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Multi-Level Electric Vehicle Charging Facilities with Limited Resources

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Abstract: In this paper, we consider electric vehicle charging facilities with limited space and power resources. We assume the facility offers a finite selection of levels, i.e., charging rates, for varying prices. Users arrive at the facility randomly, requiring a random amount of charge and possessing a random impatience factor dictating their value of time. Each user then chooses a charging rate that minimizes their total cost that includes an opportunity cost for the time required to charge associated with their impatience factor. Knowing the probability distribution of user charging demands, user impatience factors, and the number of arrivals at a charging facility, we present high-confidence bounds on the total number of active users and aggregate power use of all active users at any given time. We present a case study to illustrate the results.

Keywords: electric vehicles, stochastic modeling, resource allocation

1. INTRODUCTION

Improved affordability of electric vehicles (EVs) has catalyzed their adoption such that McKerracher et al. (2019) predict that by 2040 the United States' new vehicle sales will be comprised of 57% electric vehicles and the global passenger vehicle market will be 30% electric. With the growing numbers of electric vehicles, the demands on charging facilities will be greater. Analysis and control of the EV charging problem is therefore needed to ensure effortless operation of EV charging facilities.

This has led to research studying charging facility usage from an optimization or control system framework, e.g., in Wu et al. (2011); Zhang and Li (2015). Similarly, in Li et al. (2013), charging management is performed by solving a social welfare nonlinear optimization problem. In Gan et al. (2012), scheduling electric vehicle charging is formulated as an optimal control problem which algorithmically converges to optimal charging profiles. Bae and Kwasinski (2011) consider a spatiotemporal model for rapid charging facilities. Moradipari and Alizadeh (2019) study optimal pricing and routing schemes for a charging network where users specify their priority level while the charging network chooses between a profit and a social welfare maximizing mode.

In contrast to previously mentioned work, the present work analyzes the EV charging problem from a purely probabilistic perspective. We model the user-charging facility dynamics in a queuing framework by leveraging the knowledge of the probability distribution of users' demand and impatience factor to provide confidence intervals on a charging facility's likelihood of not exceeding specific active user and power budget levels. Confidence intervals for the number of active users and total power consumption have previously been presented in Pandit and Coogan (2018), in the case when the users can choose their charging rate arbitrarily. In contrast to Pandit and Coogan (2018), we formulate a model as a discrete choice model under stochastic demand where a user seeks to minimize the price paid to charge their vehicle from L-levels of service, i.e., L pricing functions. Each level of service offers a different charging rate, and hence the users' impatience factors will affect their level choices.

This paper is organized as follows: Section 2 formally introduces relevant parameters and formulates the problem statement for the discrete choice model, Section 3 presents the main results of this paper, Section 4 presents a case study which compares the theoretical to simulated results, and Section 5 concludes the paper. The Appendix contains proofs of some of the results presented in the paper.

2. PROBLEM FORMULATION

We consider an EV charging facility that has a finite number of charging stations for individual vehicles, receives power from a local utility operator, and does not face competition. At this facility, a user j arrives at some time a_j (in hr.) with charging demand x_j (in kW-hr.) and an impatience factor α_j (in \$/hr.). The impatience factor quantifies how much a user values their time versus money and is also called the opportunity cost factor. Throughout the paper we will make the following assumption about the aforementioned variables:

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Assumption 1. (Users). User arrivals at the charging facility are a Poisson process with parameter λ (in EVs/hr.) and hence users are arriving with no information on the current charging availability. Individual charging demand x_j and the impatience factor α_j for each user j are random variables which are independent and identically distributed (i.i.d) with probability density functions (PDF) $f_X(x)$ and $f_A(\alpha)$, respectively. Additionally, these random variables are positive and bounded so that there exists finite $0 < x_{\min} < x_{\max}$ and $0 < \alpha_{\min} < \alpha_{\max}$ such that $f_X(x)$ is supported only on $[x_{\min}, x_{\max}]$ and $f_A(\alpha)$ is supported only on $[\alpha_{\min}, \alpha_{\max}]$.

When considering a collection of i.i.d random variables indexed by subscripts, we use non-subscript variables when referring to properties that hold for any of the i.i.d random variables. For example, $\mathbb{E}[x] = \int_0^\infty \xi f_X(\xi)\,\mathrm{d}\xi$ is the expectation of each x_j.

The charging facility offers L levels of service. Each level of service $\ell \in \{1, \dots, L\}$ corresponds to a distinct charging rate $R^\ell > 0$ (in kW) and price V^ℓ (in \$/kW) that is the cost per unit energy for the service level. Thus, user j with charging demand x_j pays $x_j V^\ell$ (in \$) to receive a full charge over the time horizon x_j/R^ℓ (in hr) when choosing level of service ℓ.

