Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Semi-Implicit Euler Discretization for Homogeneous Observer-based Control: one dimensional case ⋆

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Abstract: It is well known that the implicit Euler strategy is a chattering-free implementation of sliding mode algorithms. In this paper, we propose to mix explicit and implicit discretizations in order to deal with homogeneous sliding mode control. More precisely, a semi-implicit discretization for homogeneous observer-based sliding mode control is proposed in one dimensional case with a theoretical stability proof. The effectiveness of the proposed solution is illustrated in simulation by a comparison with explicit Euler discretization.

Keywords: Homogeneous control, Sampled data, Discrete-time systems, Implicit systems, Observers, Estimators

1. INTRODUCTION

In practice, the parameters of physical processes modeling are very difficult to determine. There are some interesting contributions about the identification of these parameters, for example, see (Luspay et al., 2011; Janot et al., 2013). However, it is not possible to deal always with physical phenomena such as Coulomb friction effect. Therefore, it is interesting to investigate robust control laws; the popular sliding mode methods belong to the family of the robust control laws. The paper deals with a homogeneous finite-time controller which enforces a first-order sliding-mode for $\alpha = 0$. Due to its technical simplicity and robustness versus perturbations and parameter variations, sliding mode control (SMC) has been widely used in practical systems and recent research focuses on the implementation of explicit discretization of these sliding mode controls (see (Utkin, 1992; Yu et al., 2012)).
The main disadvantage of the explicit discretization is the chattering effect that remains even for high order SMC algorithms. Initially introduced in (Acary and Brogliato, 2010), the implicit discretization aims to replace the sign function by an *implicit projector*; recent investigations (see (Huber et al., 2013; Brogliato and Polyakov, 2015; Huber et al., 2016)) have shown very promising results that highlight a strong reduction of the chattering effect as well as robustness of the control under lower sampling frequencies and preservation of the global stability; the compensation and estimation of disturbances has been also treated in (Acary et al. (2012); Huber et al. (2014)). Experimental validations of some implicit based sliding control algorithms have been successfully performed (Huber et al., 2014; Wang et al., 2015). Recently, the implicit strategy has been extended to high order sliding mode control in (Brogliato et al., 2018, 2019) and for the explicit solution, see also (Koch et al., 2019; Barbot et al., 2020) .
Homogeneous control is a more general class of finite time methods (for appropriate homogeneous degree); regarding such type of control, to the best of authors' knowledge, the implicit discretization strategy has not yet been considered. This is due to the fact that for homogeneous control, feedback cancellation should appear with Euler implicit scheme. That is why, in this paper, a discretization strategy based on a mix of implicit and explicit Euler schemes (called "semi-implicit") is proposed for first order homogeneous sliding mode control. A mix of implicit and explicit Euler schemes has already been studied in (Polyakov et al., 2019). However, in this latter paper, the problem statement was not to track the behavior of a continuous system: indeed, only the digital framework has been considered. A property introduced by (Grizzle and Kokotovic, 1988), that states that the feedback linearizability is not preserved under sampling, has been highlighted. In (Grizzle and Kokotovic, 1988), it is also shown that many approximate discretization schemes would also result in systems that are not feedback linearizable. In other hand, the problem of system under sampling has been also treated by the way of comparing the Volterra series obtained from the continuous closed loop system and the one obtained by the system under sampling (Monaco and Normand-Cyrot, 1985).
By considering a first order perturbed continuous system, the interpretation of the implicit discretization is firstly introduced in the sequel through the so-called implicit

⋆ This work is supported by the ANR project DigitSlid ANR-18-CE40-0008-01.

projector and its main properties are highlighted in simulation. For nonlinear systems, approximation errors due to the discretization of the feedback control appear to overcome these approximation errors; a semi-implicit scheme is proposed to discretize the homogeneous first order control. This control is associated with a semi-implicit observer that estimates online the perturbation in order to cancel the latter. A Lyapunov-based analysis gives theoretical conditions for the stability of the proposed control.
The paper is outlined as follows: Section 2 states the problem. In Section 3, the considered system and the review of the main properties of the implicit projector are stated. Section 4 presents respectively the proposed semi-implicit control and the semi-implicit observer. The observer-based control is described in Section 5 and simulation results illustrate the proposed concepts in Section 6. Section 7 gives some concluding remarks and perspectives.

