varying systems interconnected over an arbitrary graph is proposed in Langbort et al. (2004) which in turn may increase the computational complexity of the analysis and synthesis problem. The work presented in this paper is based on the method in Stewart et al. (2003), where the systems under consideration are described by circulant transfer matrices, allowing to decompose the system into its modal SISO subsystems and perform controller synthesis for each spatial frequency at a time. An important issue in control of MIMO systems, which is adressed there, is the direction of the input signal when utilizing performance criteria such as the H_∞ norm. Especially distributed systems described by PDEs are often highly ill-conditioned (i.e. the smallest and largest singular values of the transfer matrix may differ in several magnitudes). This means that there are signal directions which are hard to control and may lead to a very high controller gain which might violate physical limitations of the actuators on one hand and make the closed-loop system sensitive to model uncertainties on the other. An approach to account for this problem during the synthesis procedure is the main topic of this paper.

We consider spatially-interconnected systems in discrete time-discrete space input/output form obtained by discretizing PDEs. Under certain assumption on the system or else appropriate approximations, these large-scale systems can be transformed into a family of SISO systems which are indexed by the spatial frequency. More specifically, when the global system is described by a symmetric circular transfer matrix, the system can be brought into diagonal form by a real Fourier transform, where the rows of the Fourier matrix are the spatial harmonics of the system (Stewart et al. (2003), Bamieh et al. (2002)). The singular values are then given by the magnitude of the eigenvalues of the transfer matrix, which will be called modal subsystems. Controller design can then be performed for each SISO system and therefore spatial frequency at a time. This has two major advantages compared to classical MIMO synthesis. First, the computational load of solving a set of smaller problems is less than solving the large-scale problem (especially when using methods based on linear matrix inequalities (LMIs). Second, we propose a method to extend well-known H_∞-loopshaping objectives such as reference tracking, disturbance rejection or robustness against uncertainties in temporal-frequency domain to spatially-interconnected systems in the spatio-temporal frequency domain. Using this approach, control objectives such as performance requirements or robustness against model uncertainties can be applied to specific temporal and spatial frequency regions.

The paper is structured as follows. In Section 2 the system description used is presented and the spatial frequency decomposition by a real Fourier transform is introduced. Furthermore existing results on controller synthesis by obtaining controllers for each spatial frequency at a time and recovering a global controller by the inverse Fourier transform are briefly reviewed. In Section 3 the notion of spatio-temporal loopshaping is introduced and a specific structure for spatio-temporal weighting filters in the framework of H_∞-controller design is proposed. Finally, the procedure is applied to the heat distribution in a thin metal rod and conclusions are drawn in Section 4.

2. PRELIMINARIES

In this section, spatially distributed systems in the form of discrete time and space input/output models are introduced. A real Fourier transform is then used to decouple the large scale system into a family of SISO systems for each spatial frequency.

2.1 Spatially Interconnected Systems

We are considering systems where the dynamics are described by partial differential equations (PDEs), where the involved signals depend on a temporal and a possibly multi-dimensional spatial variable. For simplicity we will assume a single spatial dimension in this paper. As mentioned before we are assuming a collocated actuator/sensor array of size N applied to the system which induces a spatial discretization into subsystems. The following notation is based on Stewart et al. (2003). Using a finite difference approximation for the spatial and temporal derivative in the involved PDE, the system can be described by the finite difference equation

$$y(k) = -\sum_{j=1}^{m_y} A_j y(k-j) + \sum_{i=1}^{m_u} B_i u(k-i), \tag{1}$$

where $y(k), u(k) \in \mathbb{R}^N$ are the output and input vector at time step k. Applying the $\mathcal{Z}$-transform to the input and output vector respectively yields

$$Y(z) = B(z)U(z) - A(z)Y(z). \tag{2}$$

By writing the matrix factors as

$$A(z) = \sum_{j=1}^{m_y} A_j z^{-j}, \quad B(z) = \sum_{i=1}^{m_u} B_i z^{-i} \tag{3}$$

the system can further be described by a linear transfer matrix model

$$Y(z) = G(z)U(z), \tag{4}$$

where the system transfer matrix

$$G(z) = [I_N + A(z)]^{-1} B(z). \tag{5}$$

describes the spatio-temporal dynamics of the global system. Starting from a PDE, the involved matrix factors $A(z), B(z) \in \mathbb{C}^{N\times N}$ are structured due to the finite difference approximation. More specifically, with zero boundary conditions the coefficient matrices of the transfer factors are given by band-diagonal Toeplitz matrices of the form

