# Adaptive Fuzzy Output-Constrained Control of Uncertain MISO Nonlinear Systems With Actuator Faults ⋆

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**Abstract:** This paper presents a novel adaptive fuzzy fault-tolerant controller for a class of uncertain multi-input single-output (MISO) nonlinear systems subject to actuator faults with user-defined time-varying asymmetric output constraints. The highlight of the proposed method is that it can tolerate the partial and total loss of effectiveness faults without the need for additional fault detection and isolation mechanism, as well as remain the system output within the predesigned time-varying output constraints during the system operation. To achieve the new results, an error transformation strategy is implemented to convert the original constrained system to an unconstrained one. For the transformed system, an effective switching function scheme is developed to search for the desired working mode by observing a switching performance index in which the impact of faulty actuators on the system can be weakened automatically. Furthermore, it is proved that the proposed adaptive fuzzy actuator fault compensation scheme can guarantee that the output is confined within the preselected bounds and all the closed-loop signals are bounded. Finally, numerical analysis confirms the merits of the proposed controller.

*Keywords:* Fault-tolerant control, Output constraints, Error transformation, Actuator faults, Uncertain MISO nonlinear systems.

## 1. INTRODUCTION

In practical control systems, it is clear that there often exist actuator faults, i.e., partial loss of effectiveness (PLOE) fault and total loss of effectiveness (TLOE) fault, which may degrade the system performance and even lead to catastrophic closed-loop system instability. Therefore, it is of great necessity to develop a fault-tolerant control (FTC) method to enhance the system reliability and safety. To address the problem of actuator faults for nonlinear systems, an active FTC strategy (He et al. (2016); Shen et al. (2018); Ijaz et al. (2019)) based on real-time information from a fault detection and isolation (FDI) unit is designed to compensate the faults by reconfiguring the controller. However, the involvement of extra FDI units not only increases the system complexity but also leads to a small probability of false alarm and missed detections. Fortunately, a passive FTC approach (Jin (2016); Wang and Zhang (2018)) based on robust control technique can make the system insensitive to actuator failure without reconstruction of the controller online. Furthermore, for passive FTC method, if the system suffers from TLOE faults, the adaptive FTC design just considering PLOE faults (Jin (2016)) will be inapplicable to handle such condition due to loss of control effort in some control channels. To avoid such cases from happening, some additional actuators are equipped to serve as backups for dealing with the TLOE faults in (Ijaz et al. (2019); Yang et al. (2015a)). Hence, it is crucial to propose an adaptive FTC scheme under multiple actuators to accommodate actuator faults without the requirement for FDI mechanism.

It is noteworthy that the FTC problem of nonlinear systems subject to output constraints or full-state constraints has attracted numerous attention because of its practical application. As a matter of fact, owing to system specifications and safety considerations, the problem of output constraints is widespread in many engineering systems (Meng et al. (2016); Li et al. (2018); Hu et al. (2018)). For instance, if the actuator fault occurs suddenly in spacecraft system (Hu et al. (2018)) during the on-orbit servicing mission, the attitudes of spacecraft will have so potentially large changes that the constraint requirements are not fulfilled, which may interfere with the system in an undesirable way or even cause fatal accident. To achieve the goal of output constraints in control designs, an error transformation strategy was first proposed (Bechlioulis and Rovithakis (2008)) and subsequently extended to serval classes of nonlinear systems (Meng et al. (2015); Fan et al. (2019)). It should be noticed that most results are only applicable to some nonlinear systems with known control gain function (Jin (2016); Li et al. (2018)), all actuators functioning healthily (Meng et al. (2016, 2015); Fan et al. (2019)) and only suffering from PLOE faults (Jin (2016)). To the authors' knowledge, how to design an adaptive

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fault-tolerant controller without FDI units for uncertain nonlinear systems with actuator faults, user-defined time-varying output constraints and external disturbance still remains an open problem.

Therefore, a novel FTC design together with an error transformation strategy is developed for uncertain MISO nonlinear systems in the presence of actuator faults, external disturbance to achieve the satisfaction of prescribed output bounds. The contributions of this paper can be summarized as: 1)A switching function method is designed to locate the desired working mode where the healthy actuators or PLOE fault actuators are activated without human's involvement. Thence, the actuator faults can be compensated and the requirement for FDI mechanism is relaxed. 2)The tracking error is always kept within the arbitrarily user-defined time-varying constraints. The guaranteed transient performance can be achieved despite actuator faults.

