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Robust Fault Detection of Nonlinear Uncertain Systems with Event-Triggered Communication ⋆

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Abstract: The robust fault detection problem for a class of nonlinear uncertain systems with event-triggered measurement communication is addressed in this study. The proposed event-triggered robust fault detection scheme has two steps. First, with the event-triggered measurement, an adaptive approximator is proposed to learn the unknown modeling uncertainty online. Second, after the learning procedure, an event-triggered residual generator is designed by integrating the adaptive approximator with the residual signal for fault detection. The adaptive threshold for fault detection decision is derived by taking the event-triggered scheme into consideration. The performance of the event-triggered fault detection system is rigorously analyzed by characterizing the event-triggered sampling error, including the stability of the adaptive approximation and the fault detectability.

Keywords: adaptive approximation, fault detection, nonlinear systems, event-triggered scheme.

1. INTRODUCTION

In recent years, considerable research attentions have been dedicated to fault diagnosis design and analysis. Various fault diagnosis methods for different system descriptions have been reported, such as the unknown input observer method, parity space method, and multiple objective optimization methods; see Ding (2008); Seliger and Frank (1992); Zhong et al. (2018). In the case of nonlinear systems, due to their additional complexity, compared to linear systems, some different approaches have been considered. For instance, with fuzzy model approximation, nonlinear system fault diagnosis can be transferred to a specialized fuzzy system fault diagnosis problem for the feasible solution and implementation, especially when optimization is considered (Li et al. (2015, 2016b)); with the Lipschitz (or sector) assumption for the nonlinearity, the linear matrix inequality method can be utilized for the residual generator/fault detection filter solution (Pertew et al. (2007)); and with adaptive estimation/approximation, the unknown nonlinear dynamics can be addressed for nonlinear system fault diagnosis (Zhang et al. (2010); Keliris et al. (2017); Reppa et al. (2014); Boem et al. (2011)).

Model-based fault diagnosis design requires the availability of input and output data of the underlying system. For networked systems, the underlying system with the associated sensors/actuator is not collocated with its monitoring/fault diagnosis module, therefore sampling and transmission are often essential. To reduce the communication burden, event-triggered sampling has been shown to be a promising approach (Heemels et al. (2012); Peng and Li (2018)), which has drawn significant attention in recent years. The event-triggered control problem has been well-studied, and several key problems like stability and minimum inter-event time have been solved for different control systems by using the small gain theorem (Liu and Jiang (2015)), input to state stability (De Persis et al. (2013); Tabuada (2007)), and impulse system descriptions (Abdelrahim et al. (2017)), respectively.

For event-triggered nonlinear system fault diagnosis, the event-triggering creates additional challenges in the fault diagnosis system design due to the (state dependent) aperiodic sampling error. Some successful solutions have been reported. In Li et al. (2016a), a fuzzy model approximation is introduced, and then an event-triggered fault detection filter is designed for the obtained fuzzy system. Another idea is to model the event-triggered scheme as a bounded time delay (Zhang et al. (2016); Pan and Yang (2018)), and use the time-delay system observer design method, as well as the linear matrix inequality technique, for the solution to the fault detection observer.

In this study, a robust fault detection system is designed for nonlinear uncertain system with event-triggered communication. Different from the reported event-triggered fault detection methods for nonlinear systems (Li et al. (2016a); Pan and Yang (2018)), in this work we provide explicit solution for a general nonlinear system, which is subject to event-triggering. Our objective is to quantify the effects of the aperiodic sampling error on the nonlinear fault detection scheme, and then propose a suitable algorithm for fault detection by considering these effects. To address the presence of modeling uncertainty, an adaptive approxiamtion method is adopted, which is

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rigorously analyzed taking into consideration the event-triggering parameters. A nonlinear observer-based residual generator is proposed, and the corresponding adaptive threshold is derived for fault detection decision by taking the event-triggered sampling scheme into consideration. By introducing an auxiliary variable to characterize the event-triggered sampling property, the fault detectability analysis under the proposed event-triggered residual generator and adaptive threshold is derived. Finally, the fault detection scheme for the event-triggered nonlinear uncertain system is established, and the event-triggered sampling effects on adaptive approximation and nonlinear system fault detection are investigated.

