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Distributed H_-/L_∞ fault detection observer design for linear systems ⋆

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Abstract: This paper studies the distributed fault detection problem for linear time-invariant (LTI) systems with distributed measurement output. A distributed H_-/L_∞ fault detection observer (DFDO) design method is proposed to detect actuator faults of the monitored system in the presence of a bounded process disturbances. The DFDO consists of a network of local fault detection observers, which communicate with their neighbors as prescribed by a given network graph. By using finite-frequency H_- performance, the residual in fault detection is sensitive to fault in the interested frequency-domain. The residual is robust against effects of the external process disturbance by L_∞ analysis. A systematic algorithm for DFDO design is addressed and the residual thresholds are calculated in our distributed fault detection scheme. Finally, we use a numerical simulation to demonstrate the effectiveness of the proposed distributed fault detection approach.

Keywords: Distributed fault detection, linear system observers, finite-frequency, L_∞ analysis.

1. INTRODUCTION

Fault diagnosis has been intensively studied to improve the reliability of modern control systems, see Isermann (2006); Ding (2008); Chen and Patton (2012); Zhang et al. (2012) and the references therein. The method of observer-based fault detection is widely-used among the existing model-based fault diagnosis techniques Xu et al. (2019). A fault detection observer is designed to make the residual have the fault sensitivity and disturbance attenuation ability simultaneously. As a result, H_-/H_∞ fault detection has been intensively studied in the past two decades (Hou and Patton, 1996; Zhong et al., 2003; Li and Yang, 2014; Wang et al., 2017b). The concept of H_-/H_∞ fault detection was first proposed and formulated as a special type of constrained optimisation problem in Hou and Patton (1996). In many practical systems fault signals have fnite-frequency domains, e.g. in the case of incipient fault. Therefore, it is important and practical to design fault detection observers in the fnite-frequency domain. The fault detection observer design problem in fnite-frequency domain was proposed for the first time (Liu et al., 2005). An effective finite-frequency domain H_-/H_∞ fault detection method was proposed for descriptor system in Wang et al. (2017b). The L_∞ performance describes the peak-to-peak gain of systems. Compared with H_∞ performance, the peak-to-peak gain is more reasonable and useful in residual evaluation and calculation (Ding, 2008). The concept of H_-/L_∞ fault detection was first proposed for continuous-time systems in Wang et al. (2017a) and was later extended to the discrete-time case in Han et al. (2018). Based on L_∞ analysis, time-varying thresholds were obtained. However, most of the existing fault detection methods developed up to now assume that measurement outputs are obtained from sensors that are centrally located.

As the size and complexity of systems increase, several practical systems have become large-scale and/or physically output distributed. For these systems, research on decentralized or distributed FDI was carried out in the literature as well (Zhang and Zhang, 2012; Li et al., 2016; Marino et al., 2017). In Huang et al. (1999) fault tolerant decentralized H_∞ control for symmetric composite systems was presented. In Sauter et al. (2006), a decentralized FDI scheme was studied for a network system. A multi-layer distributed FDI scheme was proposed for large-scale systems in Boem et al. (2017). In addition, a distributed fault detection approach for interconnected second-order systems was studied in Shames et al. (2011). The monitored plant discussed in the above literature can be separated into several interconnected subsystems. Each fault filter or observer is designed for the corresponding subsystem. For large-scale systems that do not physically consist of certain subsystems or can not be separated into several interconnected subsystems, distributed fault diagnosis was studied only in very few publications. For a single monitored discrete-time system, a distributed fault diagnosis algorithm was proposed by using average-consensus techniques in Franco et al. (2006).

