Valve Stiction Model Estimation in Closed-loop Operation

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Abstract: The estimation of valve stiction model is studied. In industrial applications, valve outputs are often not available, the stiction nonlinear block appears before a linear dynamic block which is operation in a closed-loop system. By parameterizing the valve stiction model as a form of cubic splines, an identification method is proposed using a relaxation iteration scheme. Parameter estimation for the linear part is accomplished through a two-stage procedure. Firstly, an unbiased estimation is obtained by the high-order ARX (AutoRegressive eXogenous) model. Then the ARX model is reduced to a Box-Jenkins model. The consistency of the method is established. Simulation data sets and real operation data sets are used to illustrate the method.

Keywords: Valve stiction, Hammerstein model, Identification, Relaxation

1. INTRODUCTION

In the process industry, the poor control performance is caused not only by bad control tuning but also by nonlinear characteristics of control valves. Among many types of undesired nonlinear characteristics of industrial control valves, stiction is the most common problem (Riccardo, Scali and Pannocchia, 2016). Therefore, developing a method to estimate valve stiction is necessary for later compensation measures.

Based on the input-output behavior of a sticky valve, many studies have been conducted to define and model stiction. Muller (1994) has proposed a detailed physical model. Stenman (2009) has reported a one-parameter data-driven stiction model. Besides, a data-driven model with two parameters has been proposed to describe the relationship between a controller output and a valve position by Choudhury (2005). Also, some other complicated date-driven models are established. More recently, Jelali and Huang (2010) have published a book to present a comprehensive review of the state of the art on stiction detection and quantification methodologies.

Several authors have proposed model-based approaches for stiction detection and quantification. Considering that the position of the industrial control valves is seldom available in practice, the identification method for valve stiction using a Hammerstein model is desirable. A Hammerstein model is in the form of an N-L model where a nonlinear block is embedded before a linear block. Jelali (2008) provides a procedure for quantifying valve stiction in Hammerstein systems based on global optimization technology. Pattern search methods or generic algorithms are used to fix the global minimum of the parameters of the stiction model, and the linear model parameters are identified by a least-squares estimator. Vörös (2010) presents an analytic form of nonlinear characteristic description to solve the estimation problem of cascaded systems consisting of an input backlash followed by a linear dynamic system. Furthermore, Wang and Zhang (2012) adopts a point-slope-based hysteresis model and an iterative algorithm to solve the identification problem.

In this work, we will study parametric model identification of SISO (Single-Input/Single-Output) Hammerstein model with valve stiction. The input-output data can be generated from closed-loop tests. The nonlinear part of model is parametrized in cubic splines. A relaxation iteration scheme is proposed by using the model structure in which the error is bilinear-in-parameters. The linear model estimation follows the asymptotic method (Zhu, 1998).

The rest of the paper is organized as follows: in Section 2 we will establish appropriate model structure for valve nonlinearity and formulate the identification problem of Hammerstein system. Model parameter estimation and consistency analysis is discussed in Section 3. Case studies are performed in Section 4. Section 5 contains the conclusion.

2. MODEL PARAMETRIZATION

The phenomena of valve stiction can be understood through the phase plot of the input-output behaviour as shown in Fig. 1 (Choundhury, 2005). It consists of four components: deadband, stick-band, slip-jump and the moving phase. When the valve comes to a rest or changes the direction at point A (or E) in Fig. 1, the valve sticks as it cannot overcome the force

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due to static friction. After the controller output overcomes the deadband (AB) plus the stick-band (BC) of the valve, the valve jumps to a new position (point D) and continues to move. According to the figure, the valve with stiction nonlinearity has a hysteresis behavior.



Fig. 1. Input-output behavior of viscous valve.

Several physical and data-driven models have been proposed based on the valve stiction. However, it happens that a model based on physical insight contains a number of unknown parameters. In such cases, the model structure is not suitable for identification. Here we will introduce a form of cubic spline function to represent the hysteresis nonlinearity, see, e.g., Lancaster, Šalkauskas and Kęstutis (1986).