The paper is organized as follows. Section 2 presents the model description, main system assumptions, and problem formulation. The adaptive approximation scheme and its stability analysis are given in Section 3, and the fault detection system design and its performance analysis are respectively given in Sections 4 and 5. Section 6 provides some concluding remarks.

The state variables and functions are referred to without the time argument or variable arguments when there is no ambiguity.

2. PROBLEM FORMULATION

Consider the following class of nonlinear uncertain systems,

$$\begin{cases} \dot{x}(t) = Ax(t) + f(x(t), u(t)) + \eta(x(t), u(t)) \\ \qquad + \beta(t - T_0)\phi(x(t), u(t)), \\ y(t) = Cx(t) + v(t), \end{cases} \tag{1}$$

where $x \in R^n$ is the system state, $u \in R^m$ is the control input, $y \in R^p$ is the measurement output, and v is the measurement noise. The matrix A is the known linear part of state equation, $f : R^n \times R^m \mapsto R^n$ represents the known nonlinear system dynamics, and $\eta : R^n \times R^m \mapsto R^n$ characterizes the modeling uncertainty. The pair (A, C) is assumed to be observable.

The term $\beta(t-T_0)\phi(x(t), u(t))$ is employed to characterize the change in the system dynamics as a result of a fault, where $\phi : R^n \times R^m \mapsto R^n$ is an unknown smooth vector field and $\beta(t-T_0)$ is the time profile function denoting the evolution of a system fault. In this study, both incipient and abrupt fault are considered. Thus, we have

$$\beta(t - T_0) = \begin{cases} 0, & t < T_0, \\ 1 - e^{-a(t-T_0)}, & t \geq T_0, \end{cases} \tag{2}$$

where $a \geq 0$ is the unknown fault evolution rate, and T_0 is the unknown fault occurrence time. As $a \to \infty$, $\beta(t - T_0)$ approaches a step function, which corresponds to abrupt faults, while as a becomes small, $\beta(t - T_0)$ corresponds to slowly developing faults.

To avoid waste of the communication resources, in this paper we consider an event-triggered data transmission scheme from the sensors to the fault detection scheme. Define a monotonically increasing sequence γ_k to denote the triggering time:

$$\gamma_k = \begin{cases} 0, & k = 0, \\ \sup\left\{t > \gamma_{k-1} \middle| \, \|e_s(t)\| \leq s_{th}(t)\right\}, & k \in N^+, \end{cases} \tag{3}$$

where $e_s(t) = y(t) - y_s(t)$ and $s_{th}(t) = \delta_1 \|y(t)\| + \delta_2$. Here, $y_s(t)$ is the event-triggered output, which is received by the fault detection scheme. To allow both relative and constant triggering thresholds, we employ the non-negative threshold parameters δ_1 and δ_2 (Peng and Li (2018)). For simplicity, in this study we assume that there is no transmission delay. Thus, following the definition of γ_k in (3), we have

$$y_s(t) = y(\gamma_{k-1}), \quad t \in [\gamma_{k-1}, \gamma_k). \tag{4}$$

Based on (3) and (4), one knows that a new measurement output will be sent to the fault detection scheme only when the defined event is triggered. In this study, we consider the case that the event triggering is only at the measurement output $y(t)$, not at the control input $u(t)$. The reason for this is that typically in cyber-physical systems, the fault detection scheme (monitoring module) and the control scheme are co-located (or are embedded in one software/hardware module). Therefore, the controller output (and the transmitted controller output) is directly accessible by the fault detection scheme, independent of the communication scheme between the controller and the actuator to the system. Thus, we consider the case that the control input $u(t)$ is accessible by the fault detection scheme in this study.