Motivated by the above, this paper studies the distributed fault detection problem for continuous-time LTI systems

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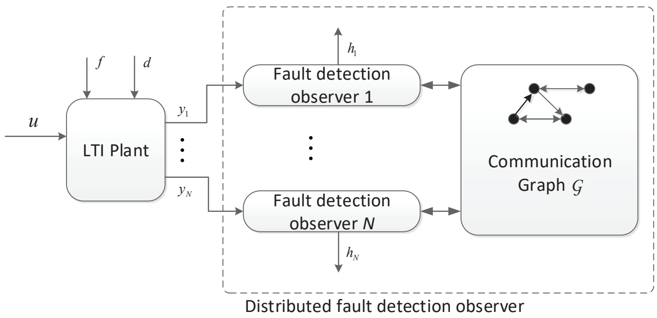

Fig. 1. Framework of distributed fault detection observer

with actuator faults. The measured output of the original plant is physically distributed and the proposed distributed H_-/L_∞ fault detection observer consists of a network of local fault detection observers with a priori given network graph (see Fig. 1 for an illustration). Each local fault detection observer has access to only a portion of the output of the known monitored system, and communicates with its neighboring fault detection observers. H_-/L_∞ criteria are used in distributed fault detection design. The local fault detection observer at each node is designed to generate a residual which is sensitive to low frequency-domain faults and robust against process disturbances. The gain matrices in the DFDO are obtained by solving linear matrix inequalities (LMI's). In addition, the residual threshold calculation is achieved by using L_∞ analysis.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Preliminaries

Notation: For a given matrix M, its transpose is denoted by M^T and M^{-1} denotes its inverse. The symmetric part of a square real matrix M is sometimes denoted by $\mathrm{Sym}(M) := M + M^T$. The rank of the matrix M is denoted by rank M. The identity matrix of dimension N will be denoted by I_N. The vector $\mathbf{1}_N$ denotes the $N \times 1$ column vector comprising of all ones. For a symmetric matrix P, $P > 0$ ($P < 0$) means that P is positive (negative) definite. For a set $\{A_1, A_2, \cdots, A_N\}$ of matrices, we use $\mathrm{diag}\{A_1, A_2, \cdots, A_N\}$ to denote the block diagonal matrix with the A_i's along the diagonal, and the matrix $\begin{bmatrix} A_1^T & A_2^T & \cdots & A_N^T \end{bmatrix}^T$ is denoted by $\mathrm{col}(A_1, A_2, \cdots, A_N)$. The Kronecker product of the matrices M_1 and M_2 is denoted by $M_1 \otimes M_2$. For a signal $x(t) \in \mathbb{R}^n$, its L_∞ norm is defined as $\|x\|_\infty = \sup_{t \geqslant 0} \|x(t)\|$, where $\|x(t)\|$ denotes the Euclidean norm of $x(t)$, i.e. $\|x(t)\| = \sqrt{x^T(t)x(t)}$.

In this paper, a weighted directed graph is denoted by $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$, where $\mathcal{N} = \{1, 2, \cdots, N\}$ is a finite nonempty set of nodes, $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is an edge set of ordered pairs of nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ denotes the adjacency matrix. The (j,i)-th entry a_{ji} is the weight associated with the edge (i,j). We have $a_{ji} \neq 0$ if and only if $(i,j) \in \mathcal{E}$. Otherwise $a_{ji} = 0$. An edge $(i,j) \in \mathcal{E}$ designates that the information flows from node i to node j. A graph is said to be undirected if it has the property that $(i,j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$ for all $i, j \in \mathcal{N}$. We will assume that the graph is simple, i.e., $a_{ii} = 0$ for all $i \in \mathcal{N}$. For an edge (i,j), node i is called the parent node, node j the child node and j is a neighbor of i. A directed path from node i_1 to i_l is a sequence of edges (i_k, i_{k+1}), $k = 1, 2, \cdots, l-1$ in the graph. A directed graph $\mathcal{G}$ is called strongly connected if between any pair of distinct nodes i and j in $\mathcal{G}$, there exists a directed path from i to j, $i, j \in \mathcal{N}$. The Laplacian $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ is defined as $\mathcal{L} := \mathcal{D} - \mathcal{A}$, where the i-th diagonal entry of the diagonal matrix $\mathcal{D}$ is given by $d_i = \sum_{j=1}^{N} a_{ij}$.

