Mechanical design optimization for a five-link walking bipedal robot

Ramil Khusainov [*] **Shamil Mamedov** [*] **Sami Sellami** [*]
Alexandr Klimchik [*]

[*] *Innopolis University, Innopolis, Russia (e-mail: {r.khusainov,*
sh.mamedov, s.sellami, a.klimchik}@innopolis.ru).

Abstract: Hybrid zero dynamics is an established theoretical framework that allows to perform dynamic legged locomotion by enforcing virtual constraints through feedback controllers. One of the major functions of the framework is finding virtual constraints that result in the most efficient locomotion in terms of energy expenditure. This paper argues that the problem of reducing such transportation costs requires the optimization of mechanical parameters along with gait parameters. Our study showed that a simultaneous optimization of mechanical and gait parameters results, on average, in threefold reduction in the energy consumption for the whole range of achievable velocities.

Keywords: Optimization, Hybrid Zero Dynamics, gait design, mechanical design, bipedal robot

1. INTRODUCTION

Stable and efficient control of bipedal walking is an intensive and advancing area of research. The goal of the bipedal robots is to perform agile and dynamic locomotion in a variety of terrains not accessible to the wheeled robots. The difficulty arises from the fact that the generation of efficient walking patterns is often computationally expensive and inherently dependent on the robot design. The foundation of optimal gaits and mechanical design is a numerical optimization which is performed either in offline simulations (for gait and mechanical parameters) (Mombaur et al., 2009; Römer et al., 2016) or within real-time optimal control (for gait parameters) (Channon et al., 1990; Hereid et al., 2016). Various metrics have been used as criteria for optimality (Koch et al., 2012), and more often than not they were related to energy consumption.

One of the well known control strategies for underactuated bipedal locomotion is the Hybrid Zero Dynamics (HZD) based control, which stabilizes the system around predefined reference trajectories. It has been successfully implemented on several platforms such as RABBIT (Chevallereau et al., 2003), ERNIE (Yang et al., 2007) and DURUS (Hereid et al., 2016). In the HZD framework, a set of virtual constraints is enforced by means of nonlinear feedback controllers yielding low-dimensional representation of the system. The concepts of virtual constraints and zero dynamics have been used in a broader context of control of underactuated systems, for example Shiriaev et al. (2010), where vitrual constraints and transverse linearization were used to stabilize periodic trajectories of the underactuated mechanical systems on zero manifold.

In HZD framework trajectory planning is posed as a non-linear program (NLP) that searches for the parameters of the virtual constraints while minimizing a given objective function e.g. cost of transportation (COT) (Buss et al., 2016; Hereid and Ames, 2017). However, energy-efficient

locomotion does not depend only on the gait parameters but also on the robot parameters (mass distribution, lengths of the links, moments of inertia): for example, changes in the lengths considerably influence the resulting COT. Thus, the optimization of mechanical parameters should be taken into account in biped design in order to achieve more efficient locomotion. To our best knowledge, so far only few research efforts addressed utilization of HZD framework in optimal mechanical design, for example Römer et al. (2016), where the only parameter that was optimized was the stiffness coefficient of a spring positioned between the femurs.

Current paper aims to develop, within an HZD framework, a methodology for generating the gait and robot parameters for a given average velocity of walking that would enable the most efficient locomotion in terms of energy consumption. The remainder of this paper is structured as follows: in section 2 problem statement is given; a brief description of the hybrid zero dynamics framework is presented in section 3; in section 4, the methodology for simultaneous gait and mechanical parameters optimization is outlined; discussion of obtained results is provided in section 5; finally, in section 6, a conclusion is drawn.

2. PROBLEM STATEMENT

The efficiency of the robot locomotion is inherently dependent on the gait and robot parameters. The former has drawn more attention in the literature than the latter, as it is usually the only way control engineers can affect energy-efficiency of a given robot. With the tremendous progress in computational efficiency, the traditional way of designing robots and then building control systems for them can be improved upon. Due to limited communication between control and mechanical engineers, building a bipedal robot might require designing and testing several prototypes that might result in costs and project time increase. We propose to use simulations in order to simultaneously search for

gait and mechanical parameters, which will minimize the COT. Hence no need for several prototypes.