$$\tilde{A}_j = \mathcal{T}(\tilde{a}_j), \quad \tilde{a}_j = \left[a_1^j, ..., a_{n_a}^j, 0, ..., 0\right]^T \in \mathbb{R}^N \tag{6}$$

exemplary. Most real word applications based on PDEs are not of infinite extent and therefore involve specific boundary conditions. Since the method in this paper heavily relies on the spatial frequency decomposition explained in the following, the system is assumed to be either of inifinite extent/periodic or spatially reversible with a boundary matrix $B = 1$ (compare Langbort et al. (2005)) such that it can be expressed using periodic boundary conditions. This essentially means, that the band-diagonal Toeplitz matrix factors can be expressed in terms of circulant matrices

$$A_j = \mathcal{T}(\bar{a}_j), \quad \bar{a}_j = \left[a_1^j, ..., a_{n_a}^j, 0, ..., 0, a_{n_a}^j, ..., a_2^j\right]^T \in \mathbb{R}^N \tag{7}$$

If the system can not be described using periodic boundary conditions, a small-gain argument can be used to determine stability of the original system under perturbation (which is essentially the difference of the original system and the circulant system approximation). Details can be found in Stewart et al. (2003) and Langbort et al. (2005) and we will assume the underlying system to fulfill this requirement for the remainder of this paper.

2.2 Spatial Frequency Decomposition

For many distributed systems and especially those that are described by discretized PDEs, the model in (1)-(5) is often ill-conditioned. More specifically, in steady state the ratio of the systems largest and smallest singular value is

$$\frac{\bar{\sigma}(G(e^{j\omega}))}{\underline{\sigma}(G(e^{j\omega}))} \gg 1, \quad \text{for} \quad \omega = 0. \tag{8}$$

This has to be taken into account when designing a controller, since this may lead to high controller gains when considering control objectives such as disturbance rejection and reference tracking, which at the same time may violate robust stability conditions. It has been shown in Stewart et al. (2003), Bamieh et al. (2002) that spatially-invariant systems can be decoupled into a family of independent modal SISO subsystems. Even more so, every symmetric circulant system with the same number of input/output channels can be diagonalized using the same real Fourier matrix (Stewart et al. (2003), Bamieh (2018))

$$F_{i,j} = \begin{cases} \sqrt{\frac{1}{N}} & i = 1 \\ \sqrt{\frac{2}{N}} \sin\left((j-1)\lambda_i\right) & i = 2, ..., p \\ \sqrt{\frac{2}{N}} \cos\left((j-1)\lambda_i\right) & i = p+1, ..., N \end{cases} \tag{9}$$

with

$$p = \begin{cases} \frac{N+2}{2} & \text{if } N \text{ is even} \\ \frac{N+1}{2} & \text{if } N \text{ is odd} \end{cases} \tag{10}$$

as

$$FG(z)F^T = \mathrm{diag}(g(\lambda_1, z), ..., g(\lambda_N, z)), \tag{11}$$

where $g(\lambda_i, z)$ are the modal subsystems or eigenvalues of $G(z)$ and

$$\lambda_i = \frac{2\pi(i-1)}{N}$$

denotes the spatial frequency of the i-th spatial mode. Since

$$FG(z)F^T = \left[I + FA(z)F^T\right]^{-1} FB(z)F^T \tag{12}$$

and

$$FB(z)F^T = F\left(\sum_{k=1}^{m_u} B_k z^{-k}\right) F^T \tag{13}$$

$$= \sum_{k=1}^{m_u} \mathrm{diag}(b_k(\lambda_1), ..., b_k(\lambda_N)) z^{-k} \tag{14}$$

exemplary, the modal subsystems for each spatial frequency $\lambda_i \in [\lambda_1, ..., \lambda_N]$ are given by

$$g(\lambda_i, z) = \frac{b(\lambda_i, z)}{1 + a(\lambda_i, z)} \tag{15}$$

with

$$b(\lambda_i, z) = \sum_{k=1}^{m_u} b_k(\lambda_i) z^{-k}. \tag{16}$$

The eigenfunction directions are equal to the singular vector directions. They are given by the harmonic functions of the spatial variable which, in turn, are given by the rows of the Fourier matrix F (Bamieh et al. (2002)). Since the singular values of a circulant system are equal to the magnitude of its eigenvalues, they can be plotted over all temporal frequencies and on a grid of all spatial frequencies λ_i. Figure 1 shows $|g(\lambda, e^{j\omega})|$ for the heat rod used as an example in Section 4. Here, the first modal subsystem (corresponding to $\lambda_1 = 0$) has integral behavior due to the central difference approximation of the spatial derivative in the underlying PDE together with the periodic boundary condition. The other modal subsystems are first order systems with a roll-off at increasing spatial frequencies λ_i.

