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A New Method for Deriving Weights in Group Fuzzy Analytic Hierarchy Process and Evaluation Measures

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Abstract: The paper discusses fuzzy comparison matrices, consistency check, weight prioritization methods and weight evaluation methods in fuzzy group analytic hierarchy process. There are various methods of weight prioritization, however, they are not critically evaluated. In the paper, two measures are introduced for the evaluation of the group weights. Then, a new method is proposed to improve the process of deriving weights and use it in an application compared to another common three methods. Our results show that the new method is a good method for deriving weights of indexes.

Keywords: Group Fuzzy AHP, Ari-Geo Method, Assessment Measure, Fuzzy Consistency Check, Triangular Fuzzy Number

1. INTRODUCTION

The Analytic Hierarchy Process (AHP) is one of multiple criteria decision-making (MCDM) methods. Chen and Hwang (1991) classified multi-criterion decision making (MCDM) methods according to the type of information and the salient features of the information which is the most common way for classifying MCDM methods by far. The Analytic Hierarchy Process (Saaty, 1980 and 1994) decomposes a complex MCDM problem into a system of hierarchies so that it can be used in single- or multi-dimensional decision-making problems. The importance of the AHP and its variants in decision making is best illustrated in the more than 1,000 references cited by Saaty (1994). It has been used in almost all the applications related with decision-making, involved on selection (Michael Angelo B.Promentilla, et al., 2018), evaluation (Pantelidis P, Pazarskis M, Karakitsiou A, et al., 2018), benefit-cost analysis (Alessio I, López Cristina, 2018), allocations (Boukherroub T, Lebel L, Ruiz A, 2015), planning and development (Crnčan, Ana; Škrtić, Zoran; Kristić, et al., 2018), priority and ranking (Singh J, Sharma S K, Srivastava R, 2019), decision making (Singh M P, Singh P, Singh P, 2019), forecasting (Zhang Y, Zhang C, Liu Y, 2016) and related fields, etc. In addition, the ease with which AHP can be used in combination with other methods is the reason why AHP is widely used, such as AHP-DBSCAN (Wang S, Wang G, Zhang J, 2019), AHP-TOPSIS (Ahmet Çalık, Sinan Çizmecioğlu, AyhanAkpınar, 2019), AHP-FCE (Huiru Z, Sen G, 2014), AHP-Entropy (Libiao B, Hailing W, Ning H, et al., 2018), Delphi-AHP (Vidal L A, Marle F, Bocquet J C, 2011), etc., which can be seen more clearly and comprehensively in the review paper by Ho W and Ma X (2017) who made a conclusion of 36 integrated AHP approaches.

AHP is designed to cope with both the rational and the intuitive judgements to decompose the goal into multiple component factors, and to form a hierarchical structure model according to the relationship between the component factors, and then to analyze them by layer to finally obtain the importance weights of alternatives or indexes.

Considering the comprehensiveness and complexity of the problems in real applications, only one decision-maker (DM) can hardly make reliable decisions. Therefore, group decision-making can take a good place in decision support systems. Furthermore, human thinking and estimations have uncertainty and vagueness, so fuzzy set theory can be a useful tool to deal with them. In this paper, group fuzzy AHP is mainly discussed from 4 aspects: 1) Fuzzy pairwise comparison matrices; 2) Consistency check; 3) Weight prioritization methods; 4) Evaluation measures for weights. To summarize the methodology, the steps of the group fuzzy AHP are given in the following: 1) A group of decision-makers identifies indexes and construct a hierarchical model. 2) Appropriate linguistic variables for the relative importance and pairwise comparison matrices of all DMs are constructed. 3) Check the consistency of comparison matrices. 4) Aggregate all the consistent comparison matrices into a group comparison matrix and check the consistency. 5) Prioritization methods are used to get the weights of indexes. 6) Evaluate the weights and make a final evaluation.

The first contribution of this paper is to introduce two assessment methods to evaluate the quality of group weights obtained by different aggregation methods and different weight prioritization methods, which is rare in many studies and applications of AHP and its extensions. In addition, considering the disadvantages of the existing matrix-aggregated approaches and weight prioritization methods, this paper proposes a new method, and proves its feasibility and effectiveness in practical application.