Consider the closed loop block diagram as shown in Fig.2 where a sticky valve is included between the process and the controller block. Typically, the controller will be a linear PID controller.



Fig. 2. Process control loop with valve stiction.

Denote u(t) as the input signal and y(t) as the output signal. We introduce $f(\cdot)$ to represent the sticky control valve. x(t) is the valve output signal which is not measured. And r(t) is the external excitation signal. The real process G(q) is described as a Box-Jenkins model (1971).

$$x(t) = f(u(t)) \tag{1}$$

$$y(t) = \frac{B(q)}{A(q)}x(t) + \frac{C(q)}{D(q)}e(t)$$
 (2)

$$v(t) = \frac{C(q)}{D(q)}e(t)$$
(3)

Where

$$A(q) = 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-1}$$
$$B(q) = b_1 q^{-1} + \dots + b_{n_b} q^{-n_b}$$

$$C(q) = 1 + c_1 q^{-1} + \dots + c_{n_c} q^{-n_c}$$
$$D(q) = 1 + d_1 q^{-1} + \dots + d_{n_d} q^{-n}$$

 q^{-1} is the unit delay operator, n_a , n_b , n_c and n_d are the orders of corresponding polynomials. The disturbance signal e(t) is assumed to be a white noise signal with zero mean and variance R. In most real industrial systems, the valve output x(t) is not available. Denote

$$Z^{N} = \{u(1), y(1), u(2), y(2), \cdots u(N), y(N)\}$$
(4)

Valve stiction belongs to a class of nonlinearity with memory. Here two different cubic splines are respectively used for the ascent and the descent paths of the hysteresis nonlinearity which indicate that signal u(t) is increasing or decreasing. The direction signal is given as

$$h(t) = \begin{cases} 1 & u(t) \ge u(t-1) \\ 0 & u(t) < u(t-1) \end{cases}$$
(5)

Denote a set of knots $\{g_1^A, g_2^A, \dots, g_{m_i}^A\}$ for increasing u(t) which are real numbers and satisfy

$$g_1^A = u_{min} < g_2^A < \dots < g_{m_1}^A = u_{max}$$
(6)

A cubic spline function for valve output is given as

$$f_{1}(u) = \sum_{k=2}^{m_{1}-1} \beta_{k}^{A} \left| u - g_{k}^{A} \right|^{3} + \beta_{m_{1}}^{A} + \beta_{m_{1}+1}^{A} u + \beta_{m_{1}+2}^{A} u^{2} + \beta_{m_{1}+3}^{A} u^{3}$$
(7)

Similarly, the descent path of the input nonlinearity can be defined as

$$g_1^D = u_{min} < g_2^D < \dots < g_{m_2}^D = u_{max}$$
(8)

$$f_{2}(u) = \sum_{k=2}^{m_{2}-1} \beta_{k}^{D} \left| u - g_{k}^{D} \right|^{3} + \beta_{m_{2}}^{D} + \beta_{m_{2}+1}^{D} u + \beta_{m_{2}+2}^{D} u^{2} + \beta_{m_{2}+3}^{D} u^{3}$$
(9)

Where $\{\beta_2^A, \beta_3^A, \dots, \beta_{m_1+3}^A\}$ and $\{\beta_2^D, \beta_3^D, \dots, \beta_{m_2+3}^D\}$ are parameters to be estimated. Here m_1 and m_2 are called the number of knots which can be seen as the order of the cubic splines. For simplicity, one can set $g_i^A = g_i^D$ $(i = 1, 2, \dots, m)$ so that $m_1 = m_2 = m$.

It is easy to find that the function $f_1(u)$ and $f_2(u)$ are smooth, moreover, the first and the second derivatives of the functions are continuous. The description of cubic splines for hysteresis nonlinearity is based on the following two assumptions.

A1: The signal u(t) and y(t) are approximately cyclo-wisesense stationary with a certain period.

A2: The ascent and the descent paths of input hysteresis nonlinearity share either the higher or the lower extreme point.