2.3 Controller Synthesis

Introducing a reference input $r(k)$ and assuming error feedback of the form $e(k) = r(k) - y(k)$ the distributed controller structure (reflecting the same structure as the plant) is

$$u(k) = \sum_{l=0}^{m_c} C_l e(k-l) - \sum_{n=1}^{m_d} D_n u(k-n), \tag{17}$$

where each coefficient matrix is in the form of a symmetric circulant matrices as in (7). The controller may be written in terms of a transfer matrix as

$$K(z) = [I + D(z)]^{-1} C(z), \tag{18}$$

with matrix factors according to (3). Again, a transformation as in (11) can be used to diagonalize the controller yielding SISO subsystem controller for each spatial frequency $\lambda_i \in [\lambda_1, ..., \lambda_N]$ as

$$k(\lambda_i, z) = \frac{c(\lambda_i, z)}{1 + d(\lambda_i, z)}. \tag{19}$$

Because the eigenvalues of symmetric circulant systems always come in pairs (see for example Olson et al. (2014)) it is even sufficient to design controllers for $\lambda_i \in [\lambda_1, ..., \lambda_p]$ and then set $k(\lambda_i, z) = k(\lambda_{2+N-i}, z)$ for $p+1 \le i \le N$.

Since both the plant and the controller are decoupled by the same Fourier transform, the design of a large MIMO controller can now be done on the set of small SISO subsystems for each spatial frequency. The MIMO controller can then be reconstructed according to Bamieh et al. (2002), Hovd et al. (1994) as

$$K(z) = F^T \mathrm{diag}(k(\lambda_1, z), ..., k(\lambda_N, z))F. \tag{20}$$

That way it is possible to simplify the synthesis procedure significantly because it is computationally more efficient to solve a set of small control problems than to solve the large control problem (especially, when using LMI based synthesis methods). In general, the MIMO controller obtained by the inverse Fourier transform is not sparsely structured, i.e. the controller may need connection links to many (possibly all) neighbouring subsystem. In Stewart et al. (2003) a simple spatial order reduction is proposed.

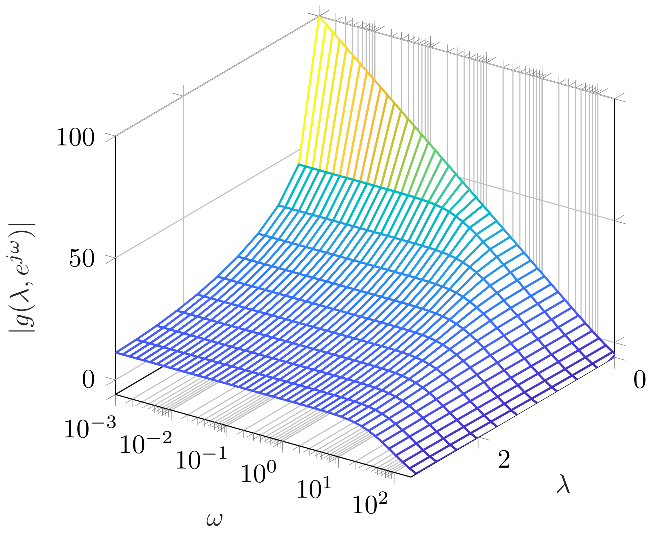

Fig. 1. $|g(\lambda, e^{j\omega})|$ in dB over all temporal and spatial frequencies for the temperature distribution of a metal rod with periodic boundary conditions

While the controller may then be also implemented in a distributed fashion, here we are not providing a stability and performance analysis; this is subject of current research.

3. SPATIO-TEMPORAL LOOPSHAPING

In general, several methods can be used to obtain the set of SISO controllers for the modal subsystems. Here we will consider H_∞-loopshaping since the idea of expressing control objectives in terms of dynamic weighting filters can be extended into multidimensional form such that the filters not only include temporal but spatial dynamics as well. We will review some approaches on classical H_∞ -loopshaping and motivate the extension of this method to the spatio-temporal frequency domain first.