In this paper, we use triangular fuzzy numbers to denote the fuzzy comparison matrices given by DMs as shown in Section 2, and introduce the methods of consistency check of group fuzzy AHP in Section 3. Then, Section 4 selects three classical fuzzy group AHP approaches that produce crisp weights as compared to a new method proposed in this paper. The Section 5 gives two criterions to evaluate the weights by different aggregation and prioritization methods. The evaluation is done on the real application from the literature in Section 6 and a conclusion and expectation in Section 7.

2. TRIANGULAR FUZZY NUMBERS

In this paper, fuzzy comparison matrices are presented by triangular fuzzy numbers:

$$\mathrm{A} = \left(\tilde{a}_{ij}\right)_{n\times n} = \left[\left(l_{ij}, m_{ij}, u_{ij}\right)\right]_{n\times n}$$

$$\tilde{a}_{ji} = \tilde{a}_{ij}^{\ -1} = \left(\frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}}\right)$$

where $\tilde{a}_{ij}$ stands for the fuzzy degree of the index i over j, l_{ij} and u_{ij} represent the lower and upper bounds of the triangular fuzzy number $\tilde{a}_{ij}$ respectively, and m_{ij} is the median value. l_{ij}, m_{ij} and u_{ij} are non-negative real numbers with $l_{ij} \le m_{ij} \le u_{ij}$ and $l_{ij}u_{ji} = m_{ij}m_{ji} = u_{ij}l_{ji} = 1$ for $i, j = 1,2,\cdots,n$. The main fuzzy arithmetic operators for two triangular fuzzy numbers follow the extension principle by Zadeh in 1975.

A linguistic variable is a variable whose values are linguistic terms (Zadeh, 1975). The concept of linguistic variable is very useful when dealing with complex or ambiguous descriptions. The values of linguistic variables can be represented by fuzzy numbers. Therefore, reasonable linguistic variables in Table 1 can be used to evaluate the relative importance of the indexes.

Table 1. Fuzzy evaluation scores for the importance

Linguistic terms	Fuzzy score
Absolutely strong (AS)	(2,5/2,3)
Very strong (VS)	(3/2,2,5/2)
Fairly strong (FS)	(1,3/2,2)
Slightly strong (SS)	(1,1,3/2)
Equal (E)	(1,1,1)
Slightly weak (SW)	(2/3,1,1)
Fairly weak (FW)	(1/2,2/3,1)
Very weak (VW)	(2/5,1/2,2/3)
Absolutely weak (AW)	(1/3,2/5,1/2)

3. CONSISTENCY CHECK

In practice, experts may also make errors or inconsistent judgments when making decisions, i.e. that there are judgments: the importance of indicator i is 3 times that of indicator j, the importance of indicator k is 2 times that of indicator j, and the importance of indicator i is 2 times that of indicator j. This is a judgment inconsistency, because the importance of indicators i and k are opposite. Therefore, it is very necessary to check the consistency of the judgment matrix given by the experts, which has a crucial impact on the reliability of the final weight. The most commonly used approach was given by Satty (1980): The consistency index (CI) for a crisp comparison matrix can be computed with the equation $\mathrm{CI} = \frac{\lambda_{max}-n}{n-1}$, where λ_{max} is the largest eigenvalue of the comparison matrix, and n is the amount of indicators. The consistency ratio (CR) is measured by the ratio $\mathrm{CR} = \frac{CI}{RI(n)}$, where $RI(n)$ is a random index that depends on the size of the matrix, as shown in Table 2. If the CR value of a comparison matrix is equal or less than 0.1, it can be acceptable, meaning the matrix is consistent. When the CR is unacceptable, the decision-maker is encouraged to redecide the pairwise comparisons.

