Stabilization of Passive Dynamical Systems with Actuator and Sensor Disturbances ⋆

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Abstract: We analyze stabilizability of passive dynamical systems subject to actuator and sensor disturbances. New sufficient conditions are given for the conventional static output feedback, which is used to stabilize passive systems, to guarantee the (integral) input-to-state stability property with respect to the disturbances. As an illustrative example application of the obtained results to robust state observer redesign is considered.

Keywords: Nonlinear control, Passivity, Input-to-state stability, Stabilization, Robust state observer.

1. INTRODUCTION

Passive dynamical systems play the central role in many theoretical control areas and applications. Passivity property is of major importance in adaptive control, see e.g. Seron et al. (1995), Jiang et al. (1996), Astolfi et al. (2008), and nonlinear observer design (Shim and Seo (2000), Shim et al. (2003)). The key nonlinear state feedback design methods explicitly or at least implicitly rely on passivity properties of dynamics, see e.g. Byrnes et al. (1991), Sepulchre et al. (1997), Khalil (2002). For instance, such effective and popular nonlinear control design philosophy as integrator backstepping (Krstić et al. (1995)) by its virtue can be considered as a feedback passivation scheme, see e.g. Sepulchre et al. (1997).

The success of passivity based designs is underpinned by the fact that many physical systems can be rendered passive by proper choice of input and output functions, see e.g. Ortega et al. (1998), Fantoni and Lozano (2002), Astolfi et al. (2008), Glazkov and Reshmin (2019), Reshmin (2019), Reshmin (2019a), Golubev et al. (2019), Glazkov and Golubev (2019).

In the present paper we analyze the zero equilibrium stabilization problem for disturbed passive nonlinear dynamical systems of the form

$$\begin{aligned} \dot{x} &= f(x, u + d_1), \\ \tilde{y} &= h(x) + d_2, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input, $y = h(x) \in \mathbb{R}^m$ is the output function to be measured, $d_1 \in \mathbb{R}^m$ and $d_2 \in \mathbb{R}^m$ stand for actuator and sensor disturbances, respectively, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^m$ are locally Lipschitz, $f(0, 0) = 0$, $h(0) = 0$. Here, as actuator and sensor disturbance models one takes unknown piecewise continuous bounded functions $d_1 = d_1(t)$ and $d_2 = d_2(t)$ of $t \geq 0$, respectively.

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Let us recall that the unperturbed dynamical system (1) given by

$$\begin{aligned} \dot{x} &= f(x, u), \\ y &= h(x), \end{aligned} \tag{2}$$

is called passive, see e.g. Byrnes et al. (1991), Khalil (2002), if there exists a continuously differentiable positive semidefinite (storage) function $V(x)$ such that for all $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ the (dissipation) inequality holds

$$\dot{V} = \frac{\partial V(x)}{\partial x} f(x, u) \leq w(y, u) \tag{3}$$

with the (supply rate) function $w(y, u) = y^{\mathrm{T}} u$. Additionally, a passive system (2) is said to be strictly passive, if $w(y, u) = y^{\mathrm{T}} u - \psi(x)$ for some continuous positive definite function $\psi(\cdot)$, and output strictly passive if $w(y, u) = y^{\mathrm{T}} u - y^{\mathrm{T}} \varrho(y)$, where $\varrho(\cdot)$ is a continuous function such that $y^{\mathrm{T}} \varrho(y) > 0$ for all $y \neq 0$ (see e.g. Byrnes et al. (1991), Khalil (2002)).

Further, a dynamical system of the form (2) is zero-state detectable (Byrnes et al. (1991)) if for any solution $x = x(t)$ of the system with $u = u(t) \equiv 0$ such that $y(t) = h(x(t)) \equiv 0$ one has $x(t) \to 0$ as $t \to +\infty$.

Then, a zero-state detectable passive system (2) with positive definite proper storage function $V(x)$ can readily be globally asymptotically stabilized by static output feedback $u = -k(y)$, with $k(y)$ being any locally Lipschitz function such that $k(0) = 0$ and $y^{\mathrm{T}} k(y) > 0$ for all $y \neq 0$ (Byrnes et al. (1991)).