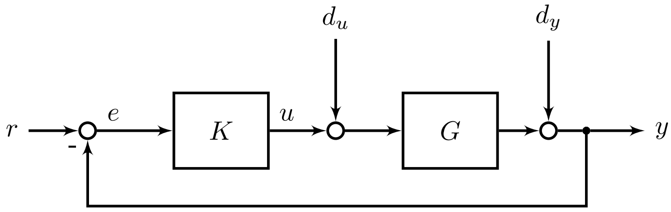

Fig. 2. Closed-loop system with disturbances

3.1 Control Objectives

Consider the closed-loop system with input and output disturbances as well as a reference input as shown in Figure 2. An illustrative example to show why the directionality of MIMO systems is important is borrowed from Stewart et al. (2003). The robust stability of a system G_p perturbed by an additive unstructured uncertainty Δ such that $G_p = G+\Delta$ is guaranteed if the nominal unperturbed system G is stable and the condition

$$\max_{d_y \neq 0} \frac{\|u\|_2}{\|d_y\|_2} = \bar{\sigma}\left([I+KG]^{-1} K\right) \leq \frac{1}{\bar{\sigma}(\Delta)} \tag{21}$$

holds. At the same time, perfect output disturbance rejection requires

$$\frac{\|e\|_2}{\|d_y\|_2} = 0 \quad \forall d_y \in \mathcal{L}_2. \tag{22}$$

Even though the tracking performance objective is often only imposed for low frequencies this would imply high controller gains for weak input directions of the system. More specifically, if the disturbance input direction is the same as the output singular vector corresponding to the smallest singular value of the system $\underline{\sigma}(G)$, perfect disturbance rejection would imply $\|u\|_2/\|d_y\|_2 = \underline{\sigma}^{-1}(G)$. It is therefore beneficial to not only separate performance and robustness requirements in terms of temporal frequencies but also to take the directions of the disturbances into account. Since the eigenvalues of circulant transfer matrices are directly linked to the singular vectors of the system, the control objectives may be separated in terms of the spatial frequencies of the system. To avoid large controller gains which aim to suppress disturbances aligning with the weak input and output directions of the plant, the performance objectives for modal subsystems of that specific spatial frequency can be relaxed. In Figure 1 it can be seen that the system dynamics have a roll-off for high temporal and spatial frequencies. This is typical for the distributed systems considered here and one can therefore conclude that performance objectives should be applied to the frequency regions of λ and z where $|g(\lambda, z)|$ is high and robustness requirements and upper bounds on controller gains should be imposed on those frequency regions where $|g(\lambda, z)|$ is low.

3.2 Spatio-temporal weighting filters

A rough summary of the control objective criteria in terms of their frequency regions is that performance objectives such as good reference tracking and disturbance rejections are usually applied to low frequencies ω, where the plant gain is usually high and the relative model uncertainty is low. More specifically, the loop gain is lower bounded for small frequency regions and the gain in the high frequency region is upper bounded. At the same time robustness requirements are confined to high frequency regions where the relative model uncertainty is large. At the same time, the gain at high spatial frequencies usually experiences a roll-off such that these input directions are hard to control and therefore robustness requirements and penalties on controller gains should also be applied to the high spatial frequency regions.

In classical H_∞-loopshaping these criteria are often imposed by applying low-pass filter to the performance output (such as the generalized 'error') and high-pass filter to the control input u, essentially shaping the sensitivity and control sensitivity of the loop. This ensures good tracking in steady state while avoiding control interaction with high frequency dynamics. When applying weighting filters with only temporal dynamics, they are valid for all input directions corresponding to a flat singular value shape over the spatial frequency axis at each temporal frequency ω and therefore oriented on the worst-case. The idea is, that a spatial roll-off can be introduces by including spatial dynamics in the weighting filters as well, which will eventually lead to weighting filters parameterized by the spatial frequency after the transformation into modal subsystems. Consider an S/KS-Design with dynamical filters W_S and W_K shaping the sensitivity and control sensitivity of the plant. When step input reference tracking for SISO systems is considered, W_S in continuous time is usually chosen as a low-pass filter of the form

$$W_S(s) = \frac{\omega_1}{M} \frac{1}{s+\omega_1}. \tag{23}$$

which (after a finite difference approximation with temporal sampling T) in discrete time corresponds to

$$W_S(z) = \frac{\frac{T\omega_1}{M} z^{-1}}{1+(\omega_1 T - 1)z^{-1}}. \tag{24}$$