Table 2. Random index (RI) of random matrices

n	3	4	5	6	7	8
$RI(n)$	0.525	0.882	1.11	1.25	1.341	1.404
n	9	10	11	12	13	14
$RI(n)$	1.451	1.486	1.514	1.536	1.555	1.570

Source: Franek and Kresta (2014)

When a fuzzy comparison matrix is denoted by triangular fuzzy numbers, Liu, et al. (2012) gave the corresponding consistency definition. A triangular fuzzy reciprocal comparison matrix is represented as $A = \left(a_{ij}\right)_{n\times n} = \left(l_{ij}, m_{ij}, u_{ij}\right)_{n\times n}$. Define three matrices: $A^L = (a_{ij}^L)_{n\times n}$, $A^M = (a_{ij}^M)_{n\times n}$ and $A^U = (a_{ij}^U)_{n\times n}$, where there are

$$a_{ij}^L = \begin{cases} l_{ij}, i < j \\ 1,\ i = j \\ u_{ij}, i > j \end{cases} \qquad a_{ij}^U = \begin{cases} u_{ij}, i < j \\ 1,\ i = j \\ l_{ij}, i > j \end{cases} \qquad a_{ij}^M = m_{ij}$$

Then, there is the following definition:

Definition If the three matrices A^L , A^M and A^U are all consistent, A is proved consistent. Otherwise, A is inconsistent.

Therefore, according to this definition, we can check the consistency of the fuzzy matrices denoted by triangular fuzzy numbers from the individual judgements and the aggregated group comparison matrix.

4. GROUP FUZZY AHP PRIORITIZATION METHODS

When m decision-makers make individual decisions, an aggregated one should be constructed to satisfy each decision maker and used for deriving weights. Three common methods are summarized below from the literature and a new one is proposed.

4.1 Max-Min combined with extend analysis method

According to the literature (Chen, 2015; Larimian, Zarabadi, & Sadeghi, 2013), take the minimum value of the lower boundaries and the maximum value of the upper boundaries and geometric mean of the medians to aggregate m individual fuzzy comparison matrices into one (A^G) with equations:

$$A^G = \left(a_{ij}{}^G\right)_{n\times n} = \left[\left(l_{ij}{}^G, m_{ij}{}^G, u_{ij}{}^G\right)\right]_{n\times n}$$

$$l_{ij}{}^G = \min_{k=1,2,\dots,m}\left\{l_{ij}^{(k)}\right\},\ m_{ij}{}^G = \sqrt[m]{\prod_{k=1}^{m} m_{ij}^{(k)}}$$

$$u_{ij}{}^G = \max_{k=1,2,\dots,m}\left\{u_{ij}^{(k)}\right\}$$

where $a_{\mathrm{ij}}^{(\mathrm{k})} = \left(l_{ij}^{(k)}, m_{ij}^{(k)}, u_{ij}^{(k)}\right)$ is decided by the kth expert.

Then, the extent analysis method proposed by Chang (1996) is introduced to derive weights from A^G. Initially, define the normalized fuzzy synthetic extents with respect to the ith index as:

$$S_i = \sum_{j=1}^{n} B_{ij} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\right]^{-1}$$

$$\sum_{j=1}^{n} B_{ij} = \left(\sum_{j=1}^{n} l_{ij}, \sum_{j=1}^{n} m_{ij}, \sum_{j=1}^{n} u_{ij}\right)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} l_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{n} u_{ij}\right)$$

Then, S_i for $i = 1,2,\dots,n$ are compared and define the degree of possibility (PD) of $S_j \geq S_i$ as:

$$\mathrm{PD}\left(S_j \geq S_i\right) = \begin{cases} 1, & if\ m_j \geq m_i \\ 0, & if\ l_i \geq u_j \\ \frac{l_i - u_j}{(m_j - u_j) - (m_i - l_i)}, & otherwise \end{cases}$$

The degree of possibility of a fuzzy number S to be greater than k fuzzy numbers S_i for $i = 1,2,\dots,k$ is calculated by:

$$\mathrm{PD}(S \geq S_1, S_2, \dots, S_k) = PD[(S \geq S_1) \cap (S \geq S_2) \cap \dots \cap (S \geq S_k)] = \min PD(S \geq S_i)$$

The weight of index i is calculated by equations:

$$\widetilde{w}_i = \min PD(S_i \geq S_k),\ w_i = \frac{\widetilde{w}_i}{\sum_{i=1}^{n}\widetilde{w}_i}$$

where $k = 1,\dots,n,$ and $k \neq i$.