Throughout this paper, the following assumptions are used:

Assumption 1. The unknown measurement noise is bounded, i.e., $\|v(t)\| \leq \bar{v}$, where $\bar{v}$ is a known constant.

Assumption 2. For well-posedness, the state vector $x(t)$ and control input $u(t)$ are bounded before and after the occurrence of system fault, i.e., there exists compact regions of interest $X \subset R^n$ and $U \subset R^m$ such that $x(t) \in X$ and $u(t) \in U$, for all $t \geq 0$.

Assumption 3. The nonlinear vector field $f(x, u)$ is locally Lipschitz with respect to x; i.e., for all $x, \hat{x} \in X$ and $u \in U$, we have

$$\|f(x, u) - f(\hat{x}, u)\| \leq L_f \|x - \hat{x}\|, \tag{5}$$

where L_f is a known Lipschitz constant.

Assumption 4. The unknown modeling uncertainty satisfies $\|\eta(x(t), u(t))\| \leq \bar{\eta}(x(t), u(t))$, where $\bar{\eta} : R^n \times R^m \mapsto R$ is an unknown κ_∞ function (Khalil (2002)); i.e., $\bar{\eta}(0) = 0$, $\bar{\eta}$ is strictly increasing, and $\lim_{r\to\infty} \bar{\eta}(r) = \infty$.

Assumption 1 characterizes the required available information for the measurement noise. This assumption is required to distinguish between the effect of system faults and noise. It is a common assumption for fault detection design (Riverso et al. (2016)). Assumption 2 requires the boundedness of the system state and control input before and after the fault occurrence with event-triggered measurements. This assumption is required, since the focus of this study is the fault detection system design, not the control system design. The fault detection system is independent of the controller structure and fault accommodation is not considered. More details of various event-triggered control schemes for nonlinear systems can be found in Liu and Jiang (2015); Tabuada (2007); Abdelrahim et al. (2017) and references therein. Assumption 3 characterizes the class of nonlinear systems under consideration; this class of systems is commonly found in robotic and power system applications. Assumption 4 implies the boundedness of the unknown modeling uncertainty when $x(t)$ and $u(t)$ are bounded, which is necessary for the adaptive approximation.

The main objective for this study is to design robust fault detection scheme for the class of nonlinear uncertain system described by (1) with event-triggered measurements defined by (3) and (4). To deal with the unknown and unstructured modeling uncertainty, an event-triggered adaptive approximator is proposed to learn the unknown modeling uncertainty $\eta(x(t), u(t))$ (see Fig. 1). After the learning procedure, the approximator will be integrated into the event-triggered observer-based residual generation and evaluation for the fault detection decision.

3. EVENT-TRIGGERED ADAPTIVE APPROXIMATION

In this section, we propose an adaptive approximation scheme to learn the modeling uncertainty, within an event-triggered measurement communication framework, for the period $t \in [0, T_l]$, where $T_l < T_0$. The stability properties of the approximation system with event-triggered measurement are also derived.

3.1 Approximation model

For the unknown modeling uncertainty $\eta(x, u)$, let its optimal approximation be $\hat{\eta}(x(t), u(t), \theta^*) : X \times U \times R^{n_\theta} \mapsto R^n$, which could be described by a Radial Basis Function network or some other network model (Farrell and Polycarpou (2006)). Here, θ^* is the optimal parameter vector defined as

$$\theta^* = \arg\left\{\min_{\theta\in\Theta}\left\{\sup_{x\in X, u\in U} \tilde{n}(t,\theta)\right\}\right\}, \tag{6}$$

where $\tilde{n}(t,\theta) = \|\eta(x(t), u(t)) - \hat{\eta}(x(t), u(t), \theta)\|_\infty$ and $\Theta \subset R^{n_\theta}$ is a given compact set. The predefined set Θ is introduced to avoid parameter drift for weight vector estimation (Farrell and Polycarpou (2006)).

