Unknown System Dynamics Estimator for Nonlinear Uncertain Systems

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Abstract: For feedback control designs, one of the fundamental problems is to handle the unknown system dynamics. In this paper, an alternative unknown system dynamics estimator (USDE) with low-pass filter operations is presented based on an invariant manifold method, in which we only need to set a scalar, the filter parameter. The convergence performance and robustness of this USDE are analysed in both the time-domain and frequency-domain. To circumvent the sensitiveness to the measurement noise, a further enhanced USDE (EUSDE) with two-layer of low-pass filters is constructed. With the proposed estimators, all time-varying components, such as unmodeled dynamics, nonlinearities and external disturbances, can be viewed as a lumped unknown system dynamics term and then effectively estimated even in the presence to fair measurement noise. The function of these estimators is the same as the well-known disturbance observer (DOB) and extended state observer (ESO). Hence, they can be easily incorporated into control schemes. Numerical simulation results are presented to show the effectiveness of the proposed estimation schemes.

Keywords: Unknown system dynamics estimator, nonlinear uncertain systems, measurement noise, filter operation, robustness.

1. INTRODUCTION

Unknown system dynamics are one of main factors that affect the stability and control performance of the closed-loop control systems. Specifically, the mathematical model used in the control designs usually cannot accurately describe the actual system behaviour due to the modelling uncertainties, external disturbances, and other environment variations. Hence, how to effectively address these unknown dynamics is a critical issue, which help to enhance the performance of feedback control schemes (Gao, 2014; Xie & Guo, 2000).

To handle the influence of these unknown system dynamics, many advanced control schemes have been developed. One well-known strategy is adaptive control (Ioannou & Sun, 1996), in which the parametric uncertainties can be estimated and directly compensated online. However, only linearly parameterized systems can be handled via adaptive control, while neural networks or fuzzy logic systems need to be used to address nonlinear uncertainties. On the other hand, robust control (X. Li, Soh, & Xie, 2017) was derived to address the worst case control designs, while its nominal performance is sacrificed to derive better robustness. Hence, the upper bounds of system unknown dynamics should be known.

In practice, the unknown nonlinearities, external disturbances, and other uncertainties can be lumped into a uniform term. Hence, if such unknown system dynamics can be precisely estimated, a feedforward control can be used to eliminate their effects on the control response (Chen, Yang, Guo, & Li, 2015). Hence, another notable strategy to handle these unknown dynamics is to estimate the lumped dynamics. This idea motivates a variety of estimation and compensation methods, such as disturbance observer (DOB) (S. Li, Yang, Chen, & Chen, 2016), extended state observer (ESO) (Han, 2009), unknown input observer (UIO) (Johnson, 1968), and equivalent input disturbance based estimator (EID) (She, Fang, Ohyama, Hashimoto, & Wu, 2008), etc. These methods have been well-recognized and widely applied in different applications during the past decades. However, an observer usually needs to be synthesized in these estimators, and thus the parameter tuning is not a trivial task in general. Moreover, although the convergence of ESO has been solved in the recent literature, the robustness of these estimators against measurement noise deserves further investigations.

With the wish to develop a simple, fast and robust estimator of unknown dynamics for generic systems, we will provide a new unknown system dynamics estimator (USDE) by further tailoring our previous work (Na, Chen, Herrmann, Burke, & Brace, 2018; Na, Herrmann, Burke, & Brace, 2015). In this framework, filter operations are applied on the measurable variables, and then an ideal invariant manifold is constructed to design the estimator. Moreover, we design an enhanced unknown system dynamics estimator (EUSDE) to improve its robustness against measurement noise, where two-layer low-pass filters are introduced. The first layer low-pass filter is used to obtain the derivative of the measurable variables, and the second layer low-pass filter is utilized to design the estimator based on an ideal invariant manifold. Simulation results show the effectiveness of the proposed USDE and EUSDE, which also indicate their satisfactory estimation performance even in presence of large measurement noise.

