Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Sliding Mode Controller Based on a Hybrid Surface for Tracking Improvement of Non-Linear Processes

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Abstract: This paper presents the synthesis of a sliding mode controller based on a hybrid surface (SMC-HS) as an alternative to improve the performance of the traditional SMC in its transient response. The combination of two sliding surfaces is carried out in order to obtain the advantages of both. Through reset actions, the use of one or other surface is prioritized. The improvement offered by the proposal is quantified by performance indexes when the new controller is tested in a non-linear, self-regulating model.

Keywords: Nonlinear Control, Sliding Mode Control, Reset Action, Robustness.

1. INTRODUCTION

Variable structure controllers are developed using non-linear control techniques. The Sliding Mode Controller (SMC) is one of these controllers (Khalil, 2019). SMC is a robust control tool with great potential that has been developed for over 45 years. It responds satisfactorily to nonlinear systems with complex dynamics that usually operate under conditions of uncertainty. Also, it is inherently insensitive to the variation of modelling parameters and disturbances once it finds the sliding mode of its scheme (Camacho, 2018).

The SMC has the task of bringing the system from an initial condition to a desired state, which is determined according to the selected sliding surface. The resulting control law is then formed by two components, the continuous or sliding part and the discontinuous or reaching part (Camacho et al., 2003).

For design purposes, PID type sliding surfaces are usually used to constantly act on the positioning error (Capito et al., 2016). With this surface, it is possible to carry out tracking and regulation within the controlled process. However, overshoot and oscillations in the output are characteristic of the integral term in the PID controller.

Hence, an alternative in order to reach the surface and maintain it by reducing the aforementioned drawbacks is the re-tuning of the SMC parameters, taking as a reference point the values initially obtained with the equations presented in Báez et al. (2017). To improve the performance of the SMC, its structure is also connected to a control by predictive and internal model schemes (Camacho, 2002; Camacho et al., 2003).

Despite the inherent potential of the SMC, external and additional development and enhancement is required, such as development of more control schemes instead of working to improve its performance by focusing on the internal structure. Then, an option is to work from the bases that form the control scheme. The surface chosen is linked to the overall operation of the SMC, so that the choice or modification of the surface will impact the improvement of the performance of the controller proposed by Camacho and Smith (2000).

In this work to eliminate the adverse effects of the integral term of the PID surface, we consider some alternatives, the use of a PD surface being the simplest one. However, a PD sliding surface is not capable of compensating for disturbances and errors of the model in the system, which represents a weakness. Therefore, the concept of control based on reset can be taken, so under a certain condition the integral term is reset to null.

A control system based on reset consists of a linear controller to which a reset mechanism of a state has been incorporated. Zeroing the integral component of the controller (or any of its states) applies only when a certain condition is achieved. The condition that triggers the reset is usually the zero crossing of the tracking error. The reset control idea is old, going back to J.C. Clegg's approach from 1958 and was refined in the 70's by Horowitz, who emphasized the capability of the reset systems to overcome the fundamental limitations that affect linear systems with delays or with poles or zeros in the right half plane (Barreiro and Baños, 2012).

The reset control concept extrapolated to SMC results in the reset of the integral term in the PID sliding surface according to the system requirements. Thus, the new sliding surface is called a Hybrid Sliding surface (HS). Mathematically, different reset criteria can be chosen, as presented by Lu and Lee (2013). In the case of the SMC-HS, a generalized reset criterion was chosen. To our knowledge, this is the first time that this concept has been used with SMC. Thus, the main result of this work is the design of a SMC based on a hybrid surface (SMC-HS) for improved tracking in nonlinear processes. The proposal is tested in the non-linear model of Camacho and Smith, (2000).

Later, SMC with a PID sliding surface is compared with the proposed approach using the performance index ITSE (Integral of Time multiplied by the Squared Error) and TVu (Total Variation of control effort)

The rest of the paper is organized as follows — Section 2 presents the fundamentals of Reset control and SMC. Section 3 shows the synthesis of the SMC-HS proposed. The nonlinear system, results and discussions are described in Section 4. Finally, conclusions are summarized in Section 5.