First, an error transformation strategy is designed to transform the output constrained problem of the original system into a traditional stability problem of an unconstrained one. Further, a novel switching function is proposed to locate the desired working mode in an automatic way and the FDI mechanism is avoided. According to the switching function scheme, a fuzzy fault-tolerant controller is established to guarantee the satisfaction of prescribed transient performance and tracking performance. Finally, semiglobal uniformly ultimate boundedness of all the closed-loop signals are ensured via Lyapunov analysis.

The remainder of this paper is structured as follows. Section 2 is devoted to actuator fault model, FLS introduction, and problem statement. The system transformation technique, a novel switching function are presented in Section 3. In Section 4, the simulation results are provided and Section 5 comes to the conclusion.

## 2. PROBLEM FORMULATION

### *2.1 Modeling and Problem Statement*

Consider the following nonlinear MISO systems:

$$\begin{aligned} \dot{x} &= f(x) + g^T(x)\,u + d(t) \\ y &= x \end{aligned} \tag{1}$$

where $x \in \mathbb{R}$ is the state of the nonlinear system. $u = [u_1, \ldots, u_m]^T \in \mathbb{R}^m$, $y$ denote the input vector and output, respectively. In order to be consistent with the actual application, the maximum control input is assumed to be limited in this paper. $f(x), g(x)$ are totally unknown smooth functions with $g(x) = [g_1(x), \ldots, g_m(x)]^T \in \mathbb{R}^m$. $d(t)$ denotes the time-varying unknown bounded disturbance such that $|d(t)| \leq \bar{d}$ with $\bar{d}$ being unknown positive constant (Yang et al. (2015b)).

Thereafter, a novel fault-tolerant control method against actuator faults subject to output constraints is proposed as in Fig. 1 to show the highlights of this paper.

Furthermore, unexpected actuator faults might occur at unknown time $t_f$ in practice during system operation, leading to the following relation between the output of actuator $u(t)$ and actuator input $\upsilon(t)$:

$$u(t) = \rho(t)\, S(t)\, \upsilon(t) + \delta(t)\,, t > t_f \tag{2}$$

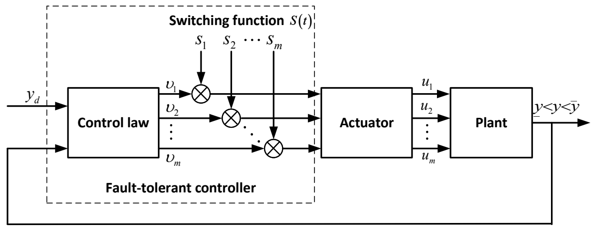


Fig. 1. An adaptive FTC method with prescribed output constraints

where $u(t) = [u_1(t), \ldots, u_m(t)]^T$, $\upsilon(t) = [\upsilon_1(t), \ldots, \upsilon_m(t)]^T$, $\rho(t) = \mathrm{diag}\{\rho_1(t), \ldots, \rho_m(t)\}$, $S(t) = \mathrm{diag}\{s_1(t), \ldots, s_m(t)\}$, $\delta(t) = [\delta_1(t), \ldots, \delta_m(t)]^T$. $0 \leq \rho_p(t) \leq 1$, $p = 1, \ldots, m$ represents the actuator efficiency. The condition $0 < \underline{\rho}_p \leq \rho_p(t) < 1$ corresponds the loss of effectiveness (LOE) fault. $\underline{\rho}_p$ is the known lower bound of $\rho_p(t)$. $\delta_p(t)$ denotes the unknown bounded time-varying bias or stuck fault. Moreover, the switching function satisfies $s_p(t) \in \{\tau_0, 1\}$ where $\tau_0 > 0$ is designed to be a sufficiently small constant which is consistent with the maximum control input of industrial systems.

The control goal is to develop a novel adaptive FTC scheme for the uncertain nonlinear system (1) such that: 1)the output $y(t)$ tracks the reference signal $y_d(t)$ while not transgressing the user-defined output constraints, that is, $\underline{y}(t) < y(t) < \bar{y}(t)$, during the entire system operation and 2)all the signals in the closed-loop system are bounded regardless of the coexistence of unknown actuator faults at unknown fault time instants, external disturbance and prescribed output constraints.

In order to obtain a feasible solution, some common assumptions are presented.

*Assumption 1.* For unknown nonlinear system (1), when $m-1$ actuators sustain stuck or outage faults, the remaining actuator undergoing LOE fault can still make the system controllable.