Assumption 2. (EV Charging Facility) Among L levels of service offered by the charging facility, a higher charging rate is more costly, i.e., if $R^i > R^k$ then $V^i > V^k$. Moreover, charging rates and prices are distinct so that $R^i \neq R^k$ for all $i \neq k$. Lastly, and without loss of generality, the charging facility's pricing functions are enumerated such that $V^1 < V^2 < \ldots < V^L$ and therefore $R^1 < R^2 < \ldots < R^L$. Define the minimum charging rate $R^{\min} := R^1$ and the maximum charging rate $R^{\max} := R^L$.

Since user j values their time at a rate α_j, they may be willing to pay for a higher level of service since it delivers a full charge faster. To this end, the total cost faced by a user with impatience factor α_j and charge demand x_j who chooses service level ℓ is given by

$$g_\ell(x_j, \alpha_j) = x_j V^\ell + \alpha_j \frac{x_j}{R^\ell}. \tag{1}$$

In (1), the first term, $x_j V^\ell$, quantifies the cost to the user resulting from their demand at arrival. The second term of (1), $\alpha_j \frac{x_j}{R^\ell}$, is the cost associated with how much a user values their time where $\frac{x_j}{R^\ell}$ is the time to charge for a particular service level ℓ. Individual users choose a level of service at a charging facility which minimizes their total cost of charging factoring in their impatience. Let $S(x_j, \alpha_j) : [x_{\min}, x_{\max}] \times [\alpha_{\min}, \alpha_{\max}] \to \{1, \dots, L\}$ be defined by

$$S(x_j, \alpha_j) = \operatorname*{arg\,min}_{\ell \in \{1,\dots,L\}} g_\ell(x_j, \alpha_j). \tag{2}$$

Then, a rational user j chooses level of service $S(x_j, \alpha_j)$ in order to minimize their total cost as formalized in Assumption 3.

We also define the values v_j and r_j to be the charging rate and cost per unit of energy chosen by user j after solving (2), i.e., $r_j = R^{S(x_j,\alpha_j)}$ and $v_j = V^{S(x_j,\alpha_j)}$. Observe that the charging times $u_j := x_j/r_j$, and α_j, constitute a collection of independent and identically distributed random variables.

Assumption 3. (Users are Rational) Each user chooses a charging rate according to (2) and leaves the charging facility once they have satisfied their charging demand. Thus, user j occupies a charger at the facility during the time interval $[a_j, a_j + u_j]$.

Charging facilities are concerned with adhering to both user capacity and energy consumption restrictions. Let $N(t)$ be the set of active users at the charging facility at time t, i.e., $N(t) = \{i : t \in [a_i, a_i + u_i]\}$, where u_i is the time to charge, and let $\eta(t) = |N(t)|$ be the cardinality of the set of active users. Moreover,

$$Q(t) = \sum_{i \in N(t)} r_i = \sum_{i \in N(t)} \frac{x_i}{u_i}$$

is the total charging rate at time t for all active users, i.e., the charging facility's power consumption.

We consider the problem in which the charging facility is interested in providing probabilistic guarantees on the number of users in the system and the total power requirements at any given time t. We thus wish to compute a high-confidence bound on the total number of of active users and their respective aggregate power draw at any given time, as is made precise in the following problem statement.

Problem Statement 1. Given an EV charging facility with L service levels satisfying Assumption 2 and EV users satisfying Assumptions 1 and 3, for any $\mathcal{M}$ number of users and $\mathcal{R}$ total charging facility power consumption rate, compute $\delta(\mathcal{M})$ and $\gamma(\mathcal{R})$ such that

$$\mathbb{P}\big(\eta(t) < \mathcal{M}\big) \geq 1 - \delta(\mathcal{M}) \tag{3}$$

and

$$\mathbb{P}\big(Q(t) < \mathcal{R}\big) \geq 1 - \gamma(\mathcal{R}). \tag{4}$$

3. MAIN RESULTS

In this section, we first introduce a proposition which formalizes the probability a randomly selected user will choose a particular level of service. Then, we present a theorem which solves the problem statement above and provides probabilistic guarantees of the form of (3)–(4).

First, we present Proposition 1 which defines the probability a cost function of the form of (1) will be the minimum within the set of broadcast levels of service.