2.2 Problem formulation

In this paper, we consider a continuous-time LTI system subject to actuator faults and disturbances represented by

$$\begin{cases} \dot{x} = Ax + Bu + Ff + Ed \\ y = Cx \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^r$ is the input, $f \in \mathbb{R}^q$ is the fault, $d \in \mathbb{R}^l$ is the disturbance, and $y \in \mathbb{R}^m$ is the measurement output. $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times r}, F \in \mathbb{R}^{n \times q}, E \in \mathbb{R}^{n \times l}, C \in \mathbb{R}^{m \times n}$ are constant matrices with appropriate dimensions. We partition the output y as $y = \mathrm{col}(y_1, \cdots, y_N)$, where $y_i \in \mathbb{R}^{m_i}$ and $\sum_{i=1}^{N} m_i = m$. Accordingly, $C = \mathrm{col}(C_1, \cdots, C_N)$ with $C_i \in \mathbb{R}^{m_i \times n}$. Here, the portion $y_i = C_i x \in \mathbb{R}^{m_i}$ is assumed to be the only information that can be acquired by node i in the DFDO.

Assumption 1. The communication graph is a strongly connected directed graph. The pair (C, A) is observable.

Assumption 2. The disturbance d is unknown but bounded, and $\|d\|_\infty$ is bounded by some known constant.

Assumption 3. The fault $f(t)$ belongs to the following low-frequency range

$$\Omega_f = \{\omega : \|\omega\| \leqslant \omega_l\} \tag{2}$$

where ω_l is a known scalar.

Remark 4. Note that Assumption 1 is a basic assumption in distributed estimation and is a sufficient conditions for the existence of a distributed observer (Park and Martins, 2017; Wang and Morse, 2018; Han et al., 2019a). The assumption is necessary to construct a distributed H_-/L_∞ fault detection observer as well in this paper. This paper is interested in faults belonging to the low-frequency range. Hence, the method mainly focuses on low-frequency faults. It is noted that in practice faults are in the low-frequency domain, see for example Zhang et al. (2012). Moreover, the low low-frequency range condition can be easily expended to middle-frequency and high-frequency range conditions.

We will design a H_-/L_∞ DFDO for the system given by (1) with the given communication network. The DFDO will consist of N local fault detection observers, and the local fault detection observer at node i has the following dynamics

$$\begin{cases} \dot{\hat{x}}_i = A\hat{x}_i + L_i(y_i - C_i\hat{x}_i) + Bu \\ \qquad + M_i \sum_{j=1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i) \\ h_i = y_i - C_i\hat{x}_i \end{cases}, \; i \in \mathcal{N} \tag{3}$$

where $\hat{x}_i \in \mathbb{R}^n$ is the state of the local observer at node i, $h_i \in \mathbb{R}_i^m$ is the residual of the local fault detection observer at node i, a_{ij} is the (i,j)-th entry of the adjacency matrix $\mathcal{A}$ of the given network, and $L_i \in \mathbb{R}^{n \times m_i}$ and $M_i \in \mathbb{R}^{n \times n}$ are gain matrices to be designed.

To analyze and synthesize the observer (3), we define the local estimation error of the i-th observer as

$$e_i := \hat{x}_i - x. \tag{4}$$

By combining (1) and (3) we find that the error of the i-th local fault detection observer is represented by

$$\begin{cases} \dot{e}_i = (A - L_i C_i)e_i - Ed - Ff \\ \qquad + M_i \sum_{j=1}^{N} a_{ij}(e_j - e_i) \qquad , \; i \in \mathcal{N}. \\ h_i = C_i e_i \end{cases} \tag{5}$$

Let $e := \mathrm{col}(e_1, e_2, \cdots, e_N)$ be the joint vector of errors and $\tilde{d} := \mathbf{1}_N \otimes d$ be the extended disturbance vector. Then we obtain the global error system

$$\begin{cases} \dot{e} = \Lambda e - M(\mathcal{L} \otimes I_n)e - \tilde{E}\tilde{d} - \tilde{F}f, \\ h_i = C_i e_i, \; i \in \mathcal{N}. \end{cases} \tag{6}$$

where

$$\Lambda = \mathrm{diag}\{A - L_1 C_1, \cdots, A - L_N C_N\},$$
$$M = \mathrm{diag}\{M_1, \cdots, M_N\},$$
$$\tilde{E} = I_N \otimes E, \; \tilde{F} = \mathbf{1}_N \otimes F,$$

It is noted that $\tilde{d}$ is bounded since d is bounded.

