Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Active Attitude Control of Ground Vehicles with Partially Unknown Model ⋆

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Abstract: We present a novel solution to the attitude control problem of ground vehicles by means of the Active Front Steering (AFS) system. The classical feedback linearization method is often used to track a reference yaw dynamics while guaranteeing vehicle stability and handling performance, but it is difficult to apply because it relies on the exact knowledge of the nonlinearities of the vehicle, in particular the tire model. In this work, the unknown nonlinearities are real-time learnt on the basis of the universal approximation property, widely used in the area of neural networks. With this approximation method, the Uniform Ultimate Boundedness (UUB) property with respect to tracking and estimation errors can be formally proven. Preliminary simulation results show good tracking capabilities when model and parameters are affected by uncertainties, also in presence of actuator saturation.

Keywords: Automotive control, Nonlinear control, Learning, Uncertain Systems, Neural networks.

1. INTRODUCTION

In modern automobiles, the wide use of electronics guarantees the improvement of safety, maneuverability and comfort via active control actions [Guiggiani (2014)], [Rajamani (2006)]. This is obtained by using devices applying forces and torques to the vehicle, with the aim of tracking a desired, safe, and feasible reference behavior [Ackermann (1995); Setlur (2006); Burgio (2006)]. The forces and torques imposed to the vehicle by these devices have to be eventually applied by means of the tires, which therefore represent the main component in this regard [Pacejka (2005)]. Hence, it is of great importance to have an accurate tire modelling, in order to appropriately impose the desired behavior. The literature on attitude control problem is rich of papers on the active control of a vehicle, where Active Front Steering (AFS) is used to impose a desired lateral behavior. The interested reader can find in [Acosta (2015)]–[Etienne (2019)] examples of design and implementations of such active control actions, often employed in combination with other actuators or taking into account the effect of saturation.

A major problem in attitude control arises from inaccurate knowledge of the parameters appearing in the mathematical model of the vehicle and, in particular, of the tire model. This affects the performance of the controller applied to track a yaw reference, while guaranteeing vehicle stability and handling performance, since most classic nonlinear techniques [Isidori (1995)] are based on the exact knowledge of system nonlinearities. In [Bianchi (2010)], the issue of model uncertainty was addressed by means of the adaptive feedback linearization technique [Sastry (1989)], but the functional form of the tire functions was assumed to be known.

In this paper, a novel solution to the attitude control problem of ground vehicles is proposed. In order to relax some of the strict model–matching restrictions, the idea is to 'learn' about the unknown nonlinearities using the universal approximation property [Cybenko (1989)], according to which smooth functions can be approximated arbitrarily well by means of sigmoidal functions with a sufficient number of parameters. This property is largely used in the area of neural networks, and was originally exploited for the control of robot manipulators with uncertain dynamics [Lewis (1998)]. In the automotive field, deep learning approaches were recently used in black–box vehicle modeling [Alexa (2014)], as well as in the control of its longitudinal and lateral dynamics [Devineau (2018)]. In the present work, the approach originally described in [Lewis (1998)] for general nonlinear systems is reformu-

⋆ This paper has been partially supported by the Project "Coordination of autonomous unmanned vehicles for highly complex performances", Executive Program of Scientific and Technological Agreement between Italy (Ministry of Foreign Affairs and International Cooperation, Italy) and Mexico (Consejo Nacional de Ciencia y Tecnología, Mexico), SAAP3.

lated for the case of vehicle dynamics, with independent neural networks for approximating front and rear tire characteristics. The classic Lyapunov–based approach is then employed to guarantee the Uniform Ultimate Boundedness (UUB) property [Khalil (2002)] with respect to estimation errors. Preliminary simulation results show good tracking capabilities in the presence of uncertain tire characteristics and parameters, and also considering actuator saturation (not taken into account in the control design method), and an additive white noise on the measured states.

The paper is organized as follows. After recalling some preliminary results in Section 2, Section 3 presents the model of a ground vehicle, the control problem statement and the nonlinear linearizing feedback that requires perfect knowledge of the tire characteristics. In Section 4 the tire characteristics are supposed to be unknown and are estimated using the universal approximator. A feedback controller is then derived that guarantees Uniform Ultimately Boundedness for the closed–loop system. Section 5 provides simulation results that show the effectiveness of the proposed control technique in a non-ideal control situation.

2. RECALLS OF KNOWN FACTS

In the following a system of the form

$$\dot{x} = f(x) + d \tag{1}$$

is considered, with $x \in \mathbb{R}^n$ the system state, $f : \mathbb{R}^n \to \mathbb{R}^n$, and d a bounded perturbation signal, such that $|d(t)| \le \bar{d}$, with $\bar{d} \in \mathbb{R}^+$ a bound. The statements below are the well known Lyapunov theorems, for asymptotic stability and uniform ultimate boundedness, and the universal approximation theorem, very popular in the context of neural networks.

