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State Estimation in the Presence of Intermittent Actuator Faults

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Abstract: The problem of intermittent, random actuator faults is important in many applications, such as in networked systems, in which there may be intermittent losses of communication between the actuators and the plant. However, state estimation of such systems is rarely addressed, with the majority of the work focusing on fault-tolerant control. In this work, the Kalman filter is modified for state estimation of systems with intermittent actuator faults when the fault rate is known. The proposed estimator is then extended to the case when the actuator fault rate is unknown using the multiple model estimation algorithm. In addition, a sketch of a proof of convergence for this technique is provided. Several simulations involving a DC motor that experiences random actuator faults demonstrate the effectiveness of the proposed techniques.

Keywords: Actuator failures, Parameter estimation, Estimation algorithms, Stochastic parameters, State estimation

1. INTRODUCTION

Many of the classic state estimation and control techniques assume that all measurements contain some state information, and actuators respond perfectly throughout operation. However, sensors can randomly drop or delay measurements, corrupt the measurement data, or fail completely. Likewise, actuators can intermittently lose communication with the system or fail in a variety of ways. The development of state estimation techniques in the presence of various types of sensor and actuator faults attempts to address these issues.

State estimation in the presence of a variety of sensor faults is addressed in work such as Nahi (1969), Hounkpevi and Yaz (2006), Liang et al. (2011), Wang et al. (2012a), and Gao et al. (2008) among others. These works cover a wide range of system types and sensor faults, including random faults with either known or unknown statistics.

The goal of many early investigations into systems with actuator failures is simply to identify the failure mode of the actuator, such as in Lane and Maybeck (1994) and Menke and Maybeck (1995). More recent investigations attempt to gain more information about the actuator failure, as in fault estimation, rather than simply identifying the failure mode. In Jiong-Sang Yee et al. (2002), an estimator is designed for systems in which multiple, simultaneous, and abrupt actuator failures can occur. The proposed estimator identifies the ineffectiveness factor of the actuator, which is included in the control to compensate for the actuator failure. In these works, intermittent actuator faults are not addressed, and it is assumed that the system states can be measured.

Systems with intermittent actuator faults are often considered for networked or cooperative systems. However, most of this work focuses on the control of such systems. An interesting application to the control of continuous-time networked systems appears in Tian et al. (2010). This work considers intermittent, random actuator faults with a known distribution. The authors propose a controller that is designed using Lyapunov techniques, where the observer gains are found by solving a set of LMIs. However, estimation of the system states and the fault distributions are not considered. Another application to networked systems appears in Peng et al. (2010). The authors develop a robust state feedback control that is proven to be stable in the sense of Lyapunov. The networked system model includes plant uncertainty of a known form, intermittent, random networked communication faults, and intermittent, random actuator faults. The communication faults and the actuator faults are of a known distribution, and all of their statistics are known. Furthermore, the authors assume all of the system states are directly measurable, a common assumption in networked systems. It is shown that as the probability of actuator faults increases, the length of the permitted time delay to maintain stability decreases. Random actuator faults in a bilinear system are considered in Wang et al. (2012b), where it is assumed that each of the actuators has a known fault probability. The authors propose a robust control that satisfies mixed general performance criteria and is found by solving a state dependent LMI at each time step. The performance of the control is verified by simulation of the inverted pendulum on a cart.

While a large portion of the literature that addresses the actuator fault problem focuses on the control of

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such systems, systems with intermittent, random actuator faults are considered less frequently than systems with true actuator failures. Furthermore, the state estimation problem for such systems is rarely addressed.

In this work, a modified Kalman filter is proposed for state estimation in the presence of random actuator faults. Section 2 presents the derivation of the modified Kalman filter for systems with random actuator faults with a known fault rate. Section 3 extends the modified Kalman filter to systems with random actuator faults with an unknown fault rate using the multiple model estimation algorithm, and a sketch of a proof of convergence is presented. Simulations using a DC motor are presented in Section 4 that demonstrate the effectiveness of the proposed techniques. The paper concludes in Section 5.

2. STATE ESTIMATION WITH ACTUATOR FAULTS WITH KNOWN STATISTICS

2.1 Problem Formulation

Consider a linear time varying stochastic system (1) in which the set of actuators randomly fault with some constant known rate. No input is applied during an actuator fault, and the input is applied as expected when no fault is present. The probability of the set of actuators faulting at time k is constant, and each fault is independent of subsequent faults.

$$\begin{aligned} x_{k+1} &= A_k x_k + \gamma_k B_k u_k + F_k v_k \\ y_k &= C_k x_k + \gamma_k D_k u_k + G_k w_k \end{aligned} \tag{1}$$

