Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Anti-windup Disturbance Rejection Control Design for Sampled Systems with Output Delay and Asymmetric Actuator Saturation Constraint ⋆

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Abstract: A novel anti-windup disturbance rejection control design is proposed for industrial sampled systems with output delay and asymmetric actuator saturation constraint. To deal with the asymmetric actuator saturation constraint as often encountered in engineering practice, the input constraint is equivalently transformed into a symmetric actuator saturation constraint for the convenience of control design. Based on the equivalent system description, a model-based extended state observer (MESO) is designed to simultaneously estimate the system state and disturbance, which becomes an anti-windup compensator when the actuator saturation occurs. In order to compensate for the delay mismatch in MESO, a generalized predictor is utilized to estimate the undelayed system output. Accordingly, a pole placement approach is given to design the feedback controller. A set-point pre-filter is designed to ensure no steady-state output tracking error, in terms of a desired transfer function for the set-point tracking. Based on the delay-dependent sector condition and generalized free-weighting-matrix (GFWM), a sufficient condition guaranteeing the stability of the closed-loop system is established in terms of linear matrix inequalities (LMIs). An illustrative example from the literature is used to demonstrate the effectiveness and advantage of the proposed control method.

Keywords: Sampled systems, Asymmetric actuator saturation, Output delay, Anti-windup design, Extended state observer (ESO)

1. INTRODUCTION

Actuator saturation widely occurs in industrial control systems. Without specific treatment, the saturation constraint may lead to severe performance degradation or even instability. In particular, asymmetric actuator saturation appears more than symmetric actuator saturation in engineering applications, e.g., a flow valve typically has an opening range from zero to 90 degree. However, a small number of references addressed this issue in the past decades. An asymmetric Lyapunov function approach was investigated to estimate the domain of attraction for a linear system subject to asymmetric actuator saturation (Li and Lin (2018)). The semi-global stabilization problem for a discrete-time linear system with asymmetric saturation was analyzed by transforming the asymmetric saturation bound into the symmetric case (Wu and Liu (2019)). A robust control design for spacecraft rendezvous systems was developed by considering the parameter uncertainties and actuator unsymmetrical saturation based on a discrete gain scheduling approach (Wang and Xue (2018)).

Besides the actuator saturation, time delay is usually involved with industrial applications, which also degrades the control performance or even provokes instability. A lot of research efforts were devoted to time-delay systems in the recent years (Fridman (2014)). Among different approaches, the LMI-based delay-dependent stability analysis has received more attentions owing to less conservativeness, such as the generalized free-weighting-matrix (GFWM) (Zhang et al. (2016)). For time-delay systems subject to symmetric saturation, a few references addressed the related control problems. The problem of global stabilization was studied for a family of discrete-

⋆ This work is supported in part by the NSF China Grants 61633006, 61903060, the Fundamental Research Funds for the Central Universities of China (DUT18ZD201), National Key RD Program of China under Grant 2017YFA0700300, and the Foundation of Key Laboratory of Advanced Process Control for Light Industrial(Jiangnan University), Ministry of Education, China, Grant No. APCLI1807.

time feedforward time-delay systems with bounded controls (Yang and Zhou (2018)). A delay-dependent polytopic approach was extended to analyze the regional stabilization problem for discrete time-delay systems with actuator saturation (Chen et al. (2018)).

Note that active disturbance rejection control(ADRC) has been well recognized as an effective methodology to deal with disturbances and system uncertainties. An anti-windup ADRC design was provided for a class of uncertain nonlinear systems subject to external disturbance (Ran et al. (2016)). An LMI-based anti-windup approach was proposed for a class of uncertain multiple input multiple output (MIMO) systems subject to actuator saturation with linear active disturbance rejection controller(LADRC) (Yu et al. (2018)). However, there is no result available in the published literature on ADRC design of sampled systems with time-delay and asymmetric actuator saturation.

In this paper, a novel anti-windup control design based on ADRC is proposed for sampled systems with output delay and asymmetric actuator saturation constraint. To deal with the asymmetric saturation, the input constraint is transformed from the asymmetric case to symmetric case for the convenience of control design. Based on the transformed system description, a model-based extended state observer (MESO) is designed to estimate not only the system state but also the overall disturbance, which could become an anti-windup compensator once the actuator saturation occurs. A generalized predictor is adopted to estimate the delay-free system output for the design of MESO. Based on the above estimation, the feedback controller is designed by specifying the desired closed-loop system poles. Meanwhile, a set-point pre-filter is given to ensure no steady-state output error. A sufficient condition is established in terms of LMIs based on the delay-dependent sector condition and GFWM for stability analysis of the closed-loop control system.