However, it is well known that stabilizability of an unperturbed system does not guarantee stabilizability of the system in presence of disturbances, see e.g. Freeman (1995). Notice that the most powerful concept when analyzing behavior of nonlinear dynamical systems subject to disturbances and model uncertainties proved to be the input-to-state stability (ISS) introduced in Sontag (1989) and promoted in later works, see e.g. Sontag and Wang (1995), Sontag and Wang (1996), Sontag (1998), Angeli et al. (2000).

The ISS properties of passive dynamical systems were investigated e.g. in Efimov (2006), Wang and Weiss (2007), Dashkovskiy et al. (2011). In Efimov (2006) it was shown that the control law $u = -k(y) + d_1$ with $k(0) = 0$ guarantees the (integral) ISS property of a control-affine strictly passive system (2) with respect to the d_1 input if the condition $y^{\mathrm{T}}k(y) \geq c\|y\|^2$ holds for some $c > 0$. Here and further in the paper $\|\cdot\|$ denotes the Euclidian norm. The last inequality by its virtue implies that a linear "L_gV" control law $u = -ky$ with some positive gain $k > 0$ is used.

In this paper, we obtain more general results that are not confined to a linear control $u = -ky$ and are more natural for nonlinear functions $k(y)$. Using nonlinear controls of the form $u = -k(y)$ to stabilize passive dynamical systems can be motivated by the fact that linear control laws may fail under some extra uncertainties. This can be easily illustrated by a well known example, see Krstić et al. (1995),

$$\dot{x} = -kx + x^2 d, \tag{4}$$

where $x \in \mathbb{R}$, k is some positive constant, $d \in \mathbb{R}$ stands for disturbances. The system (4) can be seen as the unperturbed passive system $\dot{x} = u$, $y = x$ with a linear control $u = -ky$ facing extra model uncertainty $x^2 d$. It can be easily shown that the system (4) does not have the ISS property (Krstić et al. (1995)) with respect to the d input. Still, e.g. the nonlinear choice $u = -ky - y^5$ readily results in the ISS system $\dot{x} = -kx - x^5 + x^2 d$.

The rest of the paper is organized as follows. Some ISS related results used throughout the paper are revised in section 2. Stabilization of passive dynamical systems under actuator disturbances is discussed in section 3. Section 4 contains sufficient conditions of stabilizability of passive systems under sensor disturbances given in terms of integral ISS. Application of the obtained results to robust state observer redesign is considered in section 5. Finally, the paper concludes with some remarks in Section 6.

2. ISS PRELIMINARIES

In this section, for convenience sake, let us revise the main ISS notions and theorems used later in the paper.

Consider a nonlinear dynamical system

$$\dot{x} = F(x, d), \tag{5}$$

where $x \in \mathbb{R}^n$ is the state vector, the input $d \in \mathbb{R}^m$ stands for disturbances, $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is locally Lipschitz, $F(0, 0) = 0$.

A system (5) is said to be input-to-state stable (Sontag (1989)) if there exist a class KL function $\beta(\cdot,\cdot)$ and a class K function $\gamma(\cdot)$ such that for any piecewise continuous bounded input function $d = d(t)$ of $t \geq 0$ and any initial condition $x(0) = x_0 \in \mathbb{R}^n$ the solution $x(t)$ of the system (5) exists for all $t \geq 0$ and satisfies the inequality

$$\|x(t)\| \leq \beta(\|x_0\|, t) + \gamma(\sup_{0\leq\tau\leq t} \|d(\tau)\|).$$

Theorem 1. (Sontag and Wang (1995), Sontag and Wang (1996)) A system (5) is input-to-state stable if there exists a continuously differentiable positive definite proper function $V(x)$ such that for all $x \in \mathbb{R}^n$ and $d \in \mathbb{R}^m$ holds the inequality

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x} F(x, d) \leq -\alpha_1(\|x\|) + \varrho(\|d\|),$$

where $\alpha_1(\cdot)$ and $\varrho(\cdot)$ are some class K_∞ and class K functions, respectively.