Here, $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^q$ is the input vector, and $y_k \in \mathbb{R}^p$ is the measurement vector. The system noise vector $v_k \in \mathbb{R}^n$, the measurement noise vector $w_k \in \mathbb{R}^p$, and the initial state value x_0 are independent white random variables with Gaussian densities:

$$\begin{bmatrix} x_0 \\ v_k \\ w_k \end{bmatrix} \sim N\left(\begin{bmatrix} \bar{x}_0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} X_0 & 0 & 0 \\ 0 & V_k & 0 \\ 0 & 0 & W_k \end{bmatrix} \right) \tag{2}$$

Similarly, a fault at time k is modeled as a scalar Bernoulli random variable γ_k with a value of 0 or 1:

$$\gamma_k \sim B(\xi, (1-\xi)\xi) \tag{3}$$

where ξ represents the constant success rate of the actuators; conversely, $1-\xi$ represents the fault rate of the actuators. Additionally, γ_k is ergodic and independent of x_k, v_k, and w_k.

2.2 Modified Kalman Filter Design

A modified Kalman filter (MKF) is derived to estimate the states of the system in (1) using the information about the fault rate of the actuator. Assume the estimator has the following form, which will be shown to be an unbiased one:

$$\hat{x}_{k+1} = A_k \hat{x}_k + \xi B_k u_k + K_k^o (y_k - C_k \hat{x}_k - \xi D_k u_k) \tag{4}$$

where $\hat{x}_0 = \bar{x}_0$ and K_k^o is the estimator gain. The state estimation error $e_k = x_k - \hat{x}_k$ is

$$\begin{aligned} e_{k+1} &= A_k x_k + \gamma_k B_k u_k + F_k v_k - A_k \hat{x}_k - \xi B_k u_k \\ &\quad - K_k^o (y_k - C_k \hat{x}_k - \xi D_k u_k) \\ &= (A_k - K_k^o C_k) e_k + (\gamma_k - \xi)(B_k - K_k^o D_k) u_k \\ &\quad + F_k v_k - K_k^o G_k w_k \end{aligned} \tag{5}$$

The expected value of the error dynamics is calculated using the knowledge that v_k and w_k are zero mean and ξ is the mean of γ_k

$$\begin{aligned} \mathbb{E}[e_{k+1}] &= (A_k - K_k^o C_k)\, \mathbb{E}[e_k] + \mathbb{E}[\gamma_k - \xi](B_k - K_k^o D_k) u_k \\ &\quad + F_k\, \mathbb{E}[v_k] - K_k^o G_k\, \mathbb{E}[w_k] \\ &= (A_k - K_k^o C_k) \bar{e}_k \end{aligned} \tag{6}$$

When $\hat{x}_0 = \bar{x}_0$, the initial error is $\mathbb{E}[e_0] = \bar{e}_0 = 0$, and all following errors are $\mathbb{E}[e_k] = \bar{e}_k = 0$. Therefore, this estimator is unbiased.

The gain of the estimator K_k^o is chosen to minimize the second moment of the estimation error (equivalently, the covariance of the states), resulting in a minimum variance estimate. Define the second moment of the estimation error as

$$P_k = \mathbb{E}[e_k e_k^T] = \mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \tag{7}$$

Then by substituting (5) into (7) the progression of P_k is found as

$$\begin{aligned} P_{k+1} = \mathbb{E}[&((A_k - K_k^o C_k) e_k + (\gamma - \xi)(B_k - K_k^o D_k) u_k \\ &+ F_k v_k - K_k^o G_k w_k) \\ &((A_k - K_k^o C_k) e_k + (\gamma - \xi)(B_k - K_k^o D_k) u_k \\ &+ F_k v_k - K_k^o G_k w_k)^T] \end{aligned} \tag{8}$$

After expanding, eliminating each of the independent zero mean cross terms, and combining like terms, (8) simplifies to

$$\begin{aligned} P_{k+1} = {} & A_k P_k A_k^T + F_k V_k F_k^T + \xi(1-\xi) B_k u_k u_k^T B_k^T \\ & - (A_k P_k C_k^T + \xi(1-\xi) B_k u_k u_k^T D_k^T) K_k^{oT} \\ & - K_k^o (C_k P_k A_k^T + \xi(1-\xi) D_k u_k u_k^T B_k^T) \\ & + K_k^o (C_k P_k C_k^T + \xi(1-\xi) D_k u_k u_k^T D_k^T \\ & + G_k W_k G_k^T) K_k^{oT} \end{aligned} \tag{9}$$

The optimal minimum variance estimator is obtained by minimizing (9) through the estimator gain K_k^o by completing the square. The minimal condition is $K_k^o = K_k^*$. After simplification,

$$\begin{aligned} -K_k^* &\left(C_k P_k C_k^T + \xi(1-\xi) D_k u_k u_k^T D_k^T + G_k W_k G_k^T \right) K_k^{oT} \\ &= - \left(A_k P_k C_k^T + \xi(1-\xi) B_k u_k u_k^T D_k^T \right) K_k^{oT} \end{aligned} \tag{10}$$