The paper is organized as follows. Section 2 states the control problem and provides some preliminaries. The proposed anti-windup control scheme for sampled systems with time delay is detailed in Section 3. Section 4 analyses the stability of the proposed control scheme. Section 5 gives an illustrative example to demonstrate the effectiveness of the proposed control scheme. Finally, some conclusions are drawn in Section 6.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider a sampled system of single-input-single-output (SISO) described by

$$P(z)=G(z)z^{-d}=\frac{b_{n-1}z^{n-1}+\cdots+b_1z+b_0}{z^n+a_{n-1}z^{n-1}+\cdots+a_1z+a_0}z^{-d} \quad (1)$$

where $G(z)=\frac{N(z)}{D(z)}$ is the nominal delay-free transfer function, and d is the output delay.

Denote by $x=[x_1,x_2,\cdots,x_n]^{\mathrm{T}}$ the state vector of the delay-free system shown by $G(z)$, the corresponding state-space realization of controllable canonical form can be expressed by $C_{\mathrm{m}}(zI-A_{\mathrm{m}})^{-1}B_{\mathrm{m}}$ with $C_{\mathrm{m}}\triangleq[1\ \frac{b_1}{b_0},\cdots,\frac{b_{n-1}}{b_0}]$

$$A_{\mathrm{m}}\triangleq\begin{bmatrix}0&1&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&1\\-a_0&-a_1&\cdots&-a_{n-1}\end{bmatrix},\quad B_{\mathrm{m}}\triangleq\begin{bmatrix}0\\0\\\vdots\\0\\b_0\end{bmatrix},$$

Accordingly, a state-space description of a sampled system with output delay and asymmetric actuator saturation constraint is written as

$$\begin{cases}x_1(k+1)=x_2(k)\\\vdots\\x_n(k+1)=-a_0x_1(k)-a_1x_2(k)...-a_{n-1}x_n(k)\\\qquad\qquad+b_0\mathrm{SAT}[u(k)]+b_0\omega(k)\\y(k)=C_mx(k-d)\end{cases}\quad (2)$$

where $\omega(k)$ is the system disturbance that may also represent unmodeled system dynamics or uncertainty with nonlinear characteristics. $\mathrm{SAT}(u(k))$ is the saturation function defined by

$$\mathrm{SAT}(u(k))=\begin{cases}\alpha,(u(k)>\alpha)\\u(k),(-\beta\le u(k)\le\alpha)\\-\beta,(u(k)<-\beta)\end{cases}\quad (3)$$

where $0\le\beta<\alpha$ are the asymmetric saturation bound. By taking $\sigma(k)=u(k)-\frac{\alpha-\beta}{2}$, it follows that

$$\mathrm{SAT}(u(k))=\mathrm{sat}(\sigma(k))+\frac{\alpha-\beta}{2}\quad (4)$$

where

$$\mathrm{sat}(\sigma(k))=\begin{cases}(\alpha+\beta)/2,&\text{if }\ \sigma(k)>(\alpha+\beta)/2\\\sigma(k),&\text{if }-(\alpha+\beta)/2\le\sigma(k)\le(\alpha+\beta)/2\\-(\alpha+\beta)/2,&\text{if }\ \sigma(k)<-(\alpha+\beta)/2\end{cases}$$

Based on the above transformation (Wu and Liu (2019)), it is easily seen that

$$B_m\mathrm{SAT}(u(k))=\ B_m\mathrm{sat}(\sigma(k))+B_m(\alpha-\beta)/2\quad (5)$$

By the above relation, the system described by (2) can be rewritten as

$$\begin{cases}x(k+1)=A_mx(k)+B_m\mathrm{sat}(\sigma(k))+B_m\dfrac{\alpha-\beta}{2}+B_m\omega(k)\\y(k)=C_mx(k-d)\end{cases}\quad (6)$$

Denote an extended state by $x_{n+1}=b_0\frac{\alpha-\beta}{2}+b_0\omega(k)$, an augmented state-space expression of the above system can be written as

$$\begin{cases}\tilde{x}(k+1)\ =A_{\mathrm{e}}\tilde{x}(k)+B_{\mathrm{e}}\mathrm{sat}(\sigma(k))+E_{\mathrm{e}}h(k)\\y(k)\qquad\ =C_{\mathrm{e}}\tilde{x}(k-d)\end{cases}\quad (7)$$

where $\tilde{x}(k)=[x^{\mathrm{T}}\ x_{n+1}]^{\mathrm{T}}$, $h(k)=b_0[\omega(k+1)-\omega(k)]$ and

$$A_{\mathrm{e}}\triangleq\begin{bmatrix}A_{\mathrm{m}}&\begin{bmatrix}\mathbf{0}\\1\end{bmatrix}\\\mathbf{0}&1\end{bmatrix},B_{\mathrm{e}}\triangleq\begin{bmatrix}B_{\mathrm{m}}\\0\end{bmatrix},C_{\mathrm{e}}\triangleq[C_{\mathrm{m}}\ 0],E_{\mathrm{e}}\triangleq\begin{bmatrix}\mathbf{0}\\1\end{bmatrix}.$$

where $\mathbf{0}$ represents a zero vector of compatible dimension. Based on the above transformation, an anti-windup design using ESO is developed to accommodate for the output delay and asymmetric actuator saturation, as detailed in the next section.

