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Constrained Transactive Control in Power Systems Based on Population Dynamics

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Abstract: The power system has gone through an evolutionary process towards a smart grid, this process is a challenge for the system operator, these challenges are related to implementation in real time, as well as problems with the control and stability of the system. We propose a distributed transactive control algorithm based on population games to dynamically manage the distributed generators and smart loads in the system to reach the optimum social welfare. The proposed algorithm preserve stability and guarantee optimality conditions considering several constraints on the real-time operation. Loads are modeled flexible and base loads. Stability analysis and Nash equilibrium of the proposed game is studied by means of potential games concepts. Simulation results of the proposed algorithm shows the stability and convergence of the proposed algorithm.

Keywords: Distributed control, distributed optimization, transactive control, game theory, population dynamics.

1. INTRODUCTION

Centralized controllers often suffer from serious computation, communication, and robustness issues in power systems with so many devices (Knudsen et al., 2016). Distributed control is becoming a viable strategy to handle these issues (Quijano et al., 2017). In order to be a compelling solution, distributed control has to satisfy several technical requirements. The first requirement is the proper coordination among different devices used at low levels such as traditional generation units, renewable generation units, and smart loads (Kok and Widergren, 2016). Distributed controllers have to be able to provide stability and robustness margins for the entire system in the presence of uncertainty and offering some optimality guarantee on real-time behavior (Lamnabhi-Lagarrigue et al., 2017). Finally, distributed controllers rely on a communication infrastructure to work. Several devices of the power system can communicate with each other exchanging information through a communication network integrated into the energy network (Mojica-Nava et al., 2015; Marden and Shamma, 2015).

Recently, multiple demand response strategies have proposed to deal with generation and load variations balance (Knudsen et al., 2016; Shiltz et al., 2016), among these strategies, transactive control is emerging as a strong contender for the coordinated operation of so many devices. Transactive control, also called market-based control, is a strategy that uses a market mechanism to enable actors to interact with each other through an economic signal to allocate the available resources (Bejestani et al., 2014; Shiltz et al., 2016). Most of the results of transactive control are focused on the transmission level of the electrical infrastructure (Bejestani et al., 2014; Shiltz et al., 2016). However, a scalable coordination approach to distribution systems operation could present advantages over traditional control strategies. Many methods can be used to solve the transactive problem in a centralized manner. In general, iteration methods such as projected gradient methods (Baron-prada et al., 2018; Guo et al., 2016), and many others have been extensively used (Bertsekas, 1999). However, coordination of the involved agents in this more complex large-scale networked paradigm of the smart grids is a more challenging task. Besides, when agents change locally, the central system must be reconfigured to solve the new problem. Finally, a centralized optimizer could be susceptible to a single point of failure. In response to the limitations of centralized methods, in the last decades, there has been an increasing interest in the development of decentralized optimization strategies (Nedic and Ozdaglar, 2009; Yang and Johansson, 2010; Lakshmanan and De Farias, 2008). One of the first algorithm implemented in a distributed way is the center-free algorithm (Ho et al., 1980). Since then, several distributed optimization algorithms have been proposed for distributed resource allocation problems. Most of these algorithms are based on the seminal work in (Tsitsiklis, 1984). These algorithms are based on reaching a consensus on their estimates of an optimal solution based on local information. Despite the recent success of such algorithms, some limitations arise when they have to be implemented in real-time operation such as lack of robustness to environmental variations, the dependency of the synchronization of information between agents, or the requirements of a two-time scale solution. In contrast, game-theoretical methods have emerged as a strong possibility to coordinate the operation of these networked devices (Quijano et al., 2017). Several benefits can be exploited such as real-time

⋆ The research reported in this publication was done while the author was student of the Universidad Nacional de Colombia.

adaptation and robustness to dynamic variation in environmental conditions (Marden and Shamma, 2015).