With the approximation $\hat{\eta}(x(t), u(t), \theta^*)$ and without fault, the state dynamics of system (1) can be rewritten as

$$\begin{aligned}\dot{x}(t) = Ax(t) + f(x(t), u(t)) + \hat{\eta}(x(t), u(t), \theta^*)\\ + e_\eta(x(t), u(t)),\end{aligned} \tag{7}$$

where $e_\eta(x(t), u(t)) = \eta(x(t), u(t)) - \hat{\eta}(x(t), u(t), \theta^*)$ is the minimum functional approximation error for $x(t) \in X$, $u(t) \in U$, and $\theta^* \in \Theta$.

In this study, we employ a linearly parameterized approximation network, given by

$$\begin{aligned}\hat{\eta}(x(t), u(t), \theta^*) = \Pi(x(t), u(t))\theta^*,\\ \Pi(x(t), u(t)) = \pi_1^T(x(t), u(t)) \oplus \cdots \oplus \pi_n^T(x(t), u(t)),\end{aligned} \tag{8}$$

where $\Pi : X \times U \mapsto R^{n\times n_\theta}$ is the known basis matrix and $\pi_i : X \times U \mapsto R^{n_{\theta_i}}$ is the bounded basis function vector for $i \in \{1, ..., n\}$ and $(x, u) \in X \times U$ ($\sum_{i=1}^{n} n_{\theta_i} = n_\theta$).

With the approximation model in (8) and the formulated state dynamics in (7), we next present an event-triggered adaptive approximation algorithm.

3.2 Event-triggered adaptive approximation algorithm

The proposed design is based on an event-triggered observer-based scheme. The key task is to minimize $\tilde{n}$ with the event-triggered measurement such that the accessible modeling uncertainty estimation can be used to enhance the robustness of fault detection.

Based on (7), (8), and the event-triggered measurement $y_s(t)$, we have the following estimation scheme:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + f(\hat{x}(t), u(t)) + \Pi(\hat{x}(t), u(t))\hat{\theta}(t)\\ &\quad + \Psi(t)\dot{\hat{\theta}}(t) + L(y_s(t) - \hat{y}(t)),\\ \dot{\hat{\theta}}(t) &= P_\Theta\left\{\Gamma\Psi^T(t)C^T(y_s(t) - \hat{y}(t)\right\},\\ \dot{\Psi}(t) &= A_L\Psi(t) + \Pi(\hat{x}(t), u(t)),\\ \hat{y}(t) &= C\hat{x}(t),\end{aligned} \tag{9}$$

where $\hat{x}$ is the state of the estimator, $\hat{\theta}$ is the weight vector, and $\Psi(t)$ is the intermediate variable representing the filtered version of $\Pi(\hat{x}(t), u(t))$, which is used for ensuring the stability of the adaptive approximation scheme. P_Θ denotes the projection operator to restrict the weight estimation within the predefined set Θ (Farrell and Polycarpou (2006)) and also ensure the stability of the learning algorithm in the presence of nonzero minimum approximation error $e_\eta(x(t), u(t))$. The matrix L is the observer gain for ensuring a Hurwitz A_L, where $A_L = A - LC$ (since (A, C) is observable, the existence of the required L can be met). The matrix $\Gamma > 0$ is the learning rate for weight estimation. Without loss of generality, the initial values for the estimation system (9) are set to $\hat{x}(0) = 0$, $\hat{\theta}(0) = 0$, and $\Psi(0) = 0$.

Remark 1. The adaptive approximation scheme for fault diagnosis has been studied in Zhang et al. (2010); Keliris et al. (2017). However, in these studies the input for the estimation system is continuous, which implies that there is no sampling error. In this paper, we propose an approximation estimation algorithm based on event-triggered measurements, with the aperiodic sampling error addressed in the analysis.