Theorem 1. (Lyapunov(–like) theorems, Khalil (2002)).

1. The system (1) with $d \equiv 0$ is 0–GAS (Globally Asymptotically Stable at the origin) if there exists $V(t,x) > 0$ for $x \neq 0$, such that $\dot{V} < 0$ for $x \in \mathbb{R}^n \setminus \{0\}$;
2. The system (1) with $d \equiv 0$ is 0–GES (Globally Exponentially Stable at the origin) if it is 0–GAS with exponential convergence, i.e. $|x| \le a e^{-bt}|x(0)|$, for some $a, b > 0$, for all $x(0) \in \mathbb{R}^n$;
3. The system (1) is UUB (Uniformly Ultimately Bounded) if there exists $V(t,x) > 0$ for $x \neq 0$ in a compact set $\Omega \subseteq \mathbb{R}^n$, such that $\dot{V} < 0$ for $|x| > R$, and for some $R > 0$ such that $\{x \in \mathbb{R}^n : |x| \le R\} \subseteq \Omega$. ⋄

Theorem 2. (Adapted from Cybenko (1989)). Every smooth function $f : \mathbb{R} \to \mathbb{R}$ can be approximated with arbitrarily good accuracy as

$$f(x) = W^T \sigma(Vx) + \varepsilon \tag{2}$$

for some weight vectors $W, V \in \mathbb{R}^n$, where $\sigma : \mathbb{R} \to \mathbb{R}$ is a sigmoid function. ⋄

The approximation (2) holds for all x in a compact interval Ω, and the functional estimation error ε is bounded so that $|\varepsilon| \le \varepsilon_N$, with ε_N being a known bound dependent on Ω.

The theorem above is extensively used in the context of neural networks, where n is the number of neurons of the so-called *hidden* layer in a two–layer neural network [Lewis (1998)].

In this work, we employ as odd sigmoid function, defined on the real axis, the *hyperbolic tangent*

$$\sigma(x) = \tanh(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^x + \mathrm{e}^{-x}}$$

whose range is the interval $[-1, 1]$, such that $\sigma(0) = 0$ and $\lim_{z \to +\infty} \sigma(z) = 1$. With abused notation, one can define a vector–valued sigmoid $\sigma(x)$, for $x \in \mathbb{R}^n$, whose i^{th} component is $\sigma(x_i)$, according to the scalar sigmoid defined above.

3. PROBLEM SETTING AND FEEDBACK LINEARIZATION

In what follows, the well–known single track model of a ground vehicle is considered [Guiggiani (2014)], depicted in Fig. 1. It describes the lateral and yaw dynamics, essential to design an active controller for the vehicle attitude

$$\begin{aligned} \dot{v}_y &= -v_x\omega_z + \frac{\mu}{m}\Big(F_{y,f} + F_{y,r}\Big) + \frac{\mu}{m}\Delta_d + \frac{\mu}{m}\Delta_c \\ \dot{\omega}_z &= \frac{\mu}{J_z}\Big(F_{y,f}l_f - F_{y,r}l_r\Big) + \frac{\mu l_f}{J_z}\Delta_d + \frac{\mu l_f}{J_z}\Delta_c \end{aligned} \tag{3}$$

where v_x, v_y are the vehicle longitudinal and lateral velocity (m/s), ω_z is the vehicle yaw angular velocity (rad/s), m is the vehicle mass (kg), J_z is the vehicle inertia momentum (kg m^2), l_f, l_r are the distances from the center of gravity to the front and rear axle (m), respectively, $F_{y,f}, F_{y,r}$ are the front/rear tire lateral forces (N), respectively, μ is the tire–road friction coefficient (dimensionless), Δ_d (N) is the driver input, actuated by means of the steering wheel and assumed measurable, and Δ_c (N) is the AFS control input. The interested reader can find in [Acosta (2017)], [Bianchi (2010)] additional details on the model.

In the following, v_x is assumed constant, which is a reasonable assumption when dealing with the vehicle cornering behavior.