In this paper, we propose a distributed dynamic transactive control algorithm for efficient integration of generators and smart loads in a microgrid. The transactive microgrid control is based on a population games approach which solves a social welfare optimization problem. In particular, a distributed replicator dynamics algorithm is implemented. In the proposed approach, the algorithm preserves stability while guaranteeing some optimality conditions on the real-time operation. The equilibrium of the social welfare problem characterizes the outcomes of the interaction of agents with conflicting objectives. However, there is still a hierarchy conflict between the utility of the individual and the system-level objective. The proposed population game approach dynamically obtains the socially optimal equilibrium while resolving this hierarchical conflict.

The rest of the paper is organized as follows. Section 2 describes the modeling of agents in the power system, and the social welfare optimization is presented. Section 3 describes the main conceptual framework for the distributed transactive control algorithm based on the distributed replicator dynamics. In Section 4, simulations results are presented to illustrate the effectiveness of the proposed algorithm. Finally, in Section 5, some conclusions and future work are drawn.

2. PROBLEM STATEMENT

We consider a transactive grid where there are two types of agents, generators and smart loads. The network is represented by a graph, where the nodes represent any agent in the power system.

2.1 Preliminaires: Graph Theory

Consider the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, $\mathcal{G}$ is connected, undirected and unweighted (Bullo, 2019). Let i and j be any agent of the graph $\mathcal{G}$. Furthermore, $\mathcal{V} = \{1, \dots, V\}$ denote the set of agents in the network, the nodes are represented by semicircles blues and reds as is shown in Fig. 1, V denote the total number of agents in the power network. Moreover, $\mathcal{E}$ represents the set of links between agents in the network. If $(i, j) \in \mathcal{E}$, then it is a link between i and j. The black lines in the Fig 1 represent the links. The neighborhood of i-th agent is denoted as $\mathcal{N}_i$, and is defined as the set of nodes that have communication links to the i-node.

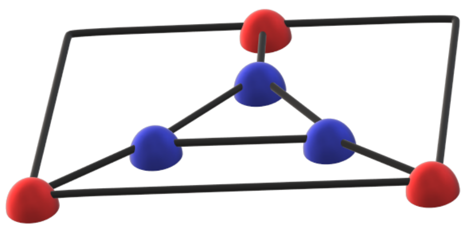

Fig. 1. Graph based on WSCC 9 bus power system with three generators (red) and three consumers (blue).

2.2 Model of Smart Loads and Generators

Smart loads are those that can adjust its consume, taking into account the information given by the network. This kind of load has several states, and it chooses one of them depending on its utility through its objective function, an example of this type of load is the air conditioning. The set of smart loads is denoted as $m \in \mathcal{M} = \{1, \dots, M\}$. Smart loads are assumed to be agents who can obtain some system variables through demand response devices. These devices are capable of making decisions about the amount of power required by each consumer to maximize their utility (Palensky and Dietrich, 2011). The consumers' utility function is defined as (Bejestani et al., 2014)

$$U(P_{d_m}) = \rho_{d_m} P_{d_m} + \frac{\beta_{d_m}}{2} P_{d_m}^2, \tag{1}$$

where $\rho_d \in \mathbb{R}^M$ and $\beta_d \in \mathbb{R}^M$ are utility coefficients, $P_d(k) \in \mathbb{R}^M$ is the vector that contains the load demanded by each smart consumer at time instant k, $P_d(k) = [P_{d_1}(k), \dots, P_{d_M}(k)]^\top$. The agent m-th has local constraints given by

$$\overline{P}_{d_m} \geq P_{d_m} \geq \underline{P}_{d_m}, \tag{2}$$

where $\underline{P}_{d_m}$ and $\overline{P}_{d_m}$ are the minimum and maximum power demanded by the load m-th respectively. $\underline{P}_{d_m}$ is also called *base load*, the base load is the minimum load that each consumer needs and it is not part of the power negotiation with the power system.