2. PROBLEM STATEMENT

Consider a first order continuous perturbed system

$$\dot{x} = p(t) + u(t) \tag{1}$$

with $x \in I\!R$ the state variable, $u \in I\!R$ the control input and $p \in I\!R$ the perturbation such that $|p(t)| < p_M$, p_M being a positive constant.
It is well-known that the main limitation of controlling system (1) by the sliding mode control (with $\lambda > p_M$)

$$u(t) = -\lambda \, \mathrm{sgn}(x(t)) \tag{2}$$

is, under classical (explicit) discretization, the chattering effect. To overcome such limitation, the implicit Euler discretization has shown very interesting performances. This implicit method is founded on the discrete projector (Acary and Brogliato (2010)) whose definition only depends on the function sign. Hereafter, to deal with homogeneous control

$$u(t) = -\lambda \, |x(t)|^{\alpha} \mathrm{sgn}(x(t)) \tag{3}$$

with $\alpha \in [0, 1[$ [1], a semi-implicit Euler discretization scheme is required due to the fact that, in (3), the sign function is multiplied by $|x(t)|^{\alpha}$ that makes the implicit approach not applicable (for detail, see in the sequel, equation (7)). This algorithm is useful for example in any scheme where there are software-in-the-loop implementations (see Figure 1).
Thanks to the implicit homogeneous properties, the structure of the proposed control, composed of a semi-implicit homogeneous control part, associated to a semi-implicit homogeneous observation part, allows reducing the chattering effect and estimating the perturbation in order to be compensated by the control. The goal of this work is therefore to investigate the use of semi-implicit approaches for control and observation of perturbed system (1).

In the next section, some recalls on the Euler implicit sliding mode discretization are presented.

3. RECALL ON EULER IMPLICIT SLIDING MODE CONTROL

Consider the system (1) without perturbation (i.e. $p(t) = 0$) and the control law (2). The associated exact discretized

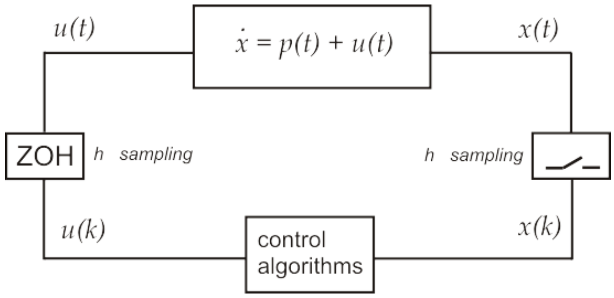

Fig. 1. Controlled system by discrete control algorithms.

system, with a sampling-time h, is controlled by the *implicit projector* $\mathcal{N}_{\lambda,h}$ that gives

$$\begin{cases} x_{k+1} = x_k + h\, u_{k+1} \\ u_{k+1} = -\lambda \, \mathrm{sgn}(x_{k+1}) \end{cases} \tag{4}$$

where the $\mathrm{sgn}(x_{k+1})$ is evaluated thanks to the operator $\mathcal{N}_{\lambda,\,h}$ with $\lambda > 0$ that is defined as

$$\begin{cases} |x_k| < \lambda h \to \mathcal{N}_{\lambda,\,h} = \dfrac{x_k}{\lambda h} & \text{(i.e. } x_{k+1} = 0) \\ |x_k| \geq \lambda h \to \mathcal{N}_{\lambda,\,h} = \mathrm{sgn}(x_k) & \text{(i.e. } x_{k+1} \neq 0) \end{cases} \tag{5}$$

Notice that, from the system point-of-view, the implicit property is characterized by the fact that the system is driven through u_{k+1} that predicts/anticipates which would be the "ideal" input to hold the system on the sliding surface diminishing thus the chattering.
The implicit projector $\mathcal{N}_{\lambda,\,h}$ (5) is a piecewise-defined function that is composed of several sub-domains depending on the value of x_k

- if $x_k \geq |\lambda h|$, then u_{k+1} belongs to the saturation mode [2] defined by $u_{k+1} = -\lambda \, \mathrm{sgn}(x_k)$,
- else u_{k+1} belongs to the linear mode and corresponds to a $1/\lambda$-contraction of $\frac{x_k}{\lambda h}$,

that allows "converging" to the sliding surface condition $x = 0$. In the sequel, Lemma 1 details the evolution of the sliding variable and gives related conditions that must satisfy u_{k+1} to keep x inside the sliding surface.