Matching terms in (10) yields the optional estimator gain:

$$\begin{aligned} K_k^* = {} & \left(A_k P_k C_k^T + \xi(1-\xi) B_k u_k u_k^T D_k^T \right) \left(C_k P_k C_k^T \right. \\ & \left. + \xi(1-\xi) D_k u_k u_k^T D_k^T + G_k W_k G_k^T \right)^{-1} \end{aligned} \tag{11}$$

The second moment of the estimation error is then

$$\begin{aligned} P_{k+1} = {} & A_k P_k A_k^T + F_k V_k F_k^T + \xi(1-\xi) B_k u_k u_k^T B_k^T \\ & - \left(A_k P_k C_k^T + \xi(1-\xi) B_k u_k u_k^T D_k^T \right) \left(C_k P_k C_k^T \right. \\ & \left. + \xi(1-\xi) D_k u_k u_k^T D_k^T + G_k W_k G_k^T \right)^{-1} \left(C_k P_k A_k^T \right. \\ & \left. + \xi(1-\xi) D_k u_k u_k^T B_k^T \right) \end{aligned} \tag{12}$$

The MKF for random actuator faults with known fault rate is defined by the estimator (4), the estimator gain (11), and the state covariance dynamics (12).

3. STATE ESTIMATION WITH ACTUATOR FAULTS WITH UNKNOWN STATISTICS

3.1 Problem Formulation

Consider a linear time invariant stochastic system (13) in which the fault rate of the actuator is unknown. In this

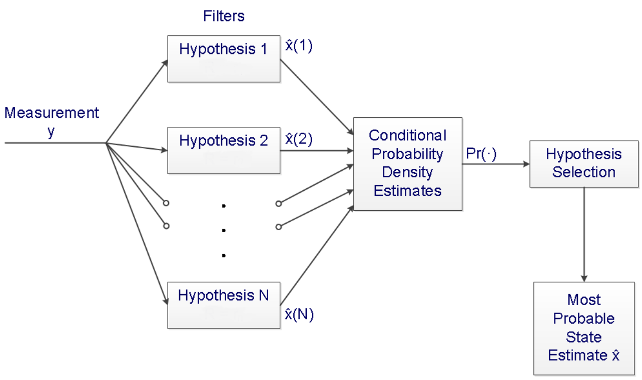

Fig. 1. Block diagram of the multiple model estimation algorithm (Stengel (1994)).

problem, the probability that the set of actuators faults at time k is an unknown constant.

$$\begin{aligned} x_{k+1} &= Ax_k + \gamma_k Bu_k + Fv_k \\ y_k &= Cx_k + \gamma_k Du_k + Gw_k \end{aligned} \tag{13}$$

In this formulation of the problem, $0 \le \xi \le 1$ is an unknown constant.

3.2 Multiple Model Estimation for Estimating Random Actuator Fault Rate

This work combines the MKF proposed in Section 2 with multiple model estimation (MME) to simultaneously estimate the states and the unknown actuator fault rate. The MME algorithm proposed in Lainiotis (1971) can estimate the parameter of a system using knowledge of the states from system measurements and some assumptions about the nature of the unknown parameter. In particular, ξ is constant or slowly varying and lies within the bounded region $0 \le \xi \le 1$. This parameter space is quantized into N possible values, and one of N estimators is designed around each possible parameter value, or hypothesis. The set of hypotheses is represented as $\Xi = \{\xi_1, ..., \xi_i, ..., \xi_N\}$. The posterior probability that $\xi = \xi_i$ given the set of measurement data up to the current time step is calculated for each hypothesis ξ_i, and the posterior probability that approaches 1 is associated with the hypothesis that is closest to the true parameter value. This structure is demonstrated in Fig. 1 (Stengel (1994)).

The posterior probability of each estimator can be found using Bayes' Rule, where $p(\cdot)$ is a probability density function:

$$p(\xi_i|Y_k) = \frac{p(y_k|Y_{k-1}, \xi_i)p(\xi_i|Y_{k-1})}{\sum_{m=1}^{N} p(y_k|Y_{k-1}, \xi_m)p(\xi_m|Y_{k-1})} \tag{14}$$