Remark 1. Compared with the transformation approach (Wu and Liu (2019)), the constant term $b_0\frac{\alpha-\beta}{2}$ arising from the transformation is lumped into the above extended state so that no additional assumption on the system matrix A_m is required any more.

3. PROPOSED ANTI-WINDUP CONTROL SCHEME

The proposed anti-windup control scheme is shown in Figure 1, where $P(z)$ is the controlled plant. F_1 and F_2 are stable filters used to predict the system output without time delay, K_0 is the feedback controller, K_{f} is a set-point pre-filter, $r(z)$ is the real set-point reference, and $\tilde{r}$ is the filtered set-point reference. An artificial symmetric saturation function $\mathrm{sat}(\cdot)$ is designed in the dash-line box where $C(z) = \frac{\alpha-\beta}{2r(z)}$ is introduced to satisfy the transformation symmetry of the input constraint.

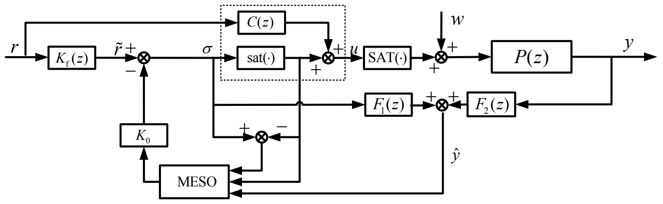

Fig. 1. Block diagram of the proposed control scheme

3.1 Anti-windup MESO design

To cope with time delay and mitigate the undesirable effects caused by actuator saturation, an anti-windup MESO is designed as

$$\begin{aligned} z(k+1) &= A_{\mathrm{e}}z(k) + B_{\mathrm{e}}\mathrm{sat}(\sigma(k)) + L_{\mathrm{o}}(\hat{y}(k) - C_{\mathrm{e}}z(k)) \\ &\quad + L_{\mathrm{AW}}[\mathrm{sat}(\sigma(k)) - \sigma(k)] \end{aligned} \tag{8}$$

where L_{AW} is the anti-windup gain, $\hat{y}(k)$ is the estimated delay-free output prediction based on a predictor.

To allow for practical application, the desired characteristic equation of the above MESO with $L_{\mathrm{AW}} = \mathbf{0}$ is specified as $|zI - (A_{\mathrm{e}} - L_{\mathrm{o}}C_{\mathrm{e}})| = (z - \omega_{\mathrm{o}})^{n+1} = 0$ where $\omega_{\mathrm{o}} \in (0, 1)$ is a tuning parameter. Using the Ackerman formula, the observer gain vector is calculated as

$$L_{\mathrm{o}} = \Psi(A_{\mathrm{e}}) \begin{bmatrix} C_{\mathrm{e}} \\ C_{\mathrm{e}}A_{\mathrm{e}} \\ \vdots \\ C_{\mathrm{e}}A_{\mathrm{e}}^{n} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \tag{9}$$

To improve the anti-windup performance, L_{AW} is designed as

$$L_{\mathrm{AW}} = [\mathbf{0} \;\; l_{aw}]^{\mathrm{T}}$$

where l_{aw} is the only parameter in the anti-windup gain.

It is suggested to initially take $\omega_{\mathrm{o}} \in [0.9, 0.99]$, $l_{\mathrm{aw}} \in [0.001, 0.0001]$. Then by monotonically increasing or decreasing these two parameters, a good trade-off between the estimation performance and anti-windup compensation of MESO and its robustness against plant uncertainties could be obtained.

3.2 Generalized predictor

In order to eliminate the impact of time delay, the generalized predictor (Liu et al. (2018)) is adopted here to obtain the delay-free output prediction shown in Fig. 1. The nominal system (1) is decomposed as

$$P(z) = G(z)z^{-d} = \tilde{G}(z)\Gamma(z)z^{-d} \tag{10}$$

where $H(z) = \frac{z-1}{z-\lambda}\frac{(1-\lambda)^q z^q}{(z-\lambda)^q}$, $\Gamma(z) = \frac{N(z)}{(z-\lambda)^m}H(z,\lambda)$,

$$\tilde{G}(z) = \frac{(z-\lambda)^m}{D(z)}H^{-1}(z,\lambda) = c_{\mathrm{g}}(zI - A_{\mathrm{g}})^{-1}b_{\mathrm{g}},$$

$[(1-\lambda)^q z^q]/(1-z)^q$ is an all-pass filter for mitigating sensitivity to measurement noise, with q a user specified order in practice with respect to the noise level. m is the number of zeros in $G(z)$.