The assumptions hold under very general conditions due to the reason that the hysteresis nonlinearity usually leads to self-sustaining oscillations with certain amplitude and period in control systems. If not, one can design and carry out identification tests to realize conditions A1 and A2. Here, the cubic spline function was chosen over other models due to two reasons: (1) it is flexible in curve-fitting yet simple in parameter estimation; (2) it has better numerical conditions in parameter estimation than other models such as polynomials.

The system (1) - (9) can be identified using the prediction error method (Ljung, 1999). Then we can write the predictor for the model structure in the following form.

$$\hat{y}(t) = \frac{D(q)B(q)}{C(q)A(q)} f[u(t)] + \left[1 - \frac{D(q)}{C(q)}\right] y(t)$$
(10)

The prediction error is given by:

$$\varepsilon(t) = y(t) - \hat{y}(t) = \frac{D(q)}{C(q)} \left[y(t) - \frac{B(q)}{A(q)} f[u(t)] \right]$$
(11)

Then parameters of the model can be determined using the test data by minimizing

$$V_{BJ} = \sum_{t=1}^{N} \varepsilon^{2}(t) = \sum_{t=1}^{N} \left\{ \frac{D(q)}{C(q)} \left[y(t) - \frac{B(q)}{A(q)} f[u(t)] \right] \right\}^{2}$$
(12)

Once parameters of the Hammerstein model are obtained, we can visualize and check the valve input-output plot for the cyclic pattern. The width of the cyclic pattern can be considered to be directly related to the amount of stiction presented in the valve.

However, direct minimization of the loss function (12) is very difficult as the prediction error in (11) has very complex nonlinear relations to model parameters, which will lead to numerical issues such as local minima and non-convergence.

3. IDENTIFICATION ALGORITHM

High model accuracy and reliable numerical solutions are desired in parameter estimation. Here the asymptotic method proposed by Zhu (2000) is followed in the identification of the Hammerstein model with valve nonlinearity. The method is based on the so-called asymptotic theory (Ljung, 1985) and it starts with a high order model estimation and then followed by a model reduction.

3.1 High Order Model Estimation

It is well known that any linear prediction error model structure can be approximated arbitrarily well by an ARX or equation error model with sufficiently high order. Let us approximate the linear part of the Hammerstein model with Box-Jenkins structure by a high order ARX model:

$$y(t) = \frac{B^{n}(q)}{A^{n}(q)}f(u(t)) + \frac{1}{A^{n}(q)}e(t)$$
(13)

Where *n* is the order of ARX model. In the section 2, we use cubic splines to approximate the stiction nonlinearity, and the basic form of cubic spline functions can be determined by different choice of knots. According to the equation (7) and (9), we could just obtain the virtual valve output signal $\hat{f}(u(t))$. Now the linear part of the Hammerstein model with Box-Jenkins structure can be approximated by the high order ARX model (13). The loss function for parameter estimation becomes

$$V_{ARX} = \sum_{t=1}^{N} \varepsilon_{ARX}^{2}(t) = \sum_{t=1}^{N} \left[A^{n}(q) y(t) - B^{n}(q) f(u(t)) \right]^{2}$$
(14)

It is easy to find that the error $\varepsilon_{ARX}(t)$ is bilinear in the parameters of $A^n(q)$, $B^n(q)$ and f(u). We can use following relaxation algorithm for parameter estimation (Narendra and Gallman, 1966).

Initialization. Set $B^n(q) = 1$, estimate $A^n(q)$ and f(u(t)) using the least-squares.