Here, y_k is the measurement at time k, Y_{k-1} is the set of all measurements through time $k-1$, and ξ_i is the hypothesis to which the i^{th} instance of the estimator is tuned. Equation (14) is calculated recursively, where $p(\xi_i|Y_{k-1})$ is the previous value of the posterior probability. The non-recursive portion can be calculated for Gaussian distributions as

$$p(y_k|Y_{k-1}, \xi_i) = (2\pi)^{-\frac{p}{2}} \left|\Omega_{k|\xi_i}^{-1}\right|^{\frac{1}{2}} e^{-\frac{1}{2}\tilde{y}_{k|\xi_i}^T \Omega_{k|\xi_i}^{-1} \tilde{y}_{k|\xi_i}} \tag{15}$$

where

$$\tilde{y}_{k|\xi_i} = y_k - \hat{y}_{k|\xi_i} \tag{16}$$

is the innovations sequence of the estimator tuned to ξ_i, and

$$\Omega_{k|\xi_i} = \mathbb{E}[\tilde{y}_{k|\xi_i}\tilde{y}_{k|\xi_i}^T] \tag{17}$$

is the design covariance for the estimator tuned to ξ_i.

The state estimate is generated by weighting the individual estimates from the set of conditional filters by its posterior probability:

$$\hat{x}_{k|k-1} = \sum_{i=1}^{N} \hat{x}_{k|k-1,\xi_i} p(\xi_i|Y_k) \tag{18}$$

where $\hat{x}_{k|k-1,\xi_i}$ is the state estimate produced by the estimator tuned to ξ_i.

3.3 Convergence Proof

Proof of convergence of the MME algorithm is provided in Anderson and Moore (2005), and a sketch of how that proof is revised in this application is provided here. It is shown that the modified Kalman filter satisfies all of the convergence criteria.

Criterion 1. A minimum variance unbiased estimator is compatible with MME if the innovations sequence $\tilde{y}_k$ is asymptotically Gaussian and asymptotically wide sense stationary.

Criterion 2. The estimator using MME converges if for $\xi_i \neq \xi_j$, either $\tilde{y}_{k|\xi_i} - \tilde{y}_{k|\xi_j}$ fails to approach zero as $k \to \infty$, or $\Omega_{k|\xi_i} \neq \Omega_{k|\xi_j}$, or both.

Sketch of Proof. First consider *Criterion 1*. It was proven that the MKF is a minimum variance unbiased estimator in the derivation of the filter in the previous section. Next, consider the statistics of the innovations sequence given by

$$\begin{aligned} \tilde{y}_k &= y_k - \hat{y}_k \\ &= C(x_k - \hat{x}_k) + (\gamma_k - \xi)Du_k + Gw_k \end{aligned} \tag{19}$$

Note that, $\tilde{y}_k$ is not Gaussian due to the presence of γ_k in the measurement. However, the limiting distribution may be used since the sequence follows the behavior of the probabilities as $k \to \infty$. The Bernoulli distributed γ_k is a special case of the binomial distribution. Then, by the Central Limit Theorem, γ_k can be approximated as Gaussian as $k \to \infty$. With this approximation, $\tilde{y}_k$ becomes a linear combination of independent Gaussian distributed random variables. Therefore $\tilde{y}_k$ can be considered asymptotically Gaussian.

Consider the statistics of $\tilde{y}_k$ as $k \to \infty$. The final part of *Criterion 1* is that $\tilde{y}_k$ is asymptotically wide sense stationary. The mean of the innovations sequence is

$$\begin{aligned} \mathbb{E}[\tilde{y}_k] &= C\,\mathbb{E}[e_k] + \mathbb{E}[\gamma_k - \xi]Du_k + G\,\mathbb{E}[w_k] \\ &= 0 \end{aligned} \tag{20}$$

which is constant for all k. Since $\tilde{y}_k$ is zero mean, the second moment and the design covariance are equal and are calculated as follows:

$$\begin{aligned} \Omega_k &= \mathbb{E}\left[\tilde{y}_k\tilde{y}_k^T\right] \\ &= CP_kC^T + \xi(1-\xi)Du_ku_k^TD^T + GWG^T \end{aligned} \tag{21}$$

Since (13) is time invariant, (12) approaches a constant value P when $k \to \infty$ and $u_k = u$ is constant (such as in steady state). Then (21) simplifies to

$$\lim_{k\to\infty} \Omega_k = CPC^T + \xi(1-\xi)Duu^TD^T + GWG^T \tag{22}$$

demonstrating that the second moment and design covariance are finite and independent of time as $k \to \infty$. Therefore, the innovations sequence of the MKF is asymptotically wide sense stationary for time invariant systems when the limit of the input is constant, and *Criterion 1* is satisfied, indicating that the MKF is compatible with the MME algorithm.