2.3 Model of Smart Loads and Generators

Furthermore, we define a generator as n. The set of generators is denoted as $\mathcal{N} = \{1, \dots, N\}$, where N is the number of generators in the network. Moreover, the cost of generation is defined as quadratic (Bejestani et al., 2014; Shiltz et al., 2016), i.e.,

$$C(P_{g_n}) = \rho_{g_n} P_{g_n} + \frac{\beta_{g_n}}{2} P_{g_n}^2 \tag{3}$$

where $\rho_g \in \mathbb{R}^N$ and $\beta_g \in \mathbb{R}^N$ are cost coefficients, $P_g(k) \in \mathbb{R}^N$ is a vector that contains the power delivered by each generator at time instant k, $P_g(k) = [P_{g_1}(k), \dots, P_{g_N}(k)]^\top$. Moreover, each generator has local constraints given by

$$\overline{P}_{g_n} \geq P_{g_n} \geq \underline{P}_{g_n}, \tag{4}$$

where $\underline{P}_{g_n}$ and $\overline{P}_{g_n}$ are the minimum and maximum power delivered by the n-th generator, respectively. We assume that $\underline{P}_{g_n}$ and $\overline{P}_{g_n}$ do not have changes through the time.

2.4 Social Welfare Optimization Problem

The social welfare problem, i.e., to maximize the benefits of the consumers and to minimize the cost of the generators. It is based on social welfare function obtained from (1), (3), and it is defined as

$$S_W(P_d, P_g) = U\left(P_d\right) - C\left(P_g\right). \tag{5}$$

The main goal is to maximize (5) as it is shown in the following optimization problem

Problem 1.

$$\underset{P}{\text{maximize}} \quad S_W(P) \tag{6a}$$

$$\text{subject to} \quad \sum_{n=1}^{N} P_{g_n} + \sum_{m=1}^{M} P_{d_m} = P_L, \tag{6b}$$

$$\overline{P}_{g_n} \geq P_{g_n} \geq \underline{P}_{g_n} \quad \forall\, n \in \mathcal{N}, \tag{6c}$$

$$\overline{P}_{d_m} \geq P_{d_m} \geq \underline{P}_{d_m} \quad \forall\, m \in \mathcal{M}, \tag{6d}$$

where $P = [P_d, P_g]^\top$, losses are assumed as a constant power demand P_L that is calculated through Kron's Algorithm as is shown in the following subsection .

2.5 Network Losses Model

Network losses in a distribution system are factors that can be determinant to reach the optimum social welfare. To get a better solution and given that it is not possible to solve the system in all possible cases of demand and power generation (Zhu, 2015), we use an estimation of this power loss given by the Kron's formula as follows

$$P_L = \sum_{k=1}^{N_k} \sum_{l=1}^{N_l} p_k B_{kl} p_l + \sum_{k=1}^{N_k} B_{k0} p_k d + B_{00}, \tag{7}$$

where $B = [B_{ll}]$, B_0, and B_{00} are the loss factors, N_l is the number of generator considered in the estimation algorithm. In order to calculate an estimation of the network losses the algorithm in (Mojica-Nava et al., 2017) is used. This algorithm can calculate loss factors which give us a closeness estimation of the losses. However, it is important to highlight that the lines will be considered but only considering active power, i.e., only the resistive component of the line is considered. As a result of this estimation algorithm, it is obtained an average constant losses power P_L to be included in the algorithm as part of the load to be dispatched.

3. DISTRIBUTED TRANSACTIVE CONTROL ALGORITHM

In this section, it is presented the main contribution of this paper. A distributed control algorithm considering distributed energy integration and flexible power demand using transactive control based on population games is developed. Transactive control is responsible for the determination of how much energy each user will consume and how much power each generator have to generate to satisfy the demand while operating the system most economically.

Constraints in (6) could be represented in the form $\mathbf{H}^\top P = P_L$, where the decision variable are stacked in a vector as $P^\top = [P_g, P_d]^\top = [P_1, \ldots, P_{N+M}]^\top$, and $\mathbf{H}$ is a vector indicating the sign depending if the entry is a generator (+) or a smart load (-). In this work, we use a distributed replicator dynamics equation (Quijano et al., 2017) to solve the social welfare problem dynamically associated with a transactive control problem between microgrids, distribution system, and consumers. This model is a dynamic resource allocation problem among V agents (M generators and N smart loads).

The feasible set is defined as the set of possible trajectories of the system restricted by Problem 1 constraints as

$$\Delta = \{P \in \mathbf{R}^V : \mathbf{H}^\top P = P_L\}.$$

Some assumptions have to be considered to assure optimal behavior and the existence of solutions of Problem 1.

