Optimal Experiment Design for AC Power Systems Admittance Estimation

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Abstract: The integration of renewables into electrical grids calls for the development of tailored control schemes which in turn require reliable grid models. In many cases, the grid topology is known but the actual parameters are not exactly known. This paper proposes a new approach for online parameter estimation in power systems based on optimal experimental design using multiple measurement snapshots. In contrast to conventional approaches, our method computes optimal excitations extracting the maximum information in each estimation step to accelerate convergence. The performance of the proposed method is illustrated with a case study.

Keywords: Power System Parameter Estimation, Optimal Experiment Design

1. INTRODUCTION

The safe and reliable operation and control of power systems with a large share of renewables requires reliable grid models. In many cases topology information is available while the line parameters are unknown or erroneous (Abur and Expósito (2004); Kusic and Garrison (2004)). This might lead to difficulties in predicting critical situations which in turn can lead to black-outs and to substantial socio-economic costs. At the same time, shutting down of critical power systems infrastructure to perform identification procedures is usually no viable option. Thus, online algorithms for determining the parameters in electrical power systems are of significant interest.

In the present paper, we consider the *stationary* AC power system parameter estimation problem, i.e. the problem of estimating line parameters of the AC power flow equations neglecting transient phenomena. [2] More specifically, we propose to tackle the problem via concepts stemming from Optimal Experimental Design (OED).

Classical static power system parameter estimation can roughly be categorized along two axis: first, methods considering only one time instant of measurements versus methods using multiple ones; and second, methods simultaneously estimating states (voltage magnitude and phase angle) *and* the parameters versus methods only considering the parameters or following a sequential state-parameter estimation procedure cf. (Abur and Expósito

(2004); Monticelli (1999); Zarco and Gómez-Expósito (2000)). Bian et al. (2011) and Slutsker et al. (1996) consider pure parameter estimation with multiple time samples, whereas Quintana and Cutsem (1988) rely on a single snapshot. This method is extended to multiple snapshots in (Van Cutsem and Quintana (1988)). Slutsker et al. (1996) present a version based on the Kalman filter including past measurements indirectly via the corresponding a posteriori state estimate and the error covariance matrix. Combined state and parameter estimation is considered in (Liu et al., 1992). An approach for combined topology/parameter estimation which seemingly does not to fit in the above categorization was recently presented in (Deka et al., 2016; Park et al., 2018).

Optimal experiment design as described by Pukelsheim (1993); Franceschini and Macchietto (2008) appears to have received only little attention in the power system context. It is so far used in the context of measurement placement only (Li et al., 2011). In contrast, OED is frequently used in the context of parameter estimation for linear and nonlinear dynamic systems, especially in the context of chemical process system identification (Körkel et al., 2004; Houska et al., 2015). Lemoine-Nava et al. (2016) introduce a method of using OED for reducing the degree of freedom of variables in the field of frame material discovery; a method for reducing the volume of high-throughput experiments based on OED was proposed by Talapatra et al. (2018). Pronzato (2008) highlights the strong relations between experimental design and control such as the use of optimal inputs to obtain precise parameter estimation.

Methods relying on a large number of samples include more information into the estimation process and thus yield a better performance compared to single snapshot

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[2] We refer to (Zhao et al. (2019)) for a recent overview on *dynamic* power system parameter estimation.

techniques. Hence we focus on multiple snapshot techniques here. There exists two approaches on how past measurements are considered: One simple approach is to incorporate the measurments of all sampling instances in one big estimation problem which grows with the number of observed measurements. This leads to potentially intractable estimation problems and therefore recursive methods have been developed considering information about past measurements as parameters in the current estimation step. In turn this leads to real-time algorithms which are able to estimate parameters while the system is running. Prominent examples for these methods are the recursive least-squares method or the Kalman filter (Ljung, 1999).

This paper proposes an approach for applying methods from optimal experimental design to the online estimation of the admittance parameters in AC power grids. Section 2 recaps the AC power grid model. The main contribution is presented in Section 3. By relying on available measurements only, i.e. voltage and power measurements of the grid, the proposed method can be categorized as a recursive online estimator. A distinctive feature of our method is that we design optimal generator power profiles—i.e. excitations—such that as much information as possible is extracted in a single estimation step while keeping the power output at the consumer constant. This strategy enables us to perform optimally designed experiments while, at the same time, ensuring that the grid remains completely functional and delivers a constant power output to the end-users. In Section 4, we compare our results to a recursive estimator with constant input on a 5-bus benchmark system. Our results indicate that the proposed method outperforms classical recursive parameter estimation techniques.