Next consider the convergence of the MME algorithm assuming *Criterion 2* is satisfied. Without loss of generality, assume the true parameter is given by $\xi = \xi_1$. For $i \neq 1$ define the sequence assuming $p(\xi_i|Y_k)$ is asymptotically Gaussian:

$$\begin{aligned} L_k &= [p(\xi_i|Y_k)][p(\xi_1|Y_k)]^{-1} \\ &= \frac{|\Omega^{-1}_{k|\xi_i}|^{1/2}\exp(-\frac{1}{2}\tilde{y}^T_{k|\xi_i}\Omega^{-1}_{k|\xi_i}\tilde{y}_{k|\xi_i})}{|\Omega^{-1}_{k|\xi_1}|^{1/2}\exp(-\frac{1}{2}\tilde{y}^T_{k|\xi_1}\Omega^{-1}_{k|\xi_1}\tilde{y}_{k|\xi_1})}L_{k-1} \end{aligned} \tag{23}$$

The natural logarithm of the progression of this sequence is evaluated when the estimator is a minimum variance estimator and $\tilde{y}_{k|\xi_1}$ is asymptotically wide sense stationary, resulting in

$$\lim_{n\to\infty} \ln\left(\frac{L_{k+n-1}}{L_{k-1}}\right) = -\alpha \tag{24}$$

for some $\alpha > 0$. Then, from the definition of L_k,

$$\lim_{k\to\infty} \frac{p(\xi_i|Y_k)}{p(\xi_1|Y_k)} = 0 \tag{25}$$

4. SIMULATION EXAMPLES

4.1 Physical System

To test the performance of the MKF for systems with known actuator fault rates and MME using the MKF for systems with unknown actuator fault rates, the discretized model of a per-unitized DC motor with random actuator faults (26) is used.

$$\begin{bmatrix} i_{k+1} \\ \omega_{k+1} \end{bmatrix} = \begin{bmatrix} 0.9802 & -0.0002 \\ 0.0094 & 0.9048 \end{bmatrix} \begin{bmatrix} i_k \\ \omega_k \end{bmatrix} + \gamma_k \begin{bmatrix} 0.0198 \\ 0.0001 \end{bmatrix} v_k \tag{26}$$

Here i_k is the armature current in Amps, ω_k is the rotational speed of the rotor in radians per second, and v_k is the commanded voltage applied to the armature in Volts. The measurement equation is given by

$$y_k = Cx_k + \gamma_k D u_k \tag{27}$$

For the system with known fault rates,

$$C = [0\ 1], \qquad D = 0 \tag{28}$$

indicating that only the rotor speed is measured. For the system with unknown fault rates,

$$C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{29}$$

indicating the applied input is measured in addition to the rotor speed. The inclusion of a nonzero D is necessary in this case to satisfy *Criterion 2* of MME. The sampling period of the system is $T = 0.01$ s. The model has been per-unitized so that rated operation is achieved when a healthy step input is applied. The intermittent actuator fault at each time step is represented by γ_k, which is a Bernoulli random variable that assumes a value of either zero (indicating an actuator fault) or one (indicating successful actuation) with mean $0 \leq \xi \leq 1$.

4.2 Known Actuator Fault Rate Results from the MKF

Seven different values of ξ are considered when ξ is available to the estimator, and 50 independent simulations are performed for each value of ξ. The seven values of ξ are: $\xi = 0.1$, $\xi = 0.3$, $\xi = 0.5$, $\xi = 0.7$, $\xi = 0.75$, $\xi = 0.85$, and $\xi = 0.9$. The peak percent errors (PE) and the mean square errors (MSE) of the estimates of i_k and ω_k averaged over the 50 simulations are presented in Table 1. Although the magnitude of the peak percent errors tends to be large, particularly at low success rates, the mean squared error remains very small. This indicates that the average error in estimation is very small, as expected from the analysis of the mean error. Furthermore, the peak error decreases as the success rate of the actuator increases. This is because fewer interruptions to the system actuation occur, resulting in smaller variations in the state values.

Table 1. Average peak percent errors in state estimation when ξ is known (50 iterations).

	$\max\lvert PE_i\rvert$	$\max\lvert PE_\omega\rvert$	MSE_i	MSE_ω
$\xi = 0.1$	151%	12.6%	1.7e−3	1.00e−3
$\xi = 0.3$	54.7%	4.8%	2.7e−3	1.00e−3
$\xi = 0.5$	29.8%	2.7%	2.9e−3	1.00e−3
$\xi = 0.7$	19.2%	1.8%	2.8e−3	1.00e−3
$\xi = 0.75$	16.4%	1.5%	2.5e−3	1.00e−3
$\xi = 0.85$	12.1%	1.2%	2.1e−3	1.00e−3
$\xi = 0.9$	9.2%	0.88%	1.7e−3	1.00e−3

4.3 Unknown Actuator Fault Rate Results from the MME

The set of simulations in *4.2* are repeated assuming the success rate of the actuator is not available to the estimator. Thus, the modified Kalman filter with multiple model estimation is used to simultaneously estimate the system states and the actuator success rate. The hypothesis set that is used in these simulations is given in (30) and consists of five hypotheses within the parameter space of ξ. Note that two of the values of ξ that are simulated, $\xi = 0.75$ and $\xi = 0.85$, are not within the hypothesis set.

$$\Xi = \{0.1, 0.3, 0.5, 0.7, 0.9\} \tag{30}$$

The peak percent errors and the mean square error of the estimates averaged over the 50 simulations are presented in Table 2. Here, the magnitude of the peak percent errors tends to be smaller than when the actuator success rate is known and decreases as the actuator success rate increases. This estimation improvement is a result of measuring the applied input to the system as in (29), which provides additional state information to the estimator. In addition, the mean squared error remains very small, indicating the average estimation error is very small.