Lemma 1. (Acary and Brogliato (2010)). Given the state variable x_k, the backward Euler implicit scheme

$$\begin{cases} x_{k+1} = x_k + h\, u_{k+1} \\ u_{k+1} = -\mathcal{N}_{\lambda,h}(x_k) \end{cases}$$

satisfies

- $x_{k+1} = 0$ if and only if both following conditions are satisfied, $\forall k$:
  (i) $|x_k| < \lambda h$;
  (ii) $u_{k+1} = -\dfrac{x_k}{h}$.
- $x_{k+1} \to 0$ if $\forall k$, the condition $|x_k| \geq \lambda h$ is satisfied.

■

The implicit projector $\mathcal{N}_{\lambda,h}$ can be considered as the key feature of implicit versions of sliding mode algorithms. Inside the projector, the coefficient λ defines the bounds of the linear mode with respect to the saturation mode and outside the projector, it has a physical meaning since it can be considered as an amplifier gain, that must be

[1] If $\alpha = 0$, the control is a sliding mode one for which the implicit scheme is applied.

[2] The $\mathrm{sgn}(x)$ function verifies: if $x > 0$, then +1; if $x < 0$ then -1; if $x = 0$ then $]-1, 1[$.

tuned in accordance with the controlled device technical specifications.

In the framework of implicit discretized control, the association of the discretized version of the controller (3) that reads:

$$u_{k+1} = -\lambda\,|x_{k+1}|^{\alpha}\,\mathrm{sgn}(x_{k+1}) \tag{6}$$

is applied to system (1) and the closed-loop reads

$$x_{k+1} = x_k - h\,\lambda\,|x_{k+1}|^{\alpha}\,\mathrm{sgn}(x_{k+1}) \tag{7}$$

In this case, if $x_{k+1} = 0$, it is not possible to evaluate the projector, that is why, in the next section, a semi-implicit Euler discretization scheme is proposed.

4. SEMI-IMPLICIT EULER DISCRETIZATION OF HOMOGENEOUS CONTROL AND OBSERVER

In this section, a new class of projector is associated to an homogeneous term. The design of a controller and an observer in the framework of the semi-implicit approach is detailed in the sequel.

4.1 Control design

The homogeneous control based on semi-implicit Euler discretization u^{SI}_{k+1} is designed in order to stabilize system (7) and is given by

$$u^{SI}_{k+1} = -\lambda\,|x_k|^{\alpha}\mathcal{N}^{SI}_{\lambda,\,h,\,\alpha} \tag{8}$$

where $\lambda > 0$ and $\alpha \in [0,\,1[$ are constant parameters tuning; the term $|x_k|^{\alpha}$ is the explicit part and the term $\mathcal{N}^{SI}_{\lambda,\,h,\,\alpha}$ constitutes the implicit part.

Remark 2. To obtain (8), $|x_{k+1}|^{\alpha}$ is replaced by $|x_k|^{\alpha}$ in order to be able to compute the projector with respect to the unperturbed system at step $'k+1'$, denoted $\tilde{x}_{k+1}$.