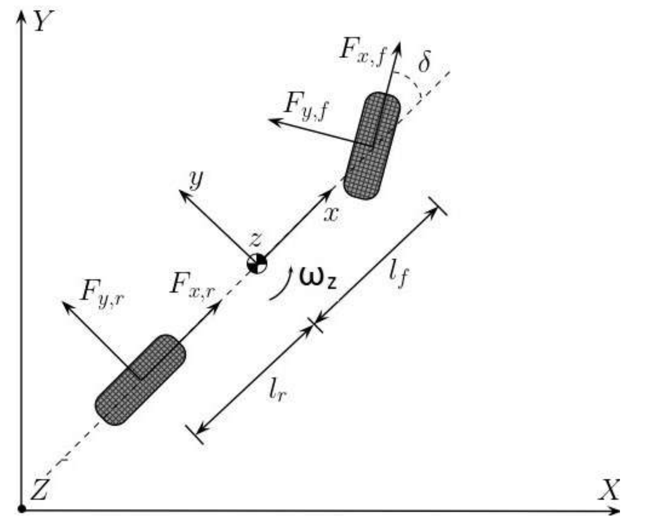

Fig. 1. Single–track vehicle model.

The control objective is to track a yaw rate reference signal $\omega_{z,\mathrm{ref}}$ for ω_z. One can assume that both states v_y, ω_z are measurable. In the case that v_y is not measurable, one can use a lateral velocity observer, as the ones implemented in [Acosta (2017)], [Borri (2017a)].

The nonlinear front and rear lateral forces

$$F_{y,f} = F_{y,f}(\alpha_f), \quad F_{y,r} = F_{y,r}(\alpha_r) \tag{4}$$

are odd functions

$$\alpha_f F_{y,f}(\alpha_f) > 0, \quad \alpha_r F_{y,r}(\alpha_r) > 0, \qquad \forall \alpha_f, \alpha_r \in \mathbb{R} \setminus \{0\} \tag{5}$$

depending on the front and rear tire slip angles

$$\alpha_f = -\frac{v_y + l_f \omega_z}{v_x}, \quad \alpha_r = -\frac{v_y - l_r \omega_z}{v_x}. \tag{6}$$

Examples of tire functions in the form (4) with property (5) are the simplified Pacejka's magic formulae [Pacejka (2005)]

$$\begin{aligned} F_{y,f}(\alpha_f) &= C_{y,f} \sin(A_{y,f} \arctan(B_{y,f} \alpha_f)) \\ F_{y,r}(\alpha_r) &= C_{y,r} \sin(A_{y,r} \arctan(B_{y,r} \alpha_r)) \end{aligned} \tag{7}$$

with global maxima $F_{yf,\mathrm{sat}} = C_{y,f} > 0$, $F_{yr,\mathrm{sat}} = C_{y,r} > 0$ at the saturation points $\alpha_{f,\mathrm{sat}}$ and $\alpha_{r,\mathrm{sat}}$, respectively.

In view of the application of the feedback linearization technique [Isidori (1995)], one can consider the lateral velocity of the so–called neutral steering point [Guiggiani (2014)]

$$v_{y,n} = v_y - \frac{J_z}{m l_f} \omega_z$$

whose dynamics is

$$\dot{v}_{y,n} = -v_x \omega_z + \frac{\mu l}{m l_f} F_{y,r} \tag{8}$$

where $l = l_f + l_r$ is the vehicle inter-axle length. Note that $v_{y,n}$ does not depend on $F_{y,f}$, neither on Δ_c, nor on Δ_d.

From the second equation in (3) and from (8), one obtains the mathematical model

$$\begin{aligned} \dot{\omega}_z &= \frac{\mu}{J_z}\Big(F_{y,f} l_f - F_{y,r} l_r\Big) + \frac{\mu l_f}{J_z}\Delta_c + \frac{\mu l_f}{J_z}\Delta_d \\ \dot{v}_{y,n} &= -v_x \omega_z + \frac{\mu l}{m l_f} F_{y,r} \end{aligned}$$

which is in the form

$$\begin{aligned} \dot{x}_1 &= f_1(x) + bu + b\Delta_d \\ \dot{x}_2 &= f_2(x) \end{aligned} \tag{9}$$

with $x_1 = \omega_z$, $x_2 = v_{y,n}$, $x = (x_1 \quad x_2)^T$, $u = \Delta_c$, and

$$f_1(x) = \frac{\mu}{J_z}\Big(F_{y,f}(k_f x) l_f - F_{y,r}(k_r x) l_r\Big)$$

$$f_2(x) = -v_x x_1 + \frac{\mu l}{m l_f} F_{y,r}(k_r x), \qquad b = \frac{\mu l_f}{J_z}.$$

Here, the slip angles have been rewritten as linear combinations of the state variables

$$\alpha_f = k_f x, \quad k_f = \begin{pmatrix} -\dfrac{l_f}{v_x} & -\dfrac{1}{v_x} \end{pmatrix}$$

$$\alpha_r = k_r x, \quad k_r = \begin{pmatrix} \dfrac{l_r}{v_x} & -\dfrac{1}{v_x} \end{pmatrix}.$$