2. FUNDAMENTALS OF RESET CONTROL AND SLIDING MODE CONTROL

2.1 Reset Control

The reset-based control has a reset mechanism built into a given state. This only applies when a certain condition is attained. The most general way in which it is applied is by triggering the reset when the zero crossing of the tracking error is detected. The fundamentals of incorporating reset to a controller were developed in 1958 by J.C. Clegg. The interest in the reset is based on the possibility it gives us to form hybrid systems, which are simple, but can overcome the limitations of linear control.

Barreiro and Baños, (2012) present a broad explanation of the bases of the reset, its conceptual generalizations, such as reset with band and variable percentages, in addition to developing the stability criteria for these type of systems.

2.2 Sliding Mode Control

The SMC requires defining a surface along which the process can slide to a desired final value. Its design consists of two stages, first the choice of the surface and then the generation of the control law based on that surface. Therefore, it is essential to specify a surface according to the response expectations of the system. The surface is where the dynamics of the process is restricted to its equations, and the behaviour of that plant will depend on the surface's robustness (Camacho, 2002).

Choosing a sliding surface is the most important step for the synthesis of the controller since its choice will represent the desired global behaviour, and it will also characterize the stability and performance in tracking (Camacho and Smith, 2000).

While the PID surface is the usual choice, considering derivative action provides a way to achieve a more highly-sensitive controller. PD responds to the rate of change of the error, which is corrected with anticipation so that the control action becomes timely without allowing the magnitude of the error to become too large. Derivative action tends to increase the stability of the system and adds damping, which, although it does not directly involve the error in steady state, improves its accuracy and does not present over-shoot (Smith and Corripio, 2006). However, the weaknesses of a derivative action, such as susceptibility to noise, are well known, but PD's benefits cannot be ignored either.

The mathematical treatment that comes with the synthesis of a SMC allows the combining of the actions of both surfaces, PID and PD, to obtain a control scheme that reflects the best qualities of each of them. The merging can be done taking into account that the objective of the combination is to restart and reactivate the integral term according to the development of the system over time (Lu and Lee, 2013).

3. SYNTHESIS OF THE SLIDING MODE CONTROLLER BASED ON A HYBRID SURFACE (SMC-HS)

The SMC is a control scheme with high potential, and with slight modifications in its structure, it can offer even better results to those already reported (Camacho and Liptak, 2018). SMC-HS seeks to improve the response in the transient state, maintaining disturbance rejection, thus achieving the benefits of both surfaces when they are combined.

Here, the mathematical development of the surfaces for the SMC-HS is indicated.

The first step to structure a SMC is to determine the sliding surface $S(t)$. In accordance with the order of the system to be controlled, the form the surface takes will be defined as expressed in (1), $S_1(t)$ for an integral type, and (2), $S_2(t)$ for a derivative one (Slotine and Li, 1991).

$$S_1(t) = \left(\frac{d}{dt} + \lambda\right)^n \int e(t)dt \qquad (1)$$

$$S_2(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e(t) \qquad (2)$$

Here, n is the order of the system, λ is a tuning parameter chosen according to the desired system dynamics, and $e(t)$ is the error at the process output.

It is evident then that the complexity of any of the surfaces is related to the order of the system. Thus, it is advisable to work with reduced order systems. Camacho and Smith (2000) prove that higher order nonlinear systems may be well approximated as First Order Plus Dead Time (FOPDT) as follows:

$$G_{p_{FOPDT}}(s) = \frac{K}{\tau s+1} e^{-t_o s} \qquad (3)$$

There, K is the gain of the system, τ is the time constant and t_o is the dead time due to the delays within the process (Smith and Corripio, 2006).

