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Using Multivariate Polynomials to Obtain DC-DC Converter Voltage Gain ⋆

Rafael F. Q. Magossi *,*** **Guilherme H. F. Fuzato** **
Daniel S. Castro *** **Ricardo Q. Machado** ***
Vilma A. Oliveira ***

* *Federal Center for Technological Education Celso Suckow da Fonseca, CEFET at Nova Friburgo, RJ, Brazil (e-mail: rafael.magossi@cefet-rj.br)*
** *Federal Institute of Education, Science and Technology of São Paulo, IFSP at Campinas, SP, Brazil (e-mail: guilherme.fuzato@ifsp.br)*
*** *Department of Electrical and Computer Engineering, Universidade de São Paulo at São Carlos, SP, Brazil (e-mail: rafael.magossi@usp.br, rquados@usp.br, voliveira@usp.br)*

Abstract: In this paper, a data driven approach is used to obtain the static gain of dc–dc power converters in terms of the duty cycle and a set of linear coefficients. A known number of measurements, dependent on the dc–dc converter topology, are used to built-in a rational function obtained by linear coefficients. This solution shows how to use measurements to determine a function to represent the static gain of dc–dc power converters in the continuous-conduction mode (CCM). To validate the proposed approach, PSIM simulations, as well as experimental results are presented. The analysis was performed with a Interleaved Boost with Voltage Multiplier (IBVM) converter. Finally, the proposed approach is shown to be an alternative to the classical scanning methods or to the conventional solution of differential equations.

Keywords: Model approximation, power cirtuits, power electronics, polynomial models, data driven.

1. INTRODUCTION

Among the most important topologies of dc–dc converters, the interleaving technique shown in Thounthong et al. (2009) and the voltage multiplier technique presented in Wu et al. (2015) have as advantage the current sharing among the phases (branches), reduced level of ripple and high voltage gain. Nonetheless, the use of these techniques also increases the number of storage devices (capacitors and inductors) as well as, the number of semiconductors in the circuit.

Using the average model proposed in Middlebrook and Cuk (1976), it is possible to obtain the dc–dc converter voltage gain in an analytic way considering the parasitic losses. In topologies with a large number of storage devices and semiconductors, finding the theoretical voltage gain becomes a challenge given that the state-space model of dc–dc converters presents high-order matrices as in Fuzato et al. (2016) or a high number of possible state matrices as in Abraham et al. (2014); Ai and Lin (2018). A simple approach is to sweep the duty cycle to obtain the static gain experimentally. However, this type of procedure needs a large number of measurements and an appropriate fitting function.

In Mohsenizadeh et al. (2013), a measurement-based approach to linear circuits was proposed and in Mohsenizadeh et al. (2016) this result was explored in terms of a state-space modeling. Using the same approach, in Oliveira et al. (2017) the authors evaluated a general model for linear dc networks, containing design parameters, multiple inputs and outputs. The outputs were described as rational function with multivariate polynomials of the parameters and linear functions of the inputs. For an unknown linear system, the unknown coefficients of these functions can be determined by taking a suitable number of measurements and solving a set of linear equations constructed from selected measurement data.

The authors in Oliveira et al. (2017) and Battacharyya et al. (2019) applied this approach to an ideal boost converter considering the duty cycle and the load as parameters, as well as the inductor current as output under the assumption that the rank of the matrices appearing in the multivariate functions was unity. However, such assumption may not be appropriate as the rank of the matrices appearing in the multivariate functions are not unity for most of the parameters considered including the duty cycle as can be checked from the dc–dc converter models presented in Fuzato et al. (2016); Deaecto et al. (2010).

In this paper, the results of Oliveira et al. (2017) and Battacharyya et al. (2019) were extended and a general

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approach for input-output model to determine the voltage gain of dc–dc converters using a known number of measurements defined in terms of the storage devices present in the converter topology is proposed. This avoids a duty cycle sweeping and exhaustive searching on to find an appropriate fitting function. In the next section we revised the concepts of measurement-based approach for a linear system.