***Assumption** 1.* It is assumed that the variables start inside the feasible set Δ, i.e., $P(0) \in \Delta$.

***Assumption** 2.* The communication graph connecting each agent of the system, generators and smart loads, in the distribution system is connected.

***Assumption** 3.* Every social welfare function $S_{Wi}(P_i)$ is assumed to be differentiable everywhere and with Lipschitz continuous derivative.

In the next subsection, it is introduced the basic concepts of population games, which are the cornerstone of the proposed algorithm.

3.1 Population Games Basics

Population games describe how a mass of agents evolve in time while they choose a strategy. Those agents follow a strategy selected from among a finite set of pure strategies. Due to the size of the population, the analysis focuses on a utility function associated with the chosen strategy. In population games, replicator dynamics has been used in several engineering application considering some benefits in implementation such as real-time adaptation and robustness to dynamic environmental uncertainties (Quijano et al., 2017; Mojica-Nava et al., 2015, 2017).

The dynamic population model describes how a pure population strategy changes through time. The replicator dynamics consider a N finite number of generators and M limited number of consumers in the system, who adopt a i-th strategy from a finite set of pure strategies. Accordingly, to achieve an appropriate performance in steady-state, the demanded power load should be the sum of all power setpoints (Mojica-Nava et al., 2015). In its general form, the distributed replicator dynamics equation can be represented as

$$\dot{P}_i = \left(\frac{1}{P_L}\right) P_i \left(F_i(P_i) \sum_{j \in \mathcal{N}_i} P_j - \sum_{j \in \mathcal{N}_i} P_j F_j(P_j) \right), \tag{8}$$

where F_i is the fitness function associated with each strategy, and $\mathcal{N}_i$ is the neighborhood of agent i sharing information through a communication network. In Quijano et al. (2017), we can find the simplex invariance for the distributed replicator dynamics, which means that if $P(0) \in \Delta$, the dynamic variable $P(t)$ evolves inside Δ and reaches an equilibrium point as is stated in Theorem 1.

The fundamental concept in the design of the population games is the appropriate selection of the fitness function such that the distributed replicator dynamic equation (8) can be used as a constrained distributed optimization algorithm (Barreiro-Gomez et al., 2017). In order to obtain fitness functions to accomplish a stable and convergence solution, we use Lemma 1, which characterizes an optimal solution to Problem (1) if the fitness functions fulfill Assumption 3 (Lamnabhi-Lagarrigue et al., 2017).

Lemma 1. A solution of Problem 1, P^* belonging to the feasible set Δ, is an optimal solution if and only if $\nabla U_i(P_i^*) = \nabla U_j(P_j^*)$ and $\nabla C_i(P_i^*) = \nabla C_j(P_j^*)$for all i, j.

Proof. Notice that ∇ stands for the Jacobian of a function. In order to relate the optimality condition in Lemma 1 with the distributed replicator dynamics (8), we introduce the Lagrangian function associated to optimization Problem 1 as

$$\mathcal{L}(P,\mu) = S_W(P) - \mu^\top \left(\mathbf{H}^\top P - \mathbf{P_L}\right), \tag{9}$$

where μ are the Lagrange multipliers, S_W is the social welfare function, and $\mathbf{H}P - P_L$ are the constraints in the system. The optimal solution (P^*, μ^*), which can be found by the Kuhn-Tucker first-order conditions for maximization establishes that P^* is a unique solution to (6) if (P^*, μ^*) is a saddle point of $\mathcal{L}(P,\mu)$ (Bertsekas, 1999; Nedic et al., 2010). ■

3.2 Distributed Replicator Dynamics and Potential Games

One of the main attributes of potential games is the existence of a single scalar-valued function, called potential function, which captures all relevant information about payoffs of the agents. Assuming there exists a continuously differentiable potential function $h : \mathbf{R}_+^V \to \mathbf{R}$, then a potential game satisfies the following relationship for the fitness function for each agent

$$\frac{\partial h(P)}{\partial P_i} = F_i(P) \quad \text{for all } \; i \in \mathcal{V}. \tag{10}$$

Equation (10) implies that the population game must satisfy the externality symmetry defined as

$$\frac{\partial F_i}{\partial P_j} = \frac{\partial F_j}{\partial P_i} \quad \text{for all } \; i,j \in \mathcal{V}. \tag{11}$$

In potential games, there is a property that relates Nash equilibrium to local maximizers of potential function (Sandholm, 2010), it is stated in the Proposition 1.