Moreover, the new projector $\mathcal{N}^{SI}_{\lambda,\,h,\,\alpha}$ is defined as follows

$$\mathcal{N}^{SI}_{\lambda,h,\,\alpha} := \begin{cases} \dfrac{|x_k|^{1-\alpha}}{\lambda h}\mathrm{sgn}(x_k) & \textbf{if } |x_k|^{1-\alpha} < \lambda h\,(\textbf{i.e. } \tilde{x}_{k+1} = 0) \\ \mathrm{sgn}(x_k) & \textbf{if } |x_k|^{1-\alpha} \geq \lambda h\,(\textbf{i.e. } \tilde{x}_{k+1} \neq 0) \end{cases} \tag{9}$$

Note that, (9) is not relevant when $h = 0$ or $\alpha = 1$. Figure

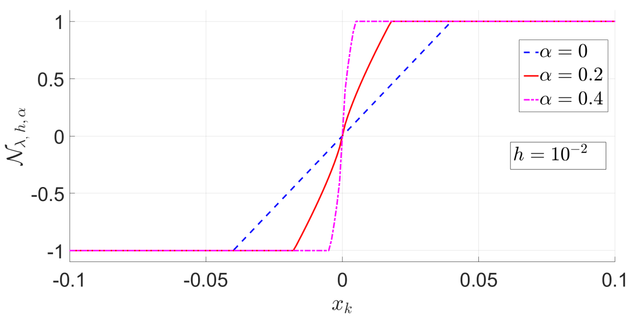

Fig. 2. Examples of representation of $\mathcal{N}^{SI}_{\lambda,h,\alpha}$ versus α.

2 illustrates the representation $\mathcal{N}^{SI}_{\lambda,h,\alpha}$ for $\lambda = 4$, $h = 10^{-2}$ and several values of α.

Remark 3. if $\lambda = 1$ and $\alpha = 0$, then $\mathcal{N}^{SI}_{1,\,h,\,0} = \mathcal{N}_{1,\,h}$ that corresponds to the original definition proposed in (Acary and Brogliato, 2010). In this case, the proposed semi-implicit projector (9) becomes the implicit projector (5).

Theorem 4. For $h > 0$, the closed loop system, composed of the system (1) under the homogeneous control based on semi-implicit Euler discretization (8) action, reads as

$$x_{k+1} = x_k + h(p_{k+1} - \lambda\,|x_k|^{\alpha}\mathcal{N}^{SI}_{\lambda,\,h,\,\alpha}) \tag{10}$$

and converges in finite-time to 0 without perturbation (p_{k+1}), and converges in finite-time to hp_{k+1} in case of perturbation p_{k+1}. ■

Proof. Consider the following candidate Lyapunov function $V = |x|$ and compute $\Delta V(x_k) := |x_{k+1}| - |x_k|$.
For $p = 0$ and an exact discretization, system (10) becomes

$$x_{k+1} = x_k - h\lambda|x_k|^{\alpha}\mathcal{N}^{SI}_{\lambda,h,\,\alpha}$$

then

- if $|x_k|^{1-\alpha} \geq h\,\lambda$ then $\Delta V = -h\lambda|x_k|^{\alpha} < 0$;
- if $|x_k|^{1-\alpha} < h\,\lambda$ then $\Delta V = -V(x_k)$, which is strictly negative for $x_k \neq 0$; given the projector $\mathcal{N}^{SI}_{\lambda,h,\alpha}$, $V(x_{k+1}) = 0$ that implies that $x_{k+1} = 0$.

It follows that, for $p = 0$, the closed loop system converges in finite time to zero.
For $p \neq 0$ and Euler discretization, in a similar way, it can be shown that $\exists\, k_1 > 0$ such that $\forall k > k_1$, $x_{k+1} = hp_k$. ■

4.2 Observer design

Given system (1), an homogeneous-based semi-implicit observer is built on the same principle as the homogeneous-based semi-implicit control; the projector aims to reconstruct the estimated state $\hat{x}$ from the error $e_k = x_k - \hat{x}_k$ including the perturbation. The proposed semi-implicit observer reads as

$$\hat{x}_{k+1} = \hat{x}_k + h(\lambda_o|e_k|^{\alpha_o}\mathcal{N}^{SI}_{\lambda_o,\,h,\,\alpha_o} + u^{SI}_{k+1}) \tag{11}$$

where $\lambda_o > 0$ and $\alpha_o \in [0,\,1[$ being constant tuning parameters. The projector $\mathcal{N}^{SI}_{\lambda_o,\,h,\,\alpha_o}$ is thus defined in a similar way than (9) by replacing λ by λ_o and α by α_o. Then, the discrete dynamics of the observation error reads as

$$e_{k+1} = e_k + h(p_k - \lambda_o|e_k|^{\alpha_o}\mathcal{N}^{SI}_{\lambda_o,\,h,\,\alpha_o}) \tag{12}$$

Equation (12) is similar to (10) considering x_k instead of e_k, that gives the following Corollary.