Proposition 1. Under Assumptions 1, 2, and 3, consider the set of L functions of two independent random variables $\big\{g_\ell(x_j, \alpha_j)\big\}_{\ell=1}^{L}$ where each g_ℓ is as in (1). Then, for $k \in \{1, \dots, L\}$,

$$\mathbb{P}\big(S(x_j, \alpha_j) = k\big) = \max\left\{0, \int_{\underline{\alpha}^k}^{\bar{\alpha}^k} f_A(\alpha)\,\mathrm{d}\alpha\right\}$$

where

$$\bar{\alpha}^k = \min\left(\alpha_{\max},\ \min_{i<k} \frac{V^k - V^i}{\frac{1}{R^i} - \frac{1}{R^k}}\right), \tag{5}$$

$$\underline{\alpha}^k = \max\left(\alpha_{\min},\ \max_{k>i} \frac{V^k - V^i}{\frac{1}{R^i} - \frac{1}{R^k}}\right). \tag{6}$$

Proposition 1 states that, when a given level of service is chosen with nonzero probability, there exists an interval, which is possibly empty, of impatience factor values for which that level of service minimizes the total cost to a user. The probability that the given level of service will be chosen is therefore computed by integrating the PDF $f_A(\alpha)$ on that interval.

Proposition 1 defines the probability a user will choose level of service k. This probability is equivalent to the probability of choosing a specific charging rate. The detailed proof of Proposition 1 is in the Appendix A.1. Since choosing a charging rate is a discrete choice, a probability mass function (PMF) can be formulated for the rates of charge.

Corollary 2. Under Assumptions 1, 2, and 3, the charging rates r_j chosen by each user j is a collection of independent and identically distributed discrete random variables each with probability mass function

$$p_r(r) = \begin{cases} \max\left\{0, \int_{\underline{\alpha}^1}^{\bar{\alpha}^1} f_A(\alpha)\,\mathrm{d}\alpha\right\}, \ r = R^1 \\ \vdots \\ \max\left\{0, \int_{\underline{\alpha}^L}^{\bar{\alpha}^L} f_A(\alpha)\,\mathrm{d}\alpha\right\}, \ r = R^L \end{cases} \tag{7}$$

where $\bar{\alpha}_k$ and $\underline{\alpha}_k$ are as in (5) and (6) for $k \in \{1, \dots, L\}$.

Note that $\mathbb{E}[r] = \sum_{\ell=1}^{L} R^\ell p_r(R^\ell)$. Moreover, the choice of charging rate r_j chosen by a user j is only a function of the impatience factor α_j. Thus r_j is independent of x_j so that $\mathbb{E}[u] = \mathbb{E}[x]/\mathbb{E}[r]$ is the expected charging time for each user j. Next, we introduce the main theorem for this paper which addresses Problem Statement 1.

Theorem 3. Consider a charging facility offering L levels of service with a minimum charging rate of $R^{\min} = R^1$ and a maximum charging rate $R^{\max} = R^L$ operating under Assumptions 1, 2, and 3. Given any $\mathcal{M} \geq 0$ number of users and $\mathcal{R} \geq 0$ total charging rate, the following statements hold at steady state for any time t:

(1) With confidence $1 - \delta(\mathcal{M})$, where

$$\delta(\mathcal{M}) = \begin{cases} \exp\left(\dfrac{-(\mathcal{M} - \lambda\mathbb{E}[u])^2}{2\left(\mathcal{M}\mathbb{E}[r^2] + \frac{R^{\max}(\mathcal{M} - \lambda\mathbb{E}[u])}{3}\right)}\right), & \text{if } \mathcal{M} > \lambda\mathbb{E}[u] \\ 1 & \text{otherwise,} \end{cases}$$

the number of users will not exceed $\mathcal{M}$, i.e., $\mathbb{P}\big(\eta(t) < \mathcal{M}\big) \geq 1 - \delta(\mathcal{M})$.

(2) With confidence $1 - \gamma(\mathcal{R})$, where

$$\gamma(\mathcal{R}) = \begin{cases} \min\left\{1, \displaystyle\sum_{m=\left\lceil \frac{\mathcal{R}}{R^{\max}} \right\rceil}^{\left\lfloor \frac{\mathcal{R}}{\mathbb{E}[r]} \right\rfloor} \exp\left(\dfrac{-\left(\mathcal{R} - m\mathbb{E}[r]\right)^2}{2\left(m\mathbb{E}[r^2] + \frac{R^{\max}(\mathcal{R} - m\mathbb{E}[r])}{3}\right)}\right) \right. \\ \left. \times\mathbb{P}\big(\eta(t) = m\big) + \delta\left(\left\lfloor \dfrac{\mathcal{R}}{\mathbb{E}[r]} \right\rfloor\right) \right\}, & \text{if } \mathcal{R} > \lambda\mathbb{E}[u]\,\mathbb{E}[r] \\ 1, & \text{otherwise,} \end{cases}$$

the total charging rate for all active users will not exceed $\mathcal{R}$, i.e., $\mathbb{P}\left(Q(t) < \mathcal{R}\right) \geq 1 - \gamma(\mathcal{R})$.

Proof.