4.2 Fuzzy geometric mean method

Referring to the literature (Beskese et al., 2015), take fuzzy geometric mean of m individual fuzzy comparison matrices to get the aggregated one (A^G) by equations:

$$A^G = \left(a_{ij}{}^G\right)_{n\times n} = \left(\sqrt[m]{a_{ij}^{(1)} \otimes a_{ij}^{(2)} \otimes \cdots \otimes a_{ij}^{(m)}}\right)_{n\times n}$$

where $a_{ij}^{(\mathrm{k})}$ for $k = 1,2,\cdots,m$ is decided by the kth expert. Then, the weights are calculated as follows:

$$\tilde{r}_i = \sqrt[n]{a_{i1}{}^G \otimes a_{i2}{}^G \otimes \cdots \otimes a_{in}{}^G},\ \widetilde{w}_i = \tilde{r}_i \oslash (\tilde{r}_1 \oplus \tilde{r}_2 \oplus \cdots \oplus \tilde{r}_n)$$

where $\tilde{r}_i$ is the fuzzy geometric mean of comparisons of index i to each index, $\widetilde{w}_i$ is the fuzzy weight of index i. Then, the method for defuzzification and normalization (Opricovic, & Tzeng, 2003) is applied:

$$w_i = \frac{\widetilde{w}_i}{\sum_{i=1}^{n}\widetilde{w}_i} = \frac{w_{il} + w_{im} + w_{iu}}{\sum_{i=1}^{n}\widetilde{w}_i}$$

where w_i is the crisp weight of index i.

4.3 Geometric mean method combined with modified extent analysis method

Take geometric mean of individual fuzzy comparison matrices to get the aggregated one (A^G) at first. Then, a modified extent analysis method was introduced to derive the weights. The extent analysis method (Chang, 1996) was improved into modified extent analysis method (Heo, Kim, & Cho, 2012), which is nowadays one of the most popular methods. Define the improved normalized synthetic extents $S_i = (l_i, m_i, u_i)$ with respect to the ith index as:

$$A^G = \left(a_{ij}{}^G\right)_{n\times n} = \left(l_{ij}{}^G, m_{ij}{}^G, u_{ij}{}^G\right)_{n\times n}$$

$$l_i = \frac{\sum_{j=1}^{n} l_{ij}{}^G}{\sum_{j=1}^{n} l_{ij}{}^G + \sum_{k=1,k\neq i}^{n}\sum_{j=1}^{n} u_{kj}{}^G},\ m_i = \frac{\sum_{j=1}^{n} m_{ij}{}^G}{\sum_{i=1}^{n}\sum_{j=1}^{n} m_{ij}{}^G}$$

$$u_i = \frac{\sum_{j=1}^{n} u_{ij}{}^G}{\sum_{j=1}^{n} u_{ij}{}^G + \sum_{k=1,k\neq i}^{n}\sum_{j=1}^{n} l_{kj}{}^G}$$

The following weight solving process is exactly the same as that in extent analysis method.

4.4 Ari-Geo method

We propose a new method in this paper. The main idea of the algorithm is that the fuzzy matrix is transformed into a non-fuzzy matrix in a reasonable and effective way after the comparison matrices are aggregated by the arithmetic mean method, and the consistency of the matrix is not changed in the process of weight calculation. The process is as follows.

Firstly, get the entry $a_{ij}{}^G$ of the group fuzzy comparison matrix through the arithmetic mean of m individual comparison matrices:

$$a_{ij}{}^{(k)} = \left[l_{ij}{}^{(k)}, m_{ij}{}^{(k)}, u_{ij}{}^{(k)}\right]$$

$$a_{ij}{}^G = \frac{1}{m}\left(a_{ij}{}^{(1)} \oplus a_{ij}{}^{(2)} \oplus \cdots \oplus a_{ij}{}^{(m)}\right)$$

Secondly, construct a matrix E:

$$\mathrm{E} = \left(e_{ij}\right)_{n\times n} = \left(\frac{u_{ij} - l_{ij}}{2m_{ij}}\right)_{n\times n}$$

where $\frac{u_{ij} - l_{ij}}{2m_{ij}}$ means the degree of vagueness of fuzzy number.