The problem of finding optimal gait parameters in the context of HZD has been covered in many papers (Westervelt et al., 2003; Hereid et al., 2016), therefore we will focus more on mechanical parameters optimization part of the framework. In general, each link of the robot can be described by a single kinematic parameter - the length of the link, l_i, and by ten dynamic parameters:

$$\pi_i = \begin{bmatrix} m_i & m_i r_{i_c}^T & I_{i_{xx}} & I_{i_{xy}} & I_{i_{xz}} & I_{i_{yy}} & I_{i_{yz}} & I_{i_{zz}} \end{bmatrix}^T.$$

The dynamic parameters include the mass of the link m_i, the first moment of inertia $m_i r_{i_c}^T$ (with $r_{i_c} \in \mathbb{R}^3$ being the position of the center of mass (COM) of the link), and the inertia tensor I with respect to the axis of rotation. If the robot is constrained to move in a plane, e.g. in the XY plane, then the number of dynamic parameters is reduced to three: $\pi_{2D_i} = \begin{bmatrix} m_i & m_i r_{i_{c,x}} & I_{i_{zz}} \end{bmatrix}^T$. Overall, for a five-link planar biped shown in Fig. 1, it is required to optimize 20 parameters subject to physical consistency constraints e.g. the center of mass of the link cannot be greater than its length. Searching for optimal parameters in a 20-dimensional space is a challenging endeavour. However, dimensionality of the problem can be reduced if it is reformulated. For that let us make several reasonable assumptions:

(1) links are uniform rods with a given linear density ρ;
(2) motors controlling the tibia are placed on the lower end of the femur (at the knee joint);
(3) motors controlling the femurs are positioned at the hip joint and attached to the torso;
(4) legs are symmetric.

To be able to change the mass, the length and the inertia of the links, we allow an arbitrary additional mass, m_{a_i}, to be attached to each link at a distance a_{m_i}, as well as the assignment of the length of each link, l_i. Thus, the problem is to search for 8 mechanical parameters, $\begin{bmatrix} m_a^T & a_m^T & l^T \end{bmatrix}^T$, also refer to Fig. 2, in addition to gait parameters, that should guarantee the most efficient locomotion.

3. HYBRID ZERO DYNAMICS FRAMEWORK

This section will present the HZD control framework that allows finding stable periodic orbits for underactuated systems with impact. Planar biped walkers are robots that move by alternating two legs in the sagittal plane. In the HZD framework they are modeled as hybrid systems consisting of a set of ordinary differential equations describing the motion when only one leg is in contact with the ground (single support/continuous phase), and an impact event described as a discrete map when the second leg touches the ground (Westervelt et al., 2003).

3.1 Swing phase model

The planar bipedal robot used in this paper consists of 5 links and 4 actuators that control the knees and hips leaving the torso passive (Fig. 1). To model the robot, let $q = \begin{bmatrix} q_1 & \ldots & q_5 \end{bmatrix}^T$ be a set of angular coordinates describing the configuration of the robot during the swing phase.
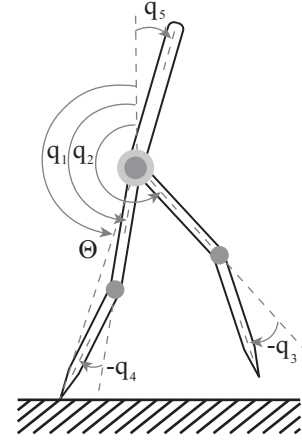

Fig. 1. Schematic representation of the robot with generalized coordinates

Using the Lagrange-Euler formulation, the dynamic model of the continuous phase can be derived as:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu, \tag{1}$$

where $D(q)$ is inertia matrix, $C(q,\dot{q})$ is the matrix of Coriolis and centrifugal forces, $G(q)$ is vector of gravitational forces and B is constant input matrix and q is the vector of generalized coordinates.

The model (1) can be written in a nonlinear state space form

$$\dot{x} = f(x) + g(x)u \tag{2}$$

where $x \in TQ := \{x := (q,\dot{q}) \mid q \in Q, \dot{q} \in \mathbb{R}\}$ and Q is a simply connected, open subset of $\begin{bmatrix} 0 & 2\pi \end{bmatrix}$.