Here, we will discuss how to design gain matrices for the H_-/L_∞ DFDO (3) so that error system (6) is internally stable while increasing the sensitivity of the low-frequency domain fault on the residual and attenuating the effect of the extended disturbance signal on the residual. The fault detection thresholds also are given by L_∞ performance. More specifically, we want to design a DFDO such that the following specifications hold:

(i) The error system (6) is internally stable, i.e., it is asymptotically stable if the extended disturbance vector $\tilde{d}$ and the fault f are zero.

(ii) The error system (6) satisfies a given H_- performance level $\beta_i > 0, i \in \mathcal{N}$, i.e., for all $t \geqslant 0$

$$\|G_{h_i f}(s)\|_- > \beta_i, \; \forall \omega \in \Omega_f, \; \forall i \in \mathcal{N} \tag{7}$$

(iii) In fault-free condition, the error system (6) satisfies a given L_∞ performance level $\gamma_i > 0, i \in \mathcal{N}$, i.e., for all $t \geqslant 0$

$$\|h_i(t)\| \leqslant \gamma_i \sqrt{V(0)\mathrm{e}^{-\alpha t} + N\|d\|_\infty^2} \tag{8}$$

where $V(0) = e(0)^T P e(0), P > 0$ is a positive definite matrix to be specified, $\alpha > 0$ is a given positive scalar and N is the number of nodes.

Remark 5. Here the L_∞ performance index in (8) is the extension of peak-to-peak gain in Ding (2008). The peak-to-peak gain mainly deals with zero initial condition and the disturbance norm is assumed as 1 in Ding (2008). The L_∞ performance index in this paper considers non-zero initial condition. The intervals of residual are able to be generated based on the disturbance bound for distributed fault detection. Similar L_∞ performance definitions have been used to achieve fault detection in our previous works (Han et al., 2018, 2019b).

Before presenting main results, let us recall the following finite-frequency H_- performance and related lemma.

Definition 6. Considering the following linear system

$$\begin{cases} \dot{x} = \mathcal{A}x + \mathcal{B}u \\ y = \mathcal{C}x + \mathcal{D}u \end{cases} \tag{9}$$

with transfer function matrix $G(s) = \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D}$.

The linear system (9) is said to have a H_- performance index β in finite frequency domain Ω, if its transfer function satisfies the following inequality:

$$\|G(s)\|_-^\Omega := \inf \inf_{\omega \in \Omega} \underline{\sigma}[G(j\omega)] > \beta \tag{10}$$

where $\underline{\sigma}$ is the minimum singular value of the transfer function , Ω is the given finite frequency domain.

Lemma 7. (*GKYP Lemma*) (Iwasaki and Hara, 2005). For a linear system 9, given a symmetric matrix Π, the following statements are equivalent:

(1) The finite-frequency inequality

$$\begin{bmatrix} G(j\omega) \\ 0 \end{bmatrix}^T \Pi \begin{bmatrix} G(j\omega) \\ 0 \end{bmatrix} < 0, \forall \omega \in \mathit{\Omega} \tag{11}$$

where $\mathit{\Omega}$ is defined in Table 1 and Π is a free chosen full rank matrix.

There exist Hermitian matrices $\mathcal{P}$ and $\mathcal{Q}$ satisfying $\mathcal{Q} > 0$ and

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ I & 0 \end{bmatrix}^T \Xi \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ I & 0 \end{bmatrix} + \begin{bmatrix} \mathcal{C} & \mathcal{D} \\ 0 & I \end{bmatrix}^T \Pi \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ I & 0 \end{bmatrix} < 0, \forall \omega \in \mathit{\Omega} \tag{12}$$

where Ξ is defined in Table 1 and $\bar{\omega}_c = (\bar{\omega}_1 + \bar{\omega}_2)/2$.