2. AVERAGE MODEL OF DC–DC CONVERTERS

In Erickson and Maksimovic (2001) the state-space matrices for all possible switching states were obtained weighting them by the converter duty cycle and considering the converter as a black-box as showed in Fig. 1.

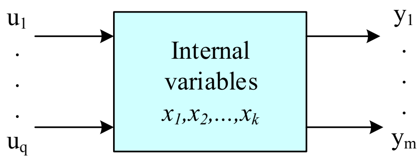

Fig. 1. Unknown dc–dc converter

In Fig. 1, the inputs are the dc power supply denoted by $u_1, u_2, \cdots, u_q$, the system parameter is the duty cycle (denoted by d), the outputs are $y_1, y_2, \cdots, y_m$, and the dc–dc converter state-space or internal variables (usually current through the inductors and voltage on the capacitors Erickson and Maksimovic (2001)) are denoted by $x_1, x_2, \cdots, x_k$. Therefore, the steady state average model of a dc–dc converter in the continuous-conduction mode (CCM), in state-space form, which has g combination of switching states and k storage devices, are represented as

$$0 = \sum_{i=1}^{g} \mathbf{A}_i \gamma(d)_i \mathbf{x} + \sum_{i=1}^{g} \mathbf{B}_i \gamma(d)_i \mathbf{u}, \quad (1)$$

$$\mathbf{y} = \sum_{i=1}^{g} \mathbf{C}_i \gamma(d)_i \mathbf{x} + \sum_{i=1}^{g} \mathbf{D}_i \gamma(d)_i \mathbf{u}, \quad (2)$$

where $\mathbf{A}_i \in R^{k\times k}$, $\mathbf{B}_i \in R^{k\times q}$, $C_i \in R^{m\times k}$, $\mathbf{D}_i \in R^{m\times q}$, $\gamma(d)_i$ denotes a function which is an affine function of the parameter d, $\mathbf{x} = [x_1, x_2, \cdots, x_k]^T$ and $\mathbf{u} = [u_1, u_2, \cdots, u_q]^T$. The average model can thus be written as

$$\begin{aligned} 0 &= \mathbf{A}(d)\mathbf{x} + \mathbf{B}(d)\mathbf{u}, \\ \mathbf{y} &= \mathbf{C}(d)\mathbf{x} + \mathbf{D}(d)\mathbf{u}, \end{aligned} \quad (3)$$

where $\mathbf{A}(d), \mathbf{B}(d), \mathbf{C}(d), \mathbf{D}(d)$ are matrices of appropriate dimensions.

3. THE PROPOSED APPROACH

In Section 2, an unknown dc–dc converter is described as the general input-output model as in Battacharyya et al. (2019). In dc–dc converters, one parameter (duty cycle) is usually to be adjusted (Erickson and Maksimovic, 2001), such that the rational function with multivariate polynomials of Battacharyya et al. (2019) is reduced to a particular case. Therefore, the following corollary for the application of the results to a dc-dc converter operating in CCM with fixed load is proposed.

Corollary 1. Suppose there is a dc–dc converter operating in CCM with k storage devices and q dc power supplies, then a desired output can be obtained in function of d using

$$y_t = \sum_{w=1}^{q} \frac{\sum_{i=0}^{k+1} \beta_{twi} d^i}{d^k + \sum_{j=0}^{k-1} \alpha_j d^j} u_w, \; t = 1, \cdots, m, \quad (4)$$

with $\alpha_j, \; \beta_{twi}$ unknown coefficients.