According to Camacho and Smith, (2000), Equation (3) can be converted to (4) as can be seen:

$$G_p(s) = \frac{K}{(\tau s+1)(1+t_o s)} \qquad (4)$$

Equation (4) can be represented in the time domain as expressed in (5).

$$\frac{d^2x(t)}{dt^2} + \frac{\tau+t_o}{\tau\, t_o}\frac{dx(t)}{dt} + \frac{1}{\tau\, t_o}\, x(t) = K\frac{U(t)}{\tau\, t_o} \qquad (5)$$

Equation (5) represents the relationship of the system output $x(t)$ to its input $U(t)$ as a function of time. The previous model represents a second order system. So, for n equal to 2 Equations (1) and (2) can be represented as follows:

$$S_1(t) = \frac{de(t)}{dt} + \lambda_1 e(t) + \lambda_0 \int e(t)dt \quad (6)$$

$$S_2(t) = \frac{de(t)}{dt} + \lambda e(t) \quad (7)$$

From (6) and (7), it stands out that the structures of the surfaces achieved take the form of a PID and PD type controller respectively. It is noted that what differentiates one surface from another is only the integral term.

For each surface, it is then possible to develop an equivalent control law $U_{eq}(t)$, as expressed in (14).

Initially, the PID sliding surface from (6) is considered. The first step is to apply the sliding condition (8) (Slotine and Li, 1991):

$$\frac{dS(t)}{dt} = 0 \quad (8)$$

Eventually, the chosen surface is derived:

$$\dot{S}_1(t) = \ddot{e}(t) + \lambda_1 \dot{e}(t) + \lambda_0 e(t) = 0 \quad (9)$$

The error is then expressed in terms of the system output and its set point $r(t)$. Considering that $e(t) = r(t) - x(t)$, (9) is rewritten:

$$\ddot{r}(t) - \ddot{x}(t) + \lambda_1 \dot{r}(t) - \lambda_1 \dot{x}(t) + \lambda_0 e(t) = 0 \quad (10)$$

The reference is supposed to be constant, so its derivatives with respect to time become zero:

$$-\ddot{x}(t) - \lambda_1 \dot{x}(t) + \lambda_0 e(t) = 0 \quad (11)$$

Substituting (11) in (5) and rearranging, $U_C(t)$ is obtained as expressed in (12):

$$U_C(t) = \frac{\lambda_0 \tau\, t_o}{K} e(t) + \frac{x(t)}{K} \quad (12)$$

$U_C(t)$ corresponds to the continuous component of the control law. The reachability control law $U_D(t)$, which is a function of the surface and constant parameters as (13) express, is added.

$$U_D(t) = K_D \frac{S(t)}{|S(t)|+\delta} \quad (13)$$

$$U_{eq}(t) = U_C(t) + U_D(t) \quad (14)$$

Constant parameters can be chosen with the expressions given by Camacho and Smith, (2000). The parameter K_D gives speed to the reachability of the system, and δ softens the response to avoid chattering problems.

$$\lambda_1 = \frac{\tau + t_o}{\tau\, t_o} \quad (15)$$

$$\lambda_o \leq \frac{\lambda_1^2}{4} \quad (16)$$

$$K_D = \frac{0.51}{|K|} \left(\frac{\tau}{t_o}\right)^{0.76} \quad (17)$$

$$\delta = 0.68 + 0.12|K| K_D \lambda_1 \quad (18)$$

In the case of the PD surface, a similar process is followed. That development results in setting the integral term to zero, so the continuous controller law is therefore given by (19).

$$U_C(t) = \frac{x(t)}{K} \quad (19)$$

In relation to the sliding control law, similar structures are found between (12) and (19). There, the last one does not include the term $\frac{\lambda_0 \tau\, t_o}{K} e(t)$. This indicates that this is the component that allows the correction of the deficiencies of a derivative surface, but which in turn implies low performance in the transient response, causing high overshoots.

Now, combining both favourable behaviours, it would be possible to have a response without overshoot for tracking tasks that is also capable of rejecting disturbances. Thus, activating and deactivating this term in the continuous and discontinuous control laws, the objectives proposed can be accomplished.