Next, define another auxiliary transfer function

$$\tilde{G}^{\star}(z) \triangleq c_{\mathrm{g}}(zI - A_{\mathrm{g}})^{-1}A_{\mathrm{g}}^{d}b_{\mathrm{g}} = \tilde{N}^{\star}(z)/\tilde{D}^{\star}(z) \tag{11}$$

According to the Lemma 1 (Liu et al. (2018)) the delay-free output prediction is given by

$$\hat{y}(z) = F_1(z)\sigma(z) + F_2(z)y(z) \tag{12}$$

where $F_1(z) = c_{\mathrm{g}}\sum_{i=1}^{d} A_{\mathrm{g}}^{i-1}b_{\mathrm{g}}z^{-i}\Gamma(z)$ and $F_2(z) = \tilde{N}^{\star}(z)/(z-\lambda)^{m+1+q}$. $\lambda \in (0, 1)$ is only a single parameter to be tuned in $F_1(z)$ and $F_2(z)$. It is recommended to initially set $\lambda \in (0.95, 0.99)$ in practice. By monotonically tuning λ, a trade-off can be obtained between the prediction performance and its robustness against the plant uncertainties.

3.3 Feedback controller design

Based on the above MESO to estimate the augmented disturbance x_{n+1} and the system state, a feedback control law is designed as

$$u(k) = \tilde{r}(k) - K_0 z(k) \tag{13}$$

where the feedback controller gain is taken as

$$K_0 = [k_1 \;\; k_2 \;\; \cdots \;\; k_n \;\; 1]/b_0 = \left[\bar{K}_0 \;\; 1/b_0\right]. \tag{14}$$

By applying the control law into (7), the characteristic equation of the delay-free state-feedback control system is

$$\begin{aligned} |zI - (A_{\mathrm{e}} - B_{\mathrm{e}}K_0)| &= (z-1)(z^n + (a_{n-1} + k_n)z^{n-1} \\ &\quad + \cdots + (a_0 + k_1)) = 0 \end{aligned} \tag{15}$$

For simplicity, all of the closed-loop poles are placed at $\omega_{\mathrm{c}} \in (0, 1)$ except for $z = 1$, i.e.,

$$z^n + (a_{n-1} + k_n)z^{n-1} + \cdots + (a_0 + k_1) = (z - \omega_{\mathrm{c}})^n \tag{16}$$

Therefore, the controller parameters can be determined as $k_i = C_n^{i-1}(-\omega_{\mathrm{c}})^{n-i+1} - a_{i-1}, i = 1, \cdots, n$ where $C_n^{i-1} = n!/[(i-1)!(n-i+1)!]$. For practical implementation, it is suggested to initially take $\omega_{\mathrm{c}} \in [0.9, 0.95]$ and then by monotonically tuning it, a good trade-off between the closed-loop control performance and its robust stability could be obtained.

3.4 Set-point pre-filter design

To improve the set-point tracking performance without steady-state output error, the set-point tracking controller $K_{\mathrm{f}}(z)$ is added as shown in Fig. 1.

Based on the above MESO design and feedback control law, the closed-loop system transfer function for set-point tracking is obtained as

$$\begin{aligned} y(z) &= K_{\mathrm{f}}(z)C_{\mathrm{m}}(zI - A_{\mathrm{m}} + B_{\mathrm{m}}\bar{K}_0)^{-1}B_{\mathrm{m}}z^{-d}r(z) \\ &= K_{\mathrm{f}}(z)T_{\mathrm{d}}(z)z^{-d}r(z) \end{aligned} \tag{17}$$

where $T_d(z)$ is the complementary sensitivity function.

The desired form for obtaining the H_2 optimal control performance is proposed as

$$T_d(z) = N(z)/(z-\omega_c)^n = T_{dA}(z)T_{dM}(z) \quad (18)$$

which can be factorized into an all-pass part, $T_{dA}(z)$, and a minimum-phase part, $T_{dM(z)}$. Correspondingly, the set-point tracking controller is designed as

$$K_f(z) = (z^{n_g}T_{dM})^{-1}[(1-\lambda_f)^{n_f}z^{n_f}/(z-\lambda_f)^{n_f}] \quad (19)$$

n_g is a positive integer chosen to keep $z^{n_g}T_{dM}(z)$ biproper. The low-pass filter $(1-\lambda_f)^{n_f}z^{n_f}/(z-\lambda_f)^{n_f}$ is adopted to modulate the control peak for practical application. n_f is the filter order specified by the user, and $|\lambda_f| < 1$ is a tuning parameter. It is suggested to initially take $\lambda_f \in (0.97, 0.99)$, and then monotonically decrease it to make a trade-off between the set-point tracking speed and control effort which may cause saturation.