Proposition 1. A potential game with the potential function h satisfies that Nash equilibrium of the potential game is equal to the solution of the Kuhn-Tucker conditions of the optimization problem maximize $h(P)$ subject to feasible set Δ.

On the other hand, we are interested in the stability of the game. A significant result in population games about stability states that a population game satisfying (10) is a stable game if the potential function h is concave (Sandholm, 2010).

Having introduced the main concepts of potential population games, it is possible to present a definition for the fitness function relating the population game as a potential game guaranteeing optimality and stability conditions. Therefore, the fitness functions are defined as follows

$$F_i(P_i) = \nabla_P \mathcal{L} = \nabla_P S_W - \mathbf{H}\mu, \tag{12}$$

which introduces an extended system including the Lagrange multipliers dynamics with fitness function defined as

$$F_r(\mu_r) = \nabla_\mu \mathcal{L} = -(\mathbf{H}^\top P - P_L), \tag{13}$$

and dynamic equation

$$\dot{\mu}_r = \mu_r \left(F_r(\mu_i) \sum_{j\in\mathcal{N}_r} \mu_j - \sum_{j\in\mathcal{N}_r} \mu_j F_j(\mu_j) \right) \tag{14}$$

where $r \in \mathcal{R} = \{1, \dots, R\}$ is the number of constraints. In the next subsection it is presented the optimality and stability analysis of the extended dynamical system.

3.3 Optimality and Stability Analysis

Consider the extended population game defined by distributed replicator dynamics (8) and Lagrange multiplier dynamics (14) with fitness functions (12) and (13), respectively. It is necessary to show that the extended population game is a potential game to guarantee optimality through Lemma 1 and Proposition 1. Then, to show the game is stable, we have to verify that the potential function h is concave and twice continuously differentiable.

Theorem 1. Let $P_i(k)$, with $i \in \mathcal{V}$, be the set points generated by (8) and (14). Then, $P_i(k)$ with $i \in \mathcal{V}$ converges to the optimal solution P_i^* with $P_i^* \in \Delta$, that is

$$\lim_{k\to\infty} P_i(k) = P_i^*$$

Proof. Since we have defined fitness functions as (12) and (13), by definition it is clear that a potential function for the population game (8) is $h(P,\mu) = \mathcal{L}(P,\mu) = S_W(P) - \mu^\top(\mathbf{H}P - P_L)$ and considering the form of the social welfare function S_W defined in (5), it can be shown that the game satisfies the externality symmetry (11). When the optimality condition in Lemma 1 is reached then $F_i(P_i) = F_j(P_j)$ for all i,j and it is noticed that in the distributed replicator dynamics (8) we have

$$\begin{aligned}
&\left(F_i(P_i) \sum_{j\in\mathcal{N}_i} P_j - \sum_{j\in\mathcal{N}_i} P_j F_j(P_j) \right) \\
&\qquad = \left(F_i(P_i) \sum_{j\in\mathcal{N}_i} P_j - \sum_{j\in\mathcal{N}_i} P_j F_i(P_i) \right) \\
&\qquad = \left(F_i(P_i) (\sum_{j\in\mathcal{N}_i} P_j - \sum_{j\in\mathcal{N}_i} P_j) \right) \\
&\qquad = 0,
\end{aligned}$$

which implies that (8) reaches an equilibrium point inside the feasible set Δ. ■

For the stability analysis, we need to verify that the potential function $h(P,\mu)$ is concave. To prove that the function is concave we check the Hessian matrix of $h(P,\mu)$ is semidefinite negative, i.e, $\nabla^2 h(P,\mu) \preccurlyeq 0$. The Jacobian is obtained as

$$\nabla h(P,\mu) = \begin{bmatrix} \nabla_P h \\ \nabla_\mu h \end{bmatrix} = \begin{bmatrix} \nabla S_W - \mu^\top \mathbf{H} \\ -(\mathbf{H}\mu + P_L) \end{bmatrix}.$$