(1) We begin by proving the first statement. First, we observe that, for a Poisson random variable Z with mean λ, for any $\mathcal{M}$, $\mathbb{P}\big(Z < \mathcal{M}\big) \geq 1 - \delta^\dagger(\mathcal{M})$ where

$$\delta^\dagger(\mathcal{M}) = \exp\left(\frac{-\left(\mathcal{M} - \lambda\right)^2}{2\left(\lambda + \frac{\mathcal{M} - \lambda}{3}\right)}\right).$$

This observation is made in Proposition 4 of Appendix A.2. This observation leads to Corollary 6 for a charging facility operating under Assumption 1. Moreover, since the arrival and service process can be seen as an $M/G/\infty$ queue, $\eta(t)$ is itself a Poisson random variable for each t with mean $\lambda\mathbb{E}[u]$ (Massey, 2002, Equation (9)). We can then apply this observation to $\mathbb{E}[\eta(t)]$ in Corollary 6 and this completes the proof of the first statement.

(2) To prove the the second statement, recall we are interested in the sum of of the charging rate of active users in the charging facility. Then, $\mathbb{P}\big(\sum_{i \in N(t)} (r_i - \mathbb{E}[r]) \geq \nu\big) = \mathbb{P}\left(Q(t) \geq \eta(t)\mathbb{E}\left[r\right] + \nu\right)$. Let $\mathcal{R} = \eta(t)\mathbb{E}[r] + \nu$ which implies $\nu = \mathcal{R} - \eta(t)\mathbb{E}[r]$. By total probability, it holds that

$$\begin{aligned} &\mathbb{P}\big(Q(t) \geq \mathcal{R}\big) \\ &\quad= \sum_{m=0}^{\infty} \mathbb{P}\big(Q(t) \geq \mathcal{R} \mid \eta(t) = m\big)\mathbb{P}\big(\eta(t) = m\big) \\ &\quad= \sum_{m=\left\lceil \frac{\mathcal{R}}{R^{\max}} \right\rceil}^{\beta} \mathbb{P}\big(Q(t) \geq \mathcal{R} \mid \eta(t) = m\big)\mathbb{P}\big(\eta(t) = m\big) \\ &\qquad + \mathbb{P}\left(\eta(t) > \beta\right) \end{aligned}$$

Notice that the lower bound of the summation is no longer zero since the probability of exceeding $\mathcal{R}$ is zero even when choosing the worst case rate for $m < \mathcal{R}/R^{\max}$. Hence, we take the lower bound of the summation to be the ceiling of $\mathcal{R}/R^{\max}$ and the upper bound to be some value β. Using Fact 5 (Bernstein's Inequality) in Appendix A.2, with $b = R^{\max}$ and $n = \eta(t)$, gives

$$\begin{aligned} &\mathbb{P}\big(Q(t) \geq \eta(t)\mathbb{E}[r] + \nu \mid \eta(t)\big) \\ &\qquad \leq \exp\left(\frac{-\nu^2}{2\left(\eta(t)\mathbb{E}[r^2] + \frac{R^{\max}\nu}{3}\right)}\right). \end{aligned}$$

Using this fact and substituting for ν,

$$\begin{aligned} &\mathbb{P}\big(Q(t) \geq \mathcal{R}\big) \\ &\leq \sum_{m=\left\lceil \frac{\mathcal{R}}{R^{\max}} \right\rceil}^{\beta} \exp\left(\frac{-\left(\mathcal{R} - m\mathbb{E}[r]\right)^2}{2\left(m\mathbb{E}[r^2] + \frac{R^{\max}(\mathcal{R} - m\mathbb{E}[r])}{3}\right)}\right) \\ &\qquad \times \mathbb{P}\big(\eta(t) = m\big) + \mathbb{P}\left(\eta(t) > \beta\right). \end{aligned}$$

Note that to apply Bernstein's inequality, $\mathcal{R} - m\mathbb{E}\left[r\right] > 0$. This implies, $m < \mathcal{R}/\mathbb{E}\left[r\right]$. Then, $\beta = \left\lfloor \mathcal{R}/\mathbb{E}\left[r\right] \right\rfloor$, i.e., the floor value of $\mathcal{R}/\mathbb{E}\left[r\right]$. Using this fact, and the result from Statement 1 of Theorem 3,

$$\mathbb{P}\big(Q(t) \geq \mathcal{R}\big) \leq \sum_{m=\left\lceil \frac{\mathcal{R}}{R^{\max}} \right\rceil}^{\left\lfloor \frac{\mathcal{R}}{\mathbb{E}[r]} \right\rfloor} \exp\left(\frac{-\left(\mathcal{R} - m\mathbb{E}[r]\right)^2}{2\left(m\mathbb{E}[r^2] + \frac{R^{\max}(\mathcal{R}-m\mathbb{E}[r])}{3}\right)} \right) \times \mathbb{P}\big(\eta(t) = m\big) + \delta\left(\left\lfloor \frac{\mathcal{R}}{\mathbb{E}[r]} \right\rfloor \right) = \gamma^{\dagger}(\mathcal{R}).$$