Table 2. Average peak percent errors in state estimation when ξ is unknown (50 iterations).

	$\max\lvert PE_i\rvert$	$\max\lvert PE_\omega\rvert$	MSE_i	MSE_ω
$\xi = 0.1$	14.1%	1.2%	1.00e−3	1.00e−3
$\xi = 0.3$	2.2%	0.19%	1.00e−3	1.00e−3
$\xi = 0.5$	1.2%	0.11%	1.00e−3	1.00e−3
$\xi = 0.7$	0.89%	0.08%	1.00e−3	1.00e−3
$\xi = 0.75$	0.75%	0.08%	1.01e−3	1.00e−3
$\xi = 0.85$	1.8%	0.17%	1.03e−3	1.00e−3
$\xi = 0.9$	0.95%	0.09%	1.01e−3	1.00e−3

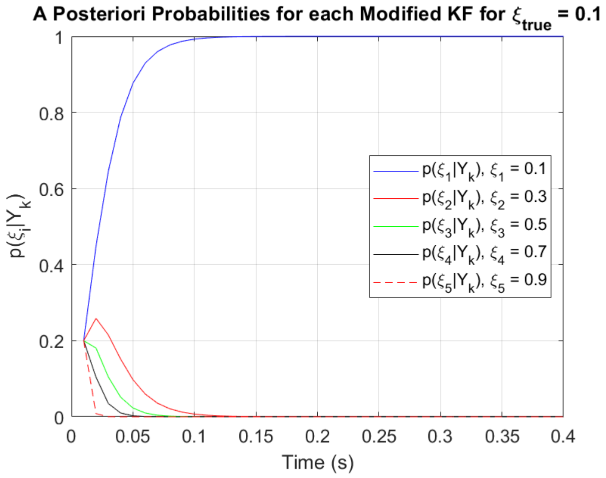

Fig. 2. The posterior probabilities for each hypothesis when $\xi_{true} = 0.1$.

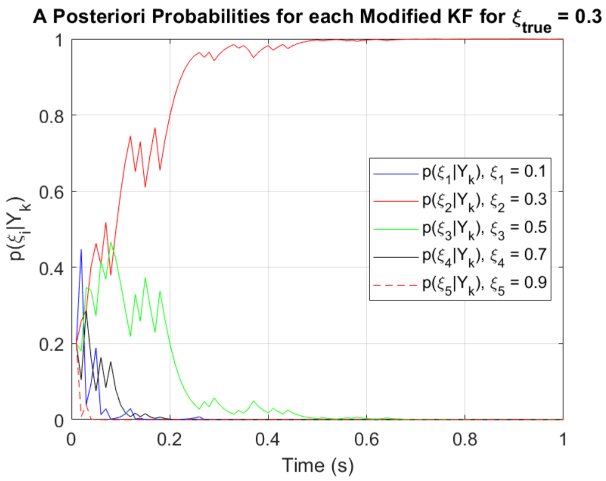

Fig. 3. The posterior probabilities for each hypothesis when $\xi_{true} = 0.3$.

An estimate of ξ can be found by examining the posterior probabilities of each of the hypotheses. Plots of the posterior probabilities when ξ is within the hypothesis set are presented in Fig. 2 through Fig. 6. Note that the posterior probability associated with the hypothesis that matches the value of ξ approaches one, while all of the other posterior probabilities approach zero. Thus, the multiple model algorithm correctly identifies the value of ξ in all of the performed simulations. In addition, the algorithm generally converges in under one second, and a favored hypothesis emerges as quickly as 0.1 to 0.2 seconds into the simulations.