3.1 Finding the unknown coefficients

This section starts by illustrating the problem of finding the unknown coefficients described in Corollary 1 with a simple example. Suppose there are a single input u, a single output y and $k = 1$. Therefore the output is

$$y = \left(\frac{\beta_0 + \beta_1 d + \beta_2 d^2}{\alpha_0 + d} \right) u. \quad (5)$$

The coefficients β_0, β_1, β_2 and α_0 for u constant, can be found using four different measurements of the parameter d denoted by $d(1), d(2), d(3)$ and $d(4)$ and output y denoted by $y(1), y(2), y(3)$ and $y(4)$. Then, solve the following linear system of equations

$$\begin{bmatrix} u & d(1)u & d(1)^2u & -y(1) \\ u & d(2)u & d(2)^2u & -y(2) \\ u & d(3)u & d(3)^2u & -y(3) \\ u & d(4)u & d(4)^2u & -y(4) \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \alpha_0 \end{bmatrix} = \begin{bmatrix} d(1)y(1) \\ d(2)y(2) \\ d(3)y(3) \\ d(4)y(4) \end{bmatrix}. \quad (6)$$

From this example, it can be concluded that the number of measurements in a system with k storage devices is $2k + 2$ (it is easy to show using mathematical induction). Therefore, a approach to obtain the static gain of a dc–dc converter as a function of d is proposed as follows:

(1) From the boost converter topology find the number of storage devices denoted as k and assigns it to the dimension of the state-space matrix $A(d)$;
(2) Count the number of input voltage sources denoted as q and assigns it to the number of columns of the state-space matrix $B(d)$;
(3) Write the output as in Corollary 1;
(4) With adequate dc voltage inputs, take $2k + 2$ measurements of the output by varying the duty cycle d in steps of $\frac{d_U - d_L}{2k+1}$, where d_L and d_U are the lower and upper bound of the duty cycle of desired operation region of the converter;
(5) Finally, write a linear system as in (6) and solve for the unknowns.

Remark 1. In case of input current sources, simply use a transformation for voltage source and apply the approach given.

Remark 2. Although in the approach given the duty cycle is varied in fixed steps equally spaced over the duty cycle range, the measurements of the output can be taken randomly over this range.

Remark 3. Note that the approach given is not dependent on the number of semiconductors, circuit losses or even connections among the elements on dc–dc converter. Therefore, when the system complexity increases, this approach is less complex as only the number of the storage devices is considered.

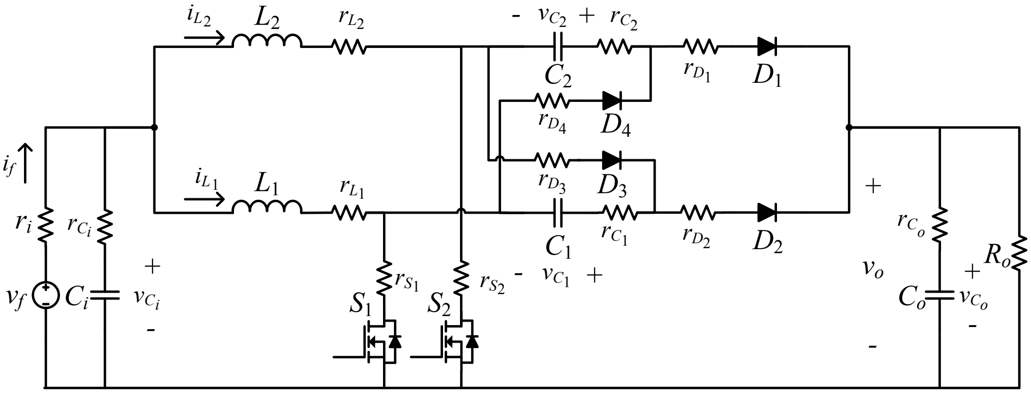

Fig. 2. IBVM converter with losses (Fuzato et al., 2016).