A system (5) is said to be integrally input-to-state stable (Sontag (1998), Angeli et al. (2000)) if there exist a class K_∞ function $\alpha(\cdot)$, a class KL function $\beta(\cdot,\cdot)$ and a class K function $\gamma(\cdot)$ such that for any piecewise continuous bounded input function $d = d(t)$ of $t \geq 0$ and any initial condition $x(0) = x_0 \in \mathbb{R}^n$ the solution $x(t)$ of the system (5) exists for all $t \geq 0$ and satisfies the inequality

$$\alpha(\|x(t)\|) \leq \beta(\|x_0\|, t) + \int_0^t \gamma(\|d(s)\|)ds.$$

Theorem 2. (Angeli et al. (2000)) A system (5) is integrally input-to-state stable if there exists a continuously differentiable positive definite proper function $V(x)$ such that for all $x \in \mathbb{R}^n$ and $d \in \mathbb{R}^m$ holds the inequality

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x} F(x, d) \leq -\alpha_2(\|x\|) + \varrho(\|d\|),$$

where $\alpha_2(\cdot)$ is a continuous positive definite function, $\varrho(\cdot)$ is a class K function.

Theorem 3. (Angeli et al. (2000)) A system (5) is integrally input-to-state stable if for $d = d(t) \equiv 0$ the origin $x = 0$ is globally asymptotically stable and there exists a continuously differentiable positive definite proper function $V(x)$ such that for all $x \in \mathbb{R}^n$ and $d \in \mathbb{R}^m$ holds the inequality

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x} F(x, d) \leq \varrho(\|d\|),$$

where $\varrho(\cdot)$ is a class K function.

Theorem 4. (Angeli et al. (2000)) A system (5) is integrally input-to-state stable if there exist a continuous output function $y = h_1(x)$, $h_1(0) = 0$, $y \in \mathbb{R}^p$ such that the system (5) considered with that output is zero-state detectable, and a continuously differentiable positive definite proper function $V(x)$ such that for all $x \in \mathbb{R}^n$ and $d \in \mathbb{R}^m$ holds the inequality

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x} F(x, d) \leq -\alpha_3(\|h_1(x)\|) + \varrho(\|d\|),$$

where $\alpha_3(\cdot)$ is a continuous positive definite function, $\varrho(\cdot)$ is a class K function.

3. STABILIZATION UNDER ACTUATOR DISTURBANCES

We start with the following lemma.

Lemma 5. Let the system (2) be zero-state detectable and output strictly passive with a positive definite proper storage function $V(x)$. If the function $\varrho(y)$ in the supply rate satisfies for all $y \in \mathbb{R}^m$ the inequality

$$y^{\mathrm{T}} \varrho(y) \geq c\|y\|^{2l}, \tag{6}$$

where $l \in \mathbb{N}$ is some natural number and $c > 0$ is a positive constant, then the system (2) is integrally ISS with respect to the input u.

Proof. Since the system (2) is output strictly passive the following dissipation inequality holds for all $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$:

$$\dot V = \frac{\partial V(x)}{\partial x} f(x,u) \le y^{\mathrm T} u - y^{\mathrm T}\varrho(y),$$

where $y^{\mathrm T}\varrho(y) > 0$ for all $y \ne 0$. Then, in view of (6) and the Young's inequality

$$ab \le \frac{1}{\varepsilon} a^p + \varepsilon^{\frac{1}{p-1}} b^{\frac{p}{p-1}} \tag{7}$$

which is valid for all $a, b \in \mathbb{R}^+ = [0, +\infty)$, $p > 1$, $\varepsilon > 0$, one gets