Table 1. $\mathit{\Omega}$ and Ξ for different frequency ranges

	Low-frequency	Middle-frequency	High-frequency
$\mathit{\Omega}$	$\|\omega\| \leq \bar{\omega}_l$	$\bar{\omega}_1 \leq \|\omega\| \leq \bar{\omega}_2$	$\|\omega\| \geq \bar{\omega}_h$
Ξ	$\begin{bmatrix} -\mathcal{Q} & \mathcal{P} \\ \mathcal{P} & \bar{\omega}_l^2\mathcal{Q} \end{bmatrix}$	$\begin{bmatrix} -\mathcal{Q} & \mathcal{P} + j\bar{\omega}_c\mathcal{Q} \\ \mathcal{P} - j\bar{\omega}_c\mathcal{Q} & \bar{\omega}_1\bar{\omega}_2\mathcal{Q} \end{bmatrix}$	$\begin{bmatrix} \mathcal{Q} & \mathcal{P} \\ \mathcal{P} & -\bar{\omega}_h^2\mathcal{Q} \end{bmatrix}$

3. MAIN RESULTS

3.1 Distributed fault detection observer design

Based on the Lemma 7, we propose the following lemma to guarantee the error system satisfy finite frequency H_- performance.

Lemma 8. Given finite frequency H_- performance index $\beta_i > 0$, if there exist a symmetric matrix $P_f \in \mathbb{R}^{Nn \times Nn}$ and positive definite matrix $Q \in \mathbb{R}^{Nn \times Nn}, Q > 0$ such that

$$\begin{bmatrix} \Theta & (\Lambda - M(\mathcal{L} \otimes I))^T Q\tilde{F} - P_f\tilde{F} \\ \star & -\tilde{F}^T Q \tilde{F} + \beta_i^2 I \end{bmatrix} < 0 \tag{13}$$

where $\Theta = \omega_l^2 Q + \mathrm{He}(P_f(\Lambda - M(\mathcal{L} \otimes I))) - C_i^T C_i$, then the error system (6) satisfies the finite frequency H_- performance.

Proof. For the i-th local fault detection observer, we define $\Pi = \mathrm{diag}(-I, \beta_i^2 I)$, then the finite frequency inequality is

$$G_{h_i f}^T(j\omega) G_{h_i f}(j\omega) > \beta_i^2 I, \; \forall \omega \in \Omega_f \tag{14}$$

We have $\|G_{h_i f}(j\omega)\| > \beta_i, \; \forall \omega \in \Omega_f$.

On the other hand, by the GKYP lemma, the finite frequency inequality (14) holds if and only if

$$\begin{bmatrix}\Lambda - M(\mathcal{L}\otimes I) & \tilde{F}\\ I & 0\end{bmatrix}^T \Xi \begin{bmatrix}\Lambda - M(\mathcal{L}\otimes I) & \tilde{F}\\ I & 0\end{bmatrix} + \begin{bmatrix}\bar{C} & 0\\ 0 & I\end{bmatrix}^T \Pi \begin{bmatrix}\bar{C} & 0\\ 0 & I\end{bmatrix} < 0 \tag{15}$$

Here, Ξ is chosen for low frequency domain and substituted into (15). Then we get the inequality (13). Hence the finite frequency H_- is satisfied.

Next, we give L_∞ performance constrain condition.

Lemma 9. Given $\alpha > 0$, if there exist positive scalar $\tau_i > 0$, positive definite matrix $P_i \in \mathbb{R}^{n\times n}, P_i > 0$, matrices $W_i \in \mathbb{R}^{n\times m_i}$, $Y_i \in \mathbb{R}^{n\times n}$ such that the following linear matrix inequalities hold

$$\begin{bmatrix}\mathrm{He}(P(\Lambda - M(\mathcal{L}\otimes I))) + \alpha P & -P\tilde{E}\\ \star & -\alpha I\end{bmatrix} < 0, \tag{16}$$

$$\tau_i C_i^T C_i - P_i < 0,\ \forall i \in \mathcal{N}, \tag{17}$$

where $P = \mathrm{diag}(P_1, \cdots, P_N)$, then the distributed fault detection observer satisfies L_∞ performance in (iii). Here $\gamma_i = \frac{1}{\sqrt{\tau_i}}$.