4. VALIDATION OF THE PROPOSED APPROACH BY SIMULATION

In this section, the efficiency of the proposed approach is illustrated using a dc–dc converter called Inteleaved Boost with Voltage Multiplier (IBVM) with losses as in Fuzato et al. (2016). The IBVM converter has shared legs and it is shown in Fig. 2 (Spiazzi et al., 2012). The rated parameter values are given in Table 1. In Fuzato et al. (2016), the voltage gain of the IBVM converter, found using the its state-space model, is given by

$$\frac{V_o}{V_i} = \frac{a_1\, d + a_2}{a_3\, d^2 + a_4\, d + a_5}, \tag{7}$$

where

$$\begin{aligned} a_1 &= -4\, R_o^2 - 4\, r_{C_o}\, R_o,\ a_2 = 4\, R_o^2 + 4\, r_{C_o}\, R_o,\ a_3 = 2\, R_o^2,\\ a_4 &= -2\, R_o\, r_C - R_o\, r_{C_o} - 2\, R_o\, r_d - R_o\, r_s - 2\, r_C\, r_{C_o}\\ &\quad - 2\, r_{C_o}\, r_D - r_{C_o}\, r_s - 4\, R_o^2,\\ a_5 &= 2\, R_o\, r_C + R_o\, r_{C_o} + 2\, R_o\, r_D + 8\, R_o\, r_f + 4\, R_o\, r_L\\ &\quad + 5\, R_o\, r_s + 2\, r_C\, r_{C_o} + 2\, r_{C_o}\, r_D\\ &\quad + 8\, r_{C_o}\, r_f + 4\, r_{C_o}\, r_L + 5\, r_{C_o}\, r_s + 2\, R_o^2. \end{aligned}$$

We can apply the data-driven approach using the following facts:

(1) The number of energy storage devices is $k = 6$;
(2) The number of input voltage sources is $q = 1$.

The rational function which relates the output voltage with the input voltage is

$$\frac{V_o}{V_i} = \frac{\phi_1}{\theta_1} \tag{8}$$

where

$$\phi_1 = \beta_0 + \beta_1 d + \beta_2 d^2 + \beta_3 d^3 + \beta_4 d^4 + \beta_5 d^5 + \beta_6 d^6 + \beta_7 d^7,$$

$$\theta_1 = \alpha_0 + \alpha_1 d + \alpha_2 d^2 + \alpha_3 d^3 + \alpha_4 d^4 + \alpha_5 d^5 + d^6.$$

Thus, 14 measurements are needed. Considering a duty cycle operation between 0.5 and 1 (according to Fuzato et al. (2016)), vary the duty cycle in steps of about 0.0357 and solve the system of equations (9).

$$\begin{bmatrix} V_i & d(1)V_i & d^2(1)V_i & \cdots & d^7(1)V_i & -V_o(1) & -d(1)V_o(1) & \cdots & -d^5(1)V_o(1) \\ V_i & d(1)V_i & d^2(2)V_i & \cdots & d^7(2)V_i & -V_o(2) & -d(2)V_o(2) & \cdots & -d^5(2)V_o(2) \\ \vdots & & & & \vdots & & \vdots & & \vdots \\ V_i & d(14)V_i & d^2(14)V_i & \cdots & d^7(14)V_i & -V_o(14) & -d(14)V_o(14) & \cdots & -d^5(14)V_o(14) \end{bmatrix} \begin{bmatrix} \beta_0 \\ \vdots \\ \beta_7 \\ \alpha_0 \\ \vdots \\ \alpha_5 \end{bmatrix} = \begin{bmatrix} d^6(1)V_o(1) \\ \vdots \\ d^6(14)V_o(14) \end{bmatrix}. \tag{9}$$

Table 1. Rated parameters of the IBVM converter used for simulation.

Parameter	Value	Parameter	Value
$L_1 = L_2 = L$	140μH	$r_{s_1} = r_{s_2} = r_s$	24mΩ
$r_{L_1} = r_{L_2} = r_L$	9mΩ	$r_{D_1} = r_{D_2} = r_D$	53mΩ
$C_1 = C_2 = C$	1μF	r_f	5mΩ
$r_{C_1} = r_{C_2} = r_C$	29mΩ	V_o	250V
C_o	470μF	V_f	24V
r_{C_o}	33mΩ	R_o	80Ω

Using $u = 10\,$V, the IBVM converter is simulated in the PSIM software. The 14 measurements acquired are also shown in Table 2. For this case, the coefficients obtained are presented in Table 3 which yields the static gain as in (8). Figure 3 shows the static gain waveforms by using (7) and (8).