3.2 Impact model

The impact model is a discrete mapping that occurs when the swing foot hits the ground. We assume that the impact is perfectly plastic, thus post impact configurations and velocities must satisfy the following equations: $q^+ = \mathcal{R}q^-$ and

$$\begin{bmatrix} D_e(q^-) & -J_e^T(q^-) \\ J_e(q^-) & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_e^+ \\ \delta F \end{bmatrix} = \begin{bmatrix} D_e(q^-)\dot{q}_e^- \\ 0 \end{bmatrix} \tag{3}$$

where δF is a vector of impulsive forces, $D_e(\cdot)$ is the inertia matrix of the unpinned model, $J_e(\cdot)$ is the extended jacobian of the swing leg end and $\mathcal{R}$ is a matrix expressing the relabeling of the generalized coordinates after impact. The solution of Eq. (3) yields to the reset map Δ (Eq.4) and an equation to compute the impact forces $\delta F = \Delta_F \dot{q}^-$. Thus the overall biped model can be expressed as a nonlinear system with impulsive effect

$$\dot{x} = f(x) + g(x)u, \qquad x \notin S$$
$$x^+ = \Delta \cdot x^-, \qquad x \in S \tag{4}$$

where $S = \{(q,\dot{q}) \in TQ \mid p_2^v(q) = 0, p_2^h(q) > 0\}$ is the impact surface and p_2 is the cartesian position of the nonstance leg.

3.3 Virtual holonomic constraints (VHC)

In the HZD framework, virtual constraints are introduced to synthesize feedback controllers that enable stable and robust locomotion. By designing virtual constraints that are invariant through impact, an invariant submanifold

is created — termed the hybrid zero dynamics surface (Westervelt et al., 2007). Virtual constraints are defined as the difference between the actual and the desired output of the robot.

$$y = q_b - h_d(\theta(q), \alpha)$$

where q_b is a vector of the actuated joints of the biped, and $h_d(\theta(q), \alpha)$ are the desired outputs usually defined in terms of Bézier Polynomials

$$h_d(\alpha, \theta) = \sum_{k=0}^{M} \alpha_k \frac{M!}{k!(M-k)!} s^k (1-s)^{1-k}, \ \ s = \frac{\theta - \theta^+}{\theta^- - \theta^+}.$$

with $\theta(q)$ being a strictly monotonic function of generalized coordinates called space phase variable or motion generator. In this paper, we have selected the angle of the line connecting the hip joint with the stance leg's foot (labeled by Θ in Fig. 1) in the capacity of the motion generator.

3.4 Stability analysis of the Hybrid Zero Dynamics

The zero dynamics can be written in a special set of local coordinates $(\xi_1, \xi_2) = (\theta, \gamma)$

$$\dot{\xi}_1 = k_1(\xi_1)\xi_2, \quad \dot{\xi}_2 = k_2(\xi_1) \quad\quad (5)$$

with γ being the generalized momentum conjugate to the unactuated degree of freedom (q_5). For $\theta^+ \leq \xi_1 \leq \theta^-$ define

$$V_{zero}(\xi_1) = -\int_{\theta^+}^{\xi_1} \frac{k_2(\xi)}{k_1(\xi)} d\xi,$$

so that in the coordinates, $(\zeta_1, \zeta_2) = (\theta, 0.5\gamma^2)$, the Poincaré return map of the HZD, $\chi : S \cap Z \to S \cap Z$, is given by

$$\chi(\zeta_2^-) = \delta_{zero}^2 \xi_2^- - V_{zero}(\theta^-),$$

with domain of definition:

$$\{\xi_2^- > 0 \mid \delta_{zero}^2 \xi_2^- - \max_{\theta^+ \leq \xi_1 \leq \theta^-} V_{zero}(\xi_1) \geq 0\}. \quad (6)$$

If $\delta_{zero} \neq 1$, and $\zeta_2^* = -\dfrac{V_{zero(\theta^-)}}{1 - \delta_{zero}^2}$ is the fixed point of χ then it is an exponentially stable equilibrium of $\zeta_2(k + 1) = \chi(\zeta_2(k))$ if $0 < \delta_{zero} < 1$.