Hence, the Hessian is obtained as

$$\nabla^2 h(P,\mu) = \begin{bmatrix} \nabla_P^2 h \\ \nabla_\mu^2 h \end{bmatrix} = \begin{bmatrix} \nabla^2 S_W \\ 0 \end{bmatrix}.$$

The problem to check the semidefiniteness of the Hessian of h reduces to check the Hessian of $\nabla^2 S_W$. Recall that S_W is as in (5), and deriving twice it is obtained the Hessian as

$$\nabla^2 S_W = \sum_{m=1}^{M} \beta_{d_m} - \sum_{n=1}^{N} \beta_{g_n}.$$

As a result to guarantee that $\nabla^2 S_W \le 0$, we obtain a relationship between the coefficients of cost functions of the generators and utility functions of the smart loads as follows

$$\sum_{m=1}^{M} \beta_{d_m} \leq \sum_{n=1}^{N} \beta_{g_n} \tag{15}$$

If condition (15) is satisfied, then the algorithm based on the DRD is stable. In this section, it has been analyzed the main features of the proposed algorithm in term of optimality and stability. In the next section, several case studies are presented to illustrate the effectiveness of the distributed replicator dynamics to solve the social welfare problem.

4. SIMULATION RESULTS

In this section, we simulate a microgrid inspired in the WSCC 9 node model, as is shown in Fig. 1. This model has three generators (represented by red nodes) and three loads (represented by blue nodes). They can change their load depending on the network state. We seek to optimize the social welfare of the agents in the microgrid, reducing the cost of generators and maximizing the utility of consumers. Agents in the microgrid have generation and consumption of power limited by (2) and (4). The power limits taken for this test as equal as the cost coefficients are shown in Table 1.

Table 1. System Parameters in Simulation

	Generators			
	$\overline{\mathbf{P}}_{\mathbf{g_i}}$	$\underline{\mathbf{P}}_{\mathbf{g_n}}$	$\beta_{\mathbf{g_n}}$	$\rho_{\mathbf{g_n}}$
1	4000	0	0.04	5
2	6000	0	0.03	3
3	7000	0	0.02	1
	Consumers			
	$\overline{\mathbf{P}}_{\mathbf{d_m}}$	$\underline{\mathbf{P}}_{\mathbf{d_m}}$	$\beta_{\mathbf{d_m}}$	$\rho_{\mathbf{d_m}}$
1	4100	3000	0.03	8
2	5200	4000	0.02	5
3	6300	5000	0.01	4

We use three distributed generators and three distributed consumers connected indistinctly in a connected graph, in other words, it is not necessary that the power system has a particular topology in order to guarantee the convergence of this algorithm.

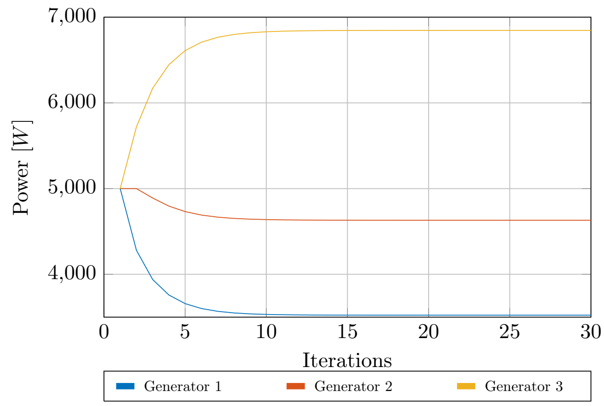

Fig. 2. Power Delivered by Generators in the Distributed Transactive Control Algorithm

In order to include the base load inside the transactive control, we assume that every consumer agent has a base load, denoted as $(\underline{P_{d_m}})$, and has to be supplied regardless of the price it has, also has a quantity of load with which you can vary your consumption that is given by $\overline{P_{d_m}} - \underline{P_{d_m}}$. Fig. 2 shows the convergence of the generators when the algorithm is executed to the optimum power set level, taking into account the constraints and load consumed by the other agents in the system.