Table 2. Output terminal voltage as a function of the duty cycle for IBVM converter.

d	V_o	d	V_o	d	V_o
0.500	39.74	0.731	72.86	0.961	295.13
0.538	43.01	0.769	84.49	1	0.003
0.577	46.86	0.807	100.40		
0.615	51.46	0.846	123.34		
0.653	57.06	0.884	158.69		
0.692	64.01	0.923	216.82		

5. EXPERIMENTAL VALIDATION

To validate the proposed approach, we used a built-in experimental bench with a IBVM converter shown in Fig. 4. In this set-up, the eZdsp TMS320F28335 from Texas Instruments is the control unit, the dc source of 60V and

Table 3. Coefficients solution of the proposed approach for IBVM converter

β_0	β_1
0.418825900963993	-2.90706424121615
β_2	β_3
8.01289149687686	-10.9527957477706
β_4	β_5
7.41164416607867	-1.97243432012344
β_6	β_7
-0.0139710095831490	0.00290375791084200
α_0	α_1
0.209907940387375	-1.66647335907726
α_2	α_3
5.47095126476853	-9.50361598868387
α_4	α_5
9.21066158485236	-4.72142135872236

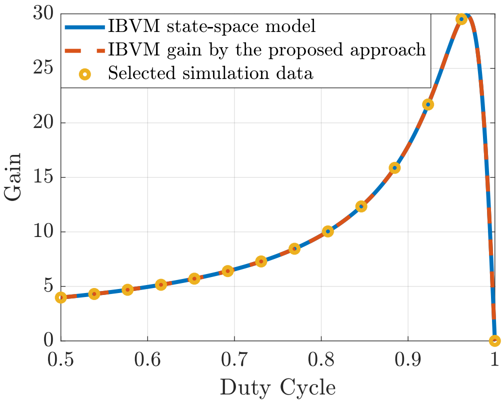

Fig. 3. Comparison between the voltage gain calculated via PSIM, using the state-space model and by the proposed approach.

80A is used as power supply and a 5054 Digital Phosphor Oscilloscope from Tektronix is employed to acquire the data to be used. Additionally, the IBVM main features are presented in Table 4.

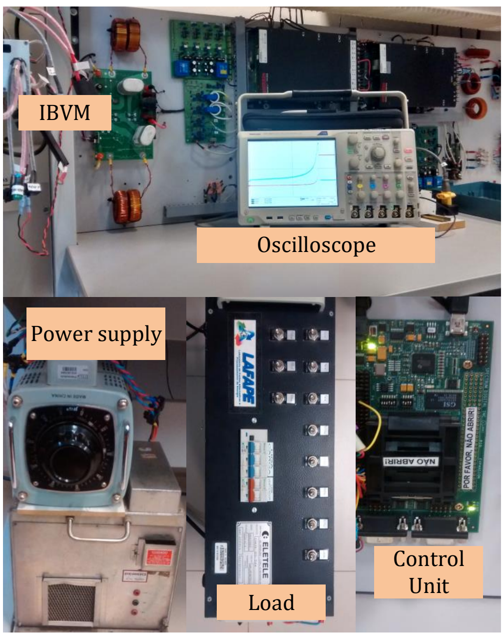

Fig. 4. The experimental test bench with a IBVM converter

Table 4. Main features of the set-up and nominal parameters for the experimental IBVM converter.

Feature	Value
Diodes	DSEI 2X31-06 C
MOSFET	IXFN140N30P
Voltage multiplier capacitors $C_1 = C_2 = C$	1μF
Load resistance	122.7Ω
Inductors $L_1 = L_2 = L$	870μH
Input capacitors C_i	1000μF
Output capacitor C_o	1360μF

Experiments were performed by varying the duty cycle linearly from 0.5 until 0.94 which was considered as the largest safe duty cycle (in this experimental test bench, the power supply did not have short-circuit control). Firstly, the duty cycle was taken in the interval [0.5 0.75] and secondly, in the interval [0.75 0.94].