Notation: For a given $a \in \mathbb{R}^n$ and $\mathcal{C} \subseteq \{1,...,n\}$, $(a_k)_{k \in \mathcal{C}} \in \mathbb{R}^{|\mathcal{C}|}$ stacks all $k \in \mathcal{C}$ elements. Similarly, for a given $A \in \mathbb{R}^{n \times l}$ and $\mathcal{S} \subseteq \{1,\ldots,n\} \times \{1,\ldots,l\}$, $(A_{i,k})_{(i,k) \in \mathcal{S}} \in \mathbb{R}^{|\mathcal{S}|}$ denotes a vector stacking elements $A_{i,k}$ for all $(i,k) \in \mathcal{S}$. Moreover, $\mathfrak{i} = \sqrt{-1}$ denotes the imaginary unit, and hence $z = \mathrm{Re}(z) + \mathfrak{i} \cdot \mathrm{Im}(z)$.

2. AC POWER GRID MODEL

Let $(\mathcal{N}, \mathcal{L}, Y)$ be a power grid with $\mathcal{N} = \{1,\ldots,N\}$ denoting the set of buses, $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of transmission lines, and $Y \in \mathbb{C}^{N \times N}$ is the sparse, complex-valued admittance matrix. The admittance matrix is defined as

$$Y_{k,l} = \begin{cases} \sum_{j \neq k} (g_{k,j} + \mathfrak{i}\, b_{k,j}) \text{ if } k = l, \\ -(g_{k,l} + \mathfrak{i}\, b_{k,l}) \quad \text{ if } k \neq l, \end{cases}$$

where $g_{k,l}$ are the line conductances and $b_{k,l}$ are the line susceptances for all transmission lines $(k,l) \in \mathcal{L}$. In general, not all buses are connected. Thus, for most real networks the matrix Y can be expected to be sparse. Hence we set $g_{k,l} = b_{k,l} = 0$ for all $(k,l) \notin \mathcal{L}$. For example, for the 5-bus network in Figure 1 the nodes 3 and 5 are not directly connected. [3]

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[3] We use the per-unit system which is a standard normalization procedure from power systems (Glover et al., 2012). The base-power is 100 MVA and the base-voltage is 230 kV. The base-frequency is
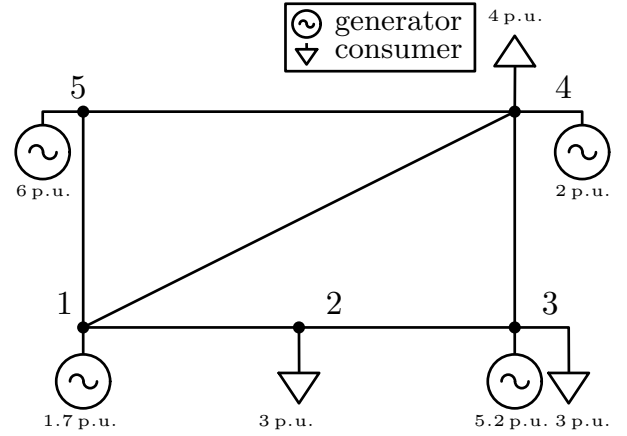

Fig. 1. Modified 5-bus system from Li and Bo (2010) with 4 generators and 3 consumers.

Let v_k denote the voltage amplitude at the k-th node and θ_k the voltage angle. Throughout this paper, we assume that the voltage magnitude and the voltage angle at the first node,

$$\theta_1 = 0 \quad \text{and} \quad v_1 = \text{const.} \,,$$

are fixed. This assumption can always be made without loss of generality, since the power flow in the network depends on the relative voltage angles $\theta_k - \theta_l$. Similarly, the voltage v_1 at the first node is regarded as the reference voltage. Thus, because θ_1 and v_1 are given, the vector

$$x = (v_2, \theta_2, v_3, \theta_3, \ldots, v_N, \theta_N)^\top$$

is the state of the power system. In the following, we use the auxiliary function