For MIMO systems one may choose diagonal filters shaping the sensitivity of each input/output channel. For each temporal frequency, this will correspond to a flat singular value plot in the spatial frequency domain since the sensitivity gain is independent of the direction. To enforce a roll-off in the spatial frequency domain, spatial dynamics are added such that A_{W_S} is not diagonal but also of the form

$$A_S = \mathcal{T}(\bar{a}_S), \tag{25}$$

$$B_S = \mathcal{T}(\bar{b}_S), \tag{26}$$

essentially leading to weighting filters with spatial dynamics. Every modal subsystem is then shaped by the modal weighting filters $w_S(\lambda_i)$. The same form can be applied to the KS-weighting filter to enforce an upper bound on control action in the high spatial frequency region. Alternatively, weighting filters can be directly chosen for the transformed system, but with an additional degree of freedom or 'tuning knob' in terms of the dependence on the spatial frequency. The benefit of this procedure will be shown on the example of controlling the temperature distribution in a thin metal rod in the following section.

4. EXAMPLE: HEAT EQUATION

In the following the temperature control of a thin metal rod of length L described by the PDE

$$\frac{\partial y(t,\sigma)}{\partial t} = \alpha \frac{\partial^2 y(t,\sigma)}{\partial \sigma^2} + u(t,\sigma), \tag{27}$$

is considered, where $y(t,\sigma)$ is the temperature at time t and position σ, $u(t,\sigma)$ is an external heat input and $\alpha = 0.1$ is the diffusivity constant corresponding to the material of the rod. Applying a uniformly distributed collocated actuator/sensor array of size N and approximating the first order temporal derivative by a forward difference and the second order spatial derivative by a central difference approximation and scaling the input signal yields the finite difference equation

$$\begin{aligned} y(k,s) = &(1-2\beta)y(k-1,s) \\ &+ \beta y(k-1,s-1) + \beta y(k-1,s+1) \\ &+ u(k-1,s) \end{aligned} \tag{28}$$

with $\beta = \alpha T/h^2$ and $T, h = L/N$ being the temporal and spatial sampling respectively. This can then be brought into a linear transfer matrix model form as (4) with the matrix factors being

$$A_1(z) = \mathcal{T}(\bar{a}_1)\, z^{-1} \tag{29}$$

$$B_1(z) = I_N\, z^{-1} \tag{30}$$

with $\bar{a}_1 = [-1+2\beta, -\beta, 0, ..., 0, -\beta]$ assuming periodic boundary conditions. The system can be brought into diagonal form according to (11)-(13) yielding the SISO subsystems corresponding to each spatial frequency component. The spatio-temporal singular value plot of is shown in Figure 1. Note that A_1 has an eigenvalue of $z = 1$ with the corresponding eigenvector $\mathbf{1}$ corresponding to integral behavior of the first modal subsystem.

4.1 Controller Design

The objective of the controller is to track a reference temperature profile of the rod. A standard S/KS generalized plant is used as shown in Figure 3. Two controllers are designed. Design 1 is done with weighting filters with only temporal dynamics and Design 2 with weighting filters with spatio-temporal dynamics. It can be seen in Figure 1 that the weakest input direction corresponds to the highest spatial frequency. In order to avoid high controller gains, we want to avoid control action with high spatial frequency

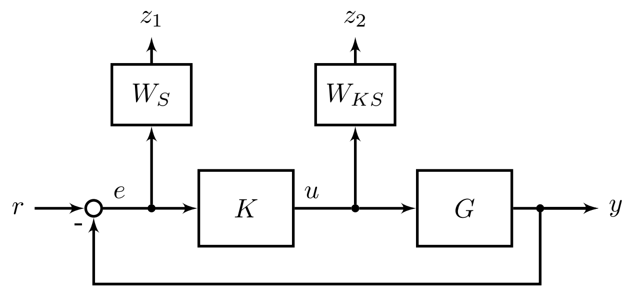

Fig. 3. Generalized plant for S/KS design

dynamics. In the classical loopshaping sense, one would have to apply a weighting filter to the control input signal u (possibly as a high pass filter to confine the result to high temporal frequency regions). By adding spatial dynamics to the weighting filter, this constraint may be relaxed in the low spatial frequency regions. Specifically, the weighting filters used to shape the control sensitivity for Design 1 and Design 2 are