*Assumption 2.* The bias fault $\delta(t)$ is upper bounded such that $\|\delta(t)\| \leq \bar{\delta}$, where $\bar{\delta} > 0$ is unknown.

*Assumption 3.* The signs of the nonlinear function $g_p(x)$, $p = 1, \ldots, m$ are known, and there exist unknown positive constants $g_m$ and $g_M$ such that $0 < g_m \leq |g_p(x)| \leq g_M < \infty$. More specially, the assumption that $0 < g_m \leq g_p(x) \leq g_M < \infty$ is given.

### *2.2 Fuzzy Logic System*

Because of the existence of unknown functions, fuzzy logic system (FLS) is utilized as an online approximator to identify the unknown nonlinear dynamics. More details can be found in (Shen et al. (2014)). Then, a lemma on the FLSs is presented as follows.

*Lemma 1. (Zhai et al. (2016))* For any continuous function $f(Z)$ defined on an arbitrary compact set $\Omega_Z$ and any given positive constant $\epsilon > 0$, there exists an FLS $W^T\phi(Z)$ such that

$$\sup_{Z \in \Omega_Z} \left| f(Z) - W^T\phi(Z) \right| \leq \epsilon$$

where $Z$ denotes the input vector to the FLS and $W = [w_1, \ldots, w_N]^T \in \mathbb{R}^N$ represents the ideal weight vector with $N$ being the total number of rules. $\phi(Z) = [\phi_1(Z), \ldots, \phi_N(Z)]^T \in \mathbb{R}^N$ is the fuzzy basis function vector .

Apparently, $W$ is bounded such that $\|W\| \le \omega_M$ (Hu et al. (2017)) and that the approximation error $\epsilon$ satisfies $|\epsilon| \le \epsilon_M$ (Hu et al. (2017)), with $\omega_M$, $\epsilon_M$ being unknown positive constants. To cope with the unknown control gain function $g_p(\cdot)$, the variable $\vartheta$ is introduced as $\vartheta = \frac{1}{g_m}\|W\|^2$.

## 3. CONTROL METHODOLOGY

### 3.1 System Transformation

The system tracking error is derived as

$$e(t) = y(t) - y_d(t) \tag{3}$$

Then, by substituting the time-varying output constraints into (3), one has

$$\underline{e}(t) < e(t) < \bar{e}(t) \tag{4}$$

where $\underline{e}(t) = \underline{y}(t) - y_d(t)$, $\bar{e}(t) = \bar{y}(t) - y_d(t)$. Hence, the output constraints are converted to the error constraints given by (4).

In order to guarantee the prescribed transient performance, an error transformation algorithm is presented to transform the origin constrained tracking error into an unconstrained one. Thereafter, a function $T(\varsigma(t), \underline{e}(t), \bar{e}(t))$ is introduced as

$$e(t) = T(\varsigma(t), \underline{e}(t), \bar{e}(t)) \tag{5}$$

$$\varsigma(t) = T^{-1}(e(t), \underline{e}(t), \bar{e}(t)) \tag{6}$$

where $\varsigma(t)$ is the transformed error, and $T(\cdot)$ denotes a smooth invertible function with satisfying $\partial T(\cdot)/\partial\varsigma > 0$. In this paper, the function $T$ is selected as

$$T = \frac{\bar{e}(t) - \underline{e}(t)}{\pi}\arctan(\varsigma(t)) + \frac{\bar{e}(t) + \underline{e}(t)}{2} \tag{7}$$

By recalling (6), the new unconstrained error $\varsigma(t)$ is obtained as

$$\varsigma(t) = \tan\left(\frac{\pi}{2}\frac{2e(t) - \bar{e}(t) - \underline{e}(t)}{\bar{e}(t) - \underline{e}(t)}\right) \tag{8}$$

By combining (5) and (7), the following fact holds on:

$$\begin{cases} \lim\limits_{\varsigma\to-\infty} e(t) = \underline{e}(t) \\ \lim\limits_{\varsigma\to+\infty} e(t) = \bar{e}(t). \end{cases} \tag{9}$$

From (9), it is noted that as long as $\varsigma(t)$ is bounded, we have

$$\underline{e}(t) < e(t) < \bar{e}(t) \tag{10}$$

Adding $y_d(t)$ to every side of (10), we get $\underline{y}(t) < y(t) < \bar{y}(t)$, which indicates that the system output can be always maintained in the predesigned region. Thus, the boundedness of the transformed error $\varsigma(t)$ is sufficient to guarantee the time-varying output constraints.