Iteration. Denote $\hat{A}_{(i)}^n(q)$, $\hat{B}_{(i)}^n(q)$, $\hat{f}_{1,(i)}(u(t))$ and $\hat{f}_{2,(i)}(u(t))$ as the estimate from iteration i, then

1) Compute $\hat{A}_{(i+1,1)}^n(q)$ and $\hat{f}_{(i+1)}(u(t))$ for fixed $\hat{B}_{(i)}^n(q)$ by minimizing

$$\sum_{t=1}^{N} \left\{ A_{(i+1,1)}^{n}(q) y(t) - \hat{B}_{(i)}^{n}(q) [h(t) f_{1,(i+1)}(u(t)) + (1-h(t)) f_{2,(i+1)}(u(t))] \right\}^{2} (15)$$

2) Compute $\hat{A}_{(i+1,2)}^n(q)$ and $\hat{B}_{(i+1)}^n(q)$ for fixed $\hat{f}_{(i+1)}(u(t))$ by minimizing

$$\sum_{t=1}^{N} \left\{ A_{(t+1,2)}^{n}(q)y(t) - B_{(t+1)}^{n}(q)[h(t)\hat{f}_{1,(t+1)}(u(t)) + (1-h(t))\hat{f}_{2,(t+1)}(u(t))] \right\}^{2}$$
(16)

Go back to 1). Stop when convergence occurs.

Both steps are linear least-squares problems for which the solutions are numerically simple and reliable. Note that $A^n(q)$ is updated twice at each iteration. Considering that two arbitrary gains may be distributed between the linear and the nonlinear part, a normalization procedure must be conducted after each iteration. For example, we could set the gain of the linear model as a fixed constant.

According to Golub and Pereyra (1973), assume that the input u(t) is persistent exciting with order greater than (2n,m) and the number of its amplitude levels is greater than the number of knots. Then the relaxation algorithm (15) and (16) minimizes the criterion in (14) locally if it converges.

The proof can also be found in the literature. Now, the estimation for the valve output signal can be referred as $\hat{x}(t) = \hat{f}[u(t)]$. Denote the process model and the disturbance model as

$$\hat{G}^{n}(q) = \frac{\hat{B}^{n}(q)}{\hat{A}^{n}(q)}, \quad \hat{H}^{n}(q) = \frac{1}{\hat{A}^{n}(q)}$$
(17)

Model reduction for the obtained high order ARX model will be discussed below.

3.2 Model Reduction For ARX Model

Employing the high order ARX model structure in equation (13), the process model $\hat{G}^n(q)$ is often over parameterized, thus leads to high variance owing to the high order *n*. The order of ARX model need to be reduced for the purpose of reducing variance error of the model.

If we know the exactly nonlinear output x(t), the order of ARX model is high enough and the nonlinear output x(t) is persist exciting, it can be proved that the high order estimates are consistent, and the errors of the transfer functions at each frequency will follow a Gauss distribution (Ljung, 1985).

Equation (18) is an expression for the asymptotic variance of the process model.

$$var[\hat{G}^{n}(e^{i\omega})] \approx \frac{n}{N} \frac{\Phi_{v}(\omega)R}{\Phi_{x}(\omega)R - |\Phi_{xe}(\omega)|^{2}}$$
(18)

Where $\phi_{v}(\omega)$ is the spectra of v(t) and $\phi_{x}(\omega)$ is the spectra of x(t), $\phi_{xe}(\omega)$ is the cross-spectrum between signal x(t) and e(t). The following log-likelihood function proposed is used for model reduction (Zhu, 1998). The reduced model is presented by $\hat{G}^{t}(e^{i\omega})$.

$$V = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \hat{G}^n(e^{i\omega}) \cdot \hat{G}^l(e^{i\omega}) \right|^2 \frac{\Phi_x(\omega)R - \left| \Phi_{xe}(\omega) \right|^2}{\Phi_v(\omega)R} d\omega \qquad (19)$$

The function must be minimized using a numerical search routine. Here we could adopt the Gauss-Newton method.