4. OPTIMIZATION

Trajectory planning is an important step in achieving efficient bipedal locomotion. Its purpose is to search for time trajectories or trajectories of the motion generator, such that their predefined cost is minimized, and they are subject to a set of constraints. We define the objective function as the integral of the instantaneous mechanical power delivered by the actuators, divided by the step length and mass multiplied by the gravity constant, called cost of transportation (Koch et al., 2012)

$$J(x) = \frac{1}{p_2^h(q_0^-)m_{tot}g} \int_0^T |\dot{q}_b(t)^T| \, |u(t)| dt \quad (7)$$

In trajectory planning for underactuated mechanical systems, and for bipedal robots in particular, the objective function (Eq.(7)) is minimized by choosing parameters of the virtual constraints, for example the coefficients of the Bezier polynomials. Extending the space of optimization parameter by a set of mechanical parameters gives the

solver more freedom to reduce COT. However, higher parameter space complicates the problem, resulting in more computational time.

In our setup, the vector of extended optimization parameters consists of the gait parameters α, the parameter ξ defining the range of the phase variable θ as $\theta \in [\pi - \xi, \pi + \xi]$, and the design parameters (Fig. 2)

$$x = [\, P \ \xi \ \alpha_2 \ \alpha_3 \ ... \ \alpha_M \,]^T$$

where $P = [\, l_t \ l_f \ a_{m_t} \ a_{m_f} \ a_{m_T} \ m_{a_t} \ m_{a_f} \ m_{a_T} \,]$. Note that the length of the link corresponding to torso is not included in P since it does not influence the dynamics of the robot.
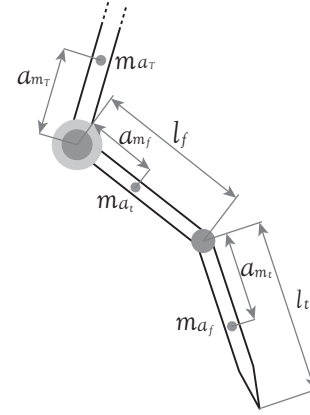

Fig. 2. Schematics of the mechanical parameters being optimized

4.1 Constraints

In classical HZD framework stable walking cannot be achieved without imposing relevant constraints during trajectory planning stage. Moreover, the constrains allow for obtaining desired styles of walking. In general, the constraints can be divided into two categories: nonlinear inequality constraints (NICs), and nonlinear equality constraints (NECs). The NICs, $h(x) \leq 0$, must be satisfied during the whole single support phase. They can be summarized as follows:

- ground reaction force experienced by the stance leg
$$h_1(x) = -F_1^N, \quad h_2(x) = |F_1^T| - \mu_s F_1^N;$$
- the minimum torso heights to achieve the desired walking style enforced by limiting the generalized coordinates
$$h_3(x) = |q_5| - \frac{\pi}{4};$$
- knee hyper-extensions and folding constraints
$$h_4(x) = q_3, \quad h_5(x) = -q_3 - \frac{\pi}{2},$$
$$h_6(x) = q_4, \quad h_7(x) = -q_4 - \frac{\pi}{2};$$
- impact forces
$$h_8(x) = -\delta F^N, \quad h_9(x) = |\delta F^T| - \mu_s \delta F^N;$$
- positivity of the post impact velocity of the swing leg
$$h_{10}(x) = -J_e(q^-)q_e^+;$$
- the stability of the fixed point if the Poincare map
$$h_{11}(x) = \delta_{zero}^2 - 1, \quad h_{12}(x) = -\delta_{zero}^2;$$

- minimum swing leg height, that enforces the robot to lift the swing leg instead of dragging it. It is expressed by the virtual barrier of height $p_{2,min}^v$ located in the middle of the step

$$h_{13}(x) = -p_2^v(\theta_c) + p_{2,min}^v, \quad \{\theta_c : p_2^h(\theta_c) = 0\}; \quad (8)$$

- the maximum torque allowed at each joint

$$h_{14:17}(x) = |u_i(x)| - u_{i,max}, \quad 1 \le i \le 4;$$

- the maximum velocities allowed at each joint

$$h_{18:21}(x) = |\dot{q}_i(x)| - \dot{q}_{i,max}, \quad 1 \le i \le 4 \quad (9)$$

The NECs, $g(x) = 0$, enforce:

- the average walking rate

$$g_1(x) = \bar{v} - \frac{p_2^h(q^+)}{T};$$

- the validity of the impact of the swing leg end with the walking surface

$$g_2(x) = p_2^v(q^-), \ g_3(x) = p_2^v(q^+);$$

- the post impact torque continuity

$$g_{3:6}(x) = u(\theta^+) - \mathcal{R}u(\theta^-), \ u = [\, u_1 \ ... \ u_4 \,]^T.$$