In the first part of the experiment, the terminal voltage of the IBVM converter was changed from 45V up to 85V according to the duty cycle variation (from 0.5 up to 0.75) as shown in Fig. 5. When the duty cycle is set to 0.5, at the beginning of the event, a peak of 8A is absorbed from the dc source and a voltage step (from 15V up to 45V) is observed at the IBVM terminals as also shown in Fig. 5. Additionally, in this figure, it was observed the current variation at the input terminals (from 1A up to 3A) of the IBVM converter as well as the well-done level of equalization between currents of the inductors.

Then, a duty cycle of 0.75 is applied to perform the second part of the experiment as shown in Fig. 6. In this figure, at the moment the event takes place, it is noticed a peak of 12A at the dc source terminals and a voltage step (from 15V up to 85V) at the IBVM terminals as well. In addition, as the duty cycle is incremented (from 0.75 up to 0.94 within 160s) the terminal voltage rises from 85V up to 185V, while the input current of the IBVM achieves 40A. Additionally, the equalization current level in both inductors was smaller than 2%.

To apply the proposed approach, the oscilloscope waveforms were stored in a data file. Assuming that the converter is operating with a duty cycle between 0.5 and 0.94, the following facts apply

(1) The storage devices was $k = 6$.
(2) The input voltage source was $q = 1$.

As $k = 6$, then the output and input voltage will be related by (8). To find the constants, 14 measurements are needed and from step 4 the duty cycle steps was set as 0.03384. Using the acquired data, the obtained measurements are shown in Table 5, and the coefficients are calculated using the approach presented before (see Table 6). In the interest range of the duty cycle, the theoretical and experimental static gain curves are shown in Fig. 7.

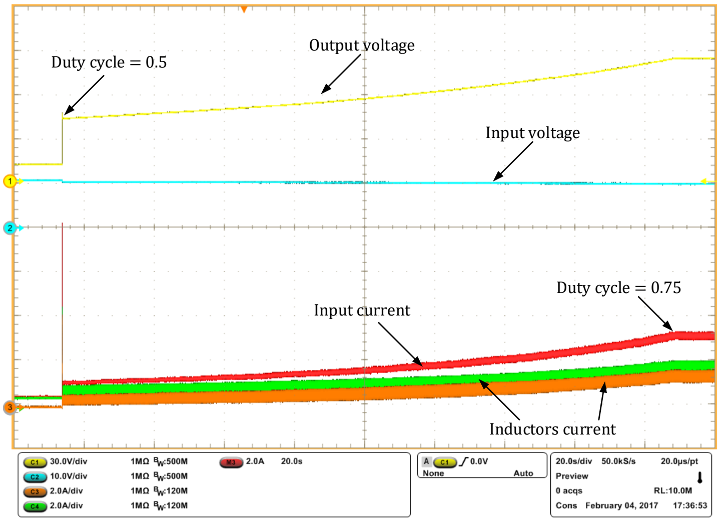

Fig. 5. IBVM converter waveforms during the duty cycle sweep between 0.5 and 0.75. The blue line is the input voltage with 10V/div, the yellow line is the output voltage with 30V/div, the orange line is the current in an inductor with 2A/div, the green line is the current in the another inductor with 2A/div and the red line is the input current with 2A/div. The time scale was 20s/div