$$P_k(x,y) = v_k^2 \sum_{l \in \mathcal{N}_k} \begin{pmatrix} g_{k,l} \\ -b_{k,l} \end{pmatrix} - v_k \sum_{l \in \mathcal{N}_k} v_l \begin{pmatrix} g_{k,l} & b_{k,l} \\ -b_{k,l} & g_{k,l} \end{pmatrix} \begin{pmatrix} \cos(\theta_k - \theta_l) \\ \sin(\theta_k - \theta_l) \end{pmatrix}$$

in order to model the active and reactive power residuum at the k-th node, where the shorthand

$$\mathcal{N}_k = \{l \in \mathcal{N} \mid (k,l) \in \mathcal{L}\}$$

denotes the set of neighbors of the k-th node. Moreover,

$$y = \begin{pmatrix} g_{k,l} \\ b_{k,l} \end{pmatrix}_{(k,l) \in \mathcal{L}} \in \mathbb{R}^{2|\mathcal{L}|}$$

denotes the admittance parameter vector, that is, a vector consisting of all non-zero components of the admittance matrix Y that are off-diagonal. Notice that $\dim y = 2|\mathcal{L}|$ grows with the number of transmission lines.

The variables p_k^d and q_k^d denote the active and reactive power demands, which are, for the sake of simplicity, assumed to be known and constant. We set $p_k^d = 0$ and $q_k^d = 0$, if there is no consumer at the k-th node. Moreover, $\mathcal{G} \subseteq \mathcal{N}$ is the set of generators in the system. The associated generator active and reactive power at the k-th node, with $k \in \mathcal{G}$, are denoted by p_k^g and q_k^g, respectively. Notice that the net active and reactive net power supply at Node k are

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unfortunately not given in the dataset (Li and Bo, 2010), but it is also not needed as conductances and susceptances are given.

$$S_k(u) = \begin{pmatrix} p_k^{\mathrm{g}} - p_k^{\mathrm{d}} \\ q_k^{\mathrm{g}} - q_k^{\mathrm{d}} \end{pmatrix} \quad \text{if} \quad k \in \mathcal{G}$$

and

$$S_k(u) = \begin{pmatrix} -p_k^{\mathrm{d}} \\ -q_k^{\mathrm{d}} \end{pmatrix} \quad \text{if} \quad k \notin \mathcal{G} .$$

In the context of this paper, we regard the active and reactive powers at all but the first generator

$$u = \begin{pmatrix} p_k^{\mathrm{g}} \\ q_k^{\mathrm{g}} \end{pmatrix}_{k \in \mathcal{G} \setminus \{1\}}$$

as an input that the grid operator can choose. Notice that the conservation of energy must hold at all nodes, which implies that the power residuum at the nodes must be equal to the supplied power,

$$\forall k \in \mathcal{N}, \qquad P_k(x, y) = S_k(u) . \tag{1}$$

In the literature equations (1) are known under the name power-flow equations (Monticelli, 1999). At this point, it is important to be aware of the fact that, because the consumer demand is assumed to be constant and given, the grid operator can choose u but needs to make sure that the active and reactive power at the first generator satisfy

$$\begin{pmatrix} p_1^{\mathrm{g}} \\ q_1^{\mathrm{g}} \end{pmatrix} = P_1(x, y) + \begin{pmatrix} p_1^{\mathrm{d}} \\ q_1^{\mathrm{d}} \end{pmatrix}, \tag{2}$$

i.e. the overall power balance holds. In the power systems literature, Node 1 is commonly called the *slack* node (Grainger and Stevenson, 1994). Typically, a large generator is connected to this node ensuring that the above power balance can always be satisfied.

Remark 1. (Considering energy storage). *Equation (2) can alternatively be satisfied by installing a battery or other storage devices at the first node, which supplies the active and reactive power p_1^{g} and q_1^{g}. The advantage of introducing a storage device is that (2) can be satisfied even if the generators temporarily do not match the consumer demand.*

In summary, the power flow equations can be written compactly as

$$P(x, y) = S(u) , \tag{3}$$

with shorthands

$$P = [P_2^\top, \ldots, P_N^\top]^\top \quad \text{and} \quad S = [S_2^\top, \ldots, S_N^\top]^\top .$$

Notice that

$$\dim(P) = \dim(x) = 2(N - 1) .$$

Moreover, the active and reactive power flow in the transmission line $(k, l) \in \mathcal{L}$ is given by

$$\Pi_{k,l}(x, y) = v_k^2 \begin{pmatrix} g_{k,l} \\ -b_{k,l} \end{pmatrix}$$
$$- v_k v_l \begin{pmatrix} g_{k,l} & b_{k,l} \\ -b_{k,l} & g_{k,l} \end{pmatrix} \begin{pmatrix} \cos(\theta_k - \theta_l) \\ \sin(\theta_k - \theta_l) \end{pmatrix},$$

which also depends on the non-zero admittance matrix coefficients y, $P_k(x, y) = \sum_{l \in \mathcal{N}_k} \Pi_{k,l}(x, y)$.