Now, the transformed closed-loop system dynamics is given by

$$\begin{aligned} \dot{\varsigma} &= \kappa\left[f(x) + g^T(x)(\rho S v + \delta) + d(t) - \dot{y}_d(t)\right] + \varsigma_d \\ y &= x \end{aligned} \tag{11}$$

where $\kappa = \partial\varsigma/\partial e > 0$ and $\varsigma_d = (\partial\varsigma/\partial\underline{e})\,\dot{\underline{e}} + (\partial\varsigma/\partial\bar{e})\,\dot{\bar{e}}$ are available as feedback signals since the signals $\varsigma, e, \underline{e}, \bar{e}, \dot{\underline{e}}$ and $\dot{\bar{e}}$ are known.

### 3.2 Switching Function Design

In this section, a novel switching function scheme is developed to minimize the influence of faulty actuators for achieving the control goal. Before proceeding, some definitions are provided.

*Definition 1.* According to $m$ actuators, we number $m$ actuators in sequence and the total number of fault patterns is $M = 2^m - 2$. The $\mu$th actuator fault pattern $\sigma_{(\mu)}$, $\mu = 1, \ldots, M$ is defined as

$$\sigma_{(\mu)} = \left\{\sigma_{\mu 1}, \sigma_{\mu 2}, \ldots, \sigma_{\mu l_\mu}\right\} \tag{12}$$

where $\sigma_{\mu 1}, \ldots, \sigma_{\mu l_\mu} \in \{1, \ldots, m\}$ represents $p$th actuator. Therefore, after the actuator fault occurrence time $t_f$, the corresponding switching functions $s_{\sigma_{\mu 1}} = \cdots = s_{\sigma_{\mu l_\mu}}$ are set $\tau_0$ to minimize impact of the faulty actuators.

*Definition 2.* For a system with $m$ actuators, the number of working modes is $L = 2^m - 1$ and an overall working mode set is generated as

$$\begin{aligned} \varPi &= \left\{\hat{S}_1, \hat{S}_2, \ldots, \hat{S}_L\right\} \\ &= \{\mathrm{diag}\{\tau_0, \tau_0, \ldots, \tau_0, 1\}, \mathrm{diag}\{\tau_0, \tau_0, \ldots, 1, \tau_0\}, \ldots, \\ &\qquad \mathrm{diag}\{1, 1, \ldots, 1, 1\}\} \end{aligned} \tag{13}$$

*Definition 3.* For the fault pattern $\sigma_{(\mu)}$, the undesired working mode set comes that

$$\bar{\varPi}_{(\mu)} = \left\{\bar{S}_{\mu 1}, \bar{S}_{\mu 2}, \ldots, \bar{S}_{\mu \bar{L}_\mu}\right\} \tag{14}$$

where $\bar{L}_\mu$ stands for the number of undesired working modes, and each mode $\bar{S}_{\mu\alpha} = \mathrm{diag}\{\bar{s}_{\mu\alpha 1}, \ldots, \bar{s}_{\mu\alpha m}\}$, $\alpha = 1, \ldots, \bar{L}_\mu$ satisfies

(1) Only $\rho_p \neq 0$, $p = 1, \ldots, m$ actuator faults model is considered:

$$\bar{\varPi}_{(\mu)} = \emptyset$$

(2) The $\rho_p = 0$ actuator fault model is included:

$$\bar{s}_{\mu\alpha p} = \begin{cases} 1 & \rho_p = 0 \\ \tau_0 \text{ or } 1 & \text{else} \end{cases}$$

*Definition 4.* According to the overall and undesired working mode sets, the set containing all the desired working modes is given by

$$\varPi_{(\mu)} = \varPi - \bar{\varPi}_{(\mu)}. \tag{15}$$

Based on the aforementioned definitions, an adaptive automatic switching scheme is employed to avoid the complexity of FDI mechanism. And the switching performance index $\theta$ is defined as

$$\dot{\theta} = k_\theta N(\varsigma), \theta(0) = 0.001 \tag{16}$$

where $k_\theta$ is a positive constant, $N(\varsigma)$ is a smooth "dead-zone" type function which is designed as

$$N(\varsigma) = \begin{cases} 0, |\varsigma| \le \hbar \\ (|\varsigma| - \hbar)^2, |\varsigma| > \hbar \end{cases} \tag{17}$$

$\hbar$ is a constant to be determined later. And the switching function is described as

$$S(t) = S(\theta(t)) = \hat{S}_\eta \in \varPi \tag{18}$$

where the switching function index $\eta$ is presented by

$$\eta = \lceil \mathrm{mod}\,(\theta, L) \rceil \tag{19}$$

$\lceil\cdot\rceil$ is the ceiling function, and $\mathrm{mod}(\cdot)$ is the residual operator.