In view of tracking a desired yaw velocity $x_{1,\mathrm{ref}} = \omega_{z,\mathrm{ref}}$, we define the tracking error $e = \omega_z - \omega_{z,\mathrm{ref}} = x_1 - x_{1,\mathrm{ref}}$, and consider the linearizing feedback [Isidori (1995)]

$$u = \frac{-ke + \dot{x}_{1,\mathrm{ref}} - f_1(x)}{b} - \Delta_d \tag{10}$$

with $k > 0$. The error dynamics becomes

$$\dot{e} = f_1(x) + bu + b\Delta_d - \dot{x}_{1,\mathrm{ref}} = -ke$$

which is 0–GES. It is reasonable to assume that $\dot{x}_{1,\mathrm{ref}}$ incorporates an addend $b\Delta_d$, which is equal to the one in $\dot{x}_1$, so that both the control input u and the e dynamics turn out to be independent of Δ_d.

Since the relative degree with respect to x_1 is $r = 1 < n = 2$, one has to consider the resulting zero dynamics

$$\dot{x}_2|_{x_1 \equiv 0} = f_2(x)|_{x_1 \equiv 0} = \frac{\mu l}{m l_f} F_{y,r}(k_r x)|_{x_1 \equiv 0}.$$

It is easy to check that this dynamics is 0–GAS. In fact, considering the Lyapunov candidate $\mathcal{V}(x_2) = x_2^2/2$, one has

$$\dot{\mathcal{V}} = \frac{\mu l}{m l_f} x_2 F_{y,r}(k_r x)|_{x_1 \equiv 0} < 0, \qquad \text{for } x_2 \neq 0$$

since

$$F_{y,r}(k_r x)|_{x_1 \equiv 0} = F_{y,r}(-x_2/v_x) = -F_{y,r}(x_2/v_x)$$

and thanks to property (5).

4. CONTROL DESIGN FOR THE PARTIALLY UNKNOWN MODEL

The control (10) requires the perfect knowledge of the tire characteristics, which is an assumption that is not usually fulfilled in practical cases. Hence, following [Lewis (1998)], in this section the tire characteristics are estimated making use of sigmoid functions in the form of the universal approximator (2)

$$\begin{aligned} \hat{F}_{y,f} &= \hat{W}_f^T \sigma\Big(\hat{V}_f k_f x\Big) \\ \hat{F}_{y,r} &= \hat{W}_r^T \sigma\Big(\hat{V}_r k_r x\Big) \end{aligned}$$

with $\hat{W}_i, \hat{V}_i \in \mathbb{R}^{n_i}$ the weight vectors, $i = f, r$, with n_f, n_r being the number of hidden-layer neurons of the front and rear neural networks, respectively.

Furthermore, the presence of an additional bounded disturbance d is considered in the yaw equation of (9)

$$\begin{aligned} \dot{x}_1 &= f_1(x) + bu + b\Delta_d + d \\ &= \frac{\mu}{J_z}\Big(F_{y,f}(k_f x) l_f - F_{y,r}(k_r x) l_r\Big) + b(u + \Delta_d) + d \end{aligned} \tag{11}$$

with $|d| \leq \bar{d} \in \mathbb{R}^+$. This disturbance can take into account further environmental disturbances and model uncertainties in the yaw dynamics.

Considering a Taylor series expansion, one can write the functional estimation errors $\tilde{F}_{y,i} = F_{y,i} - \hat{F}_{y,i}$, $i = f, r$, as follows [Lewis (1998)]

$$\tilde{F}_{y,i} = \tilde{W}_i^T\left(\hat{\sigma}_i - \hat{\sigma}_i' \hat{V}_i k_i x\right) + \hat{W}_i^T \sigma\left(\tilde{V}_i k_i x\right) + w_i \tag{12}$$

where w_i denote the higher–order terms,

$$\hat{\sigma}_i = \sigma_i\left(\hat{V}_i k_i x\right), \quad \hat{\sigma}_i' = \sigma_i'\left(\hat{V}_i k_i x\right), \quad i = f, r$$

and

$$\tilde{W}_i = W_i - \hat{W}_i, \qquad \tilde{V}_i = V_i - \hat{V}_i, \quad i = f, r \tag{13}$$

are the weight errors with respect to the unknown real weights W_i, V_i. The higher–order terms are bounded according to

$$w_i(t) \leq C_{i,0} + C_{i,1}|\tilde{\Theta}_i| + C_{i,2}|e||\tilde{\Theta}_i| \tag{14}$$

for computable positive constants $C_{i,0}$, $C_{i,1}$, $C_{i,2}$, where the aggregate errors $\tilde{\Theta}_i := \Theta_i - \hat{\Theta}_i = \begin{pmatrix}\tilde{W}_i^T & \tilde{V}_i^T\end{pmatrix}^T = \begin{pmatrix}W_i^T & V_i^T\end{pmatrix}^T - \begin{pmatrix}\hat{W}_i^T & \hat{V}_i^T\end{pmatrix}^T$ are defined with respect to the unknown real weights.