$$\begin{aligned} \dot V &\le -c\|y\|^{2l} + y^{\mathrm T} u \le -c\|y\|^{2l} + \|y\|\|u\| \\ &\le -c\|y\|^{2l} + \frac{1}{\varepsilon}\|y\|^{2l} + \varepsilon^{\frac{1}{2l-1}}\|u\|^{\frac{2l}{2l-1}} \\ &= -\big(c - \frac{1}{\varepsilon}\big)\|y\|^{2l} + \varepsilon^{\frac{1}{2l-1}}\|u\|^{\frac{2l}{2l-1}} \\ &= -\alpha_3(\|y\|) + \varrho(\|u\|) \end{aligned}$$

for all $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $\varepsilon > 0$. Here, if $\varepsilon > \frac{1}{c}$ the functions $\alpha_3(s) = (c - 1/\varepsilon)s^{2l}$ and $\varrho(s) = \varepsilon^{\frac{1}{2l-1}} s^{\frac{2l}{2l-1}}$, $s \ge 0$, belong to class K_∞.

Hence, according to theorem 4 the system (2) is integrally ISS with respect to the input u. ■

Consider now the system (1) with $d_2(t) \equiv 0$ written as below

$$\begin{aligned} \dot x &= f(x, u + d_1), \\ \tilde y &= h(x) = y. \end{aligned} \tag{8}$$

Theorem 6. Let the system (2) be zero-state detectable and passive with a positive definite proper storage function $V(x)$. Then the control law $u = -k(y)$, where $k(y)$ is a locally Lipschitz function such that $k(0) = 0$ and for all $y \in \mathbb{R}^m$ the inequality

$$y^{\mathrm T} k(y) \ge c\|y\|^{2l} \tag{9}$$

holds with some natural number $l \in \mathbb{N}$ and positive constant $c > 0$, makes the system (8) integrally ISS with respect to the input d_1.

Proof. From passivity property of the system (2) follows that time derivative of the storage function $V(x)$ along solutions of the system (8) with control law $u = -k(y)$ (i.e. the system (2) with control $u = -k(y) + d_1$) for all $x \in \mathbb{R}^n$ and $d_1 \in \mathbb{R}^m$ satisfies

$$\begin{aligned} \dot V &= \frac{\partial V(x)}{\partial x} f(x, -k(y) + d_1) \\ &\le y^{\mathrm T}(-k(y) + d_1) = -y^{\mathrm T}k(y) + y^{\mathrm T} d_1. \end{aligned}$$

Here, in view of the inequality (9) holds $y^{\mathrm T}k(y) > 0$ for all $y \ne 0$.

Therefore, the system (8) with control $u = -k(y)$ is output strictly passive and, by lemma 5, is integrally ISS with respect to the input d_1. ■

Theorem 7. Let the system (2) be strictly passive with a positive definite proper storage function $V(x)$. Then the control law $u = -k(y)$, with $k(y)$ being a locally Lipschitz function such that $k(0) = 0$ and for all $y \in \mathbb{R}^m$ the inequality (9) holds with some natural number $l \in \mathbb{N}$ and positive constant $c > 0$, makes the system (8) integrally ISS with respect to the input d_1. If additionally the $\psi(\cdot)$ function in the supply rate is proper, i.e. $\psi(x) \to +\infty$ as $\|x\| \to +\infty$, the control law $u = -k(y)$ makes the system (8) ISS with respect to the input d_1.

Proof. By strict passivity property of the system (2) the time derivative of the storage function $V(x)$ along solutions of the system (8) with control law $u = -k(y)$ for all $x \in \mathbb{R}^n$ and $d_1 \in \mathbb{R}^m$ can be estimated from above as

$$\begin{aligned} \dot V &= \frac{\partial V(x)}{\partial x} f(x, -k(y) + d_1) \le y^{\mathrm T}(-k(y) + d_1) - \psi(x) \\ &= -y^{\mathrm T}k(y) + y^{\mathrm T}d_1 - \psi(x). \end{aligned}$$