Actually, the overall working mode set serves as a working mode pool where there is at least one desired working mode. When the TLOE faults occur, the switching function index $\theta$ being the function of the transformed error $\varsigma$ will move continuously to find a working mode $\Pi_{(\mu)}$ in the working mode pool $\Pi$. In the whole search process, there are two scenarios that the system operates in the undesired working mode and in the desired working mode. If $S(t)$ takes a working mode $\bar{\Pi}_{(\mu)}$ in the event of actuator fault(s), sufficiently weak control effort of all actuators can hardly actuate the control system such that the deteriorating tracking performance will be generated. Hence, the index $\theta$ will drive the index $\eta$ to the next integer continuously until the $\Pi_{(\mu)}$ is located. Conversely, when a working mode $\Pi_{(\mu)}$ is picked, the healthy actuators or the actuators suffering from PLOE faults will be activated and force the output to move towards the reference signal. Thence, the index $\theta$ will remain unchanged, that is, the system stays in the desired working mode all the time, to guarantee the predefined tracking performance and transient performance. By this means, the index $\theta$ can vary continuously and the proposed switching function approach eventually locates a working mode $\Pi_{(\mu)}$ in the overall working mode set $\Pi$ to maintain an acceptable system performance.

### *3.3 Adaptive Fault-Tolerant Control Law*

The adaptive control law $\upsilon_p$ is designed as

$$\upsilon_p = -\frac{1}{\underline{\rho}_p \kappa}\left[k_0\varsigma + \frac{1}{2r^2}\varsigma\hat{\vartheta}\phi^T(z)\,\phi(z)\right] \tag{20}$$

where $\underline{\rho} = \min\left\{\underline{\rho}_p, p = 1, \ldots, m\right\}$ with $0 < \underline{\rho}_p < 1$ being a known constant in (2) and $k_0, r$ are positive constants.

The adaptive law for $\hat{\vartheta}$ is tuned as

$$\dot{\hat{\vartheta}} = \frac{l}{2r^2}\tau_0\varsigma^2\phi^T(z)\,\phi(z) - \gamma\hat{\vartheta} \tag{21}$$

where $l, \gamma$ are positive design parameters, and $\hat{\vartheta}(0) > 0$ is the arbitrarily chosen initial estimate of variable $\vartheta$, which can always keep $\hat{\vartheta}$ positive. Define the estimation error as $\tilde{\vartheta} = \vartheta - \hat{\vartheta}$.

### *3.4 Stability Analysis*

Eventually, according to the previous analysis, the main results of this paper are stated in the following theorem.

*Theorem 1.* Consider the transformed closed-loop system (11) under Assumption 1-3, let fault-tolerant control law be provided by (20) with adaptive law (21). Then, the system output $y(t)$ can track its reference signal $y_d(t)$ without violation of the time-varying output constraints, $\underline{y}(t) < y(t) < \bar{y}(t)$, despite actuator faults. Furthermore, all the closed-loop signals remain bounded.

*Proof:* Consider the following Lyapunov function candidate

$$V = \frac{1}{2}\varsigma^2 + \frac{g_m}{2l}\tilde{\vartheta}^2. \tag{22}$$

The time derivative of $V$ is given by

$$\begin{aligned}\dot{V} =& \varsigma\kappa\left[f(x) + g^T(x)(\rho S\upsilon + \delta) + d(t) - \dot{y}_d(t)\right]\\ &+ \varsigma\varsigma_d - \frac{g_m}{l}\tilde{\vartheta}\dot{\hat{\vartheta}}.\end{aligned} \tag{23}$$

Based on Assumption 2 and 3, Young's inequality is employed, and we have

$$\varsigma\kappa g^T(x)\,\delta \le \frac{1}{2}\varsigma^2\kappa^2 + \frac{1}{2}mg_M^2\bar{\delta}^2 \tag{24}$$

$$\varsigma\kappa d(t) \le \frac{1}{2}\varsigma^2\kappa^2 + \frac{1}{2}\bar{d}^2. \tag{25}$$