Similar to Corollary 6, as a result of Bernstein's inequality, the bound $\gamma^{\dagger}(\mathcal{R})$ is less than 1 for some interval of $\mathcal{R} \in (\Gamma_a, \infty)$ where it attains the value of 1 if $\mathcal{R} \leq \Gamma_a$. To find the exact interval for when $\gamma^{\dagger}(\mathcal{R}) = 1$ requires finding a specific value of $\mathcal{R}$; however, we know that Γ_a must be greater than or equal to $\mathbb{E}[\eta(t)]\,\mathbb{E}[r]$ as a result of using Bernstein's inequality on $Q(t)$. Hence, by using the min function finding the exact Γ_a can be avoided, i.e.,

$$\gamma(\mathcal{R}) = \begin{cases} \min\left\{1, \gamma^{\dagger}(\mathcal{R})\right\} & \mathcal{R} > \mathbb{E}[\eta(t)]\,\mathbb{E}[r] \\ 1, & \text{otherwise.} \end{cases}$$

Now, recalling $\mathbb{E}[\eta(t)] = \lambda \mathbb{E}[u]$ (Massey, 2002, Equation (9)) and $\mathbb{P}\big(Q(t) < \mathcal{R}\big) = 1 - \mathbb{P}\big(Q(t) \geq \mathcal{R}\big) \geq 1 - \gamma(\mathcal{R})$ completes the proof.

Theorem 3 quantifies the likelihood a charging facility under stochastic user arrivals and charging demand with discrete levels of service will stay within (or exceed) a specified threshold of user capacity and active user rate consumption.

3.1 Main Result Discussion

A charging facility operator whose facility operates under the construct of Section 2 and Assumptions 1, 2, and 3 can utilize Proposition 1 and Theorem 3 to properly estimate a high-confidence bound on the number of active users using its facilities and their power consumption.

Computing the high-confidence bounds depends on $\mathbb{E}[u]$ and $\mathbb{E}[r]$. As an example, consider an EV charging facility operator with capacity for 40 vehicles simultaneously and would like to ensure with high probability that a charger is available for each arriving user. Therefore, the facility operator would like to quantify the likelihood the number of active users will exceed a specified threshold. Here, an operator can use Statement 1 from Theorem 3 to get such a bound.

For instance, suppose the operator offers two levels of service with $R^1 = 30$, $R^2 = 40$, $V^1 = 5.2$, and $V^2 = 5.4$. Each arriving user chooses a level of service according to (2). For these numerical values, the total user cost (1) as a function of the impatience factor is illustrated in blue in the left plot of Fig. 1. The resulting theoretical upper bound on the number of active users for various confidence bounds is illustrated in blue in the upper right plot of Fig. 1. Notice that the theoretical bound predicts that, for $\mathcal{M} = 40$ active users there is only an 80% confidence that 40 will not be exceeded by $\eta(t)$, i.e., the number of active users.

If the operator wishes to achieve a higher level of confidence that the facility capacity will not be exceeded, the operator can increase the charging rates to $(R^1)^+ = 50$ and $(R^2)^+ = 70$ while maintaining V^1 and V^2 the same. The new total cost functions (1) as a function of the impatience factor are illustrated in red in the left plot of Fig. 1. Notice that the new theoretical bound, $1-\delta^{+}_{\mathcal{M}}(\mathcal{M})$, has increased the confidence that the number of active users will not exceed 40; however, this occurs at the expense of higher total active user charging rates. Hence, a charging facility operator can use Theorem 3 to adjust the individual level of service charging rates to manage the number of active users. A similar exercise can be conducted for a case when the facility total charging rate is of concern.

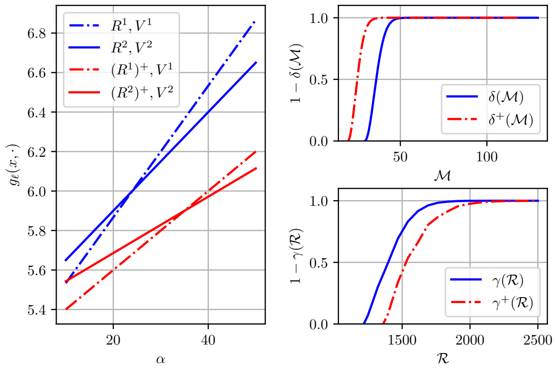

Fig. 1. An illustration of the change in the theoretical bounds after an increase in the vehicle charge rates. The lines in blue indicate the initial costs and bounds prior to a rate increase while red indicates those same values post charge rate increase.