3.3 Stability analysis

Next, the stability and learning capabilities of the event-triggered adaptive approximation scheme will be analyzed. Let $e_x(t) \triangleq x(t) - \hat{x}(t)$, $e_y(t) \triangleq y(t) - \hat{y}(t)$, and $\tilde{\theta}(t) \triangleq \hat{\theta}(t) - \theta^*$ be the estimation errors of system state, measurement output, and weight vector, respectively. The following theorem describes the stability properties of the event-triggered adaptive approximation scheme given by (9).

Theorem 1. Under Assumptions 1-4, for $t \le T_l$, $(x, u) \in X \times U$, and $\theta^* \in \Theta$, the event-triggered adaptive approximation scheme given by (9) ensures that the state estimation error $e_x(t)$, measurement output estimation error $e_y(t)$, and weight vector estimation error $\tilde{\theta}(t)$ are all bounded.

Proof. Based on (7) and (9), one can get the dynamics of state estimation error when $t \leq T_l$:

$$\begin{aligned}\dot{e}_x = & Ae_x + \tilde{f}(t) + \Pi(x(t), u(t))\theta^* \\ & - \Pi(\hat{x}(t), u(t))\hat{\theta}(t) + e_\eta(x(t), u(t)) \\ & - \Psi(t)\dot{\hat{\theta}}(t) - L(y_s(t) - \hat{y}(t)),\end{aligned} \tag{10}$$

where $\tilde{f}(t) = f(x(t), u(t)) - f(\hat{x}(t), u(t))$. According to the event-triggered scheme defined in (3) and (4), we have $y_s(t) - \hat{y}(t) = Ce_x(t) + v(t) - e_s(t)$. Referring to the expression of $\dot{\Psi}(t)$ in (9), it follows that

$$\begin{aligned}& \Pi(x(t), u(t))\theta^* - \Pi(\hat{x}(t), u(t))\hat{\theta}(t) - \Psi(t)\dot{\hat{\theta}}(t) \\ & = \tilde{\Pi}(t)\theta^* + A_L\Psi(t)\tilde{\theta}(t) - \frac{d\{\Psi(t)\tilde{\theta}(t)\}}{dt},\end{aligned} \tag{11}$$

where $\tilde{\Pi}(t) = \Pi(x(t), u(t)) - \Pi(\hat{x}(t), u(t))$. Finally, we obtain

$$\begin{aligned}\dot{e}_x = & A_L(e_x + \Psi(t)\tilde{\theta}(t)) + \tilde{f}(t) \\ & + \tilde{\Pi}(t)\theta^* + e_\eta(x(t), u(t)) \\ & - L(v(t) - e_s(t)) - \frac{d\{\Psi(t)\tilde{\theta}(t)\}}{dt}.\end{aligned} \tag{12}$$

Further, based on (12) and using $\Psi(0) = 0$, the state solution of e_x can be obtained:

$$\begin{aligned}e_x(t) = & e^{A_Lt}e_x(0) + \int_0^t e^{A_L(t-\tau)}\Big(\tilde{f}(t) \\ & + \tilde{\Pi}(t)\theta^* + e_\eta(x(t), u(t)) \\ & - L\big(v(t) - e_s(t)\big)\Big)d\tau - \Psi(t)\tilde{\theta}(t).\end{aligned} \tag{13}$$

Based on Assumptions 1 and 3 and referring to the Bellman-Gronwall Lemma (Ioannou and Sun (1995), Lemma 3.3.7), we have

$$\|e_x(t)\| \leq E_x(t) + \alpha_L L_f \int_0^t E_x(\tau)e^{-q_\alpha(t-\tau)}d\tau, \tag{14}$$

where

$$\begin{aligned}E_x(t) = & \alpha_L e^{-qt}\|e_x(0)\| + \left\|\Psi(t)\tilde{\theta}(t)\right\| + \int_0^t \alpha_L e^{-q(t-\tau)} \\ & \times\Big(\left\|\tilde{\Pi}(t)\theta^*\right\| + \|e_\eta(x(t), u(t))\| + \|L\bar{v}\| + \|e_s(t)\|\Big)d\tau.\end{aligned}$$