The linearizing feedback (10) becomes

$$\hat{u} = \frac{-ke + \dot{x}_{1,ref} - \hat{f}_1(x) + \dfrac{\mu l_f}{J_z} u_{R,f} - \dfrac{\mu l_r}{J_z} u_{R,r}}{b} - \Delta_d \tag{15}$$

where

$$\hat{f}_1(x) = \frac{\mu}{J_z}\left(\hat{F}_{y,f}(k_f x) l_f - \hat{F}_{y,r}(k_r x) l_r\right) \tag{16}$$

and

$$\begin{aligned} u_{R,f} &= -k_{z,f}\left(|\hat{\Theta}_f| + \Theta_{m,f}\right) e \\ u_{R,r} &= k_{z,r}\left(|\hat{\Theta}_r| + \Theta_{m,r}\right) e \end{aligned} \tag{17}$$

are robustifying terms, with $k_{z,i} > 0$ and $\Theta_{m,i}$ being known bounds on the unknown target weight norms.

The following theorem is the main result of this section, and guarantees Uniform Ultimately Boundedness (UUB) property of the closed–loop system, provided that the nominal feedback linearization control (10) is replaced by the control law (15), where the tire functions are approximated via sigmoid functions.

Theorem 3. Consider the system in (11), with $u(t) = \hat{u}(t)$ as in (15), with the robustifying terms (17) such that $k_{z,i} > C_{i,2} > 0$, $i = f, r$. Let the weight update laws of the universal approximator be given by

$$\dot{\hat{W}}_f = \frac{\mu l_f}{J_z}\Big(M_f e(\hat{\sigma}_f - \hat{\sigma}'_f \hat{V}_f k_f x) - \lambda_f |e| M_f \hat{W}_f\Big) \tag{18}$$

$$\dot{\hat{W}}_r = -\frac{\mu l_r}{J_z}\Big(M_r e(\hat{\sigma}_r - \hat{\sigma}'_r \hat{V}_r k_r x) + \lambda_r |e| M_r \hat{W}_r\Big) \tag{19}$$

$$\dot{\hat{V}}_f = \frac{\mu l_f}{J_z}\Big(N_f k_f x e \hat{W}_f^T \hat{\sigma}'_f - \lambda_f |e| N_f \hat{V}_f\Big) \tag{20}$$

$$\dot{\hat{V}}_r = -\frac{\mu l_r}{J_z}\Big(N_r k_r x e \hat{W}_r^T \hat{\sigma}'_r + \lambda_r |e| N_r \hat{V}_r\Big) \tag{21}$$

with $M_i > 0$, $N_i > 0$, and $\lambda_i > 0$ being design parameters, $i = f, r$. Then, the closed–loop system is UUB with respect to the tracking error $e(t)$ and the weight errors $\tilde{\Theta}_i$, with bounds given by

$$\delta_e := \frac{D_f + D_r}{k}, \qquad \delta_i := \frac{C_{3,i}}{2} + \sqrt{\frac{J_z\,(D_f + D_r)}{\mu l_i \lambda_i}}, \tag{22}$$

for $e(t)$ and $\tilde{\Theta}_i$, respectively, where the constants

$$\begin{aligned} C_{3,i} &:= \Theta_{m,i} + \frac{C_{i,1}}{\lambda_i}, \\ D_i &:= \frac{\mu l_i \lambda_i}{J_z}\frac{C_{3,i}^2}{4} + \frac{\bar{d}}{2} + \frac{\mu l_i}{J_z} C_{i,0}, \end{aligned} \tag{23}$$

are defined for $i = f, r$.

Proof 1. Consider the Lyapunov candidate

$$\begin{aligned} \mathcal{V} = \frac{1}{2}e^2 &+ \frac{1}{2}\tilde{W}_f^T M_f^{-1}\tilde{W}_f + \frac{1}{2}\tilde{V}_f^T N_f^{-1}\tilde{V}_f \\ &+ \frac{1}{2}\tilde{W}_r^T M_r^{-1}\tilde{W}_r + \frac{1}{2}\tilde{V}_r^T N_r^{-1}\tilde{V}_r \end{aligned} \tag{24}$$

and its time derivative

$$\begin{aligned} \dot{\mathcal{V}} = e\dot{e} &+ \tilde{W}_f^T M_f^{-1}\dot{\tilde{W}}_f + \tilde{V}_f^T N_f^{-1}\dot{\tilde{V}}_f \\ &+ \tilde{W}_r^T M_r^{-1}\dot{\tilde{W}}_r + \tilde{V}_r^T N_r^{-1}\dot{\tilde{V}}_r. \end{aligned}$$