4.2 Implementation

The algorithm for simultaneous optimization of gait and mechanical parameters was implemented in *MATLAB* environment. First, the equations of motion for both pinned and unpinned models were computed. Then, assuming invariance of the hybrid zero manifold, the equations of the low-dimensional representation of the system (HZD, Eq.(5)) were also calculated along with the discrete map equations. All the equations were obtained using *Symbolic Math Toolbox^{TM}*. Apart from symbolic computations, the toolbox allows to generate functions from symbolic expressions with the resulting code being optimized for better performance. This feature was extensively used to generate functions for $J(x)$, $g(x)$ and $h(x)$.

After the preliminary part of generating code for evaluating objective function and constraints, the main part of the algorithm, namely the optimization, starts. The workflow of the optimization procedure is described in Fig.(3). Firstly, the coefficients of the Bézier polynomials, α, are randomly generated within a predefined interval but they are constrained to satisfy the hybrid zero dynamics stability conditions given by (6) (otherwise some of the constraints are not defined at initial point as a result the optimization crushes). The parameter defining the θ range, ξ, and the mechanical parameters P are also randomly chosen from a bounded interval. After that, *Matlab* NLP solver *fmincon* is called with initial guess x_{0_i}. Jacobian and Hessian of neither objective function nor constraints were provided. Instead they were computed by forward finite difference method.

If at the current iteration the local minimum is found, then the final cost (J_i) is compared to the minimum cost of previous iterations (J_{min}). The parameters are saved if J_i is less than J_{min}, otherwise they are discarded. By means of this approach, we aim at finding the global minimum within a feasible set.

5. RESULTS AND DISCUSSION

Due to the non-convex nature of the optimization problem, for each velocity in the range $0.1 - 2$ m/s, the optimization
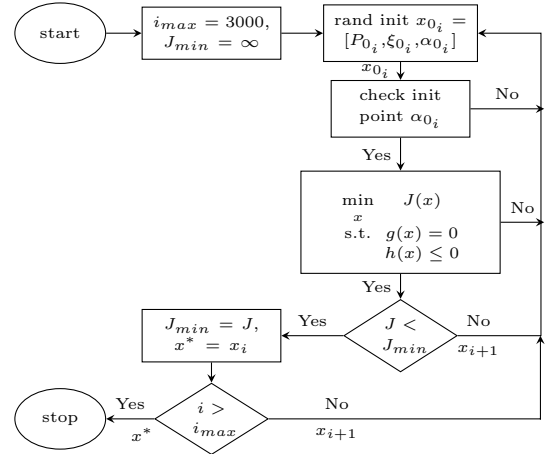

Fig. 3. Block diagram of the algorithm for the simultaneous gait and robot parameters optimization

was run 3000 times with parameters randomly initialized within a range given in Table 1. From all the points, usually only 15% converged to a local minimum. Among them the parameters corresponding to the lowest cost were selected for each velocity, and analysed. The following statements sum up the main results of the simultaneous gait and mechanical parameters optimization:

- the robot tends to have high vertical position of the center of mass (COM) while keeping the step length relatively small. In other words, parameters l_t and l_f grow infinitely large if not bounded. To explain this result, let us make qualitative analysis using a simpler model of the inverted pendulum with length L and mass $2m$. In
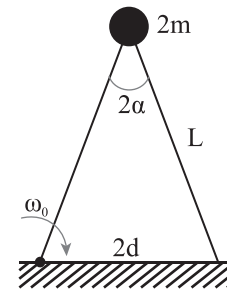

Fig. 4. Robot modeled as inverted pendulum

addition, let $2d$ be the step length, 2α be the angle between the legs at impact, and ω_0 be the angular velocity of the robot immediately before the impact (Fig. 4). Assuming the impact to be absolutely inelastic, we can find the difference in the energy of the robot before and after the impact

$$\Delta E = \frac{1}{2} 2mL^2\omega_0^2 sin^2(2\alpha) = 4m\omega_0^2 d^2(1 - d^2/L^2).$$

In order to derive the COT, let us make another assumption: the mechanical energy of the robot is lost only during the impact, then

$$J = \frac{\Delta E}{4mgd} = \frac{\omega_0^2 d}{g}(1 - d^2/L^2), \quad (10)$$

where the angular velocity ω_0 corresponding to a certain step duration T can be calculated numerically by integrating the equation of motion of the inverted pendulum -

Table 1. Model parameters.