To obtain the best order of the reduced model, a frequency domain criterion ASYC shall be applied (Zhu, 2001). The basic idea of the criterion is to choose the best order, which makes the frequency domain difference between the high order model and the reduced model approximately equal to the variance of the high order model.

$$\left|\hat{G}^{n}(e^{i\omega}) \cdot \hat{G}^{l}(e^{i\omega})\right|^{2} \approx \frac{n}{N} \frac{\Phi_{v}(\omega)R}{\Phi_{x}(\omega)R - \left|\Phi_{xe}(\omega)\right|^{2}}$$
(20)

Then the best order can be obtained through minimizing the following ASYC.

$$ASYC = \int_{-\pi}^{\pi} \left| \hat{G}^{n}(e^{i\omega}) \cdot \hat{G}^{l}(e^{i\omega}) \right|^{2} - \frac{n}{N} \frac{\Phi_{v}(\omega)R}{\Phi_{x}(\omega)R - |\Phi_{xe}(\omega)|^{2}} d\omega$$
(21)

The selection of *m* and *l* should be discussed. Denote V_{oe} as the output-error criterion evaluated on the estimation data, the following final output-error criterion can be used to determine the degree *m* of cubic splines. And order selection for the reduced linear model can be similarly derived.

$$FOE(m) = \frac{N + (2n+m)}{N - (2n+m)} V_{oe}$$
(22)

The final model we obtained is a Box-Jenkins model.

$$\hat{G}^{l}(q) = \frac{\hat{B}^{l}(q)}{\hat{A}^{l}(q)} \qquad \hat{H}^{l}(q) = \frac{\hat{C}^{l}(q)}{\hat{D}^{l}(q)}$$
(23)

3.3 Consistency Analysis

In the following some theoretical analysis will be carried out on the consistent estimation for parameters of Hammerstein model. First, two key assumptions are given below. A3 For the given system, valve stiction does exist. The true process has the same parameters as the model (1)-(9).

$$y(t) = \frac{B(q)}{A(q)} \left[h(t)f_1(u(t)) + [1 - h(t)]f_2(u(t)) \right] + \frac{C(q)}{D(q)}e(t)$$
(24)

A4 The input u(t) is strongly persistently exciting with order (2n,m). For any non-zero filters L(q) and K(q), we have

$$\lim_{N \to \infty} \inf \frac{1}{N} \sum_{t=1}^{N} E[L(q)u(t) + K(q)y(t)]^2 > 0$$
 (25)

Theorem 1. Assume that conditions A3-A4 are true, then when the amount of data N goes infinity, there exists

$$\hat{\beta}_{j}^{A} \rightarrow \beta_{j}^{A} \qquad j = 2, 3, \cdots m_{1} + 3 \tag{26}$$

$$\hat{\beta}_k^D \to \beta_k^D \qquad k = 2, 3, \cdots m_2 + 3 \tag{27}$$

Where, $\hat{\beta}_{i}^{A}$ and $\hat{\beta}_{k}^{D}$ represent optimal estimation for model parameters of cubic splines, β_{j}^{A} and β_{j}^{D} represent the true parameters.

Proof.

If the optimal estimated sticky parameters $\hat{\beta}_{j}^{A}$ and $\hat{\beta}_{k}^{D}$ are equal to corresponding true parameters β_{j}^{A} and β_{j}^{D} , there is no doubt that the estimated valve output $\hat{x}_{opt}(t)$ is equal to the true valve output x(t).

$$\hat{x}_{opt}(t) = x(t) \tag{28}$$

Therefore, prediction error model will remain consistency using closed-loop data $\hat{x}_{opt}(t)$ and y(t), provided that orders of the model are correct. So we have

$$\hat{G}(q) = G^0(q) \tag{29}$$

In this case, we now get

$$V_N^0 \to EV_N = E[y(t) - \hat{y}(t)]^2 = Ev^2(t)$$
 (30)

Where, $\hat{y}(t)$ is the prediction output.