3. OPTIMAL EXPERIMENT DESIGN FOR ADMITTANCE ESTIMATION

Next, we introduce a repeated Optimal Experiment Design (OED) and parameter estimation procedure for estimating the admittance matrix in AC power networks.

3.1 Maximum Likelihood Parameter Estimation

Throughout this paper we assume that the power flow over the transmission lines as well as the system state x can be measured. Therefore, we consider the measurement function $M : \mathbb{R}^{2(N-1)} \times \mathbb{R}^{2|\mathcal{L}|} \to \mathbb{R}^m$, $m = 2(|\mathcal{L}| + N - 1)$

$$M(x, y) = \left[x^\top, (\Pi_{k,l}(x, y))_{(k,l) \in \mathcal{L}}^\top \right]^\top .$$

If the associated measurement error has a Gaussian distribution with zero mean and given variance $\Sigma \in \mathbb{R}^{m \times m}$, $\Sigma \in \mathbb{S}_{++}^m$, the associated maximum likelihood estimation problem for the unknown admittance coefficients y reads

$$\min_{x,y} \quad \frac{1}{2} \| M(x, y) - \eta \|_{\Sigma^{-1}}^2 + \frac{1}{2} \| y - y^- \|_{\Sigma_0^{-1}}^2 \tag{4}$$
$$\text{s.t.} \quad P(x, y) = S(u).$$

Here, we assume that $y^- \in \mathbb{R}^m$ is a given initial parameter estimate with given variance $\Sigma_0 \in \mathbb{S}_{++}^m$ and η are (possibly noisy) measurements associated with $M(x, y)$.

3.2 Fisher Information

The power flow equation, $P(x, y) = S(u)$, has in general multiple solutions. For example, this equation is invariant under voltage angle shifts. However, if the sensitivity matrix [4]

$$\frac{\partial}{\partial x} P(x, y)$$

has full rank at an optimal solution (x, y) of (4) for a given u, then we can use the implicit function theorem to show that a locally differentiable parametric solution $x^\star(y, u)$ of the equation $P(x, y) = S(u)$ exists. Moreover, this solution satisfies

$$\frac{\partial}{\partial y} x^\star(y, u) = - \left[\frac{\partial}{\partial x} P(x, y) \right]^{-1} \frac{\partial}{\partial y} P(x, y)$$

and, the Fisher information matrix (Pukelsheim (1993)) of the admittance estimation problem (4) reads

$$\mathcal{F}(x, y, u) = \Sigma_0^{-1} + T(x, y, u) \Sigma^{-1} T(x, y, u)^\top , \tag{5}$$

where the shorthand

$$T(x, y, u) = \frac{\partial}{\partial y} M(x, y) + \frac{\partial}{\partial x} M(x, y) \frac{\partial}{\partial y} x^\star(y, u)$$

is used. The inverse of the Fisher information matrix, $\mathcal{F}(x, y, u)^{-1}$, can be interpreted as a linear approximation of the variance matrix of the posterior distribution of the parameter y (Telen et al. (2013)). Because this variance depends on the generator power inputs u, these inputs can be used to improve the expected quality of the estimate by using an optimal experiment design procedure, as outlined below.

[4] Conditions under which the matrix $\frac{\partial}{\partial x} P(x, y)$ has full-rank can be found in (Hauswirth et al., 2018), where linear indendence constraint qualifications for AC power flow problems are discussed in a more general setting.

Algorithm 1 Optimal Experiment Design for Power System State Estimation

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Input: Initial guess y^- and variance $\Sigma_0^- \succ 0$, a termination tolerance $\epsilon > 0$, and an initial generator set-point u^-.

Repeat:

1) *Experiment Design:* solve the OED problem (6) and denote the optimal solution for the control input by u.

2) *Collection of Measurements:* set the active and reactive power at the generators to u and take a measurement η.

3) *Maximum Likelihood Estimation:* solve the maximum likelihood estimation problem (4) for given u and denote the optimal solution by (x, y).