From (11), (15), (16), the tracking error dynamics rewrites:

$$\begin{aligned} \dot{e}(t) &= \dot{x}_1(t) - \dot{x}_{1,ref}(t) \\ &= f_1(x(t)) + b\hat{u}(t) + b\Delta_d(t) + d(t) - \dot{x}_{1,ref}(t) \\ &= -ke(t) + \frac{\mu l_f}{J_z}(\tilde{F}_{y,f}(k_f x(t)) + u_{R,f}(t)) \\ &\qquad - \frac{\mu l_r}{J_z}(\tilde{F}_{y,r}(k_r x(t)) + u_{R,r}(t)) + d(t). \end{aligned}$$

Since $\dot{\tilde{W}}_i = -\dot{\hat{W}}_i$, $\dot{\tilde{V}}_i = -\dot{\hat{V}}_i$, from (13), one can obtain (time dependencies are omitted for the sake of a more compact notation):

$$\begin{aligned} \dot{\mathcal{V}} = -ke^2 &+ \frac{\mu l_f}{J_z} e\left(\tilde{F}_{y,f}(k_f x) + u_{R,f}\right) \\ &- \frac{\mu l_r}{J_z} e\left(\tilde{F}_{y,r}(k_r x) + u_{R,r}\right) + ed \\ &- \tilde{W}_f^T M_f^{-1}\dot{\hat{W}}_f - \tilde{V}_f^T N_f^{-1}\dot{\hat{V}}_f \\ &- \tilde{W}_r^T M_r^{-1}\dot{\hat{W}}_r - \tilde{V}_r^T N_r^{-1}\dot{\hat{V}}_r, \end{aligned}$$

and by substituting the Taylor expansion of estimation errors in (12) and the update laws (18)–(21), one further gets

$$\begin{aligned} \dot{\mathcal{V}} = -ke^2 &+ e\left(d + \frac{\mu l_f}{J_z}(w_f + u_{R,f}) - \frac{\mu l_r}{J_z}(w_r + u_{R,r})\right) \\ &+ \frac{\mu l_f}{J_z} e\left(\tilde{W}_f^T\left(\hat{\sigma}_f - \hat{\sigma}'_f \hat{V}_f k_f x\right) + \hat{W}_f^T \hat{\sigma}'_f \tilde{V}_f k_f x\right) \\ &- \frac{\mu l_r}{J_z} e\left(\tilde{W}_r^T\left(\hat{\sigma}_r - \hat{\sigma}'_r \hat{V}_r k_r x\right) + \hat{W}_r^T \hat{\sigma}'_r \tilde{V}_r k_r x\right) \\ &- \frac{\mu l_f}{J_z}\tilde{W}_f^T M_f^{-1}\left(M_f e\left(\hat{\sigma}_f - \hat{\sigma}'_f \hat{V}_f k_f x\right) - \lambda_f |e| M_f \hat{W}_f\right) \\ &- \frac{\mu l_f}{J_z}\tilde{V}_f^T N_f^{-1}\left(N_f k_f x e \hat{W}_f^T \hat{\sigma}'_f - \lambda_f |e| N_f \hat{V}_f\right) \\ &+ \frac{\mu l_r}{J_z}\tilde{W}_r^T M_r^{-1}\left(M_r e\left(\hat{\sigma}_r - \hat{\sigma}'_r \hat{V}_r k_r x\right) + \lambda_r |e| M_r \hat{W}_r\right) \\ &+ \frac{\mu l_r}{J_z}\tilde{V}_r^T N_r^{-1}\left(N_r k_r x e \hat{W}_r^T \hat{\sigma}'_r + \lambda_r |e| N_r \hat{V}_r\right). \end{aligned}$$

The previous expression simplifies to:

$$\begin{aligned} \dot{\mathcal{V}} = -ke^2 &+ e\left(d + \frac{\mu l_f}{J_z}(w_f + u_{R,f}) - \frac{\mu l_r}{J_z}(w_r + u_{R,r})\right) \\ &+ \frac{\mu l_f}{J_z}\lambda_f |e| \tilde{\Theta}_f^T \hat{\Theta}_f + \frac{\mu l_r}{J_z}\lambda_r |e| \tilde{\Theta}_r^T \hat{\Theta}_r. \end{aligned}$$

A bound on the quantity $\tilde{\Theta}_i^T \hat{\Theta}_i$ is obtained by means of the following chain of inequalities

$$\tilde{\Theta}_i^T \hat{\Theta}_i \leq \Theta_{m,i}|\tilde{\Theta}_i| - |\tilde{\Theta}_i|^2 = |\tilde{\Theta}_i|\left(\Theta_{m,i} - |\tilde{\Theta}_i|\right),$$

where the bound $\Theta_i \leq \Theta_{m,i}$ has been exploited.