Based on the reset action, terms such as the integral can be restarted for a smoother response according to the convenience of the process (Barreiro and Baños, 2012). At any set point change, the controller's action will have greater sensitivity with the PD action, and once a reset value ε is reached, the change of surface will occur again. Thus, it will give the PID action the opportunity to stay robust against disturbances.

In Fig. 1, the block diagram shows the composition of the proposed controller. Both $U_C(t)$ and $U_D(t)$ are modified to obtain the hybrid behaviour of a PID and a PD surface. The terms that are modified in both cases, and as stated in the previous equations, are found in blocks with dashed lines. The change in the surface behaviour will then depend on condition C and its passage from 0 to 1 according to the system requirement.

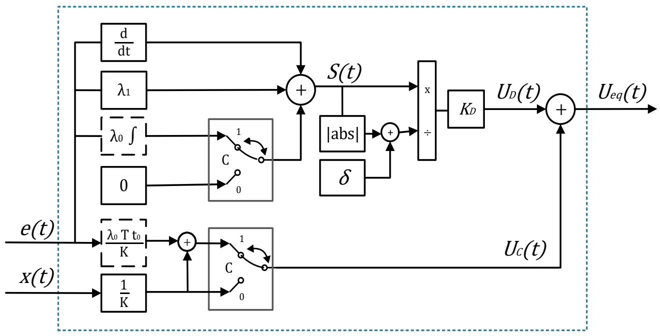

Fig. 1. SMC-HS block diagram scheme.

The flowchart of Fig. 2. describes how the condition C changes. C equal to zero implies giving the SMC a PD nature, while one is for SMC PID nature, which is generally the desired action.

Only if the set point changes to improve the tracking characteristics, the change to PD is sought. Thus, the initial condition of the system will be C equal to one. If a change of set point is registered, C will become zero. Then, C will return to its initial condition when the error becomes less than an ε value, which is defined by the process operator.

Considering the process response, ε can be defined at a certain percentage, such as 2% or 5% of the output error $e(t)$. The effect of increasing or reducing the tolerance of ε is explained below with the example.

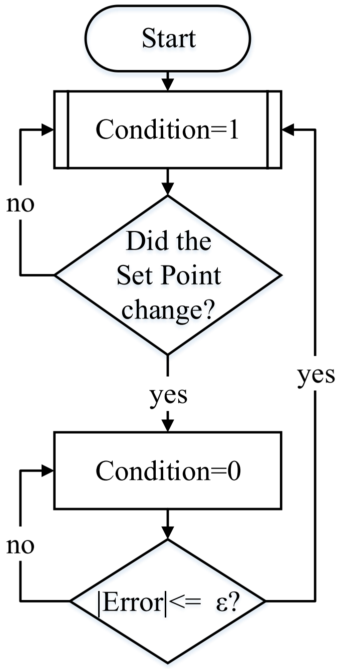

Fig. 2. Condition *C* Flow Chart.

The general scheme of the controller is shown in Fig. 3. The SMC-HS depends on the system error, its output, and the set point value.

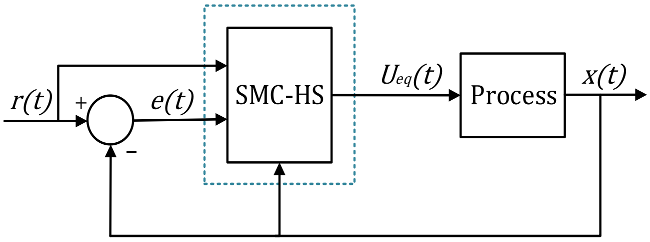

Fig. 3. General scheme of the SMC-HS.

A formal treatment of the stability problem is out of the scope of this work. For that reason, it is not indicated in detail, but it is mentioned so that the reader can notice that there are foundations about hybrid control, and those must be followed.