Then, calculate the comparison matrix $\mathrm{Q} = \mathrm{M} \times \mathrm{E}$, where $\mathrm{M} = (m_{ij})_{n\times n}$ comprises all the medians in triangular fuzzy numbers.

Subsequently, calculate a matrix $\mathrm{P} = (p_{ij})_{n\times n}$ as converting Q into a matrix whose diagonal elements are all equal to 1, from which it can be seen P satisfies reciprocity.

Also, define a new matrix $\mathrm{R} = (r_{ij})_{n\times n}$, where

$$r_{ij} = \sqrt[n]{\prod_{k=1}^{n} p_{ik} \cdot p_{kj}}$$

where it could be seen that $r_{ij} = r_{ik} \cdot r_{kj}$, which indicates that the matrix is consistent.

Consequently, the weights of indexes are obtained by equations:

$$\widetilde{w}_i = \sqrt[n]{\prod_{k=1}^{n} r_{ik}}, \quad \omega_i = \frac{\widetilde{w}_i}{\sum_{k=1}^{n} \widetilde{w}_k}$$

From the whole process of solving, it can be seen that this method has two advantages: one is that the method of defuzzification is more reasonable than that in Section 4.2; the other is that the consistency of the group matrix is not affected during the transformation, which guarantees the rationality of weights of the indexes.

5. MEASURES FOR EVALUATING GROUP FUZZY AHP METHODS

To compare the quality of the weights derived by different methods, suitable measures are needed. Srdjevic (2005) proposed two estimate measures which are general and applicable to all prioritization methods: (a) generalized Euclidean distance (ED), and (b) minimum violations (MV). Grošelj (2014) generalized these two criterions to the group case: GED and GMV. Petra and Lidija (2018) further generalized the measures to the fuzzy group case: FGED and FGMV. The computational formulas were given:

$$\mathrm{FGMV} = \frac{1}{m}\sum_{k=1}^{m}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} I_{ij}{}^{(k)}$$

$$I_{ij}{}^{(k)} = \begin{cases} 1, \text{if } w_i > w_j \ and\ l_{ij}^{(k)} < 1 \\ 1, \text{if } w_i < w_j \ and\ u_{ij}^{(k)} > 1 \\ 0.5, \text{if } w_i = w_j \ and\ a_{ij}{}^{(k)} \neq (1,1,1) \\ 0.5, \text{if } w_i \neq w_j \ and\ a_{ij}{}^{(k)} = (1,1,1) \\ 0, otherwise \end{cases}$$

$$\mathrm{FGED} = \frac{1}{m}\sum_{k=1}^{m}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\left(\left(\Delta l_{ij}^{(k)}\right)^2 + \left(\Delta m_{ij}^{(k)}\right)^2 + \left(\Delta u_{ij}^{(k)}\right)^2\right)}$$

$$\Delta l_{ij}^{(k)} = l_{ij}^{(k)} - \frac{w_i}{w_j}, \Delta m_{ij}^{(k)} = m_{ij}^{(k)} - \frac{w_i}{w_j}, \Delta u_{ij}^{(k)} = u_{ij}^{(k)} - \frac{w_i}{w_j}$$

where the meanings of variables are the same as that in the previous chapters. It can be seen that when the experts' judgements are against the related ratios of weights of indexes, the FGMV counts all violations associated with the order reversals (denoted by $I_{ij}{}^{(k)}$), and FGED measures the average distances between the judgments in fuzzy comparison matrices of all individuals and the related ratios of weights from the group comparison matrix.

6. CASE STUDY

We select a real application from Kaya and Kahraman (2011). The authors defined six indexes ($C_1 \sim C_6$) and provided pairwise comparison matrices ($A_1 \sim A_3$) from three decision-makers. In order to assess the weight of each index, the experts were asked to divide different degrees of importance between indexes into 9 terms and each term is denoted by a triangular fuzzy number as in Table 1. Tables 3 give the results of the pairwise comparisons of indexes made by three experts. In the next step, using Tables 1 and 3, the aggregated group fuzzy comparison matrices for the index weights are obtained by four different methods, namely Fuzzy-Geo, Max-Min-EA, Geo-MEA and Ari-Geo methods as in Tables 4.