Next, taking into account the inequalities (7) and (9) yields

$$\begin{aligned} \dot V &\le -c\|y\|^{2l} + y^{\mathrm T} d_1 - \psi(x) \\ &\le -c\|y\|^{2l} + \|y\|\|d_1\| - \psi(x) \\ &\le -c\|y\|^{2l} + \frac{1}{\varepsilon}\|y\|^{2l} + \varepsilon^{\frac{1}{2l-1}}\|d_1\|^{\frac{2l}{2l-1}} - \psi(x) \\ &= -\big(c - \frac{1}{\varepsilon}\big)\|y\|^{2l} - \psi(x) + \varepsilon^{\frac{1}{2l-1}}\|d_1\|^{\frac{2l}{2l-1}} \\ &\le -\psi(x) + \varepsilon^{\frac{1}{2l-1}}\|d_1\|^{\frac{2l}{2l-1}}. \end{aligned} \tag{10}$$

for all $x \in \mathbb{R}^n$, $d_1 \in \mathbb{R}^m$ and $\varepsilon > 1/c$.

Since $\psi(x)$ is positive definite there exists a class K function $\tilde\psi(\cdot)$, see Khalil (2002), such that for all $x \in \mathbb{R}^n$ $\psi(x)$ can be estimated from below as

$$\psi(x) \ge \tilde\psi(\|x\|). \tag{11}$$

Hence, the inequality (10) can be written as

$$\dot V \le -\tilde\psi(\|x\|) + \varrho(\|d_1\|), \tag{12}$$

where

$$\varrho(s) = \varepsilon^{\frac{1}{2l-1}} s^{\frac{2l}{2l-1}},\ s \ge 0$$

belongs to class K_∞. Thus, by theorem 2 the system (8) with control $u = -k(y)$ is integrally ISS with respect to the input d_1.

If additionally the $\psi(\cdot)$ function has the property $\psi(x) \to +\infty$ as $\|x\| \to +\infty$ then the $\tilde\psi(\cdot)$ function in the inequalities (11) and (12) can be chosen to belong to K_∞, see Khalil (2002). Hence, according to theorem 1 the system (8) with control $u = -k(y)$ is ISS with respect to the input d_1. ■

4. STABILIZATION UNDER SENSOR DISTURBANCES

Let us first analyze the system (1) with $d_1(t) \equiv 0$ given by

$$\begin{aligned} \dot x &= f(x, u), \\ \tilde y &= h(x) + d_2. \end{aligned} \tag{13}$$

The following result holds.

Theorem 8. Let the system (2) be zero-state detectable and passive with a positive definite proper storage function $V(x)$. Then the control law $u = -k(\tilde y) = -k(y + d_2)$, where $k(\tilde y)$ is a locally Lipschitz function such that $k(0) = 0$ and for all $\tilde y \in \mathbb{R}^m$ the inequalities

$$\tilde y^{\mathrm T} k(\tilde y) \ge c_1\|\tilde y\|^{2l},\ \|k(\tilde y)\| \le c_2\|\tilde y\|^{2l-1} \tag{14}$$

hold with some natural number $l \in \mathbb{N}$ and positive constants $c_1 > 0$, $c_2 > 0$, makes the system (13) integrally ISS with respect to the input d_2.

Proof. Due to passivity of the system (2) time derivative of the storage function $V(x)$ along solutions of the system

(13) with control law $u = -k(y + d_2)$ for all $x \in \mathbb{R}^n$ and $d_2 \in \mathbb{R}^m$ can be written as

$$\dot{V} = \frac{\partial V(x)}{\partial x} f(x, -k(y + d_2)) \le -y^{\mathrm{T}} k(y + d_2)$$
$$= -(y + d_2)^{\mathrm{T}} k(y + d_2) + d_2^{\mathrm{T}} k(y + d_2).$$