The remaining two simulations consider the estimate of ξ when the true value of ξ is not within the hypothesis set. Plots of the posterior probabilities when ξ is not within the hypothesis set are presented in Fig. 7 and Fig. 8. In this scenario, the multiple model algorithm converges to the hypothesis that is closest to the true value of ξ. The convergence time of the multiple model estimation algorithm is slightly longer because the algorithm needs additional time to eliminate the incorrect hypotheses. Despite the increase in convergence time, the algorithm consistently identifies the hypothesis that is closest to the

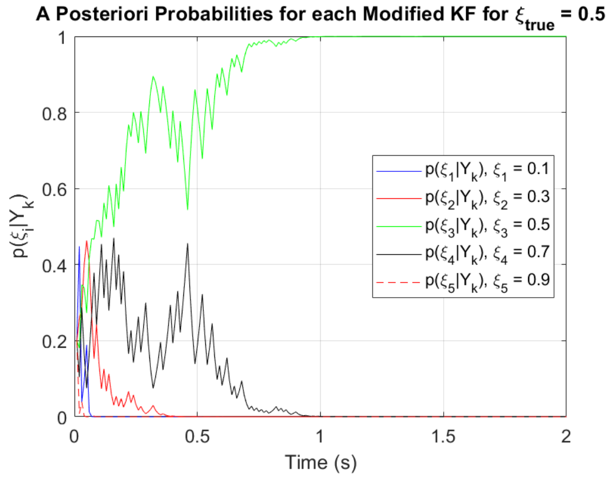

Fig. 4. The posterior probabilities for each hypothesis when $\xi_{true} = 0.5$.

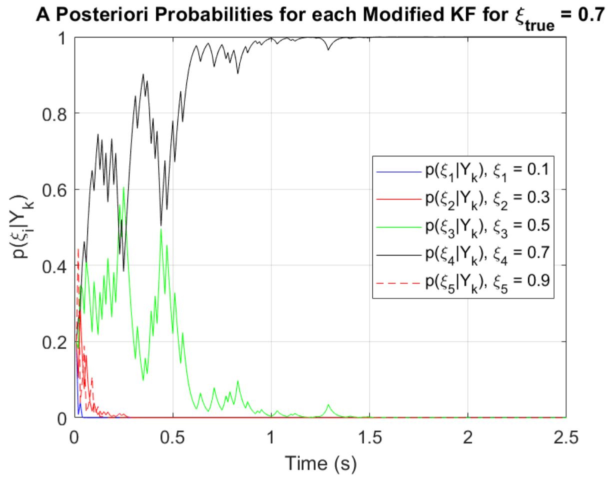

Fig. 5. The posterior probabilities for each hypothesis when $\xi_{true} = 0.7$.

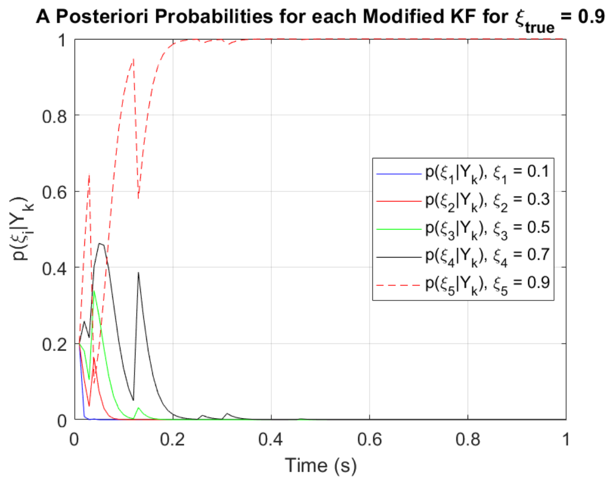

Fig. 6. The posterior probabilities for each hypothesis when $\xi_{true} = 0.9$.

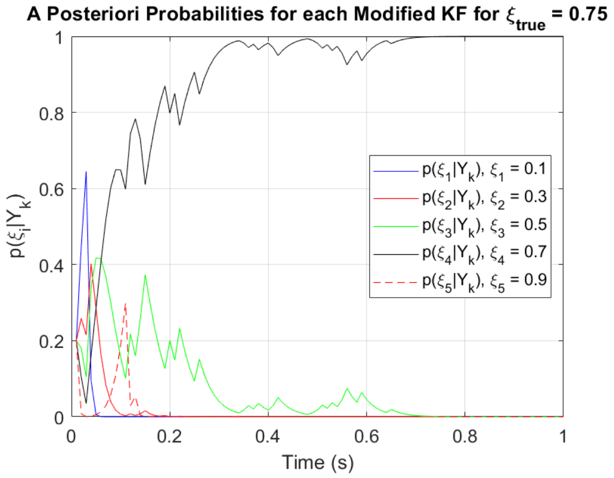

Fig. 7. The posterior probabilities for each hypothesis when $\xi_{true} = 0.75$. Note ξ_{true} is not within the hypothesis set.