4) *Termination Check:* If the trace of the variance is sufficiently small, $\mathrm{Tr}\left([\mathcal{F}(x, y, u)]^{-1}\right) < \epsilon$, break and return the parameter estimate y as well as its approximate variance $[\mathcal{F}(x, y, u)]^{-1}$.

5) *Update Step:* Set $y^- \leftarrow y$, $\Sigma_0 \leftarrow [\mathcal{F}(x, y, u)]^{-1}$. Moreover, we set $u^- \leftarrow u$ and return to Step 1).

3.3 Optimal Experiment Design for AC Power Networks

Next, we develop an OED procedure for improving the accuracy of admittance estimation. Although there are many OED design objectives possible, we focus on the A-design criterion, because the trace of a matrix can be efficiently evaluated and differentiated without much computational overhead (Telen et al., 2013, 2014)). Now, the OED problem at hand reads

$$\min_{x,u} \ \mathrm{Tr}\left([\mathcal{F}(x, y^-, u)]^{-1}\right) + \rho\|u - u^-\|_2^2$$

$$\text{s.t.} \begin{cases} P(x, y^-) = S(u) \\ \underline{u} \le u \le \overline{u} \\ \underline{x} \le x \le \overline{x} \\ \underline{u}_1 \le P_1(x, y^-) + \begin{pmatrix} p_1^{\mathrm{d}} \\ q_1^{\mathrm{d}} \end{pmatrix} \le \overline{u}_1 \end{cases} \quad (6)$$

Here, y^- denotes the current parameter estimate and u^- denotes the old generator set point used for regularization with regularization parameter $\rho \ll 1$. Moreover, the lower and upper bounds $\underline{u}$, $\overline{u}$ are introduced in order to enforce upper and lower bounds on the generator power. Similarly, the lower and upper bounds $\underline{x}$, $\overline{x}$ are used to model physical limitations on the voltage magnitude and angle. The proposed OED approach to power system state estimation is summarized in Algorithm 1.

Remark 2. (Termination in finitely many steps).
Because we assume measurements of the power flow over all transmission lines, it is clear that all admittance coefficients are observable. In other words, the matrix $T(x, y, u)$ always has full-rank—independent of how one chooses u. Thus, it follows from (5) and from the update of Σ_0 in Step 5) of Algorithm 1 that the Fisher information is strictly monotonically increasing, which, in turn, implies that Algorithm (1) terminates after a finite number of iterations.
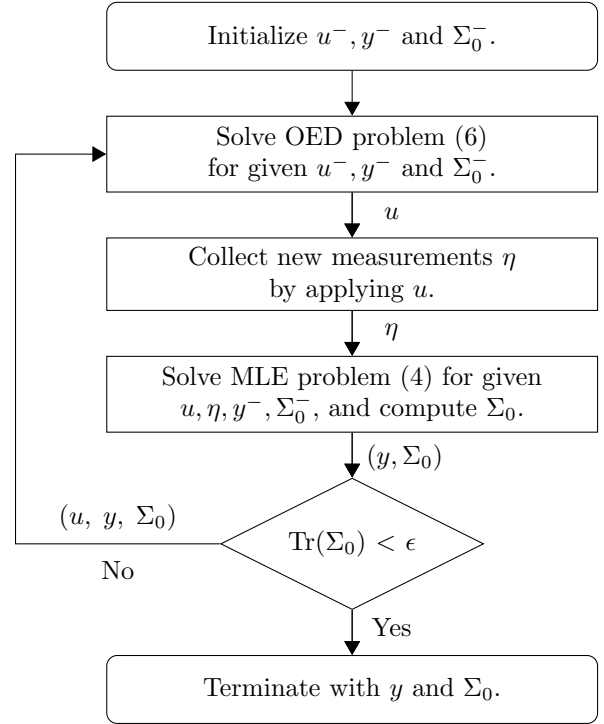

Fig. 2. Flow chart of Algorithm 1.

Remark 3. (Ensuring power balance). Note that though the algorithm varies the active and reactive power set-points p_k^g and q_k^g, the consumers are not affected by the proposed estimation method as power balance is enforced via the power flow equations (3).

4. NUMERICAL RESULTS

Next we illustrate the performance of Algorithm 1 drawing upon the modified 5-bus system from (Li and Bo, 2010) shown in Figure 1.