Substituting (24) and (25) into (23) generates

$$\begin{aligned}\dot{V} \le& \varsigma\left[\kappa f(x) + \varsigma\kappa^2 - \kappa\dot{y}_d + \varsigma_d\right] + \varsigma\kappa g^T(x)\,\rho S\upsilon\\ &- \frac{g_m}{l}\tilde{\vartheta}\dot{\hat{\vartheta}} + \frac{1}{2}mg_M^2\bar{\delta}^2 + \frac{1}{2}\bar{d}^2\end{aligned} \tag{26}$$

Due to the approximation property of FLSs, the unknown function $f(x)$ is handled by FLSs and suppose that there exists an ideal fuzzy approximation on a compact set $\Omega_z$ such that

$$Q(z) = \kappa f(x) + \varsigma\kappa^2 - \kappa\dot{y}_d + \varsigma_d = W^T\phi(z) + \epsilon \tag{27}$$

where $z = \left[x, \varsigma, e, \underline{e}, \dot{\underline{e}}, \bar{e}, \dot{\bar{e}}, \dot{y}_d\right]^T$ is the input vector to FLS. Furthermore, according to Young's inequality, we can get

$$\begin{aligned}\varsigma\left[W^T\phi(z) + \epsilon\right] \le& \frac{1}{2r^2}g_m\varsigma^2\vartheta\phi^T(z)\,\phi(z) + \frac{1}{2}r^2\\ &+ \frac{1}{2}\varsigma^2 + \frac{1}{2}\epsilon_M^2.\end{aligned} \tag{28}$$

Substituting (27), (28) into (26) yields

$$\begin{aligned}\dot{V} \le& \frac{1}{2}\varsigma^2 + \frac{1}{2r^2}g_m\varsigma^2\vartheta\phi^T(z)\,\phi(z) + \varsigma\kappa g^T(x)\,\rho S\upsilon\\ &- \frac{g_m}{l}\tilde{\vartheta}\dot{\hat{\vartheta}} + D\end{aligned} \tag{29}$$

where $D = (1/2)\left(mg_M^2\bar{\delta}^2 + \bar{d}^2 + r^2 + \epsilon_M^2\right)$.

Based on the mechanism of switching function scheme, we shall consider two scenarios for the system.

(i) When the desired working mode is chosen at time $t_i$, i.e., $S(\theta(t_i)) = S(\theta) \in \Pi_{(\mu)}$, it implies that the control effort of healthy or faulty actuators for bias and LOE fault can be delivered to reach the system. Therefore, one has

$$\begin{aligned}\dot{V} \le& -k_1\varsigma^2 + \frac{1}{2r^2}g_m\varsigma^2\left(\vartheta - \hat{\vartheta}\right)(1 - \tau_0)\,\phi^T(z)\,\phi(z)\\ &+ \frac{g_m\gamma}{l}\tilde{\vartheta}\hat{\vartheta} + D\\ \le& -k_1\varsigma^2 + \frac{g_m}{2r^2}\varsigma^2\vartheta\phi^T(z)\,\phi(z) - \frac{g_m\gamma}{2l}\tilde{\vartheta}^2 + \frac{g_m\gamma}{2l}\vartheta^2 + D\\ \le& -\left(k_1 - \frac{1}{2r^2}\omega_M^2\right)\varsigma^2 - \frac{g_m\gamma}{2l}\tilde{\vartheta}^2 + \frac{g_m\gamma}{2l}\vartheta^2 + D\\ \le& -C_1V + E_1\end{aligned} \tag{30}$$

where $k_1 = k_0g_m - 1/2$, $C_1 = \min\left\{2k_1 - \left(1/r^2\right)\omega_M^2, \gamma\right\}$, $E_1 = \left(g_m\gamma/2l\right)\vartheta^2 + D$.

By combining (22) and (30), one has

$$\frac{1}{2}\varsigma^2 \le \frac{E_1}{C_1} + V(t_i)\,e^{-C_1(t-t_i)} \tag{31}$$

Furthermore, we can obtain

$$|\varsigma| \le \sqrt{\frac{2E_1}{C_1}} + \sqrt{2}V^{\frac{1}{2}}(t_i)\,e^{-\frac{C_1}{2}(t-t_i)}. \tag{32}$$

The parameter $\hbar$ can be designed as $\hbar \geq \sqrt{2E_1/C_1}$. Thence, for $|\varsigma| > \hbar$, the switching performance index $\theta$ is obtained as

$$\dot{\theta} = k_\theta (|\varsigma| - \hbar)^2 \leq 2k_\theta V(t_i) e^{-C_1(t-t_i)} \tag{33}$$

which means that $\theta$ converges to a positive constant.