Proof. Take the following Lyapunov function for error system (6)

$$V(e_1, \cdots, e_N) := \sum_{i=1}^{N} e_i^T P_i e_i, \tag{18}$$

The derivative of $V(e)$ is

$$\begin{aligned}\dot{V}(e) =& e^T(P(\Lambda - M(\mathcal{L}\otimes I)) + (\Lambda - M(\mathcal{L}\otimes I))^T P)e\\ &+ e^T P\tilde{E}\tilde{d} + \tilde{d}^T\tilde{E}^T Pe\end{aligned} \tag{19}$$

We get the following inequality by(16)

$$\begin{aligned}\dot{V} &\leqslant -\alpha V + \alpha\tilde{d}^T\tilde{d}\\ &\leqslant -\alpha V + \alpha\|\tilde{d}\|_\infty^2\end{aligned} \tag{20}$$

Hence the error system is internally stable.

$$\begin{aligned}V(e(t)) &\leqslant V(0)\mathrm{e}^{-\alpha t} + \alpha\|\tilde{d}\|_\infty^2 \int_0^t \mathrm{e}^{-\alpha(t-\tau)}\mathrm{d}\tau\\ &\leqslant V(0)\mathrm{e}^{-\alpha t} + (1-\mathrm{e}^{-\alpha t})N\|d\|_\infty^2\\ &\leqslant V(0)\mathrm{e}^{-\alpha t} + N\|d\|_\infty^2\end{aligned} \tag{21}$$

where $V(0) = e^T(0)Pe(0)$.

By the inequality (17), we have

$$\begin{aligned}\|h_i(t)\|^2 &\leqslant \frac{1}{\tau_i} e_i^T(t)P_i e_i(t)\\ &\leqslant \frac{1}{\tau_i} e^T(t)Pe(t)\\ &\leqslant \frac{1}{\tau_i}(V(0)\mathrm{e}^{-\alpha t} + N\|d\|_\infty^2)\end{aligned} \tag{22}$$

Here $\gamma_i = \frac{1}{\sqrt{\tau_i}}$. Hence L_∞ performance index (8) is satisfied. The conditions (i) and (iii) are both satisfied.

Based on lemmas 8 and 9, we give the following theorem to design H_-/L_∞ distributed fault detection observer.

Theorem 10. Given positive scalars $\alpha > 0$, $\beta_i > 0$, $\delta_1 > 0$, $\delta_2 > 0$, matrix V, if there exist positive scalar $\tau_i > 0$, symmetric matrix $P_f \in \mathbb{R}^{Nn\times Nn}$ and positive definite $Q \in \mathbb{R}^{Nn\times Nn}, Q > 0$ $P_i \in \mathbb{R}^{n\times n}, P_i > 0$ matrices $G_i \in \mathbb{R}^{n\times n}$, $W_i \in \mathbb{R}^{n\times m_i}$, $Y_i \in \mathbb{R}^{n\times n}$ such that the inequality (17) and the following inequalities hold

$$\begin{bmatrix}\Phi_{11} & \Phi_{12} & \Phi_{13}\\ \star & \Phi_{22} & \Phi_{23}\\ \star & \star & \Phi_{33}\end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix}\Psi_{11} & \Psi_{12} & \Psi_{13}\\ \star & \Psi_{22} & \Psi_{23}\\ \star & \star & \Psi_{33}\end{bmatrix} < 0 \tag{24}$$

where

$\Phi_{11} = \omega_l^2 Q + \delta_1\mathrm{He}(\Gamma) - C_i^T C_i$
$\Phi_{12} = -\delta_1 G\tilde{F} + \Gamma V$
$\Phi_{13} = -\delta_1 G + \Gamma^T + P_f$
$\Phi_{22} = \beta_i^2 I + \mathrm{He}(V^T G\tilde{F})$
$\Phi_{23} = -V^T G - \tilde{F}^T G^T$
$\Phi_{33} = -Q - G - G^T$
$\Psi_{11} = \delta_2\mathrm{He}(\Gamma) + \alpha P$
$\Psi_{12} = -\delta_2 G\tilde{F}$
$\Psi_{13} = -\delta_2 G + \Gamma^T + P$
$\Psi_{22} = -\alpha I$
$\Psi_{23} = -\tilde{E}^T G^T$
$\Psi_{33} = -G - G^T$
$\Gamma = \mathrm{diag}(G_1 A - W_1 C_1, \cdots, G_N A - W_N C_N) - Y(\mathcal{L}\otimes I)$
$P = \mathrm{diag}(P_1, \cdots, P_N)$, $G = \mathrm{diag}(G_1, \cdots, G_N)$, $W = \mathrm{diag}(W_1, \cdots, W_N)$, $Y = \mathrm{diag}(Y_1, \cdots, Y_N)$, then the error system satisfies condition (i)-(iii). The gain matrices of distributed fault detection observer are given as