$q_\alpha = q - \alpha_L L_f$, and $\alpha_L e^{-qt} \geq \left\|e^{A_Lt}\right\|$. Based on Assumptions 2 and 4, $e_\eta(x(t), u(t))$ is bounded. Also, with the projection operator, both θ^* and $\tilde{\theta}(t)$ are bounded. With bounded $\Pi(t)$, it is known that $\tilde{\Pi}(t)$ is bounded for $(x, u) \in X \times U$ and $t \leq T_l$. Recalling the event-triggered threshold design for (3) with bounded δ_1, δ_2, and $y(t)$, we have bounded sampling error when event-triggered communication is employed. Finally, bounded $e_x(t)$ and $e_y(t)$ can be ensured when $q_\alpha > 0$. □

The above theorem summarizes the stability properties of the event-triggered approximation system, and quantifies the effect of the event-triggered sampling error on the stability of the approximator. From (14), it can be seen that the event-triggered sampling error has a direct contribution on the state estimation error and also the approximation performance. Since a smaller state estimation error indirectly reflects a better approximation performance, within the adaptive approximation time interval, smaller threshold for triggering could be adopted with smaller δ_1 and δ_2. Another interpretation for using smaller δ_1 and δ_2 during learning is that it corresponds to lower confidence on the data/model due to the unknown modeling uncertainty.

4. EVENT-TRIGGERED FAULT DETECTION

After the learning period, the estimated modeling uncertainty, represented by the adaptive approximator, is integrated into the residual generator.

According to the adaptive approximator in (9), an observer based residual generator with event-triggered communication scheme is proposed as follow:

$$\begin{aligned}\dot{\hat{z}}(t) = & A\hat{z}(t) + f(\hat{z}(t), u(t)) + \Pi(\hat{z}(t), u(t))\hat{\theta}(T_l) \\ & + L_z(y_s(t) - \hat{y}(t)),\end{aligned} \tag{15}$$

where $\hat{z}$ is the state of residual generator, $\hat{z}(0) = 0$, and L_z is the observer gain for ensuring $A_z = A - L_zC$ is Hurwitz. Let $e_z(t) = x(t) - \hat{z}(t)$ and $\hat{y}(t) = C\hat{z}(t)$, the residual signal $r(t)$ is based on the accessible measurement (since $y(t)$ is not available now), where

$$r(t) = y_s(t) - \hat{y}(t) = Ce_z(t) + v(t) - e_s(t). \tag{16}$$

Next, we consider the design of the adaptive threshold for $r(t)$. Based on (16), Assumption 1, and the event-triggered scheme in (3), it yields

$$\begin{aligned}\|r(t)\| & \leq \|Ce_z(t)\| + \bar{v} + \delta_1\|y(t)\| + \delta_2 \\ & \leq (1 + \delta_1)(\|Ce_z(t)\| + \bar{v}) + \delta_2 + \delta_1\|\hat{y}(t)\|.\end{aligned} \tag{17}$$

For the analysis of the residual signal, the dynamics of $e_z(t)$ can be obtained:

$$\begin{aligned}\dot{e}_z = & A_ze_z + \tilde{f}_z(t) + \eta_z(t) - L_z(v - e_s(t)) \\ & + \beta(t - T_0)\phi(x(t), u(t)),\end{aligned} \tag{18}$$

where $\tilde{f}_z(t) = f(x(t), u(t)) - f(\hat{z}(t), u(t))$ and $\eta_z(t) = \eta(x(t), u(t)) - \Pi(\hat{z}(t), u(t))\hat{\theta}(T_l)$. For the fault-free case the solution of (18) is given by:

$$\begin{aligned}e_z(t) = & e^{A_z(t-T_l)}e_z(T_l) + \int_{T_l}^t e^{A_z(t-\tau)}\Big(\tilde{f}_z(\tau) \\ & + \eta_z(\tau) - L_z\big(v(\tau) - e_s(\tau)\big)\Big)d\tau.\end{aligned} \tag{19}$$

For the learning performance, the following assumption is made.