Parameters		Value	unit
ρ	rod linear density	0.12	kg/m
m_{mot}	motor mass	0.15	kg
m_{sus}	suspension mass	2	kg
l_f	femur length	0.1 - 0.5	m
l_t	tibia length	0.1 - 0.5	m
m_{a_T}	torso additional mass	0.5 - 20	kg
m_{a_f}	femur additional mass	0 - 20	kg
m_{a_t}	tibia additional mass	0 - 20	kg
a_{m_T}	m_{a_T} position	0.05 - 0.7	m
a_{m_f}	m_{a_f} position	0 - 0.5	m
a_{m_t}	m_{a_t} position	0 - 0.5	m
u_{max}	maximum torque	6	N*m
$\dot{q}_{max}$	maximum angular velocity	6	rad/s
$p^v_{2,min}$	barrier height	0.05	m
ξ	range of θ	0.001 - $\pi/3$	rad
g	gravity	9.81	m/s
μ_s	coefficient of static friction	0.6	-
M	degree of Bézier polynomials	6	-
N	grid size	100	-

$\omega_0 = \omega_0(L, d, T)$. Analysis of J shows that the COT tends towards zero as L increases for fixed d and T, as ω_0 in Eq.(10) tends to zero.

To avoid having an infinitely tall biped robot we introduced the upper bounds on the femur and tibia lengths (Table 1). As a result, for all velocities, the optimal values for l_t and l_f are equal to their upper limit.

• The optimal value of the additional mass attached to the tibia, m_{a_t} is equal to zero for the whole velocity range. This result is predictable as the tibia of the swing leg is the fastest moving link and adding additional mass to it increases the mechanical power, and thus increases the COT.
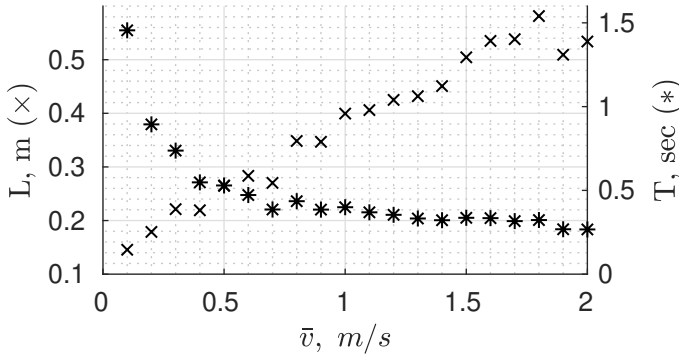

Fig. 5. Relationship between the step length, L, and average walking velocity, $\bar{v}$.

• On average, the step length proportionally increases with the speed, while the step duration inversely proportionally decreases (Fig. 5). This result is apparent from the equation for average velocity - $\bar{v} = L/T$.

• Fig. 6 shows that for low average velocities, the parameter m_{a_f} behaves rather chaotically, however location at which it is added, a_{m_f}, is almost constant and close to zero, meaning that m_{a_f} is added to the hip (Fig. 7). For higher velocities, the same mass m_{a_f} monotonically decreases towards zero, whereas the length a_{m_f} increases. The parameter m_{a_T} increases with $\bar{v}$ up to 0.5 m/s and then decreases approaching small nonzero value. The lo-
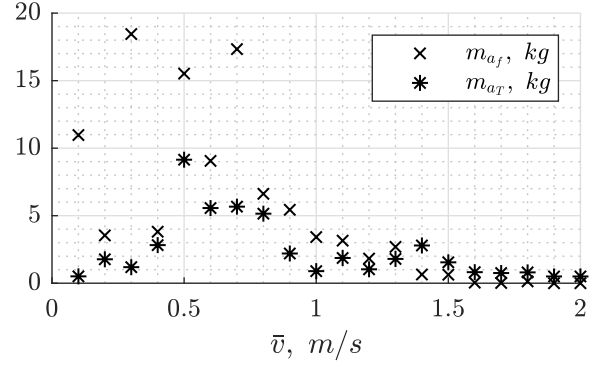