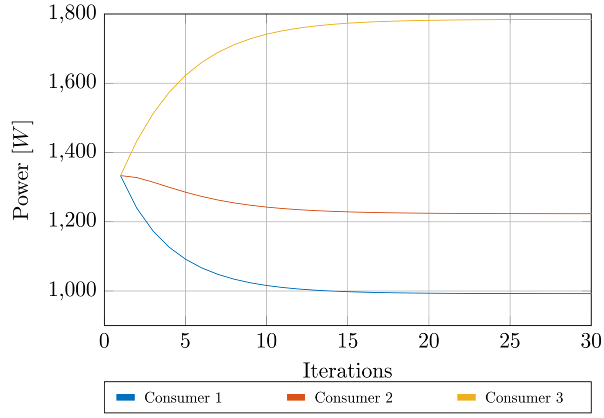

Fig. 3. Power Delivered by Consumers in the Distributed Transactive Control Algorithm

In Fig 3 is shown how the consumers adjust their loads in order to reach the maximum utility in their consume. As is shown, several consumers prefer to rise their loads while others reduce their consume. The change of the load is connected with the valuation that each consumer has about the power in each time instant. This valuation is represented in the utility coefficients in (1).

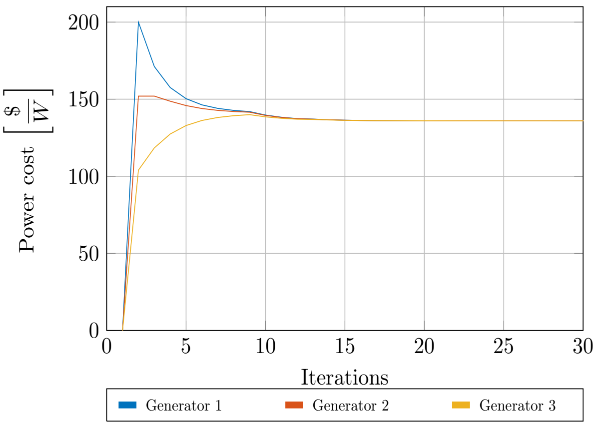

Fig. 4. Generators Fitness Function in the Distributed Transactive Control Algorithm

Furthermore is evidenced in Fig 4 and Fig 5, that generators converge to the same value of the fitness function as equal as the consumers. Then, the marginal utilities and cost are the same for each type of agents, i.e., all generators converge to a marginal cost, and all consumers converge to a marginal utility. This marginal utility implies that generators with lower power cost deliver more power to the network than generators with higher power cost, making that the network reaches the optimum state.

5. CONCLUSION AND FUTURE WORK

A distributed replicator dynamics protocol has been proposed to solve a social welfare optimization problem be-

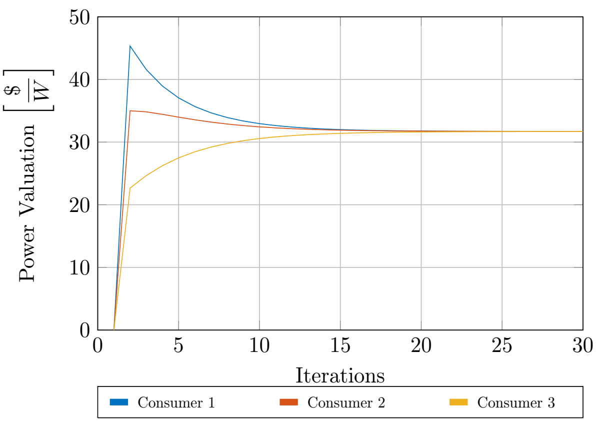

Fig. 5. Consumers Fitness Function in the Distributed Transactive Control Algorithm

tween distributed generators and smart loads in a transactive control framework. The proposed algorithm considers utility functions of generator and consumers in order to reach the optimum social welfare dynamically while maintaining some system constraints. Distributed transactive controllers are capable of handling problems with distributed information on distribution systems. As future work, this algorithm could include a dynamical calculation of losses in the system and an algorithm running in parallel to estimate the energy valuation of each consumer agent.

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