Corollary 5. The estimation error e_{k+1} with the dynamics (12), converges in finite-time

- to zero when system (1) is perturbation-free ($p = 0$) and exact discretization;
- to hp_k when $p \neq 0$ and Euler discretization.

■

5. SEMI-IMPLICIT EULER DISCRETIZATION FOR OBSERVER-BASED CONTROL

In this section, the observer (11) is included in the closed loop driven by the proposed homogeneous-based semi-implicit control (10). This observer-based control scheme allows reducing the control effort in order to reduce the perturbation effect. This is done by introducing a term close to the perturbation value thanks to the estimation error on x_k. From Corollary 5, one has

$$e_{k+1} = hp_{k+1}, \tag{13}$$

that gives

$$p_{k+1} = \frac{e_{k+1}}{h}$$

However, due to the causality principle and the presence of e_{k+1}, it is not possible to use this information in the control. Then, states

$$p_k = \frac{e_k}{h} \tag{14}$$

Now, let us compute the error between system (10) and observer (11)

$$e_{k+1} = e_k + hp_{k+1} - h(\lambda_o |e_k|^{\alpha_o} \mathcal{N}^{SI}_{\lambda_o,\, h,\, \alpha_o}). \tag{15}$$

From (15), it can be seen that an error due to the perturbation on hp_{k+1} occurs. In order to cancel this error, the control (10) is modified to include the information coming from the observer, which leads to the *semi-implicit discretized homogeneous observer-based* control as

$$\begin{cases} \hat{x}_{k+1} = \hat{x}_k + h(\lambda_o |e_k|^{\alpha_o} \mathcal{N}^{SI}_{\lambda_o,\, h,\, \alpha_o} + \bar{u}^{SI}_{k+1}) \\ e_{k+1} = e_k - h(\lambda_o |e_k|^{\alpha_o} \mathcal{N}^{SI}_{\lambda_o,\, h,\, \alpha_o}) + \; h(p_{k+1} - p_k) \\ \bar{u}^{SI}_{k+1} = -\lambda \, |x_k|^{\alpha} \mathcal{N}^{SI}_{\lambda,\, h,\, \alpha} - \dfrac{e_k}{h} \end{cases} \tag{16}$$

where the term $\frac{e_k}{h}$ comes from (14). Therefore, the observer-based control reads as a difference between the control projector $\mathcal{N}^{SI}_{\lambda,\, h,\, \alpha}$ and the observer projector $\mathcal{N}^{SI}_{\lambda_o,\, h,\, \alpha_o}$ such as

$$\bar{u}^{SI}_{k+1} = -\lambda \, |x_k|^{\alpha} \mathcal{N}^{SI}_{\lambda,\, h,\, \alpha} + \lambda_o \, |e_k|^{\alpha_o} \mathcal{N}^{SI}_{\lambda_o,\, h,\, \alpha_o} \tag{17}$$

From this observation-based control (17), the following theorem is obtained

Theorem 6. The closed loop system, composed of the system (1) controlled by the observer-based control (17), and for which the dynamics reads as

$$x_{k+1} = x_k + h\lambda \, |x_k|^{\alpha} \mathcal{N}^{SI}_{\lambda,\, h,\, \alpha}(x_k) - h(p_{k+1} - p_k) \tag{18}$$

converges in a set bounded by $|h(p_{k+1} - p_k)|$. ■

Proof. Equation (17) shows that the observation error is not function of the input and converges in finite time to a set bounded by $|p_k|$, then from (18) x_k converges in a set bounded by $|h(p_{k+1} - p_k)|$. If $p(t)$ is constant during the sampling-time h, which means that $p_{k+1} = p_k$, then from (18), x_k converges to zero in finite time. ■

6. NUMERICAL RESULTS

In this section, the well founded properties of the proposed observer-based homogeneous semi-implicit control are illustrated through numerical examples. Consider the system (1) with the discrete controller (17) applied such that $h = 10^{-3}$ s, and $x(0) = 0.45$. To ensure a faster dynamic of the observer than the control, consider $\lambda_o \gg \lambda$, set $\lambda = 1$ and $\lambda_o = 6$ and set $\alpha_o = \alpha$.