$$L_i := G_i^{-1}W_i,\ M_i := G_i^{-1}Y_i, i \in \mathcal{N}. \tag{25}$$

Proof. Pre- and post-multiply the inequality (23) with the following matrix and its transpose

$$\begin{bmatrix}I & 0 & (\Lambda - M(\mathcal{L}\otimes I))^T\\ 0 & I & \tilde{F}\end{bmatrix}$$

From $W_i = G_i L_i$, $Y_i = G_i M_i$, the inequality (13) holds. Hence the finite frequency H_- is satisfied.

Similarly, Pre- and post-multiply the inequality (24) with the following matrix and its transpose

$$\begin{bmatrix}I & 0 & (\Lambda - M(\mathcal{L}\otimes I))^T\\ 0 & I & \tilde{E}\end{bmatrix}$$

Then the inequality (16) holds with $W_i = G_i L_i$, $Y_i = G_i M_i$. Hence the L_∞ performance is satisfied and the system is internally stable.

3.2 Distributed fault detection scheme

For the residual evaluation, one of the commonly used approaches is the so-called threshold method (Ding, 2008). In this paper, we adopt the following logical relationship for fault detection

$$\begin{aligned}H_i(t) \le H_{thi}(t)\ ,\forall i \in \mathcal{N} &\Longrightarrow \text{fault free}\\ H_i(t) > H_{thi}(t)\ ,\exists i \in \mathcal{N} &\Longrightarrow \text{fault occurs}\end{aligned} \tag{26}$$

where the residual evaluation function at each node is defined as the 2-norm of the vector h_i, namely $H_i(t) = \|h_i(t)\|$. Different from the widely-used constant threshold, a time-varying threshold is obtained by L_∞ analysis. Therefore we adopt the following time-varying threshold

$$H_{thi}(t) = \gamma_i\sqrt{\lambda_{\max}\bar{e}_0^2\mathrm{e}^{-\alpha t} + N\|d\|_\infty^2}$$

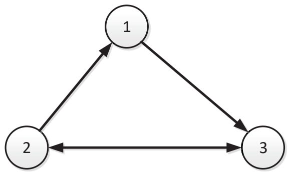

Fig. 2. The communication graph among nodes

where $\bar{e}_0 \in \mathbb{R}$ denotes the upper bound of $\|e(0)\|$, $\lambda_{\max}$ is the maximum eigenvalue of $P \in \mathbb{R}^{n\times n}$, $P > 0$ which is obtained by Theorem 10.

Based on the previous lemmas and theorem we have the following algorithm to design H_-/L_∞ DFDO:

Algorithm 1 Distributed fault detection

1: For each $i \in \mathcal{N}$, solve the LMI's (23) and (24) for all $i \in \mathcal{N}$ and get $P_i,\ G_i,\ W_i, Y_i$.
2: Define
$$L_i := G_i^{-1}W_i, M_i := G_i^{-1}Y_i, i \in \mathcal{N}$$
3: Calculate the local residual signal h_i at each node i using local fault detection observer (3).
4: Calculate the local time-varying threshold H_{thi}.
5: Make the fault detection decision by comparing the residual evaluation function $H_i(t)$ with time-varying threshold $H_{thi}(t)$ at each node i.

4. SIMULATION EXAMPLE

In this section, we will use a numerical example borrowed from Saif and Guan (1992) to illustrate the effectiveness of our approach. Consider a linear system (1) with coefficient matrices given by

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -8 & 1 & -1 & -1 & -2 & 0 \\ 4 & -0.5 & 0.5 & 0 & 0 & -4 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 2 \end{bmatrix},\ E = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 2\,0\,0\,2\,0\,0 \\ 2\,0\,1\,0\,0\,0 \\ \hline 3\,0\,1\,0\,0\,1 \\ \hline 0\,0\,0\,2\,1\,0 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix},\ F = B.$$

The communication network is given by the strongly connected digraph in Fig. 2. The Laplacian of this graph is given by

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

It can be seen that none of the local systems (C_i, A) is observable, but (C, A) is an observable pair. We will apply the conceptual Algorithm 1 to design a distributed H_-/L_∞ fault detection observer.