Then, by the inequalities (7) and (14) one gets

$$\begin{aligned}\dot{V} &\le -c_1\|y + d_2\|^{2l} + \|d_2\|\|k(y + d_2)\| \\ &\le -c_1\|y + d_2\|^{2l} + c_2\|d_2\|\|y + d_2\|^{2l-1} \\ &\le -c_1\|y + d_2\|^{2l} + \frac{c_2}{\varepsilon}\|d_2\|^{2l} + c_2\varepsilon^{\frac{1}{2l-1}}\|y + d_2\|^{2l} \\ &= -\big(c_1 - c_2\varepsilon^{\frac{1}{2l-1}}\big)\|y + d_2\|^{2l} + \frac{c_2}{\varepsilon}\|d_2\|^{2l} \\ &\le \frac{c_2}{\varepsilon}\|d_2\|^{2l} = \varrho(\|d_2\|)\end{aligned}$$

for all $x \in \mathbb{R}^n$, $d_2 \in \mathbb{R}^m$ and $\varepsilon < \left(\frac{c_1}{c_2}\right)^{2l-1}$. Here

$$\varrho(s) = \frac{c_2}{\varepsilon} s^{2l}, \; s \ge 0$$

is a class K_∞ function.

Notice that from the conditions (14) follows that $y^{\mathrm{T}}k(y) > 0$ for all $y \neq 0$. Thus, if $d_2 = d_2(t) \equiv 0$ the system (13) with control $u = -k(y)$ is globally asymptotically stable at $x = 0$ (Byrnes et al. (1991)). Hence, by theorem 3 the system (13) with control $u = -k(y + d_2)$ is integrally ISS with respect to the input d_2. ■

Consider now the system (1) with $d_1(t) \not\equiv 0$ and $d_2(t) \not\equiv 0$. Then, the above theorems 6, 8 can be combined as below.

Theorem 9. Let the system (2) be zero-state detectable and passive with a positive definite proper storage function $V(x)$. Then the control law $u = -k(\tilde{y}) = -k(y+d_2)$, where $k(\tilde{y})$ is a locally Lipschitz function such that $k(0) = 0$ and for all $\tilde{y} \in \mathbb{R}^m$ the inequalities (14) hold with some natural number $l \in \mathbb{N}$ and positive constants $c_1 > 0$, $c_2 > 0$, makes the system (1) integrally ISS with respect to the inputs d_1 and d_2.

Proof. By passivity of the system (2) the dissipation inequality (3) is rewritten for the system (1) with control $u = -k(y + d_2)$ as follows

$$\begin{aligned}\dot{V} &= \frac{\partial V(x)}{\partial x} f(x, -k(y + d_2) + d_1) \\ &\le -y^{\mathrm{T}} k(y + d_2) + y^{\mathrm{T}} d_1 \\ &= -(y + d_2)^{\mathrm{T}} k(y + d_2) + d_2^{\mathrm{T}} k(y + d_2) \\ &\quad + (y + d_2)^{\mathrm{T}} d_1 - d_2^{\mathrm{T}} d_1\end{aligned}$$

for all $x \in \mathbb{R}^n$, $d_1 \in \mathbb{R}^m$ and $d_2 \in \mathbb{R}^m$. Then, in view of the inequalities (7) and (14) one has

$$\begin{aligned}\dot{V} &\le -c_1\|y + d_2\|^{2l} + \|d_2\|\|k(y + d_2)\| \\ &\quad + \|y + d_2\|\|d_1\| + \|d_2\|\|d_1\| \\ &\le -c_1\|y + d_2\|^{2l} + c_2\|d_2\|\|y + d_2\|^{2l-1} \\ &\quad + \|y + d_2\|\|d_1\| + \|d_2\|\|d_1\| \\ &\le -c_1\|y + d_2\|^{2l} + \frac{c_2}{\varepsilon_1}\|d_2\|^{2l} + c_2\varepsilon_1^{\frac{1}{2l-1}}\|y + d_2\|^{2l} \\ &\quad + \frac{1}{\varepsilon_2}\|y + d_2\|^{2l} + \varepsilon_2^{\frac{1}{2l-1}}\|d_1\|^{\frac{2l}{2l-1}} \\ &\quad + \frac{1}{\varepsilon_3}\|d_2\|^{2l} + \varepsilon_3^{\frac{1}{2l-1}}\|d_1\|^{\frac{2l}{2l-1}}.\end{aligned}$$