$$W_{KS,1} = 1, \quad W_{KS,2}(\lambda_i) = 1 + c(\lambda_i - \lambda_p). \tag{31}$$

where $c \in (0, \frac{1}{\lambda_p})$ is a scalar constant, which can be used to scale the amount of penalty relaxation on the control sensitivity at low spatial frequencies. Here we select it such that the control sensitivity is penalized the same way in high spatial frequency regions for both designs ($W_{KS,2}(\lambda_p) = W_{KS,1}$) while the constraint is relaxed for lower spatial frequency regions in Design 2. Another thing to consider is the integral behaviour of the first modal subsystems. The shaping filter for the sensitivity of Design 1 is chosen as the zero-order-hold discretization of $W_{S,1} = (100)/(10s{+}1)$. Using the modal decomposition for controller synthesis allows to treat first modal subsystem separately. The sensitivity weighting filters used here for Design 2 are the zero-order-hold discretizations of

$$W_{S,2}(\lambda_i) = \begin{cases} 1 & \text{if } i = 1 \\ \frac{10}{10s+1} & \text{if } i = 2, ..., p \end{cases} \tag{32}$$

The synthesis for each modal subsystem is then performed using the `hinfsyn` command of the Robust Control Toolbox in Matlab Balas et al. (2005). The MIMO controller with the same interconnection structure as the plant is then obtained by back-transformation using the modal controllers as in (20).

4.2 Results

The controller is evaluated in time domain by simulating the closed loop response to a reference temperature profile input. The input reference is a unit pulse $r(t = 10:70) = 1$ applied to subsystems 8 to 11 and $r = 0$ everywhere else as shown in Figure 4.

Figure 5 shows the corresponding output response of the closed loop with the controller using spatially varying weighting filter $W_KS(\lambda_i)$ in Design 2. It can be seen that the reference is tracked well for subsystems 9 and 10, while the spatial coupling of subsystems 7/8 and 11/12 is controlled to a satisfying degree. This is shown more clearly in Figure 6 which shows the response of subsystems at spatial location $s = 10$, $s = 8$ and $s = 7$ for a controller obtained with spatially constant weighting filter $W_{KS,1}$

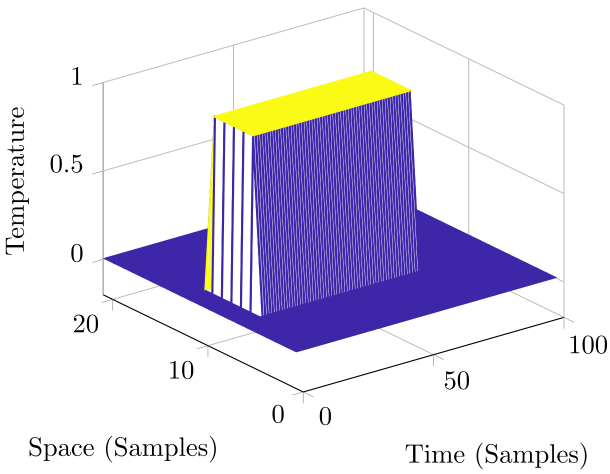

Fig. 4. Reference Input for Simulation

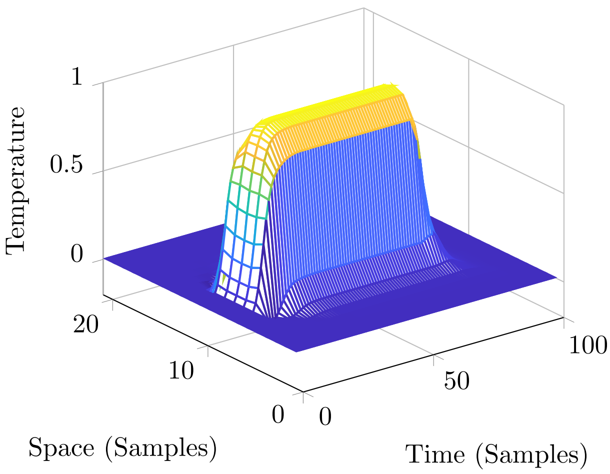

Fig. 5. Output response of the closed loop with the controller obtained by Design 2