Table 3-1. Pair-wise comparisons made by DM 1

A_1	C_1	C_2	C_3	C_4	C_5	C_6
C_1	E	SS	VS	FS	FS	SS
C_2	SW	E	FS	E	SS	E
C_3	VW	FW	E	E	SW	E
C_4	FW	E	E	E	SW	SW
C_5	FW	SW	SS	SS	E	SW
C_6	SW	E	E	SS	SS	E
$CR(A_1{}^L) = 0.0118$, $CR(A_1{}^M) = 0.0084$, $CR(A_1{}^U) = 0.0091$						

Table 3-2. Pair-wise comparisons made by DM 2

A_2	C_1	C_2	C_3	C_4	C_5	C_6
C_1	E	E	FS	FS	SS	E
C_2	E	E	FS	SS	FS	FS
C_3	FW	FW	E	E	E	E
C_4	FW	SW	E	E	E	SW
C_5	SW	FW	E	E	E	E
C_6	E	FW	E	SS	E	E
$CR(A_2{}^L) = 0.0132$, $CR(A_2{}^M) = 0.0073$, $CR(A_2{}^U) = 0.0098$						

Table 3-3. Pair-wise comparisons made by DM 3

A_3	C_1	C_2	C_3	C_4	C_5	C_6
C_1	E	FS	VS	AS	FS	FS
C_2	FW	E	FS	FS	SS	SS
C_3	VW	FW	E	SS	SW	E
C_4	AW	FW	SW	E	SS	SS
C_5	FW	SW	SW	SW	E	SS
C_6	FW	SW	E	SW	SW	E
$CR(A_3{}^L) = 0.0004$, $CR(A_3{}^M) = 0.0063$, $CR(A_3{}^U) = 0.0153$						

Through the consistency check method given in section 3, we can get that the three aggregation matrices ($B_1 \sim B_3$) are all consistent.

In our study, we select another three different approaches as compare to the newly proposed one for deriving crisp index weights from group comparison matrices. The selected methods are applied to the initial data of the application (Kaya & Kahraman, 2011) and the results are then evaluated, which are presented in Table 5.

The results show very similar weights for the last two methods, and the ranking of indexes is: $C_1 > C_2 > C_6 > C_5 > C_4 > C_3$. While, the weights by the first method has a little difference, especially on the ranking of indexes C_5 and C_6. The second method has obviously the biggest difference. On the other hand, the results of the FGMV measure for all methods are equal so that the FGMV is an invalid selection for the evaluation of methods in the case. However, the results of the FGED show that the third and fourth methods have the best evaluations, and the second is the worst. They use different approaches for obtaining the group comparison matrix. The second method uses the maximum and the minimum bounds, while the others use geometric mean or arithmetic mean of individual comparison matrices. So, the worse evaluation of the max-min method means that it may be a bad approach for aggregating matrices and deriving weights. From the results, we can also see that the newly proposed method has a good result, which confirmed the advantages referred in Section 4.4.

7. CONCLUSIONS

Fuzzy group AHP has a wide application. The new method presented in this paper is easy to understand and simple to calculate, and the experimental results also show that it can be a good method for deriving weights of indexes. In group fuzzy AHP, the reasonable representation of fuzzy judgments, the reasonable consistency check method, the effective weight prioritization method and the weight evaluation method are all important parts. Up to now, there are still many controversies about the consistency check of fuzzy matrices. There are various methods of weight prioritization, however, they are not critically evaluated. Our study chooses two assessment methods, but the quality of the methods is closely related to the selection of evaluation methods. In the future research, more extensive evaluations should be performed based on data not only from the real-world applications but also from the theoretical models.