We can now plug in the robustifying input defined in (17), the Taylor error bounds in (14) and the disturbance bound $\bar{d}$, which lead to:

$$
\begin{aligned}
\dot{\mathcal{V}} \leq & -ke^2 + \left(\bar{d} + \frac{\mu}{J_z} l_f C_{f,0} + \frac{\mu}{J_z} l_r C_{r,0}\right)|e| \\
& + \frac{\mu}{J_z} l_f \left(C_{f,1}|\tilde{\Theta}_f| + C_{f,2}|e||\tilde{\Theta}_f|\right)|e| \\
& + \frac{\mu}{J_z} l_r \left(C_{r,1}|\tilde{\Theta}_r| + C_{r,2}|e||\tilde{\Theta}_r|\right)|e| \\
& - \frac{\mu l_f}{J_z} k_{z,f}\left(|\hat{\Theta}_f| + \Theta_{m,f}\right)e^2 - \frac{\mu l_r}{J_z} k_{z,r}\left(|\hat{\Theta}_r| + \Theta_{m,r}\right)e^2 \\
& + \frac{\mu l_f}{J_z}\lambda_f |\tilde{\Theta}_f|\left(\Theta_{m,f} - |\tilde{\Theta}_f|\right)|e| \\
& + \frac{\mu l_r}{J_z}\lambda_r |\tilde{\Theta}_r|\left(\Theta_{m,r} - |\tilde{\Theta}_r|\right)|e|.
\end{aligned}
$$

Since $k_{z,i} > C_{i,2}$, one further gets

$$
\begin{aligned}
\dot{\mathcal{V}} \leq -|e| \Bigg(& k|e| + \frac{\mu l_f}{J_z}\lambda_f\left(|\tilde{\Theta}_f|^2 - \left(\Theta_{m,f} + \frac{C_{f,1}}{\lambda_f}\right)|\tilde{\Theta}_f|\right) \\
& - \left(\frac{\bar{d}}{2} + \frac{\mu l_f}{J_z} C_{f,0}\right) + \frac{\mu l_r}{J_z}\lambda_r\Bigg(|\tilde{\Theta}_r|^2 \\
& - \left(\Theta_{m,r} + \frac{C_{r,1}}{\lambda_r}\right)|\tilde{\Theta}_r|\Bigg) - \left(\frac{\bar{d}}{2} + \frac{\mu l_r}{J_z} C_{r,0}\right)\Bigg).
\end{aligned}
$$

Completing the squares, we finally obtain:

$$
\begin{aligned}
\dot{\mathcal{V}} \leq -|e| \Bigg[& k|e| + \frac{\mu l_f \lambda_f}{J_z}\left(|\tilde{\Theta}_f| - \frac{C_{3,f}}{2}\right)^2 - D_f \\
& + \frac{\mu l_r \lambda_r}{J_z}\left(|\tilde{\Theta}_r| - \frac{C_{3,r}}{2}\right)^2 - D_r\Bigg],
\end{aligned}
\tag{25}
$$

with $C_{3,i}$ and D_i are defined in (23). Note that the term in square brackets in the right-hand side of (25) defines a compact set. So it is readily seen that $\dot{\mathcal{V}} < 0$ if $|e| > \delta_e$ or if $|\tilde{\Theta}_f| > \delta_f$ or if $|\tilde{\Theta}_r| > \delta_r$, where δ_e, δ_f, δ_r are defined in (22). Hence, according to Theorem 1, the system is UUB with respect to the error space $\left(|e|, |\tilde{\Theta}_f|, |\tilde{\Theta}_r|\right)$, which concludes the proof.

Remark 4. The adaptation rules and the structure of the controller are derived from a Lyapunov-based analysis, which consists of a standard backpropagation term, a novel feed-forward term coming from a certain Taylor series expansion and an error modification term to keep the parameters bounded in the absence of persistent excitation. In terms of a neural network, the structure of the controller may be seen as composed of 2 two-layer neural networks to approximate the nonlinearities of front and rear axles and a limiting term that keeps the control signal bounded regardless of the neural network weight estimates. $\diamond$

5. NUMERICAL SIMULATIONS

In this section, we provide simulation results of the proposed control technique, showing the performance of the controller with adapted weights in the presence of actuator saturation. Simulations are carried out on a extended vehicle model resulting in some unmodeled dynamics with respect to the model (11) used in the control design, including four independent wheels and tires, a first-order actuator dynamics and a constant actuation delay of $0.1s$, of the same order of magnitude as the AFS time constant. In addition, a 15% percentage variation of parameters μ, m, J_z is introduced with respect to nominal values, as well as an additive white Gaussian noise with amplitude 0.02 on both state variables.