4. STABILITY ANALYSIS

The state-space representations of the subsystems in Fig.1 can be defined by the following forms, respectively,

$$\text{Process: } \begin{cases} x(k+1) = A_m z(k) + B_m \mathrm{sat}(\sigma(k)) \\ y(k) = C_m x(k-d) \end{cases} \quad (20)$$

where the asymmetric constraint is transformed into a symmetric case for consideration.

$$\text{MESO: } \begin{cases} z(k+1) = A_e z(k) + B_e \mathrm{sat}(\sigma(k)) + L_o[\hat{y}(k) \\ -C_e(k)z(k)] + L_{AW}(\mathrm{sat}(\sigma(k)) - \sigma(k)) \\ \sigma(k) = \tilde{r} - K_0 z(k) \end{cases} \quad (21)$$

$$F_1: \begin{cases} x_{F_1}(k+1) = A_{F_1}x_{F_1}(k) + B_{F_1}\sigma(k) \\ y_{F_1}(k) = C_{F_1}x_{F_1}(k) \end{cases} \quad (22)$$

$$F_2: \begin{cases} x_{F_2}(k+1) = A_{F_2}x_{F_1}(k) + B_{F_2}C_m x(k-d) \\ y_{F_2}(k) = C_{F_2}x_{F_1}(k) + D_{F_2}C_m x(k-d) \end{cases} \quad (23)$$

By setting $\xi = [x^T(k)\ z^T(k)\ x_{F_1}^T(k)\ x_{F_2}^T(k)]^T$, the closed-loop system is obtained as:

$$\xi(k+1) = A\xi(k) + A_d\xi(k-d) + (B - RL_{AW})\Psi(\sigma(k)) \quad (24)$$

where $\Psi(\sigma(k)) = K\xi(k) - \mathrm{sat}(K\xi(k))$, $K = [0\ -K_0\ 0\ 0]$,

$$A = \begin{bmatrix} A_m & -B_mK_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_e - L_oC_e - B_eK_o & L_oC_{F_1} & L_oC_{F_2} \\ \mathbf{0} & -B_{F_1}K_o & A_{F_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & A_{F_2} \end{bmatrix}$$

$$A_d = \begin{bmatrix} \mathbf{0} & \mathbf{0\ 0\ 0} \\ L_oD_{F_2}C_m & \mathbf{0\ 0\ 0} \\ \mathbf{0} & \mathbf{0\ 0\ 0} \\ B_{F_2}C_m & \mathbf{0\ 0\ 0} \end{bmatrix}, B = \begin{bmatrix} -B_m \\ -B_e \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad E = \begin{bmatrix} \mathbf{0} \\ I \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}.$$

The following lemma is briefly introduced for further stability analysis of the closed-loop system.

Lemma 1. (Delay-dependent generalized sector condition). If the inequality $\|K\xi(k) - G\xi(k) - \tilde{V}\sum_{k-d}^{k-1}\xi(i)| \le u_0$ hold for $k \ge 0$, then for any diagonal matrix $S > 0$, the following inequality holds, $\Psi(u(k))^T S[\Psi(u(k)) - G\xi(k) - \tilde{V}\sum_{k-d}^{k-1}\xi(i)] \le 0$

To facilitate the delay-dependent stability analysis, the following Lyapunov functional is adopted,

$$\begin{aligned} &V(\xi_k) = V_1(\xi_k) + V_2(\xi_k) + V_3(\xi_k), \\ &V_1(\xi_k) = \xi_2^T(k)P\xi_2(k),\ V_2(\xi_k) = \sum_{k-d}^{k-1}\xi^T(i)Q\xi(i), \\ &V_3(\xi_k) = \sum_{i=-d}^{-1}\sum_{j=k+i}^{k-1}\eta_2^T(j)R\eta_2(j), \\ &\xi_2(k) = \left[\xi^T(k)\ \sum_{k-d}^{k-1}\xi^T(i)\right]^T,\ \eta(j) = \xi(j+1) - \xi(j). \end{aligned} \quad (25)$$

Based on the above lemma, the following theorem is given for stability analysis:

Theorem 1. If there exist symmetric positive definite matrices P,Q,R, symmetric matrices T,Z,X, any matrices L, M, N, Y, and positive variables S, $\bar{\gamma}$ satisfying

$$\begin{bmatrix} \bar{X} & \Theta \\ * & \bar{T} + R \end{bmatrix} \ge 0 \quad (26)$$

$$\begin{bmatrix} \tilde{Q} & \bar{N}^T \\ * & u_0^2\bar{\gamma} \end{bmatrix} \ge 0, \quad (27)$$

$$\Phi \triangleq \Upsilon_1 + \Upsilon_2 + \Upsilon_3 < 0 \quad (28)$$

where $\bar{N} = [SK - \tilde{G}\ \mathbf{0}\ -\bar{V}\ \mathbf{0}]$, $\epsilon_1 = [I\ \mathbf{0}\ \mathbf{0}\ \mathbf{0}]$, $\epsilon_2 = [\mathbf{0}\ I\ \mathbf{0}\ \mathbf{0}]$, $\epsilon_3 = [\mathbf{0}\ \mathbf{0}\ I\ \mathbf{0}]$,

$$\bar{X} = \begin{bmatrix} X & Y \\ * & Z \end{bmatrix}, \Theta = \begin{bmatrix} L & M \\ * & N \end{bmatrix}, \bar{T} = \begin{bmatrix} \mathbf{0} & T \\ * & T \end{bmatrix},$$