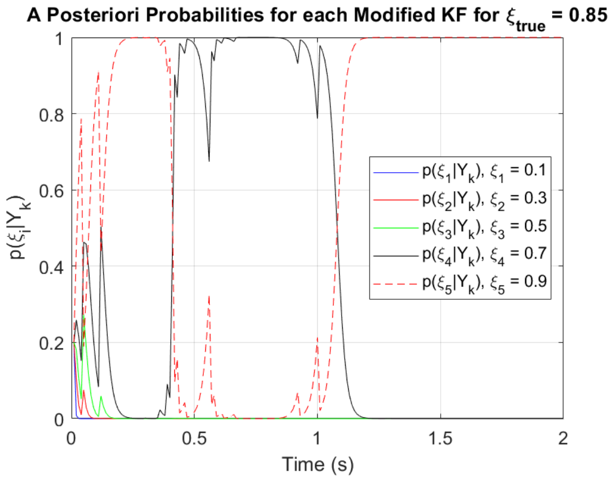

Fig. 8. The posterior probabilities for each hypothesis when $\xi_{true} = 0.85$. Note ξ_{true} is not within the hypothesis set.

true value of ξ. The error in estimating ξ can be decreased by selecting a more finely quantized hypothesis set.

5. CONCLUSION

In this work, a modified Kalman filter is proposed that estimates system states in the presence of intermittent actuator faults with a known fault rate. The proposed filter is extended to systems with intermittent actuator faults when the fault rate is unknown by combining it with the multiple model estimation algorithm. A simulation study is presented that considers a wide range of fault rates of the actuator. It is demonstrated that the modified Kalman filter estimates the system states reasonably well when the fault rate is known. In addition, the peak percent error in the estimate decreases as the success rate of the actuator increases. When the actuator success rate is unknown, the modified Kalman filter using multiple model estimation simultaneously estimates the system states and the success rate of the actuator. Furthermore, the peak percent error in the state estimate is smaller with the addition of the multiple model estimation algorithm due to the additional state information provided by the measurement of the applied input. Future work includes investigating how different control methods (such as closed-loop control) affect convergence of the multiple model estimation algorithm and extending the modified Kalman filter to multiple fault rates for multiple actuators.

REFERENCES

Anderson, B. and Moore, J. (2005). *Optimal Filtering*. Dover Publications, Inc., Mineola, NY.

Gao, Z., Breikin, T., and Wang, H. (2008). Reliable observer-based control against sensor failures for systems with time delays in both state and input. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, 38(5), 1018–1029.

Hounkpevi, F.O. and Yaz, E.E. (2006). Estimation for random sensor failure with unknown failure rate. In *Forty-Fourth Annual Allerton Conference*, 1443–1449.

Jiong-Sang Yee, Bin Jiang, Jian Liang Wang, and Yan Li (2002). Discrete-time actuator fault estimation design for flight control application. In *7th International Conference on Control, Automation, Robotics and Vision, 2002. ICARCV 2002.*, volume 3, 1287–1292 vol.3.

Lainiotis, D. (1971). Optimal adaptive estimation: Structure and parameter adaption. *IEEE Transactions on Automatic Control*, 16(2), 160–170.

Lane, D.W. and Maybeck, P.S. (1994). Multiple model adaptive estimation applied to the lambda urv for failure detection and identification. In *Proceedings of 1994 33rd IEEE Conference on Decision and Control*, volume 1, 678–683 vol.1.

Liang, J., Wang, Z., and Liu, X. (2011). Distributed state estimation for discrete-time sensor networks with randomly varying nonlinearities and missing measurements. *IEEE Transactions on Neural Networks*, 22(3), 486–496.

Menke, T.E. and Maybeck, P.S. (1995). Sensor/actuator failure detection in the vista f-16 by multiple model adaptive estimation. *IEEE Transactions on Aerospace and Electronic Systems*, 31(4), 1218–1229.

Nahi, N. (1969). Optimal recursive estimation with uncertain observation. *IEEE Transactions on Information Theory*, 15(4), 457–462.

Peng, C., Yang, T.C., and Tian, E.G. (2010). Brief paper: Robust fault-tolerant control of networked control systems with stochastic actuator failure. *IET Control Theory Applications*, 4(12), 3003–3011.

Stengel, R.F. (1994). *Optimal Control and Estimation*, 402 – 407. Dover Publications, Inc., Mineola, NY.

Tian, E., Yue, D., and Peng, C. (2010). Brief paper: reliable control for networked control systems with probabilistic sensors and actuators faults. *IET Control Theory Applications*, 4(8), 1478–1488.

Wang, X., Yaz, E.E., Jeong, C.S., and Yaz, Y.I. (2012a). A resilient extended kalman filter for discrete-time nonlinear stochastic systems with sensor failures. In *2012 American Control Conference (ACC)*, 4783–4788.

Wang, X., Yaz, E.E., Long, J., Schneider, S.C., and Yaz, Y.I. (2012b). Robust state dependent control of discrete time nonlinear systems with random actuator faults. In *5th Annual Dynamic Systems and Control Conference (DSCC) joint with 11th Motion and Vibration Conference (MOVIC)*.