The vehicle and wheel parameters are equal to $m = 1550$ kg, $l_f = 1.17$ m, $l_r = 1.43$ m, $J_z = 2300\ \text{kg}\,\text{m}^2$, $\mu = 1$.

The tire lateral force functions are given by the Pacejka's Magic Formulae (7), with the following parameters

$$
\begin{array}{lll}
A_{y,f} = 1.81, & A_{y,r} = 1.68 & B_{y,f} = 7.2, \\
B_{y,r} = 11 & C_{yf} = 8854\ \text{N}, & C_{yr} = 8394\ \text{N}.
\end{array}
$$

For the estimation of tire forces, the weight vectors have been chosen of dimension 5 (i.e. 5 neurons have been used for each force in the hidden layer of the neural network structure). The design parameters have been set equal to

$$
\begin{array}{lllll}
n_f = 5, & n_r = 5, & k = 5 & M_f = 20, & N_f = 20, \\
\lambda_f = 1 & M_r = 20, & N_r = 20, & \lambda_r = 1. &
\end{array}
$$

The reference yaw rate is the output of a reference generator model, whose input is the driver-imposed force Δ_d, which is assumed to have a globally asymptotically stable equilibrium point at the origin.

Unlike many other neural network structures, an offline learning or training phase is not needed because it is performed during the simulation, with the weight values being initialized at some presumable values and then tuned on-line by means of the update laws (18)–(21) as the system tracks the desired yaw rate. As the updating of the weights take place, the tracking performance improves (in other words, this corresponds to the neural network learning the approximation function). This is obtained by means of a learning maneuver consisting in a ramp steer with a longitudinal velocity of 28 m/s, which is stopped as soon as the AFS goes into saturation. After that, the weight values remain approximately fixed at the value reached at the end of the learning maneuver, and a more demanding test maneuver is considered, consisting in a double step steer (see the input driver in Fig. 2, top panel) of 100° with longitudinal velocity of 35 m/s. Such a maneuver is very hard and the control actuator reaches the saturation zone (Fig. 2, bottom panel). Fig. 3 shows the comparison in terms of yaw rate between the feedback linearization controller and the one based on the approximation and learning controller, which keeps the vehicle stable and ensures good tracking with a bounded error.

6. CONCLUSIONS

This work has investigated the problem of active attitude control for ground vehicles via AFS in the case of unknown or partially known tire functions. In uncertain cases, control techniques based on perfect knowledge of the underlying model, such as feedback linearization, are not guaranteed to work properly. In order to overcome this issue, approximation and learning is proposed for the unknown functions, and the closed–loop error boundedness is proven by means of Lyapunov techniques. Preliminary simulation results have been obtained by closing the loop on an non–ideal vehicle model, including model uncertainties and unmodeled dynamics, and confirm that the approach is promising for future investigations.

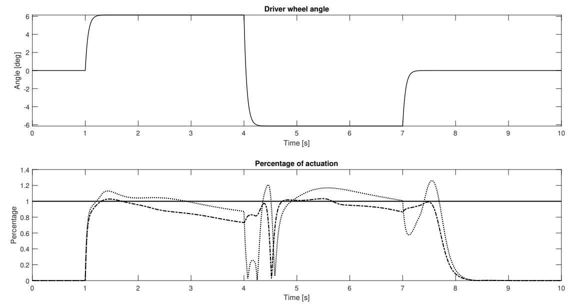

Fig. 2. Driver wheel angle [rad] vs time [s] and Percentage of actuation [adimensional] vs time [s]: AFS controlled vehicle (dash-dotted line) and AFS controlled vehicle with adaptation by learning (dotted line)

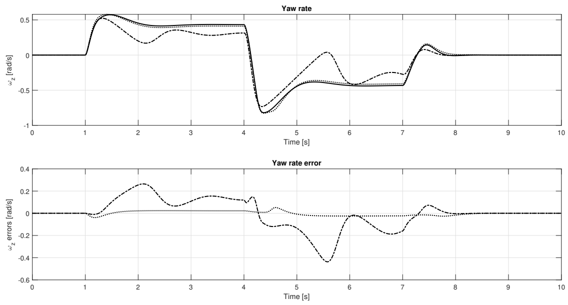

Fig. 3. ω_z [rad/s] vs time [s]: nominal reference (solid line), AFS controlled vehicle (dash-dotted line) and AFS controlled vehicle with adaptation by learning (dotted line). ω_z error [rad/s] vs time [s]: AFS controlled vehicle (dash-dotted line) and AFS controlled vehicle with adaptation by learning (dotted line).

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