6.1 Control versus piecewise constant perturbation

Properties of observer-based explicit/semi-implicit controls are compared for different values for α and perturbation p defined as the following

$$\begin{array}{ll} 0 \le t < 3, & p(t) = 0 \\ 3 \le t < 6, & p(t) = 0.1 \\ 6 \le t < 9, & p(t) = -0.2 \end{array}$$

Figures 3 and 4 respectively display the results for the homogeneous semi-implicit control and the estimation error obtained by the observer. Figure 5 gives a focus

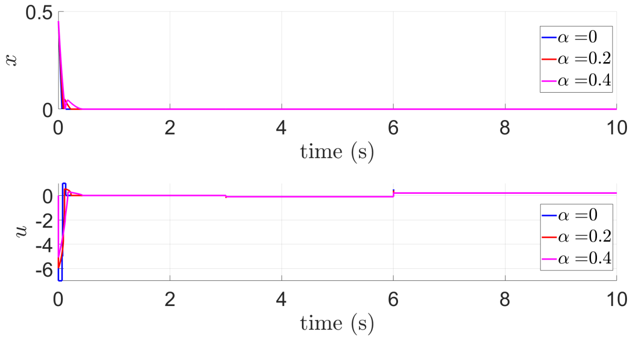

Fig. 3. **Observer-based semi-implicit control - Piecewise perturbation.** State variable x (top) and control input u (bottom) versus time (s), for different values of α.

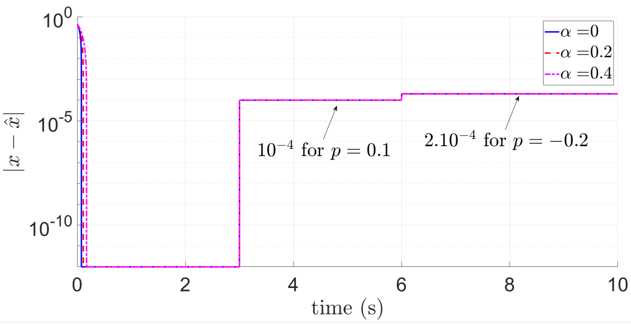

Fig. 4. **Observer-based semi-implicit control - Piecewise perturbation.** Estimation error $|x - \hat{x}|$ versus time (s), for different values of α.

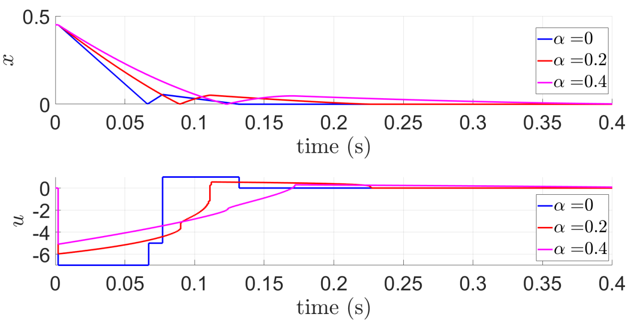

Fig. 5. **Observer-based semi-implicit control - Piecewise perturbation - Focus on transient.** State variable x (top) and control input u (bottom) versus time (s), for different values of α.

on the initial transient of the control. It can be concluded that the closed loop system is efficient, and that the choice of α influences the response-time. The estimation solution is also efficient. The performances of the proposed semi-implicit observer-based control (Table 1) are compared with those of explicit observer-based control (Table 2). From these tables, it can be seen: that the value of this the corresponding values of the error $\varepsilon = |x - \hat{x}|$, the variance and the energy picture [3] of u; concerning the semi-implicit, this latter informs on the chattering quantity and the energy consumption: the range of the variance of u (Table 1) shows that the corresponding control effort is minimized compared to the explicit control solution (Table 2) and the accuracy is improved.