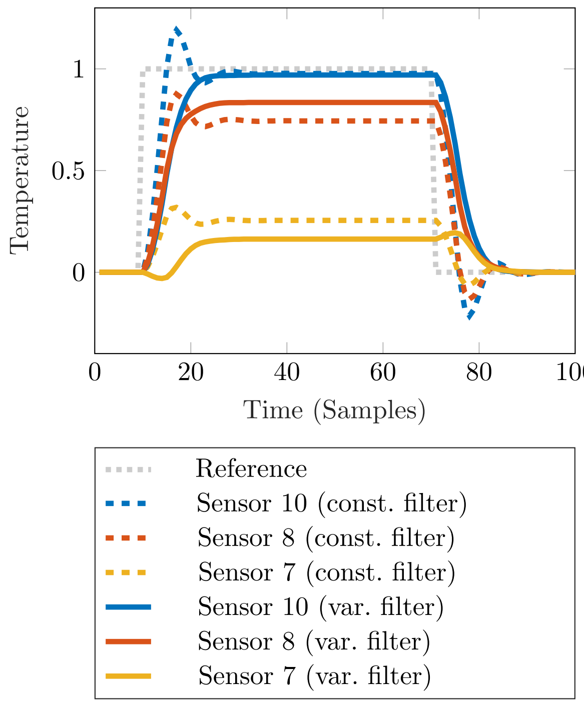

Fig. 6. Output response of three different sensors for Design 1 (dashed) and Design 2 (solid)

(dashed) in Design 1 and weighting filter depending on the spatial frequency $W_{KS,2}(\lambda_i)$ in Design 2. It can be seen that the performance of the controller obtained with filters with spatial dynamics is better in terms of the steady state error, overshoot as well as the coupling to neighbouring subsystems. This is on one hand due to the fact that the first modal subsystem is handled separately and that the penalty on the control action is relaxed for low spatial frequencies in the second design.

5. CONCLUSION

A spatio-temporal loopshaping technique was investigated based on a modal decomposition of discretized PDE systems modeled by circulant transfer matrices. The H_∞-loopshaping was extended such that weighting filters include spatial dynamics, essentially shaping the loop at each modal frequency individually. Constraints on the loop such as performance criteria can be relaxed or strengthened in certain spatial frequency regions. It was shown on the example of an actuated heat rod that performance could be improved compared to standard H_∞-synthesis using weighting filters that are flat in the spatial frequency domain.

REFERENCES

Balas, G., Chiang, R., Packard, A. and Safonov,M. (2005). Robust control toolbox. *For Use with Matlab, User's Guide*, Version 3, 2005.

Bamieh, B., Paganini, F. and Dahleh, M.A. (2002). Distributed control of spatially invariant systems. *IEEE Transactions on Automatic Control*, 47.7,2002.

Bamieh, B. (2018). Discovering the Fourier Transform: A Tutorial on Circulant Matrices, Circular Convolution, and the DFT. *arXiv preprint arXiv:1805.05533*

D'Andrea, R. and Dullerud, G.E. (2003). Distributed Control for Spatially Interconnected Systems. *IEEE Transaction on Automatic Control*, 48.9, 2003.

Hovd, M. and Skogestad, S. (1994). Control of symmetrically interconnected plants. *Automatica*, 30.6, 1994.

Langbort, C., Chandra, R.S. and D'Andrea, R. (2004). Distributed Control Design for Systems Interconnected Over an Arbitrary Graph. *IEEE Transaction on Automatic Control*, 49.9, 2004.

Langbort, C. and D'Andrea, R. (2005). Distributed control of spatially reversible interconnected systems with boundary conditions. *SIAM journal on control and optimization*, 44.1, 2005.

Olson, B., Shaw, S., Shi, C., Pierre, C. and Parker, R. (2014). Circulant Matrices and Their Application to Vibration Analysis. *Applied Mechanics Reviews*, American Society of Mechanical Engineers, 66.4, 2014.

Stein, G. and Gorinevsky, D. (2005). Design of surface shape control for large two-dimensional arrays. *IEEE Transactions on Control Systems Technology*, 13.3, 2005.

Stewart, G.E., Gorinevsky, D.M. and Dumont, G.A. (2003). Two-dimensional loop shaping. *Automatica*, 39.5, 2003.