4.1 Implementation and data

The problem data is obtained from the `MATPOWER` dataset (Zimmerman et al., 2011). As discussed in Section 2, we use bus 1 as reference with fixed v_0 and θ_0.

We consider measurements $M(x(\bar{y}, u^\star), \bar{y}) + \chi$ with additive Gaussian noise χ, which has zero mean and covariance $10^{-4} I$. Here, $\bar{y}$ denotes the ground truth of y (i.e. the line parameters from the `MATPOWER` dataset) and I is the identity matrix. Furthermore, we use $\rho = 8 \cdot 10^{-4}$ as regularization parameter in Step 1 of Algorithm 1. We initialize y^- with a non-zero vector with small norm to avoid numerical difficulties. Apart from evaluating the total variance of the OED estimator $\mathrm{Tr}(V)$, we also compute the mean relative error of the estimated parameters y via

$$\mathrm{MRE_g} = \frac{1}{|\mathcal{L}|} \sum_{(k,l)\in\mathcal{L}} \frac{|g_{k,l} - \bar{g}_{k,l}|}{|\bar{g}_{k,l}|},$$

$$\mathrm{MRE_b} = \frac{1}{|\mathcal{L}|} \sum_{(k,l)\in\mathcal{L}} \frac{|b_{k,l} - \bar{b}_{k,l}|}{|\bar{b}_{k,l}|}.$$

Figure 3 shows the mean relative error $\mathrm{MRE_g}$ and $\mathrm{MRE_b}$ over the iteration index k for two different methods: For OED and for OED with a constant input u generated in the first iteration of Algorithm 1.
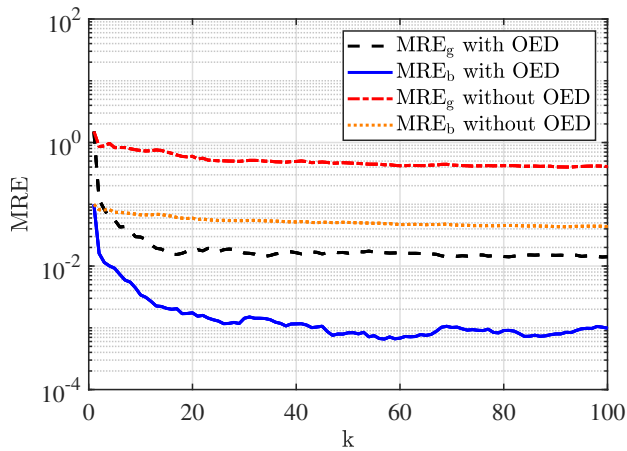

Fig. 3. Mean relative errors $\mathrm{MRE_b}$ (solid line) and $\mathrm{MRE_g}$ (dashed line) as obtained by Algorithm 1. The dotted line and dash-dotted correspond to the corresponding mean relative errors that are obtained by running the estimation without optimally exciting the active and reactive power at the generators.

The latter approach is similar to a standard recursive least-squares method. One can see that with the one-shot estimate after the first iterate, the relative error is around 80%. The error can be reduced to a level of around 1% after a couple of iterations. One can see that the performance of using an optimal input yields a considerably better performance. Moreover, the strong decrease in $\mathrm{MRE_g}$ and $\mathrm{MRE_b}$ after several iterations in both methods underlines the importance of techniques using multiple snapshots.

Table 1. Line parameter estimation results [S].

Index	Conductance true value	Conductance estimate	Susceptance true value	Susceptance estimate
$(1,2)$	3.523	3.515	-35.235	-35.233
$(1,4)$	3.257	3.274	-32.569	-32.546
$(1,5)$	15.470	15.364	-154.703	-154.832
$(2,3)$	9.168	9.725	-91.676	-91.023
$(3,4)$	3.334	3.319	-33.337	-33.347
$(4,5)$	3.334	3.351	-33.337	-33.329

Table 1 shows the ground truth $\bar{y}$ and the OED estimation result after 100 iterations. One can see that in all cases the relative error is below 6%, the $\mathrm{MRE_g}$ is 1.41% and the $\mathrm{MRE_b}$ is 0.0985%. Notice that the maximum absolute error is 0.557 Siemens.