2. UNKNOWN SYSTEM DYNAMICS ESTIMATOR

2.1 Problem formulation

Consider an uncertain system as

$$\dot{x}(t) = Ax(t) + Bu(t) + F(x,t), x(0) = 0, \tag{1}$$

where $x(t) = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$ is the system state, $x(0)$ is the initial state; $u(t) \in \mathbb{R}^m$ is the control input; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are the known system matrix and control input matrix; $F(x,t) \in \mathbb{R}^m$ is the lumped unknown system dynamics to be estimated.

To estimate the system uncertainties $F(x,t)$, it is shown in (1) that $F(x,t)$ can be written as

$$F(x,t) = \dot{x}(t) - Ax(t) - Bu(t), \tag{2}$$

which means that $F(x,t)$ can be derived by the system state $x(t)$, its derivative $\dot{x}(t)$, and control signal $u(t)$. However, we cannot directly apply this estimator in the practice since the derivative signal $\dot{x}$ is usually not measurable (S. Li et al., 2016). To address the above issue, we will provide a novel estimation method for the unknown system dynamics $F(x,t)$ in system (1) even in the presence of measurement noise.

2.2 Unknown system dynamics estimator

In this section, we will present a new USDE to *online* estimate $F(x,t)$ in (1). For this purpose, the following assumptions are made:

Assumption 1*: The state variables $x(t)$ and control signal $u(t)$ in (1) are measurable.*

Assumption 2: *The unknown dynamics $F(x,t)$ and their derivative are bounded, i.e., $\sup_{t\geq 0} \| \dot{F}(x,t) \| \leq \hbar$ holds for a positive constant $\hbar > 0$.*

Before introducing the estimator, we define the following filtered variables x_f, u_f of x, u as

$$\begin{cases} \kappa\dot{x}_f + x_f = x, \ \ x_f(0) = 0, \\ \kappa\dot{u}_f + u_f = u, \ \ u_f(0) = 0, \end{cases} \tag{3}$$

where $\kappa > 0$ is a positive constant.

Then, an ideal invariant manifold (Astolfi & Ortega, 2003) can be constructed to inspire the design of estimator. Hence, we give the following lemma:

Lemma 1. *Consider system (1) and filtered variables in (3), the variable*

$$\vartheta = F - ((x - x_f)/\kappa - Ax_f - Bu_f), \tag{4}$$

is ultimately bounded for any finite $\kappa > 0$, and exponentially converges to a small residual set. Moreover, the fact

$$\lim_{\kappa\to 0}\{\lim_{t\to\infty}[F - ((x - x_f)/\kappa - Ax_f - Bu_f)]\} = 0, \tag{5}$$

is true, so that $F - ((x - x_f)/\kappa - Ax_f - Bu_f) = 0$ is an invariant manifold for any finite $\kappa > 0$.

Proof. Considering system (1) and definitions in (3)-(5), the time derivative of ϑ is given by

$$\dot{\vartheta} = \dot{F} - (\frac{\dot{x} - \dot{x}_f}{\kappa} - A\dot{x}_f - B\dot{u}_f) = -\frac{1}{\kappa}\vartheta + \dot{F}. \tag{6}$$

Select a Lyapunov function as $V_\vartheta = \vartheta^T\vartheta/2$, and apply Young's inequality on its derivative $\dot{V}_\vartheta$, it follows that

$$\dot{V}_\vartheta = -\frac{1}{\kappa}\vartheta^T\vartheta + \vartheta^T\dot{F} \leq -\frac{1}{\kappa}V_\vartheta + \frac{\kappa}{2}\hbar^2. \tag{7}$$