$$\tilde{Q} = [\epsilon_1^T, \epsilon_3^T]P[\epsilon_1^T, \epsilon_3^T]^T + \frac{1}{d}\epsilon_3^T Q\epsilon_3 + \frac{2}{d(d+1)}\epsilon_r^T R\epsilon_r$$

$$\epsilon_r = \begin{bmatrix} dI & \mathbf{0} & -I & \mathbf{0} \\ dA - dI - I & dA_d + I & \mathbf{0} & d(B - EK_{AW}) \end{bmatrix},$$

$$\Upsilon_1 = F_1^T PF_1 - F_2^T PF_2 + \mathrm{He}(\Gamma P(F_1 - F_2)) + d[\epsilon_1^T, \epsilon_s^T]R[\epsilon_1^T, \epsilon_s^T]^T,$$

$$F_1 = \begin{bmatrix} A - I & A_d & \mathbf{0} & B - EK_{AW} \\ \mathbf{0} & -I & I & \mathbf{0} \end{bmatrix}, F_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -I & \mathbf{0} & I & \mathbf{0} \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & dI & \mathbf{0} \end{bmatrix}, \Upsilon_2 = \begin{bmatrix} Q & \mathbf{0} & \mathbf{0} & \tilde{G}^T - \bar{V}^T \\ \mathbf{0} & -Q & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & (d+1)\bar{V}^T \\ * & \mathbf{0} & * & -2S \end{bmatrix},$$

$$\begin{aligned} \Upsilon_3 = &\epsilon_1^T T\epsilon_1 - \epsilon_2^T T\epsilon_2 + 2\epsilon_f^T[M(\epsilon_1 - \epsilon_2) + N(\epsilon_1 + \epsilon_2 - 2\epsilon_3) \\ &- L((d+1)\epsilon_3 - \epsilon_1)] + d\epsilon_f^T X\epsilon_f + \frac{d(d-1)}{3(d+1)}\epsilon_f^T Z\epsilon_f, \end{aligned}$$

$$\epsilon_f = [\epsilon_1; \epsilon_2; \epsilon_3], \epsilon_s = [A - I\ A_d\ \mathbf{0}\ B - EK_{AW}]$$

then the closed-loop system (24) is locally asymptotic stable with respect to the initial state satisfying $V(\xi_0) \le S^2/\bar{\gamma}$.

Proof. Let

$$\zeta(k) = \left[\xi^T(k)\ \xi^T(k-d)\ \frac{1}{d+1}\sum_{k-d}^{k}\xi^T(i)\ \Psi^T(\sigma(k))\right]^T,$$

the forward difference $V(\xi_k)$ can be expressed as:

$$\Delta V(\xi_k) = \Delta V_1(\xi_k) + \Delta V_2(\xi_k) + \Delta V_3(\xi_k), \quad (29)$$

$$\Delta V_1(\xi_k) = \zeta^{\mathrm{T}}(k)[F_1^{\mathrm{T}} P F_1 - F_2^{\mathrm{T}} P F_2 + \mathrm{He}(\Gamma P(F_1 - F_2))]\zeta(k), \quad (30)$$

$$\Delta V_2(\xi_k) = \zeta^{\mathrm{T}}(k) \begin{bmatrix} Q & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -Q & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \zeta(k), \quad (31)$$

$$\begin{aligned} \Delta V_3(\xi_k) &= d\eta_2^{\mathrm{T}}(k) R \eta_2(k) - \sum_{k-d}^{k-1} \eta_2^{\mathrm{T}}(j) R \eta_2(j) \\ &= \zeta^{\mathrm{T}}(k)[\epsilon_1; \epsilon_s]^{\mathrm{T}} dR[\epsilon_1; \epsilon_s]\zeta(k) + \Delta V_b(\xi_k), \end{aligned} \quad (32)$$

$$\Delta V_b(\xi_k) = -\sum_{k-d}^{k-1} \eta_2^{\mathrm{T}}(j) R \eta_2(j). \quad (33)$$

Let

$$f(k) = [\xi^{\mathrm{T}}(k), \xi^{\mathrm{T}}(k-d), v_1^{\mathrm{T}}(k)]^{\mathrm{T}}, v_1(k) = \frac{1}{d+1} \sum_{k-d}^{k} \xi(k),$$

with the help of the GFWM Lemma (Zhang et al. (2016)), we could obtain that

$$\begin{aligned} \Delta V_b(\xi_k) \leq & \xi^{\mathrm{T}}(k) T \xi(k) - \xi^{\mathrm{T}}(k-d) T \xi(k-d) \\ & + 2f(k)\{M[\xi(k) - \xi(k-d)] \\ & + L[(d+1)\delta(k, k-d) - \xi(k)] \\ & + N[\xi(k) + \xi(k-d) - 2\delta(k, k-d)]\} \\ & + df^{\mathrm{T}}(k)[X + \frac{(d-1)}{3(d+1)} Z] f(k) \\ & - \sum_{k-d}^{k-1} \eta_3^{\mathrm{T}}(j) \begin{bmatrix} \bar{X} & \Theta \\ * & \bar{T} + R \end{bmatrix} \eta_3(k) \end{aligned}, \quad (34)$$

where $\eta_3 = [\eta_1^{\mathrm{T}}, \eta_2^{\mathrm{T}}]^{\mathrm{T}}, \delta(k, k-d) = \sum_{k-d}^{k} \frac{x(k)}{d+1}$.