Table 4-1. Group fuzzy comparison matrix for the weights by arithmetic mean method

B_1	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(1,1.17,1.5)	(1.33,1.83,2.33)	(1.33,1.83,2.33)	(1,1.33,1.83)	(1,1.17,1.5)
C_2	(0.72,0.89,1)	(1,1,1)	(1,1.5,2)	(1,1.17,1.5)	(1,1.17,1.67)	(1,1.17,1.5)
C_3	(0.43,0.56,0.78)	(0.5,0.67,1)	(1,1,1)	(1,1,1.17)	(0.78,1,1)	(1,1,1)
C_4	(0.44,0.58,0.83)	(0.72,0.89,1)	(0.89,1,1)	(1,1,1)	(0.89,1,1.17)	(0.78,1,1.17)
C_5	(0.56,0.78,1)	(0.61,0.89,1)	(1,1,1.33)	(0.89,1,1.17)	(1,1,1)	(0.89,1,1.17)
C_6	(0.72,0.89,1)	(0.72,0.89,1)	(1,1,1)	(0.89,1,1.33)	(0.89,1,1.17)	(1,1,1)
$CR({B_1}^L) < 0.1$, $CR({B_1}^M) < 0.1$, $CR({B_1}^U) < 0.1$						

Table 4-2. Group fuzzy comparison matrix for the weights by max-min method

B_2	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(1,1.14,2)	(1,1.82,2.5)	(1,1.78,3)	(1,1.31,2)	(1,1.14,2)
C_2	(0.5,0.88,1)	(1,1,1)	(1,1.5,2)	(1,1.14,2)	(1,1.14,2)	(1,1.14,2)
C_3	(0.4,0.55,1)	(0.5,0.67,1)	(1,1,1)	(1,1,1.5)	(0.68,1,1)	(1,1,1)
C_4	(0.33,0.56,1)	(0.5,0.88,1)	(0.67,1,1)	(1,1,1)	(0.67,1,1.5)	(0.67,1,1.5)
C_5	(0.5,0.76,1)	(0.5,0.88,1)	(1,1,1.5)	(0.67,1,1.5)	(1,1,1)	(0.67,1,1.5)
C_6	(0.5,0.88,1)	(0.5,0.88,1)	(1,1,1)	(0.67,1,1.5)	(0.67,1,1.5)	(1,1,1)
$CR({B_2}^L) < 0.1$, $CR({B_2}^M) < 0.1$, $CR({B_2}^U) < 0.1$						

Table 4-3. Group fuzzy comparison matrix for the weights by geometric mean method

B_3	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1)	(1,1.14,1.44)	(1.31,1.82,2.32)	(1.26,1.78,2.29)	(1,1.31,1.82)	(1,1.14,1.44)
C_2	(0.69,0.88,1)	(1,1,1)	(1,1.5,2)	(1,1.14,1.44)	(1,1.14,1.65)	(1,1.14,1.44)
C_3	(0.45,0.55,0.76)	(0.5,0.67,1)	(1,1,1)	(1,1,1.14)	(0.76,1,1)	(1,1,1)
C_4	(0.44,0.56,0.79)	(0.69,0.88,1)	(0.88,1,1)	(1,1,1)	(0.87,1,1.17)	(0.78,1,1.14)
C_5	(0.55,0.76,1)	(0.61,0.88,1)	(1,1,1.32)	(0.85,1,1.15)	(1,1,1)	(0.78,1,1.14)
C_6	(0.69,0.88,1)	(0.69,0.88,1)	(1,1,1)	(0.88,1,1.28)	(0.88,1,1.28)	(1,1,1)
$CR({B_3}^L) < 0.1$, $CR({B_3}^M) < 0.1$, $CR({B_3}^U) < 0.1$						

Table 5. Fuzzy evaluation matrix for the alternatives

Methods	Indexes						Assessment	
	C_1	C_2	C_3	C_4	C_5	C_6	FGMV	FGED
Fuzzy Geo	0.250	0.204	0.122	0.133	0.148	0.143	10	1.795
Max-min, EA	0.366	0.253	0.061	0.078	0.122	0.120	10	7.090
Geo, Modified EA	0.222	0.187	0.139	0.143	0.153	0.155	10	1.586
Ari-Geo	0.220	0.186	0.140	0.144	0.153	0.157	10	1.585

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