Figure 4 shows $\mathrm{Tr}(V(u, y^-))$ over the iterates k. One can see a monotonic decrease up to a level of 10^{-1} within 100 iterations. The blue solid line represents $\mathrm{Tr}(V(u, y^-))$ using Algorithm 1, and the red dashed line corresponds to the traditional recursive least square method.
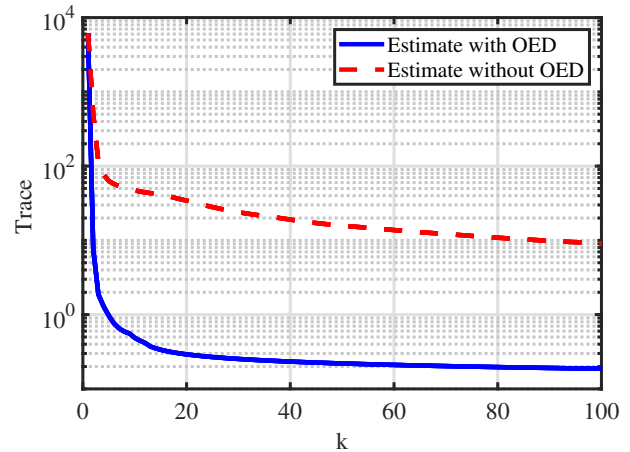

Fig. 4. Total variance of the estimation error $\mathrm{Tr}(V(u, y^-))$ obtained by Algorithm 1 (solid line) and by running the estimation without optimally exciting the active and reactive power at the generators (dashed line).
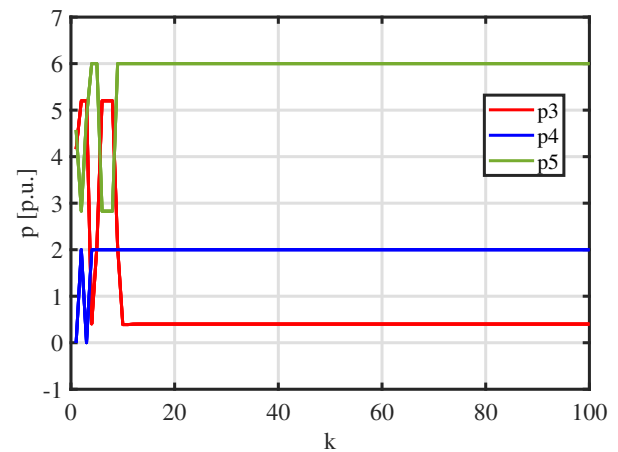

Fig. 5. Optimal active power inputs for all generators as obtained by Algorithm 1.

Figures 5 and 6 show the optimal input for active and reactive power of the three controllable generators in the 5-bus system. One can see that after after around 15 iterations, the input remains constant because of the second term in Step 1) of Algorithm 1. As the Fisher information matrix $\mathcal{F}$ is strictly monotonically increasing its inverse is strictly monotonically decreasing. With that, the second term in problem (6) starts dominating as Algorithm 1 proceeds and thus the change in the optimal input decreases.

5. CONCLUSION

This work proposed an approach to online power system parameter estimation for the AC grids based on techniques from optimal experimental design. Our simulation of a 5-bus AC power system shows that the optimal generator excitation as computed by the proposed method leads to a considerably higher estimation accuracy of the system parameters compared to a recursive least-squares estimation without such excitations. Specifically, we are able to decrease the mean relative error to less than 1% in a cou-
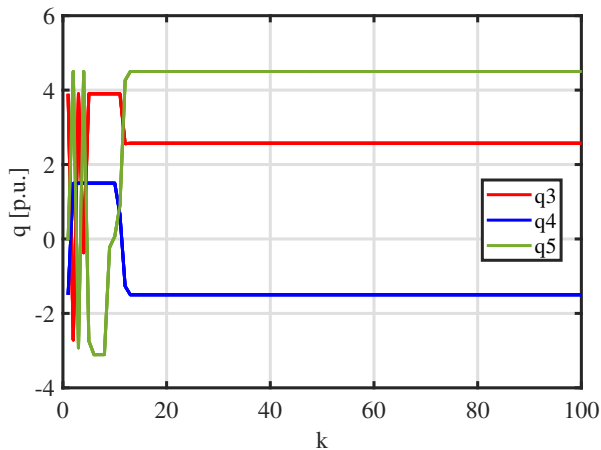
Fig. 6. Optimal reactive power inputs for all generators as obtained by Algorithm 1.

ple of iterations when considering Gaussian measurement noise with a variance of 10^{-4} for all the components of the measurement function M.

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