4. CASE STUDY

We present a case study which illustrates the theoretical results of Theorem 3 compared to a simulated Monte Carlo results. We consider a charging facility system which broadcasts $L = 5$ pricing functions. [2]

Satisfying Assumption 1, α has a truncated normal distribution in this case study. We use the notation $\mathcal{N}_{\text{trunc}}$ to denote a truncated normal distribution. Similarly, satisfying Assumption 1, we define the demand x to have a uniform distribution where $x \in [x_{\min}, x_{\max}]$. Given the case study parameters, we illustrate the charging facility's 5 pricing functions in Fig. 2. In the left plot, all of the charging facility's pricing functions are plot on $[\alpha_{\min}, \alpha_{\max}]$ with the truncated normal distribution $\mathcal{N}_{\text{trunc}}(30, 20)$ superimposed while x, i.e., charging demand, is fixed. As stated in Proposition 1, the probability a user chooses a level of service can be computed by integrating the PDF $f_A(\alpha)$ on the interval during which a particular (1) is minimum. To illustrate this fact, in the right-hand plot of Fig. 2, the second pricing function from this case study is singled out. Here, the region of the truncated normal distribution which corresponds to when $g_2(x, \alpha)$ is a minimum is shaded for a fixed x.

The simulation is run for $T = 100$ hrs with $\lambda = 20$ EVs/hr with a total of 1000 Monte Carlo draws. For the set of pricing functions illustrated in Fig. 2, we obtain

[2] The code for this case study is available at `https://github.com/gtfactslab/ifac_charging_facility`

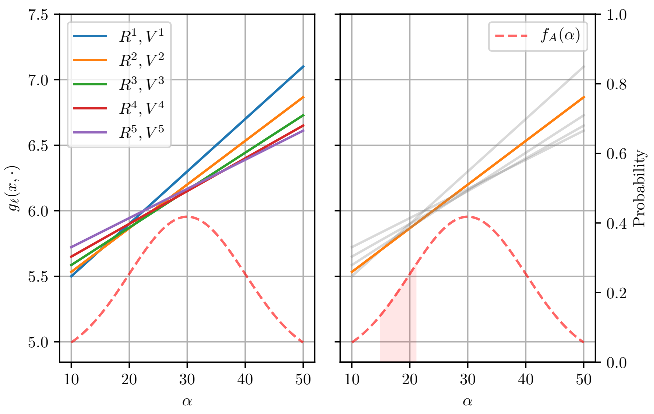

Fig. 2. The charging facility's 5 pricing functions on the interval $[\alpha_{\min}, \alpha_{\max}]$ for a fixed value of x. Here, the truncated normal distribution, $f_A(\alpha)$ which governs the α statistics is superimposed. The region of the domain $[\alpha_{\min}, \alpha_{\max}]$ over which $g_2(\cdot, \alpha)$ is a minimum.

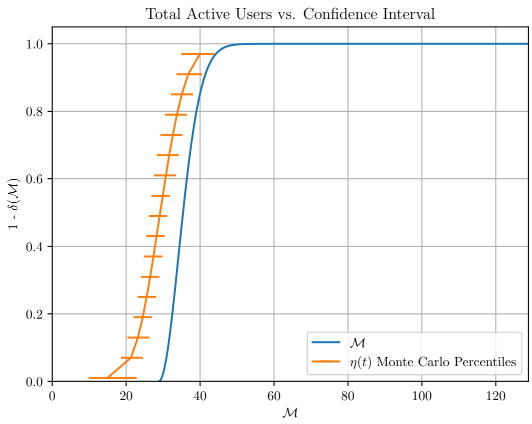

Fig. 3. A plot of the theoretical upper bound from Theorem 3 on the total number of active users in the charging facility versus Monte Carlo results. The error bars represent the min. and max. values attained in a particular percentile for all Monte Carlo draws while the curve itself is the mean.

$\mathbb{E}[u] = 1.44$ (hr.) and $\mathbb{E}[r] = 38.15$ (kW). First, we illustrate the high-confidence bound on the number of users in the charging facility, i.e., $1 - \delta(\mathcal{M})$, as a function of $\mathcal{M}$. The second set of data points in Fig. 3 illustrate the $1 - \delta(\mathcal{M})$ percentile average across all the Monte Carlo runs. Here, the error bars illustrate the minimum and maximum values attained across all the Monte Carlo runs. Empirically, we see the theoretical bound on the number of active users in the charging facility for a specified levels of service provides operators with the ability to adequately quantify the likelihood $\eta(t)$ will exceed some threshold.