Fig. 6. Relationship between added masses to the hip, m_{a_f} and the torso, m_{a_T} and average walking velocity, $\bar{v}$.
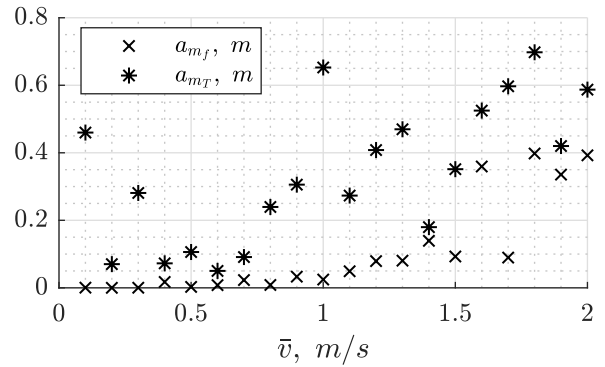

Fig. 7. Relationship between the distance at which the masses are added (a_{m_f}, a_{m_T}) and average walking velocity, $\bar{v}$.

cation of m_{a_T} on average shows the opposite behavior, namely it decreases up to velocities equal to 0.5 m/s and then increases. To sum up these results, for low velocities large amount of mass is added mostly to the hip. As $\bar{v}$ increases, the total amount of additional mass goes down, while a_{m_f} and a_{m_T} move further away from the hip, thus moving the COM of the robot forward.

• For low average velocities a heavy robot weight and constraints on the knee joint torques result in almost a fully straightened stance leg.

• For the swing leg knee joint, the constraint corresponding to joint space velocities (Eq. (9)) is active even for relatively small $\bar{v}$ (0.2 m/s). It occurs because the swing leg tends to fully straighten back after it crosses the barrier given by Eq. (8).

• The NIC on the maximum torques (Eq. (9)) becomes active in the hip joint of the supporting leg only for $\bar{v}$ greater than 1.3 m/s.

• Barrier constraint (Eq. (8)), is active for all the gaits in the whole range of $\bar{v}$. If one removes $h_{13}(x)$ from the list of constraints the robot tends to drag the swing leg.

• As $\bar{v}$ increases, the robot tends to lean the torso forward in order to shift the COM position forward. It especially becomes noticeable for $\bar{v}$ greater than 1 m/s.

Finally, to demonstrate the advantage of simultaneous gait and mechanical parameters optimization over gait optimization only the following comparative analysis was conducted. We took a robot with no additional masses
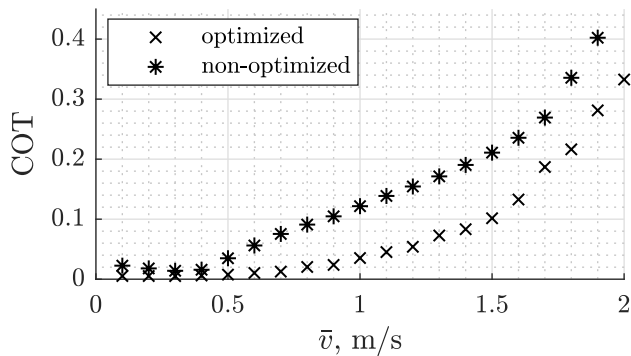
Fig. 8. Relationship between the minimal cost of transportation and average walking velocity, $\bar{v}$, for optimized and non-optimized mechanical parameters.

added to the femur and the tibia, and a small mass (10 g) added to the torso at a distance of 1 cm (in order to avoid the inertia matrix $D(q)$ to become singular), and carried out the trajectory optimization for each velocity in the set $\{0.1, 0.2, \ldots, 2\}$. Then, for the same velocities, we performed simultaneous mechanical design and trajectory optimization. Objective functions for each scenario and each velocity are shown on Fig. 8. Comparison of objective functions substantiates the benefits of using simultaneous optimization, more specifically there is a decrease in the COT by three times on average, for velocities greater than 0.4 m/s. However, for velocities lower than that, simultaneous optimization does not reduce the objective function, in contrast, it leads to its increase. This observation, combined with the difficulty of interpreting results obtained for mechanical parameters (see Fig. 6 and 7) questions the validity of the results for velocities below 0.4 m/s. On the other hand, in the real world average walking velocity of bipedal robots is definitely more than that.