Assumption 5. For $(x, u) \in X \times U$ and for $t \in [T_l, T_0]$, the difference between the model uncertainty and its approximation is bounded; i.e., $\|\eta_z(t)\| \leq \bar{\eta}(\hat{z}(t), u(t))$, where $\bar{\eta} : R^n \times U \mapsto R^+$ is a known bounding function.

Based on Assumptions 1-5 and referring to the event-triggered scheme for dealing with the sampling error in (17), we have

$$\|e_z(t)\| \leq E_z(t) + L_C\alpha_z \int_{T_l}^t e^{-q_z(t-\tau)}\|e_z(\tau)\|d\tau, \tag{20}$$

where

$$E_z(t) = \alpha_z e^{-q_z(t-T_L)} \|e_z(T_L)\| + \alpha_z \int_{T_l}^{t} e^{-q_z(t-\tau)} \times \left(\bar{\eta}(\hat{z}(t), u(t)) + L_v \bar{v} + \delta_2 + \delta_1 \|\hat{y}(t)\| \right) d\tau,$$

$L_C = L_f + \delta_1 \|C\|$, $L_v = \|L_z\| + \delta_1$, and α_z and q_z are positive scalars such that $\left\|e^{A_z t}\right\| \le \alpha_z e^{-q_z t}$.

Applying the Bellman-Gronwall Lemma to (20) yields $\|e_z(t)\| \le \bar{e}_z(t)$, where

$$\bar{e}_z(t) = E_z(t) + L_C \alpha_z \int_{T_l}^{t} E_z(\tau) e^{-(q_z - L_C \alpha_z)(t-\tau)} d\tau. \quad (21)$$

To keep the boundedness of $\bar{e}_z(t)$, it is required that $q_z - L_C \alpha_z > 0$.

Taking $\|e_z(t)\| \le \bar{e}_z(t)$ into (17), the adaptive threshold for fault detection can be obtained as

$$\begin{aligned} \|r(t)\| &\le \bar{r}(t) \\ &= (1+\delta_1)(\|C\| \bar{e}_z(t) + \bar{v}) + \delta_2 + \delta_1 \|\hat{y}(t)\|. \end{aligned} \quad (22)$$

Finally, the fault detection logic is designed to be:

$$\begin{cases} \|r(t)\| \le \bar{r}(t), & \textit{fault free}, \\ \|r(t)\| > \bar{r}(t), & \textit{faulty}. \end{cases} \quad (23)$$

The fault detection procedure is summarized in the following algorithm.

Algorithm 1.

i) Construct the approximation system (9) by selecting the design approximator parameters L and Γ, and the basis function matrix Π;
ii) Run the adaptive approximation scheme for $t \in [0, T_l]$;
iii) Activate the event-triggered residual generator in (15), with the $\hat{\theta}(T_l)$ generated from the approximation scheme;
iv) Based on the fault detection logic in (23), generate the fault detection decision.

Due to the event-triggered scheme, we notice that the sampling error affects both the stability and the adaptive threshold. Since L_f and C are system parameters, the parameter δ_1 in the event-triggered scheme will affect the convergence of the state estimation error bound of the residual generator. Therefore, the allowed maximum of δ_1 equals to $\frac{q_z - L_f \alpha_z}{\alpha_z \|C\|}$. In other words, the gain for the relative triggering threshold is required to be upper bounded for a successful fault detection. Comparing to the standard non-event-triggered fault detection case (Zhang et al. (2010); Keliris et al. (2017)), the measurement matrix C now affects the convergence of the adaptive threshold for the event-triggered fault detection. From (16) and (17), we notice that the "folding effect" of the event-triggered measurement brings in the term $\hat{y}(t)$ since $y(t)$ is not available. To guarantee a successful event-triggered fault detection, the inaccuracy of the measurement information should be limited ($e_s(t)$ should be bounded). Overall, the event-triggered fault detection scheme has a higher requirement on system and the residual generator, comparing to the non-event-triggered results.