Similarly, a charging facility may be required to adhere to power rate constraints for all of its active users at some time t. Because of this, we are interested in illustrating the second statement of Theorem 3. Fig. 4 shows the upper bound $\gamma(\mathcal{R})$ on the total power draw of the active users. Similarly, the average total power consumption and the minimum and maximum values are presented in Fig. 4. Here, we see that the theoretical bound provides a

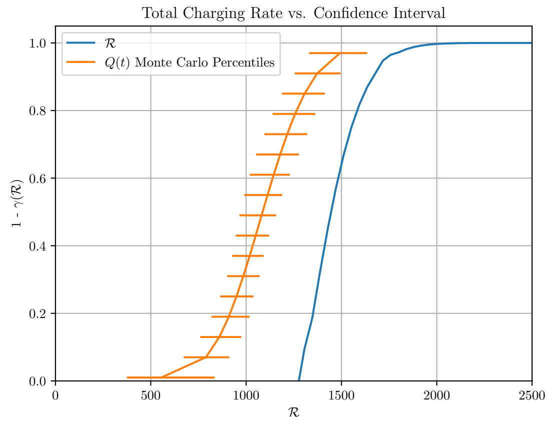

Fig. 4. A plot of the theoretical upper bound from Theorem 3 on the total rate demanded by active users (kW). Here, $1 - \gamma(\mathcal{R})$ is compared to the Monte Carlo percentile mean values. The error bars represent the min. and max. values attained in a particular percentile for all Monte Carlo draws.

conservative quantification of the amount of power draw of the active users in the charging facility. This bound is more conservative than the bound on the total number of active users because it is dependent on the bound from Statement 1 of Theorem 3 which is itself not an exact account of the number of active users in the charging facility.

5. CONCLUSION

We studied the problem of computing high-confidence bounds for charging facilities that work under the construct of Section 2 and Assumption 1, 2, and 3. Specifically, we consider the case when charging facilities present users with L-discrete choices for levels of service, i.e., pricing functions. We derived a theoretical bound which gives a bound on the likelihood the total charging rate or total number of active users in the charging facility will exceed some threshold. We illustrated how a charging facility operator may use the main result for modifying the number of users in the charging facility. Lastly, we presented a case study which illustrates the theoretical results and the compares high-confidence bounds to simulated results.

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Appendix A. PROOFS

A.1 Proof of Proposition 1

Proof. Under Assumptions 1, 2, and 3, consider a set of L functions of two independent positive random variables $\{g_\ell(x_j,\alpha_j)\}_{\ell=1}^{L}$ of the form $g_\ell(x_j,\alpha_j) = x_jV^\ell + \alpha_j\frac{x_j}{R^\ell}$ Denote the difference $g_k(x_j,\alpha_j) - g_i(x_j,\alpha_j) = x_j\big(\Delta V^{ki} + \alpha_j\Delta R^{ki}\big)$ where $\Delta V^{ki} = V^k - V^i$, $\Delta R^{ki} = \frac{1}{R^k} - \frac{1}{R^i}$ and $V^i < V^k$ and $\frac{1}{R^i} > \frac{1}{R^k}$ for all $i < k$. Define $\mathbb{P}\big(\min\{g_\ell(x_j,\alpha_j)\}_{i=1}^{L} = g_k(x_j,\alpha_j)\big) = \mathbb{P}\big(S(x_j,\alpha_j) = k\big)$ For a function $g_k(x_j,\alpha_j)$ to be the minimum function, then $g_k(x_j,\alpha_j) < g_i(x_j,\alpha_j)$ for all $i \neq k$. It follows that

$$\mathbb{P}\big(g_k(x_j,\alpha_j) < g_i(x_j,\alpha_j)\big) = \mathbb{P}\left(x_j\left(\Delta V^{ki} + \alpha_j\Delta R^{ki}\right) < 0\right) .$$

we are specifically interested in $\Delta V^{ki} + \alpha_j\Delta R^{ki} < 0$. The domain over which function g_k is a minimum is $[\underline{\alpha}^k, \bar{\alpha}^k]$. Since the functions have an ordering, i.e., $V^1 < V^2 < \ldots < V^L$, we can define $\bar{\alpha}^k = \min\big(\alpha_{\max}, \min_{i<k}\frac{V^k - V^i}{\frac{1}{R^i} - \frac{1}{R^k}}\big)$ and $\underline{\alpha}^k = \max\big(\alpha_{\min}, \max_{k>i}\frac{V^k - V^i}{\frac{1}{R^i} - \frac{1}{R^k}}\big)$. Since we are interested in when $\Delta V^{ki} + \alpha_j\Delta R^{ki} < 0$, we can use these as the bound of integration for the PDF of α whose definite integral obtains the probability a function will be a minimum. A function may never be a minimum on the domain $[\alpha_{\min}, \alpha_{\max}]$, for this case we add the $\max\{\cdot\}$ function. This completes the proof.