(ii) When an undesired working mode is chosen at time $t_j$, it indicates that the healthy actuators are turned off and the control effort of faulty actuators is minimized such that the system dynamics is not influenced. Hence, one has

$$\begin{aligned}\dot{V} \leq & k_2\varsigma^2 + \frac{g_m}{2r^2}\varsigma^2\vartheta(1-\tau_0)\phi^T(z)\phi(z) + \frac{g_m\gamma}{l}\tilde{\vartheta}\hat{\vartheta} + D \\ \leq & k_2\varsigma^2 + \frac{g_m}{2r^2}\varsigma^2\vartheta\phi^T(z)\phi(z) - \frac{g_m\gamma}{2l}\tilde{\vartheta}^2 + \frac{g_m\gamma}{2l}\vartheta^2 + D \\ \leq & \left(k_2 + \frac{1}{2r^2}\omega_M^2\right)\varsigma^2 + \frac{g_m\gamma}{2l}\tilde{\vartheta}^2 + \frac{g_m\gamma}{2l}\vartheta^2 + D \\ \leq & C_2 V + E_2\end{aligned} \tag{34}$$

where $C_2 = \max\left\{2k_2 + (1/r^2)\omega_M^2, \gamma\right\}$, $E_2 = E_1$, $k_2 = 1/2 - k_0 g_m \tau_0 > 0$.

According to the above stability analysis of undesired working mode, there are two possibilities: $\int_{t_j}^{+\infty}\dot{\theta}dt \leq 1$ still holds on $[t_j, +\infty)$, and the condition $\int_{t_j}^{+\infty}\dot{\theta}dt > 1$ achieves and consequently, the switching performance index $\theta$ will increase until a right working mode $\Pi_{(\mu)}$ is found.

Next, based on the automatic switching mechanism, the $S(t)$ can enter the desired working mode a time $t_i$ and remains in it all the time, that is, the following inequality is obtained as

$$\int_{t_i}^{+\infty}\dot{\theta}dt \leq 1. \tag{35}$$

For $0 \leq t < t_i$, the undesired working modes will be chosen, one has $\dot{V} \leq C_2V + E_2$. Then, we have

$$\begin{aligned}V(t_i) \leq & e^{C_2 t_I}V(0) + \frac{E_2}{C_2}e^{C_2 t_i} - \frac{E_2}{C_2} \\ \leq & e^{C_2 t_i}V(0) + \frac{E_2}{C_2}e^{C_2 t_i}.\end{aligned} \tag{36}$$

Substituting (36) into (33) yields

$$\begin{aligned}\dot{\theta} \leq & 2k_\theta V(t_i) e^{-C_1(t-t_i)} \\ \leq & k_\theta\left(2V(0) + \frac{2E_2}{C_2}\right)e^{-C_1 t + (C_1+C_2)t_i}\end{aligned} \tag{37}$$

Thereafter, taking the integration of (37) on $[t_i, +\infty)$ generates

$$\begin{aligned}\int_{t_i}^{+\infty}\dot{\theta}(t)\,dt \leq & \int_{t_i}^{+\infty} k_\theta\left(2V(0) + \frac{2E_2}{C_2}\right)e^{-C_1 t + (C_1+C_2)t_i}dt \\ = & k_\theta \frac{2C_2V(0) + 2E_2}{C_1C_2}e^{C_2 t_i}.\end{aligned} \tag{38}$$

Choose $k_\theta \leq C_1C_2/[(2C_2V(0) + 2E_2)\mathcal{N}]$ with $\mathcal{N} > 0$ being a design parameter. Obviously, there must exist a sufficiently large $\mathcal{N}$ such that $e^{C_2 t_i} \leq \mathcal{N}$, that is, (35) is satisfied.

Combining (i) and (ii), there exists the time $t_i$ such that the switching function can stay within a desired working mode and never comes out, i.e., $\int_{t_i}^{+\infty}\dot{\theta}(t)\,dt \leq 1$. According to Barbalat's Lemma and (33), one has $\lim_{t\to\infty} N(\varsigma) = 0$, i.e., $\lim_{t\to\infty}|\varsigma| \leq \hbar$. Furthermore, referring to the standard Lyapunov analysis and error transformation technique, the aforementioned analysis demonstrates uniformly ultimately boundedness of tracking error $e$. The other variables $y = x, \hat{\vartheta}$ and the control input $u$ are also bounded. Therefore, all the signals in the closed-loop system are guaranteed to be bounded.