6. CONCLUSION

In this paper, a method for the simultaneous optimization of the gait and mechanical parameters for a five-link planar robot within the hybrid zero dynamics framework was presented. The optimization results demonstrate that, firstly, the optimal mechanical parameters of the robot that yields to the least energy consumption strongly depend on the robot's average velocity of walking. To be more precise, at low velocities, energy-efficient robots should be heavier with the majority of the mass concentrated on the hip. Whereas, for higher velocities, the robot should be lighter, with the COM of the torso being located away from the hip such that forward inclination in the torso allows to shift the COM of the whole robot ahead hence reaching higher velocities and reducing the COT. Secondly, simultaneous optimization of both the gait and mechanical parameters of the robot results in, on average, three times lower COT compared to only gait optimization results.

In future, improvements could be applied to the algorithm. First, the model of the robot can be changed by adding springs in parallel with the motors as to obtain a more energy efficient motion by exploiting the energy conservation properties of the springs. Second, in the optimization, instead of finite difference algorithm for finding gradients,

algorithmic differentiation can be used. Third, the choice of NLP solvers should not be limited by MATLAB's built-in functions, as other solvers may increase the convergence rate. Fourth, the objective function can be modified by adding a measure of robustness to disturbances.

ACKNOWLEDGEMENTS

REFERENCES

Buss, B.G., Hamed, K.A., Griffin, B.A., and Grizzle, J.W. (2016). "Experimental Results for 3D Bipedal Robot Walking Based On Systematic Optimization of Virtual Constraints". *Ame Cont Conf (ACC) Boston, USA.*

Channon, P.H., Hopkins, S.H., and Pham, D.T. (1990). *"Simulation and optimization of gait for a bipedal robot"*, chapter vol.14, pp. 463–467,. Math. Comput. Model.

Chevallereau, C., Abba, G., Aoustin, Y., Plestan, F., Westervelt, E., Wit, C., and Grizzle, J. (2003). "Rabbit: A testbed for advanced control theory". *IEEE Contr. Syst. Mag. vol. 23, no. 5, pp. 57–79.*

Hereid, A., Kolathaya, S., and Ames, A.D. (2016). "Online Optimal Gait Generation for Bipedal Walking Robots using Legendre Pseudospectral Optimization". *IEEE in 5th Conf Decision and Control.*

Hereid, A. and Ames, A.D. (2017). FROST: Fast robot optimization and simulation toolkit. *IEEE International Conference on Intelligent Robots and Systems*, 2017-Septe, 719–726. doi:10.1109/IROS.2017.8202230.

Koch, K.H., Mombaur, K.D., and Souères, P. (2012). Studying the effect of different optimization criteria on humanoid walking motions. In I. Noda, N. Ando, D. Brugali, and J.J. Kuffner (eds.), *Simulation, Modeling, and Programming for Autonomous Robots*, 221–236. Springer Berlin Heidelberg, Berlin, Heidelberg.

Mombaur, K., Scheint, M., and Sobotka, M. (2009). *"Optimal Control and Design of Bipedal Robots with Compliance"*. at-Automatisierungstechnik . no 7. pp 349–359, vol. 57 edition.

Römer, U.J., Kuhs, C., Krause, M.J., and A.Fidlin (2016). "Simultaneous optimization of gait and design parameters for bipedal robots". *IEEE Int Conf on Robot and Auto (ICRA), Stockholm, Sweden.*

Shiriaev, A.S., Freidovich, L.B., and Gusev, S.V. (2010). Transverse linearization for controlled mechanical systems with several passive degrees of freedom. *IEEE Transactions on Automatic Control*, 55(4), 893–906.

Westervelt, E.R., Grizzle, J.W., Chevallereau, C., Choi, J.H., and Morris, B. (2007). *"Feedback control of dynamic bipedal robot locomotion"*. CRC press Boca Raton, 2007.

Westervelt, E.R., Grizzle, J.W., and Koditschek, D.E. (2003). "Hybrid Zero Dynamics of Planar Biped Walkers". *IEEE trans on auto cont, vol. 48, no. 1.*

Yang, T., Westervelt, E.R., Schmiedeler, J.P., and Bockbrader, R.A. (2007). "Design and control of the planar bipedal robot ERNIE". In *in ASME .Int Mech Eng Cong and Exp*, pp. 1217–1223. Seattle, WA.