α	Var_u	$\mathfrak{E}_u$	$\lvert\varepsilon\rvert_{p=0}$	$\lvert\varepsilon\rvert_{p=0.1}$	$\lvert\varepsilon\rvert_{p=-0.2}$
0	0	0.1602	$< 10^{-8}$	10^{-4}	2.10^{-4}
0.2	0	0.1602	$< 10^{-8}$	10^{-4}	2.10^{-4}
0.4	0	0.1602	$< 10^{-8}$	10^{-4}	2.10^{-4}

Table 1. Control input and estimation error properties obtained by the observer-based semi-implicit control (piecewise constant perturbation).

α	Var_u	$\mathfrak{E}_u$	$\lvert\varepsilon\rvert_{p=0}$	$\lvert\varepsilon\rvert_{p=0.1}$	$\lvert\varepsilon\rvert_{p=-0.2}$
0	3201	4	10^{-3}	10^{-3}	$1.2.10^{-3}$
0.2	0.2	0.16	$7.4.10^{-5}$	1.10^{-5}	$3.2.10^{-4}$
0.4	0.2	0.16	$3.1.10^{-6}$	$3.1.10^{-3}$	$1.7.10^{-2}$

Table 2. Control input and estimation error properties obtained by the observer-based explicit control (piecewise constant perturbation).

Finally, Figures 6 and 7 present the results by using the explicit version of both the homogeneous controller and observer. Clearly, the implicit approach allows to get a more accurate closed loop system with less chattering.

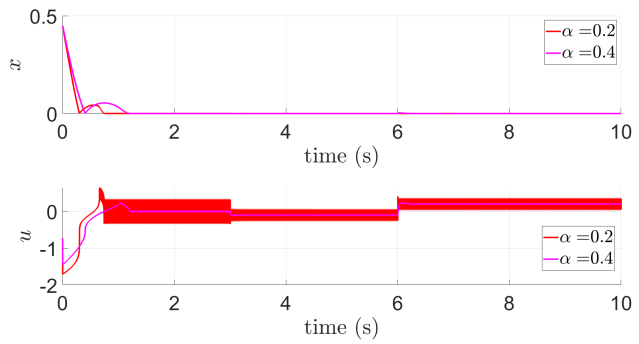

Fig. 6. **Observer-based explicit control - Piecewise perturbation.** State variable x (top) and control input u (bottom) versus time (s), for different values of α.

6.2 Control versus sine perturbation

Properties of observer-based explicit/semi-implicit controls are compared for different α versus the perturbation

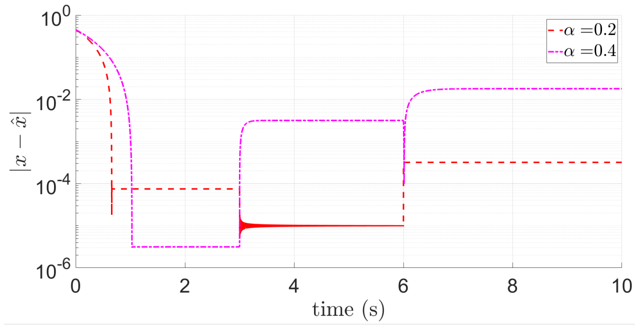

Fig. 7. **Observer-based explicit control - Piecewise perturbation.** Estimation error $|x - \hat{x}|$ versus time (s), for different values of α.