With the assumption in (26), the above inequality can be reduced as

$$\begin{aligned} \Delta V_b(\xi_k) \leq & e_1 T e_1 - e_2 T e_2 + 2e_f^{\mathrm{T}}\{M[e_1 - e_2] \\ & - L[(d+1)e_3 - e_1] + N[e_1 + e_2 - 2e_3]\} \\ & + de_f^{\mathrm{T}} X e_f + \frac{d(d-1)}{3(d+1)} e_f^{\mathrm{T}} Z e_f. \end{aligned} \quad (35)$$

On the other hand, under the assumption $\|K\xi(k) - G\xi(k) - \tilde{V}\sum_{k-d}^{k-1}\xi(i)\| \leq u_0$, it follows from Lemma 1 that

$$-2\Psi(K\xi(k))^{\mathrm{T}} S[\Psi(K\xi(k)) - G\xi(k) - \tilde{V}\sum_{k-d}^{k-1}\xi(i)] \geq 0 \quad (36)$$

Combining (30), (31), (32), (35) with (29), we have

$$\Delta V(\xi_k) \leq \zeta^{\mathrm{T}}(k) \Phi \zeta(k) \quad (37)$$

By setting $SG = \tilde{G}, S\tilde{V} = \bar{V}$, the inequalities (26) and (28) ensure the negative definite of $\Delta V(\xi_k)$.

For the L-K functional (25), using Jensen inequalities (Tarbouriech et al. (2004)), it follows that

$$V(\xi_k) \geq \bar{\zeta}^{\mathrm{T}}(k) \tilde{\Theta} \bar{\zeta}(k) \quad (38)$$

where $\bar{\zeta}(k) = \left[\xi(k) \ \xi(k-d) \ \sum_{k-d}^{k-1} \xi(i) \ \Psi(u(k)) \right]$.

Define a vector $\tilde{N} = \left[(K-G) \ \mathbf{0} \ -\tilde{V} \ \mathbf{0} \right]$, and assume the following inequality holds:

$$(1/(\gamma u_0^2)) \tilde{N}^{\mathrm{T}} \tilde{N} \leq \tilde{\Theta} \quad (39)$$

Then, for any initial condition satisfying $V(0) \leq \gamma^{-1}$, it follows that

$$\begin{aligned} (1/(\gamma u_0^2)) \bar{\zeta}^{\mathrm{T}}(k) \tilde{N}^{\mathrm{T}} \tilde{N} \bar{\zeta}(k) \leq \bar{\zeta}^{T}(k) \tilde{\Theta} \bar{\zeta}(k) \\ \leq V(\xi_k) \leq V(\xi_0) \leq \gamma^{-1} \end{aligned} \quad (40)$$

which means that the condition $|K\xi(k) - G\xi(k) - \tilde{V}\sum_{k-d}^{k-1}\xi(i)| = |\tilde{N}\bar{\zeta}(k)| \leq u_0$ can be guaranteed. Using Schur complementery, the inequality (39) is equal to

$$\begin{bmatrix} \tilde{Q} & \tilde{N}^{\mathrm{T}} \\ * & u_0^2 \gamma \end{bmatrix} \geq 0, \quad (41)$$

Pre- and post-multiplying diag$[I, I, I, S]$, and letting $\gamma S^2 = \bar{\gamma}$, the inequality (27) could be obtained.This completes the proof.

5. SIMULATION RESULTS

Consider a second-order process with time delay (Tan and Fu (2015)),

$$P(s) = \frac{2}{(3s+1)(s+1)} e^{-0.4s}$$

With a sampling period $T = 0.02$(s) for control implementation, the corresponding discrete-time model is obtained as $P(z) = \frac{0.0001322z + 0.000131}{z^2 - 1.974z + 0.9737} z^{-20}$.

To make comparison with LADRC (Tan and Fu (2015)), the observer bandwidth and the feedback controller bandwidth are tuned as $\omega_o = 0.9139$ and $\omega_c = 0.9418$, respectively. Accordingly, the gain vectors of MESO and feedback controller are computed as

$$L_0 = [0.1114 \ 0.1193 \ 0.00032],$$

$$K_0 = [-662.4063 \ 687.2931 \ 7634.4596].$$

In consideration of the negative zero $z_1 = -0.9912$ in the plant model, the set-point controller is designed by the formulae in (19) with$\lambda_{\mathrm{f}} = 0.96$ and $n_{\mathrm{f}} = 4$, $K_{\mathrm{f}} = \frac{0.0194(z-0.9418)^2 z^3}{(z-0.96)^4(z+0.9912)}$.