The studies before Horowitz's suffered from the lack of a rigorous approximation to the stability problem. Beker et al. (2004) present some considerations about stability. Thus, this problem is attacked by postulating a Lyapunov candidate function $V(x)$, which is normally quadratic, as $V(x) = x^T P(x)$.The Lyapunov candidate function in this scheme is the sliding surface $S(t)$. Following Lyapunov's considerations:

$$\dot{V}(x) = \left(\frac{\partial V}{\partial x}\right)^T Ax < 0, x \neq 0 \qquad (20)$$

$$\Delta V(x) = V(A_r x) - V(x) \leq 0, x \in M \qquad (21)$$

Where A, A_r and M are defined in Barreiro and Baños, (2012). The stability condition consists in the usual condition (20) for the continuous mode, to which (21) is added as a restriction so that V does not grow in the resets.

The issue of asymptotic stability is considered by bounding solutions and checking if they converge around A. This approach is also presented by Goebel et al. (2009).

4. RESULTS AND DISCUSSION

In order to test the performance of the proposed approach, a mixing tank is used as a model of a nonlinear system. The example is taken from Camacho and Smith, (2000).There, the description of the self-regulating process is given. Also, the equations of the mathematical model are presented, as well as the design parameters.

The parameters of the proposed FOPDT approach are found after performing the reaction curve described in Smith and Corripio (2006). The correspondent transfer function based on the parameters obtained for the model presented in (3) is expressed in (22).

$$G_{p_1}(s) = \frac{-0.87542}{2.4749s+1} e^{-4.3749s} \qquad (22)$$

The ratio $\frac{t_o}{\tau}$, called the controllability relationship, is greater than one representing a dead time dominant process.

To carry out the different tests, the operating ranges of the process must be considered. The transmitter is considered as operating within a range of 100 to 200 ° F. In addition, the operating range of the valve is 0 to 1. So, those are the limits that the control action may have.

At initial conditions, the temperature of the liquid in the mixing tank is 150 °F. At the times 10, 150, 350 and 600 min, the temperature set point of the tank is varied to 160, 165, 155, and 170 ° F respectively.

Moreover, disturbances in the flow of hot water occur. It changes from 250 lb/min to 225 lb/min and 200 lb/min at times 250 min and 500 min respectively. Those changes are shown in Fig.4.

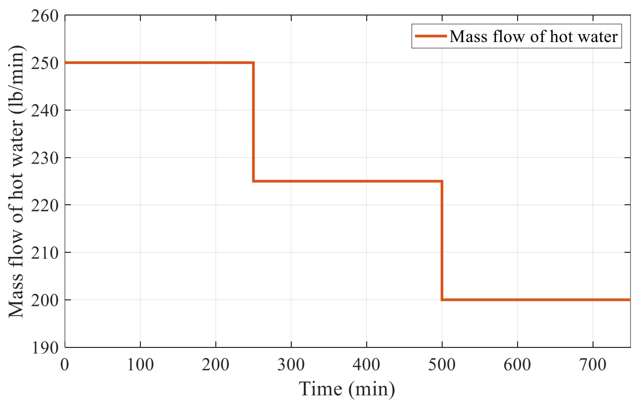

Fig. 4. Mass flow of hot water disturbances

The chosen nonlinear example not only has a delay due to the transmitter's distance from the process, but also has a delay that changes and increases as the hot water flow disturbances decrease, as shown in Fig. 5.

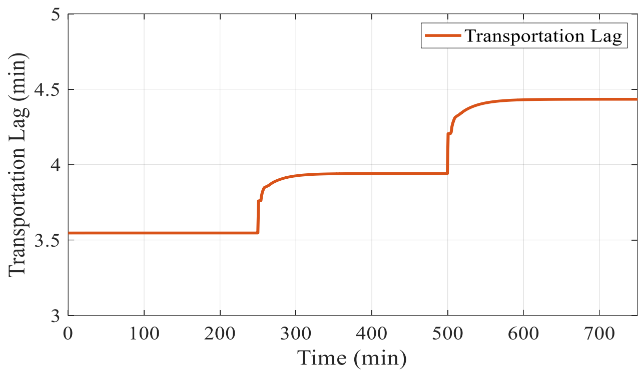

Fig. 5. Delay increase as a function of time.