$p(t) = 0.3\sin(2t)$. The perturbation frequency is chosen such that the Shannon's theorem is satisfied. Figures 6 and 7 present respectively the simulation results for the observer-based implicit control and the evolution of the error between the observer and the control. Figures 10-

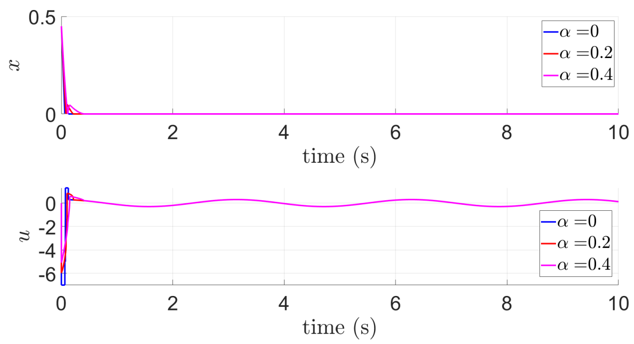

Fig. 8. **Observer-based semi-implicit control - Sine perturbation.** State variable x (top) and control input u (bottom) versus time (s), for different values of α.

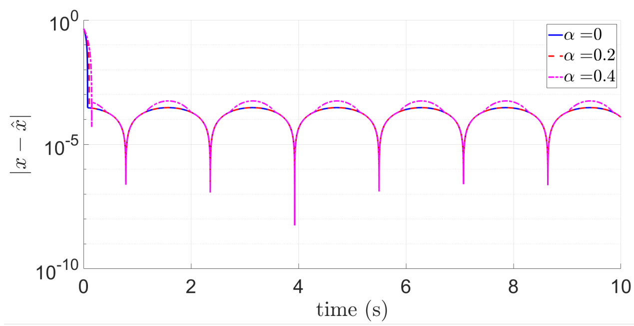

Fig. 9. **Observer-based semi-implicit control - Sine perturbation.** Estimation error $|x - \hat{x}|$ versus time (s), for different values of α.

11 present respectively the results obtained by the semi-implicit observer-based control, and the estimation error of x, for different values of α. Compared to the homogeneous explicit observer-based control (Figures 6-10), the proposed homogeneous semi-implicit observed-based control is chattering-free and shows a good perturbation rejection.

[3] The variance of u is given by $Var_u = \sum_i |u_{k+1} - u_k|$ and the "energy" is given by $\mathfrak{E}_u = h\sum_k (u_k)^2$.

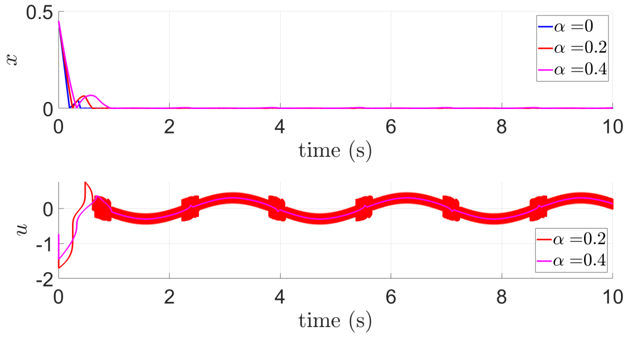

Fig. 10. **Observer-based explicit control - Sine perturbation.** State variable x (top) and control input u (bottom) versus time (s), for different values of α.

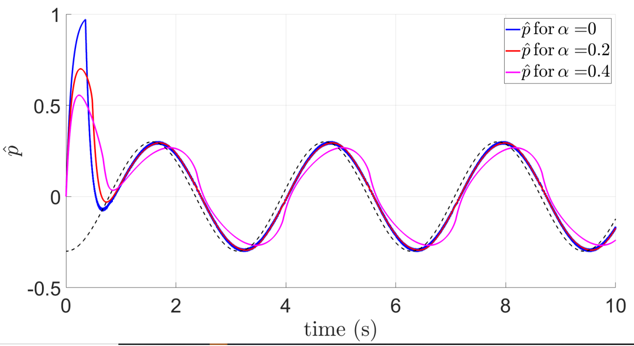

Fig. 11. **Observer-based explicit control - Sine perturbation.** Estimation error $|x - \hat{x}|$ versus time (s), for different values of α.

7. CONCLUSION

This work has investigated the use of semi-implicit discretization approach for the control and observation of perturbed systems. Having reviewed the main properties of the implicit projector based control, homogeneous semi-implicit discretization has been introduced to control and observe perturbed systems. Finally, an homogeneous observed-based semi-implicit control is proposed.

Future works include investigations of second order perturbed system as well as experimental validations on a pneumatic test-bed.

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