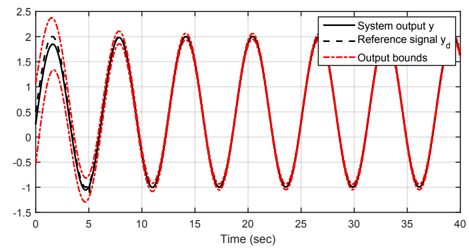


Fig. 2. Output tracking performance of the system

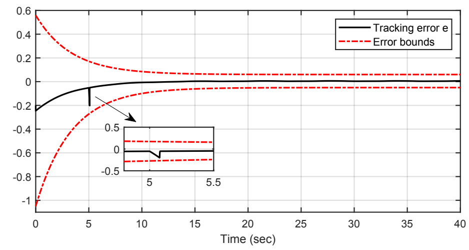


Fig. 3. Tracking error $e$ with performance requirement

## 4. SIMULATION STUDY

In this section, a simulation study is provided to evaluate the control performance of the adaptive FTC method for a first-order nonlinear system given by

$$\begin{aligned}\dot{x} &= 0.5\sin(x)e^{-x} + (2+\cos(x))(u_1+u_2) + d(t) \\ y &= x\end{aligned} \tag{39}$$

with the external disturbance $d(t) = 0.06\sin(0.5t)$, where $x$ is the state, and $u_1, u_2, y$ denote the control input and output of the system, respectively.

Suppose that the actuator #2 suffers from a stuck fault at $t_f = 5s$ with $u_2(t) = 0.1\sin(t), t > t_f$. According to the Definition 1-4, the desired working mode consists of two working modes $S_{21} = \mathrm{diag}\{1, \tau_0\}$ with $\tau_0 = 10^{-6}$, $S_{22} = \mathrm{diag}\{1,1\}$. Moreover, the reference signal is selected as $y_d(t) = 0.5 + 1.5\sin(t)$ and the prescribed output bounds are described by $\underline{y}(t) < y(t) < \bar{y}(t)$ with $\underline{y}(t) = -e^{-0.3t} - 0.05 + y_d$, $\bar{y}(t) = 0.5e^{-0.3t} + 0.06 + y_d$. The initial conditions are given as $k_\theta = 20, \hbar = 0.06, k_0 = 150, r = 10, \gamma = l = 1$.

The system output performance of nonlinear system (39) in Fig. 2 demonstrate that it can track the reference signal regardless of actuator faults and external disturbance. The simulation result in Fig. 3 illustrates that the tracking error can always reside within the predefined time-varying error constraints. Additionally, as observed in Fig. 4, the transformed error is bounded in the simulation interval such that the prescribed performance can be guaranteed in terms of $e(t)$ given by in (10). From Fig. 5, when actuator #2 undergoes stuck fault, the index $\theta$ changes with sudden increase to drive the index $\eta$ to reach 2 before violating the tracking error bounds.

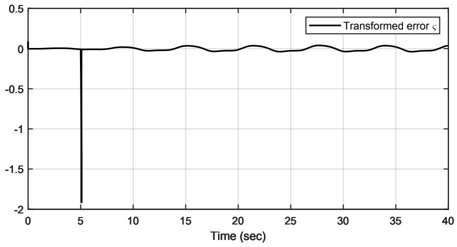


Fig. 4. Transformed error $\varsigma$

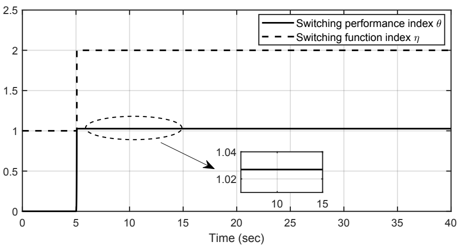


Fig. 5. Switching index $\theta$ and $\eta$

## 5. CONCLUSION

In this paper, an novel fuzzy FTC method is proposed to accommodate actuator faults for uncertain nonlinear MISO systems while forcing the output to reside in user-defined output constraints. By employing an error transformation technique, the stability of the equivalently unconstrained system is sufficient to address the output constraints of the original system. The requirements for a *prior* knowledge of system dynamics, FLS approximation error and external disturbance have been relaxed. The desired working mode can be found without the need for FDI design via the designed switching function scheme. In the desired working mode, the actuator faults compensation, the desired tracking performance and the time-varying output bounds can be guaranteed simultaneously. All the signals of the closed-loop system is proved to be bounded. The problem of FTC design with output constraints for nonlinear systems subject to infinite number of actuator faults will be investigated in the future work.

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