The response of the process can be seen in Fig. 6 and the corresponding control signals in Fig. 7. The effects of the disturbances that hot flow has on the system can also be seen in Fig. 6.

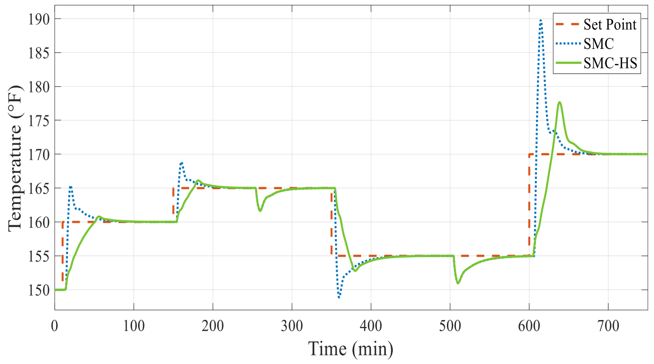

Fig. 6. System response to set point changes and disturbances.

The results demonstrate that the system improves its tracking task with the PD surface feature without being affected in the regulation process when disturbances appear in the operation. Therefore, despite the adversities, the proposed approach maintains the robustness of original SMC from Camacho and Smith (2000). Thus, the improvement of the transient response is attained.

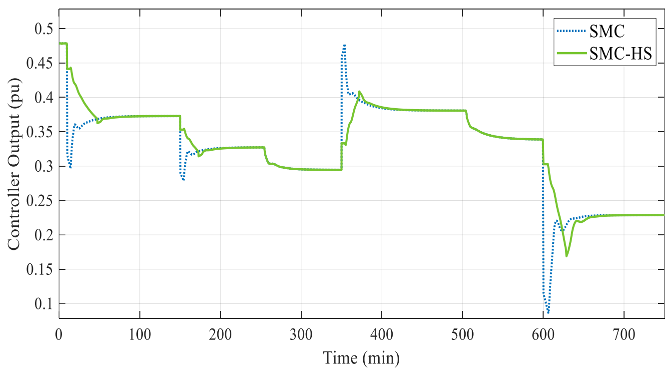

Fig. 7. Controller Output Comparison.

It is important to mention that in each modification of the surface, the integral term is restarted and remains constant as shown in Fig. 8. The SMC without the hybrid surface tends to generate peaks due to the accumulation of the integral term.

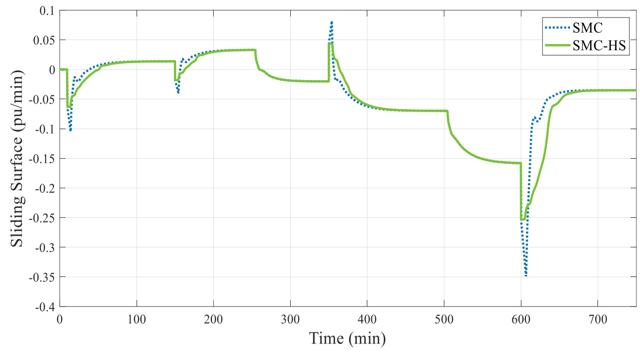

Fig. 8. Evolution in time of the PID surface vs the Hybrid surface.

The evolution of integral term in the sliding surface can be seen in Fig. 9. where it is noted that the algorithm used for the SMC-HS manages to avoid spikes when set point changes occur.

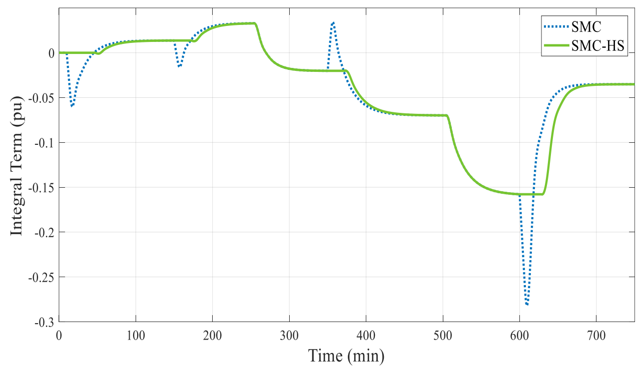

Fig. 9. Evolution of the integral term of the PID surface vs the hybrid surface.