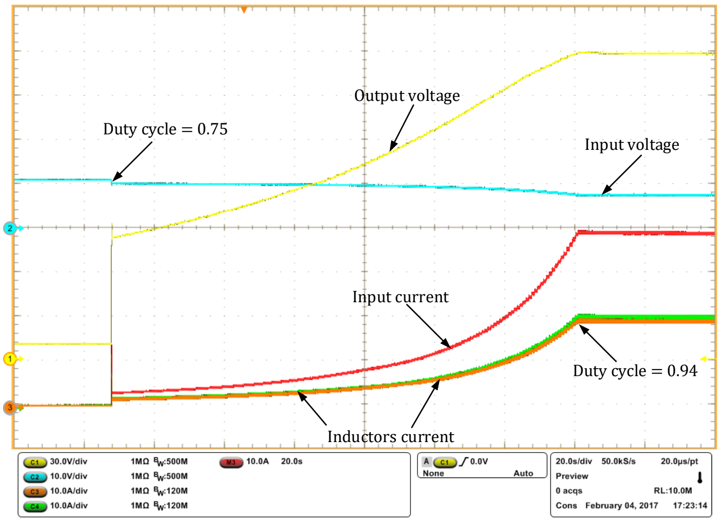

Fig. 6. IBVM converter waveforms during the duty cycle sweep between 0.75 and 0.94. The blue line is the input voltage with 10V/div, the yellow line is the output voltage with 30V/div, the orange line is the current in an inductor with 10A/div, the green line is the current in the another inductor with 10A/div and the red line is the input current with 10A/div. The time scale was 20s/div.

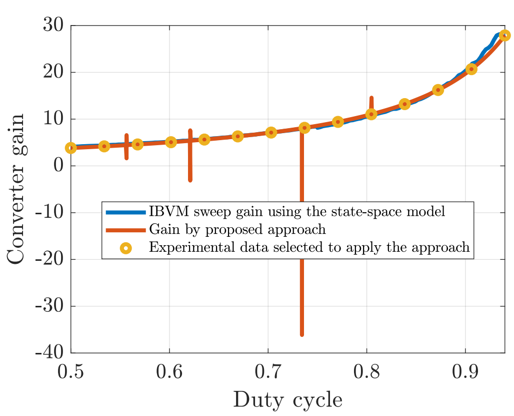

Fig. 7. Gain obtained by the proposed approach with the duty cycle d in the interval [0.5 0.94] compared with the IBVM experimental sweep gain

Table 5. Experimental IBVM converter data selected to apply the proposed approach

d	V_i	V_o	d	V_i	V_o
0.5000	10.37	39.69	0.7369	9.93	80.83
0.5338	10.27	42.91	0.7707	9.92	93.28
0.5676	10.28	47.14	0.8046	9.91	109.36
0.6015	10.27	52.01	0.8384	9.61	127.00
0.6353	10.15	57.14	0.8723	9.46	153.58
0.6692	10.10	63.70	0.9061	8.57	177.35
0.7030	10.05	71.62	0.9400	7.55	210.69

The roots of (8) are 1.051, 0.7343, 0.621, 0.8048, 0.8732 and 0.5565. Such roots appears due measurements accuracy.

Table 6. Coefficients solution of experimental IBVM converter with losses

β_0	β_1
0.325330924152482	-2.38503624140844
β_2	β_3
7.34142608218681	-12.7648951835241
β_4	β_5
14.6277409698532	-12.0708245097862
β_6	β_7
6.70059330792797	-1.77796504129334
α_0	α_1
0.187442387609786	-1.51982767723232
α_2	α_3
5.08964966872625	-9.01056179212412
α_4	α_5
8.89410804906614	-4.64086702807968

6. CONCLUDING REMARKS

In this work, the behavior of the static gain of a dc–dc converters under degradation caused by the parasitic resistances and the load demand is studied using a data driven approach. The simulations and experimental results showed that the proposed approach can be applied to a dc–dc converter operating in CCM without the need to perform a duty cycle sweeping to obtain the voltage gain. In the proposed approach, the duty cycle interval was chosen as equally spaced. A priori knowledge about the dc–dc converter can help in choosing the measurements points of interest. Differing from fitting methods which exhaustively try to find the best order of the polynomials of rational functions, in the proposed approach, the order and the number of measurements required are known in advance. Future works include a regularization technique to avoid singularities which can appear because of the measurements accuracy.

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