Finally,

$$\begin{aligned}\dot{V} &\le -\big(c_1 - c_2\varepsilon_1^{\frac{1}{2l-1}} - \frac{1}{\varepsilon_2}\big)\|y + d_2\|^{2l} \\ &\quad + \big(\varepsilon_2^{\frac{1}{2l-1}} + \varepsilon_3^{\frac{1}{2l-1}}\big)\|d_1\|^{\frac{2l}{2l-1}} + \big(\frac{c_2}{\varepsilon_1} + \frac{1}{\varepsilon_3}\big)\|d_2\|^{2l} \\ &\le \big(\varepsilon_2^{\frac{1}{2l-1}} + \varepsilon_3^{\frac{1}{2l-1}}\big)\|d_1\|^{\frac{2l}{2l-1}} + \big(\frac{c_2}{\varepsilon_1} + \frac{1}{\varepsilon_3}\big)\|d_2\|^{2l} \\ &= \varrho_1(\|d_1\|) + \varrho_2(\|d_2\|)\end{aligned}$$

for all $x \in \mathbb{R}^n$, $d_1 \in \mathbb{R}^m$, $d_2 \in \mathbb{R}^m$ and any ε_1, ε_2 such that the condition $c_1 - c_2\varepsilon_1^{\frac{1}{2l-1}} - 1/\varepsilon_2 > 0$ is satisfied. Here
$\varrho_1(s) = \big(\varepsilon_2^{\frac{1}{2l-1}} + \varepsilon_3^{\frac{1}{2l-1}}\big)s^{\frac{2l}{2l-1}}$, $\varrho_2(s) = \big(\frac{c_2}{\varepsilon_1} + \frac{1}{\varepsilon_3}\big)s^{2l}$, $s \ge 0$
are class K_∞ functions.

Moreover, the inequalities (14) imply that $y^{\mathrm{T}}k(y) > 0$ holds for all $y \neq 0$. Hence, if $d_1 = d_1(t) \equiv 0$ and $d_2 = d_2(t) \equiv 0$ the system (1) with control $u = -k(y)$ is globally asymptotically stable at $x = 0$ (Byrnes et al. (1991)). Then, by theorem 3 the system (1) with control $u = -k(y+d_2)$ is integrally ISS with respect to the inputs d_1 and d_2. ■

5. ROBUST STATE OBSERVER REDESIGN

One of possible applications of the results in section 3 and 4 is state observer performance analysis and robust redesign under measurements noise. It can be shown (see e.g. Shim and Seo (2000), Shim et al. (2003), Golubev et al. (2005)) that many popular nonlinear state observer design techniques, e.g. observer with linear error dynamics (Krener and Respondek (1985)), observer for systems with globally Lipschitz nonlinearities (Thau (1973)), high-gain observer (Gauthier et al. (1992)), observer for systems with sector nonlinearities (Arcak and Kokotović (2001)) and, in particular, observer design in Shim and Seo (2000), Shim et al. (2003), can be considered as state estimation error dynamics passivation with further stabilization of the resultant passive system by a static output feedback.

For instance, let us discuss state observer design for a dynamical system of the form

$$\begin{aligned}\dot{x} &= Ax + \varrho(y, u), \\ y &= Cx,\end{aligned} \tag{15}$$

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^p$ is the system output to be measured, $u = u(t) \in \mathbb{R}^m$ is a known input function of $t \ge 0$ which is piecewise continuous and bounded, $A \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{p\times n}$, pair (A, C) is detectable, $\varrho : \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}^n$ is locally Lipschitz.

State observer for the system (15) is constructed as follows

$$\dot{\hat{x}} = A\hat{x} + \varrho(\tilde{y}, u) + k(C\hat{x} - \tilde{y}), \tag{16}$$

where $\tilde{y} = y + d$ is the measured output, $d = d(t)$ stands for measurements noise which is considered to be unknown piecewise continuous bounded function of $t \ge 0$, $k(\cdot)$ is a locally Lipschitz function to be designed, which satisfies $k(0) = 0$.