A.2 Proof of observation in Theorem 3

The proof of statement 1 of Theorem 3 relies on the following observation.

Proposition 4. Let Z be a Poisson random variable with mean λ. Then, for any $\mathcal{M} > \lambda \geq 0$, $\mathbb{P}\big(Z < \mathcal{M}\big) \geq 1 - \delta(\mathcal{M})$ where

$$\delta(\mathcal{M}) = \exp\left(\frac{-\left(\mathcal{M}-\lambda\right)^2}{2\left(\lambda + \frac{\mathcal{M}-\lambda}{3}\right)}\right).$$

As an initial step to proving Proposition 4, we first recall Bernstein's inequality.

Fact 5. (Bernstein's Inequality, Wainwright (2019)). Given n independent, zero-mean random variables X_i such that, for some $b > 0$, $\nu > 0$, $0 \leq X_i \leq b$ for all $1 \leq i \leq n$. Then, almost surely, it holds that

$$\mathbb{P}\left(\sum_{i=1}^{n}(X_i - \mathbb{E}[X_i]) \geq \nu\right) \leq \exp\left(\frac{-\nu^2}{2\left(\sum_{i=1}^{n}\mathbb{E}[X_i^2] + \frac{b\nu}{3}\right)}\right). \quad \text{(A.1)}$$

Finally, we apply the Fact 5 to prove Proposition 4.

Proof. [Proof of Proposition 4.] Recall from the Poisson limit theorem (Durrett, 2019, Theorem 3.6.1) that a Poisson random variable Z with mean λ can be seen as a sum of n Bernoulli random variables $X_i \leq 1$ with mean p, where p is such that $np \to \lambda$ when $n \to +\infty$. In other words, $\sum_{i=1}^{n} X_i \to Z$ as $n \to +\infty$. Here, we see that we can now apply Fact 5 to find a bound on the value of a Poisson random variable which is approximated as the sum of Bernoulli random variables.

Let $X = \sum_{i=1}^{n} X_i$ and $\mathbb{E}[X] = \sum_{i=1}^{n}\mathbb{E}\left[X_i\right] = np$. Since Fact 5 applies to zero-mean random variables, let $X^0 = X - \mathbb{E}[X] = \sum_{i=1}^{n} X_i - \sum_{i=1}^{n}\mathbb{E}\left[X_i\right]$ be a zero-mean sum of Bernoulli random variables where $\mathbb{E}[X^0] = 0$. Then, we can apply Fact 5 with $b = 1$ and letting $\mathcal{M} = \nu + \mathbb{E}\left[X\right] = \nu + np$. Since we can approximate a Poisson random variable via the Poisson limit theorem, by letting $n \to +\infty$, we get

$$\mathbb{P}\left(Z \geq \mathcal{M}\right) \leq \exp\left(\frac{-\left(\mathcal{M}-\lambda\right)^2}{2\left(\lambda + \frac{\mathcal{M}-\lambda}{3}\right)}\right).$$

This proves the proposition.

A direct corollary of Proposition 4 arises when dealing with a charging station whose arrivals are a Poisson process, then the number of active users in the charging facility has a Poisson distribution, i.e., $\lambda = \mathbb{E}[\eta(t)]$.

Corollary 6. Consider a charging facility operating under Assumption 1 where $\eta(t)$ is the number of active users in the charging facility at time t and $\mathbb{E}\left[\eta(t)\right]$ is the mean number of users in the charging facility. For any $\mathcal{M} \geq 0$,

$$\mathbb{P}\left(\eta(t) < \mathcal{M}\right) \geq 1 - \delta\left(\mathcal{M}\right)$$

where

$$\delta\left(\mathcal{M}\right) = \begin{cases} \exp\left(\dfrac{-(\mathcal{M} - \mathbb{E}\left[\eta(t)\right])^2}{2\left(\mathbb{E}\left[\eta(t)\right] + \frac{\mathcal{M}-\mathbb{E}[\eta(t)]}{3}\right)}\right) & \mathcal{M} > \mathbb{E}\left[\eta(t)\right] \\ 1 & \text{otherwise.} \end{cases}$$

Proof. The prove this corollary recall the fact the number of users in a charging facility has a Poisson distribution with $\lambda = \mathbb{E}[\eta(t)]$ and directly substituting into Proposition 4 yields Corollary 6. Then, observe that if $\mathcal{M} \leq \mathbb{E}\left[\eta(t)\right] = \lambda\mathbb{E}[u]$, Bernstein's inequality can not be applied and hence $\delta(\mathcal{M}) = 1$ gives an upper bound for the sought probability.