5. FAULT DETECTABILITY ANALYSIS

Since the "folding effect" of event-triggered communication on $r(t)$ has been addressed in (17), the key for fault detectability analysis is to measure the fault effect on the residual signal. Obviously, $e_z(t)$ in (19) contains the fault-free case. When $t \ge T_0$, we can get the contribution of fault vector on the estimation error by consider the fault input. To show the fault affecting channel, based on (18), we introduce the auxiliary state $e_f(t)$, where

$$\dot{e}_f(t) = A_z e_f(t) + \beta(t - T_0)\phi(x(t), u(t)), \quad (24)$$

and $e_f(T_0) = 0$. Directly, one can find that $e_f = e_z$, when $e_z(T_0) = 0$ and $\tilde{f}_z = \eta_z = v - e_s = 0$ in (18). Hence, he existence of the auxiliary system (24) can be ensured by (18)). Based on (24), we have the state solution of $e_f(t)$ as

$$e_f(t) = \int_{T_0}^{t} e^{A_z(t-\tau)} \beta(\tau - T_0)\phi(x(\tau), u(\tau)) d\tau. \quad (25)$$

Let $e(t)$ be the state estimation error of residual generator when $t \ge T_0$. Combine (1) with (15), we have $e(t) = e_z(t) + e_f(t)$, where $e_z(t)$ is given in (19). After the occurrence of the fault, the residual signal, contributed by both $e_z(t)$ in (19) and $e_f(t)$ in (25), can be reformulated based on (16) to be

$$r(t) = y_s(t) - \hat{y}(t) = Ce(t) + v(t) - e_s(t). \quad (26)$$

To quantify the event-triggered sampling error $e_s(t)$, we can rewrite the norm inequality in (3) to be a vector equality:

$$\begin{aligned} e_s(t) &= \mu_1 \delta_1 y(t) + \mu_2 v_\delta \\ &= \mu_1 \delta_1 (Ce(t) + v(t)) + \mu_1 \delta_1 \hat{y}(t) + \mu_2 v_\delta, \end{aligned} \quad (27)$$

where $\mu_1 \in [-1, 1]$, $\mu_2 \in [-1, 1]$, and $v_\delta \in R^n$ is an auxiliary vector satisfying $\|v_\delta\| = \delta_2$. Replacing $e_s(t)$ in (26) by (27) yields

$$\begin{aligned} r(t) &= (1 + \mu_1\delta_1)(Ce(t) + v(t)) + \mu_1\delta_1\hat{y}(t) + \mu_2 v_\delta \\ &= (1 + \mu_1\delta_1)Ce_f + (1 + \mu_1\delta_1)(Ce_z(t) + v(t)) \\ &\quad + \mu_1\delta_1\hat{y}(t) + \mu_2 v_\delta. \end{aligned} \quad (28)$$

Recalling the fact that $\|e_z(t)\| \le \bar{e}_z(t)$ and based on the triangular inequality, we present the following Theorem to summarize the property of the fault detection scheme.

Theorem 2. Let Assumptions 1-5 hold. For $t \ge T_0$, if $\|(1 + \mu_1\delta_1)Ce_f(t)\| \ge 2\bar{r}(t)$ is satisfied, the occurrence of the system fault will be detected.

6. CONCLUSIONS

In this study, we address the problem of how to design and analyze robust fault detection scheme for continuous-time nonlinear uncertain systems subject to event-triggered sampling. The proposed method has two parts. First, the event-triggered adaptive approximation has been proposed to deal with the unknown modeling uncertainty. Second, fault detection logic has been established with event-triggered measurement transmission, supported by the adaptive approximation result, the nonlinear observer-based residual generation, and the adaptive detection threshold. The performances of the event-triggered adaptive approximation and event-triggered fault detection design have been analyzed.

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