In Fig. 10, the change of condition *C* between high and low is presented, depending on whether it is at one or zero, correspondingly. According to the logic established for the SMC-HS, it can be seen how the condition goes to zero at each reference change, and the rest of the time is kept at one.

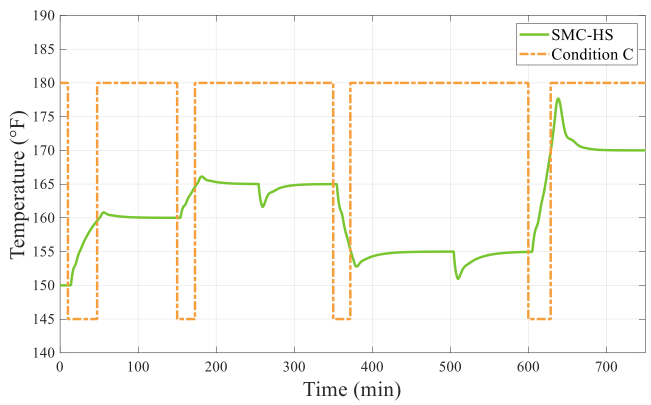

Fig. 10. System response based on the change of condition *C*.

The bandwidth at zero increases as a lower value of ε is chosen. On the other hand, the greater the tolerance of error, which means increasing ε, the bandwidth of zero will be reduced.

By reducing the zero band, the hybrid controller will take a behaviour as SMC with PID surface. It is then the value of ε which determines the band of action of the hybrid surface. For this case, ε equal to 5% of the error was chosen.

It is important to highlight that the controller output is kept limited when the reset term appears due to the hybrid surface. The reset term by itself does not have the characteristic of eliminating the steady state error in response for modelling errors and disturbances, in order to eliminate them the integral term should be present.

The indexes and criteria to contrast the performance of the proposed controller are the maximum overshoot, ISE, ITSE and TVu. Maximum overshoot (Mp) is presented for the changes at minute 10 for Mp1, and 600 for Mp2. Meanwhile, the settling time is around 100 minutes, independent of the control scheme.

To clarify the understanding of these indexes and criteria, they are presented in a radial graph. For the 5 parameters, the smallest values are those of the SMC-HS. Although both controllers have very close values in their ISE and ITSE indexes, the overall performance of the SMC-HS has a better compliance regarding the transitory characteristics, as Fig. 11 presents.

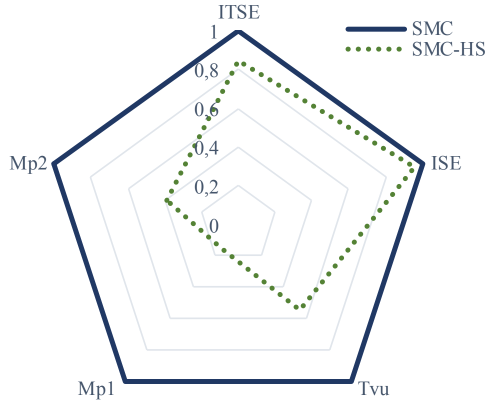

Fig. 11. Radial graph for parameter comparison.

5. CONCLUSION

A SMC based on a Hybrid Sliding Surface was designed considering the reset of the integral term.

This approach improves the tracking at the response of a nonlinear process without excessive overshooting in comparison with the original SMC that was proposed by Camacho and Smith (2000). Also, the proposal keeps the same response when disturbances appear and dead time varies.

The controller output is kept bounded when the reset term is included. A formal treatment of the stability should be considered in future works.

ACKNOWLEDGEMENTS

Authors want to thank to PIJ 17-10 project of Escuela Politécnica Nacional, Ecuador, for their support to join the conference.

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