Given the nominal delay $d_0 = 20$, the prediction filters are designed by the formula in (12) with $\lambda = 0.986$ and $m = 1$,

$$F_1(z) = c_{\mathrm{g}} \sum_{i=1}^{d_0} A_{\mathrm{g}}^{i-1} b_{\mathrm{g}} z^{-i} \Gamma(z)$$

where

$$A_g = \begin{bmatrix} 2.9736 & -2.9472 & 0.9737 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, b_g = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$c_g = \begin{bmatrix} 1 & -1.972 & 0.9722 \end{bmatrix}, \Gamma = \frac{(0.0001322z + 0.000131)(z-1)}{(z-0.986)^2},$$

and $F_2(z) = \frac{1.0347(z-0.9891)(z-0.9827)}{(z-0.986)^2}$.

Under the asymmetric saturation bound $\alpha = 1$ and$\beta = 0.5$, the anti-windup gain vector is tuned as $L_{\mathrm{AW}} = [0 \ 0 \ -0.00035]$.

Three groups of the controller setting in the reference (Tan and Fu (2015)) are tuned for comparison according to the design formulae given therein, i.e.,

(a) $b = 4/3$, $\omega_c = 1$, $\omega_o = 10$, subject to the above input saturation bound;

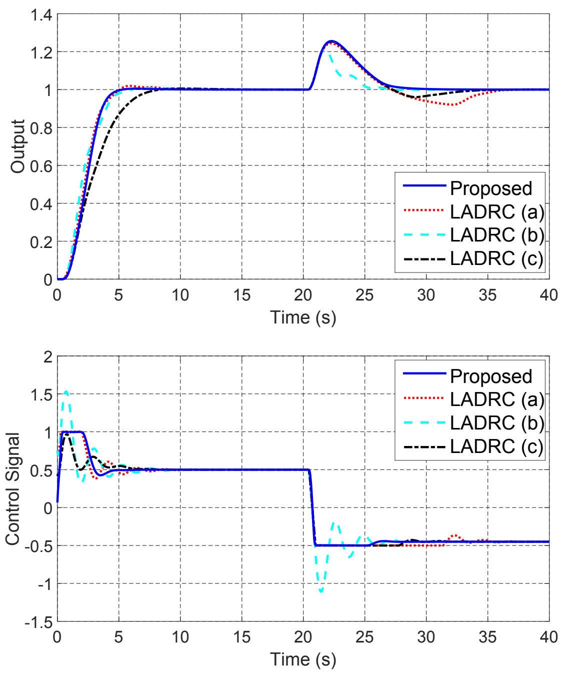

Fig. 2. System responses by using different methods

(b) $b = 4/3$, $\omega_c = 1$, $\omega_o = 10$, subject to no input saturation bound;
(c) $b = 4/3$, $\omega_c = 10/13$, $\omega_o = 10$, subject to the above input saturation bound.

Note that group (c) is tuned to have a slower set-point tracking response so as to avoid the input saturation for comparison.

For control test, a unit step reference is added to the set-point at $t = 0$(s) and a load disturbance with a magnitude of 0.95 is added to the process input at $t = 20$(s). The control results are shown in Fig. 2.

It is seen that under the similar set-point tracking speed and the input saturation bound, there is no overshoot in the set-point response along with obviously faster recovery of disturbance response (with respect to the similar peaks) by the proposed anti-windup design, compared with group (a) of LADRC. The result by group (b) of LADRC demonstrates that no overshoot under the similar set-point tracking speed with the proposed method can only be obtained when there is no input saturation. Otherwise, an evidently slower set-point tracking response is required for implemention to avoid the input saturation, as illustrated by the control result of group (c) of the reference. Meanwhile, the disturbance response under group (c) of LADRC is evidently slower than the proposed anti-windup design.

6. CONCLUSIONS

In this paper, an anti-windup ADRC design has been proposed for samples systems with output delay and asymmetric actuator saturation constraint. Based on a transformation from the asymmetric saturation to symmetric case, a modified MESO has been designed to estimate the system state and disturbance, along with anti-windup compensation. Meanwhile, a generalized predictor is adopted to predict the undelayed output to compensate the delay mismatch in MESO. Both the observer and feedback controller gain vectors are analytically derived by specifying the desired characteristic roots of MESO and the closed-loop system poles, respectively. The anti-windup gain can be monotonically tuned to improve the anti-windup performance. To guarantee no steady-state output error, a set-point pre-filter is introduced in terms of the desired closed-loop transfer function for set-point tracking. A sufficient condition for holding the control system stability are derived by using the delay-dependent sector condition and GFWM. The application to a benchmark example from the literature has well demonstrated the effectiveness and merit of the proposed control scheme.

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