On the contrary, if the optimal estimated sticky parameters $\hat{\beta}_{j}^{A}$ and $\hat{\beta}_{k}^{D}$ are not equal to true parameters β_{j}^{A} and β_{j}^{D} , then the estimated valve output $\hat{x}_{opt}(t)$ will be biased. The following relation holds

$$V_{N}^{0} \rightarrow EV_{N} = E\left[G^{0}(q)x(t) + v(t) - \hat{G}(q)\hat{x}(t)\right]^{2}$$

$$= E\left\{G^{0}(q)x(t) - \left[G^{0}(q) + \Delta G(q)\right] \cdot \left[x(t) + \Delta x(t)\right] + v(t)\right\}^{2}$$

$$= E\left\{v(t) - \left[G^{0}(q)\Delta x(t) + \Delta G(q)x(t) + \Delta G(q)\Delta x(t)\right]\right\}^{2}$$

$$= Ev^{2}(t) + E\left[G^{0}(q)\Delta x(t) + \Delta G(q)x(t) + \Delta G(q)\Delta x(t)\right]^{2}$$

$$\geq Ev^{2}(t)$$
(31)

Therefore when N tends to infinity, the loss function gets to its minimum if and only if the following equations are true.

$$\Delta G(q) = 0 \tag{32}$$

$$\Delta x(t) = 0 \tag{33}$$

This is contrary to the conclusion that the valve output is biased, namely the equation (26-27) is established. The optimal parameter estimation for Hammerstein model is consistent.

4. DEMONSTRATION

4.1 Simulation Study

A second order Box-Jenkins process is investigated. The process is given as

$$y(t) = \frac{q^{-1} + 0.5q^{-2}}{1 - 1.5q^{-1} + 0.7q^{-2}} x(t) + \frac{\alpha}{1 - 0.9q^{-1}} e(t)$$

The transfer function with a PI controller in a feedback closed-loop configuration is used here for simulation.

$$C(q) = \frac{0.31 - 0.3q^{-1}}{1 - q^{-1}}$$

The nonlinear part of the process is simulated by adopting the two-parameter model in (Choudhury, 2005) with S = 20 J = 2. The white noise signal e(t) has a zero mean and a variance of 1. The excitation signal is a GBN signal. Adjust the variance of v(t) so that the noise-to-signal ratio at output is 5% in power. Generate one data set of 1500 samples for model estimation and the other data set of 1500 samples for model validation. Part data is presented in Fig. 3.



Fig. 3. Measured controller output and process output.

Estimation results are shown as follows. The model is simulated using the validation data set. The measured output and the process output of the first 100 data are shown in Fig. 6.



Fig. 4. Estimated stiction nonlinearity.



Fig. 5. Step response of the linear model and the process.



Fig. 6. Model fit of the process output.

The fitness can be calculated by:

$$Fitness = 100 \left(1 - \frac{\|y(t) - \hat{y}(t)\|_2}{\|y(t) - E\{y(t)\}\|_2} \right)$$
(34)

Fitness compares the simulated output errors with those obtained using the empirical mean as the model output. Here, the model fitness for validation data is 94.37%. The simulation shows the effectiveness of the method.

4.2 Industrial Example

An international database is available online for academic research on the detection and diagnosis of oscillations and control valve stiction (Jelali and Huang, 2010). Here the proposed method will be applied to cdata.chemicals.loop23 in the database. The valve position is not available. The controller output and the process output are presented in Fig. 7. The sampling period is 10 sec.



Fig. 7. Measured controller output and process output.



Fig. 8. Estimated stiction nonlinearity.



Fig. 9. Step response of the linear model.



Fig. 10. Model fit of process output.

Here the estimated length of stiction is nearly equal to the reference value proposed in the book (Jelali and Huang, 2010). The model fitness is 75.67%, which is higher than the previous work (Wang and Zhang, 2012).

An important remark is that the model fitness should be calculated on a data set which has not been used for model estimation. In this case we did not distinguish like this due to the reason that the length of the given data set is limited. However, it is not a recommended practice.

5. CONCLUSION

The estimation of valve stiction based on a Hammerstein model in closed-loop system is studied. A form of cubic spline function is used to derive the identification algorithm based on a relaxation scheme. Model order is determined based on a frequency domain criterion ASYC. The consistency of the method is established. Effectiveness of the proposed method has been shown in demonstration. It is easy to generalize the method to other processes with hysteresis nonlinearity.

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