The state estimation error $e = \hat{x} - x$ dynamics are given by

$$\dot{e} = Ae + [\varrho(y + d, u) - \varrho(y, u)] + k(Ce - d). \tag{17}$$

Consider first the case when $d = d(t) \equiv 0$ and rewrite the system (17) as

$$\begin{aligned} \dot{e} &= Ae + v, \\ y_1 &= Ce, \end{aligned} \tag{18}$$

where $v = k(y_1)$ can be seen as a control input. Then, the observer design problem can be reformulated as the control problem to stabilize the zero equilibrium $e = 0$, $v = 0$ of the system (18) by a static output feedback $v = k(y_1) = k(Ce)$ (Shim and Seo (2000)).

Since the pair (A, C) is detectable there exists a matrix $L \in \mathbb{R}^{n\times p}$ such that $A + LC$ is Hurwitz. Consider the storage function candidate $V(e) = e^{\mathrm{T}}Pe > 0$, where $P = P^{\mathrm{T}} > 0$ satisfies the Lyapunov equation

$$(A + LC)^{\mathrm{T}}P + P(A + LC) = -Q$$

with some positive definite matrix $Q = Q^{\mathrm{T}} > 0$. Its time derivative along solutions of the system (18) can be written as

$$\begin{aligned} \dot{V} &= \dot{e}^{\mathrm{T}}Pe + e^{\mathrm{T}}P\dot{e} = e^{\mathrm{T}}A^{\mathrm{T}}Pe + v^{\mathrm{T}}Pe + e^{\mathrm{T}}PAe + e^{\mathrm{T}}Pv \\ &= e^{\mathrm{T}}(A + LC)^{\mathrm{T}}Pe + e^{\mathrm{T}}P(A + LC)e - 2e^{\mathrm{T}}PLCe \\ &+ 2e^{\mathrm{T}}Pv = -e^{\mathrm{T}}Qe - 2e^{\mathrm{T}}PLCe + 2e^{\mathrm{T}}Pv \\ &= -e^{\mathrm{T}}Qe - 2e^{\mathrm{T}}PLy_1 + 2e^{\mathrm{T}}Pv. \end{aligned}$$

The choice

$$v = Ly_1 + \frac{1}{2}L_1\tilde{v},$$

where $\tilde{v} \in \mathbb{R}^p$ is the new control input, $L_1 = P^{-1}C^{\mathrm{T}}$, yields

$$\dot{V} = -e^{\mathrm{T}}Qe + e^{\mathrm{T}}C^{\mathrm{T}}\tilde{v} = -e^{\mathrm{T}}Qe + y_1^{\mathrm{T}}\tilde{v}.$$

Hence, the system

$$\begin{aligned} \dot{e} &= (A + LC)e + \frac{1}{2}L_1\tilde{v}, \\ y_1 &= Ce \end{aligned} \tag{19}$$

is strictly passive.

Notice that the choice $\tilde{v} = \tilde{v}(t) \equiv 0$ results in the linear function $k(y_1) = Ly_1 = LCe$ and linear state estimation error dynamics $\dot{e} = (A + LC)e$ which are associated with a conventional state observer (16) design for $d = d(t) \equiv 0$.

Still, similarly to Shim and Seo (2000) one can try to choose a nontrivial nonlinear function $\tilde{v} = -\tilde{k}(y_1)$, where $\tilde{k}(\cdot)$ satisfies conditions of the theorem 9, to guarantee the integral input-to-state stability of the error dynamics (17) which take the form

$$\begin{aligned} \dot{e} &= (A + LC)e + [\varrho(y + d, u) - \varrho(y, u)] - Ld \\ &- \frac{1}{2}L_1\tilde{k}(Ce - d). \end{aligned} \tag{20}$$

Notice that, in particular, if the function $\varrho(y, u)$ is globally Lipschitz in y uniformly in u, then the error system (20) with $\tilde{k}(\cdot) \equiv 0$ is input-to-state stable with respect to the d input, see Khalil (2002).

6. CONCLUSION

In this paper, stabilization of passive dynamical systems under actuator and sensor disturbances was investigated. New sufficient conditions for stabilizability of passive systems with disturbances have been given in terms of integral input-to-state stability. Application of the obtained results to robust state observer redesign was discussed.

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