We choose $\alpha = 3$. H_- performance index is chosen as $\beta_i = 1$, By Algorithm 1, the L_∞ performance index are calculated as $\gamma_1 = 0.2183$, $\gamma_2 = 0.3025$ and $\gamma_3 = 0.2564$. The local observer gain matrices are calculated.

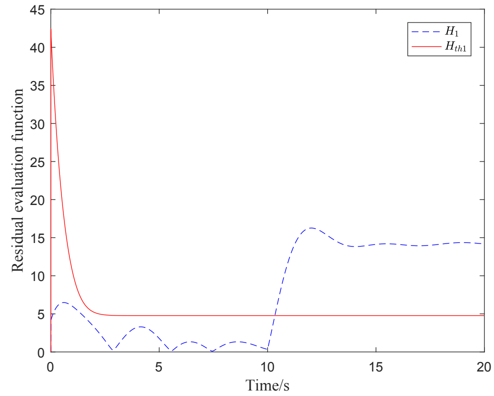

Fig. 3. The residual evaluation function and its threshold at node 1

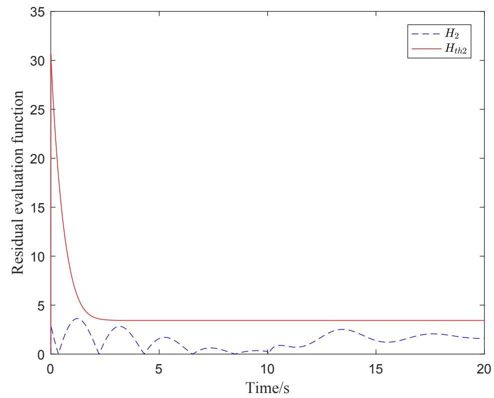

Fig. 4. The residual evaluation function and its threshold at node 2

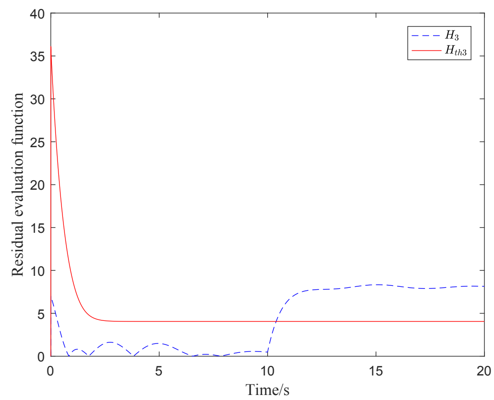

Fig. 5. The residual evaluation function and its threshold at node 3

In the simulation, the disturbance is chosen as random noise with bound $\|d\|_\infty = 0.6$. In addition, we take the following actuator fault:

$$f(t) = \begin{cases} 0 & 0s \leqslant t < 10s \\ 3 & 10s \leqslant t \leqslant 20s \end{cases} \tag{27}$$

where the time units are seconds.

In the simulation, the initial state of the observed system is taken as $x(0) = [1\ 3\ -2\ -3\ -1\ 2]^T$. For each local fault detection observer the initial state is taken to be zero.

Figs. 3–5 show the residual evaluation functions and their time-varying thresholds associated with each local fault detection observer. It can be seen that the residual evaluation functions at nodes 1 and 3 exceed their thresholds when the fault occurs.

5. CONCLUSION

In this paper, we have presented a distributed observer-based fault detection scheme for LTI systems with a bounded process disturbance. A network of local fault detection observers are built at each measurement node. The information among the local fault detection observers is exchanged by a known strongly connected directed graph. The local fault detection observer at each node is designed to detect the actuator fault of the monitored system. By using H_-/L_∞ analysis, the residuals in distributed fault detection are sensitive to interested frequency-domain fault and robust against external bounded disturbances. The local residual thresholds are calculated by L_∞ analysis in our DFDO. A numerical simulation illustrates the effectiveness of the proposed H_-/L_∞ DFDO design method.

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