This result indicates that $V_\vartheta(t) \leq e^{-t/\kappa}V_\vartheta(0) + \kappa^2\hbar^2/2$ holds and thus ϑ will exponentially converge to a small compact set defined by $\| \vartheta(t) \| \leq \sqrt{2e^{-t/\kappa}V_\vartheta(0) + \kappa^2\hbar^2}$, which follows that $\lim_{t\to\infty} \| \vartheta(t) \| = \kappa\hbar$. Consequently, for $\kappa \to 0$ and/or $\hbar = 0$, the fact $\lim_{\kappa\to 0}\lim_{t\to\infty} \| \vartheta(t) \| = 0$ holds. $\diamond$

According to the invariant manifold (4), a feasible unknown system dynamics estimator of $F(x,t)$ is given as

$$\hat{F} = \frac{x - x_f}{\kappa} - Ax_f - Bu_f. \tag{8}$$

Then, we write the system (1) in the Laplace domain as

$$F(s) = (sI - A)X(s) - BU(s), \tag{9}$$

where s is the Laplace operator and I is the unit matrix, $F(s), X(s), U(s)$ are the Laplace transform of variables $F(t)$, $x(t), u(t)$, respectively. Then, Eq.(8) can be further written as

$$\hat{F}(s) = \frac{1}{\kappa s + 1}[(sI - A)X(s) - BU(s)]. \tag{10}$$

Substituting (9) into (10), we have

$$\begin{aligned} \hat{F}(s) &= \frac{1}{\kappa s + 1}F(s) = F_f(s), \\ \hat{F}(t) &= L_s^{-1}\{F_f(s)\} = F_f(t), \end{aligned} \tag{11}$$

where $L_s^{-1}\{\cdot\}$ defines the inverse Laplace transform, and $F_f(t)$ denotes the low-pass filtered version of $F(t)$ given by $\kappa\dot{F}_f + F_f = F, F_f(0) = 0$. This means that the estimated dynamics $\hat{F}$ are the filtered version of the unknown system dynamics F, i.e., $\hat{F} = F_f$. Consequently, the estimation error $\tilde{F}$ is obtained by

$$\tilde{F} = F - \hat{F} = \left(1 - \frac{1}{\kappa s + 1}\right)F = \frac{\kappa s}{\kappa s + 1}[F]. \tag{12}$$

From the above equation, the estimation error can be minimized by setting the filter coefficient κ sufficiently small and it can also be vanishing for constant unknown dynamics, i.e., $\dot{F}=0$. Hence, the convergence property of the estimation error can be summarized by:

Theorem 1. For system (1) with unknown system dynamics estimator (8), the estimation error $\tilde{F}$ converges to a compact set defined by $\|\tilde{F}(t)\| \le \sqrt{2e^{-t/\kappa}V_{\tilde{F}}(0)+\kappa^2\hbar^2}$, which shows that $\lim_{t\to\infty}\hat{F}(t)=F(t)$ as $\kappa\to 0$ and/or $\hbar\to 0$.

Proof. Based on (12), the estimation error dynamics are given in the time-domain as

$$\dot{\tilde{F}}=\dot{F}-\dot{\hat{F}}=-\frac{1}{\kappa}\tilde{F}+\dot{F}. \tag{13}$$

We first select a Lyapunov function as $V_{\tilde{F}}=\tilde{F}^T\tilde{F}/2$, then calculate its derivative as

$$\dot{V}_{\tilde{F}}=-\frac{1}{\kappa}\tilde{F}^T\tilde{F}+\tilde{F}^T\dot{F}\le -\frac{1}{\kappa}V_{\tilde{F}}+\frac{\kappa}{2}\hbar^2, \tag{14}$$

which follows that $\|\tilde{F}(t)\| \le \sqrt{2e^{-t/\kappa}V_{\tilde{F}}(0)+\kappa^2\hbar^2}$, and thus $\lim_{t\to\infty}\|\tilde{F}(t)\|=0$ as $\kappa\to 0$ and/or $\hbar\to 0$. ◇

Moreover, to further reveal the mechanism behind this proposed estimator, we present the frequency response of estimator error (12) under different filter parameters, which is given in Fig.1.

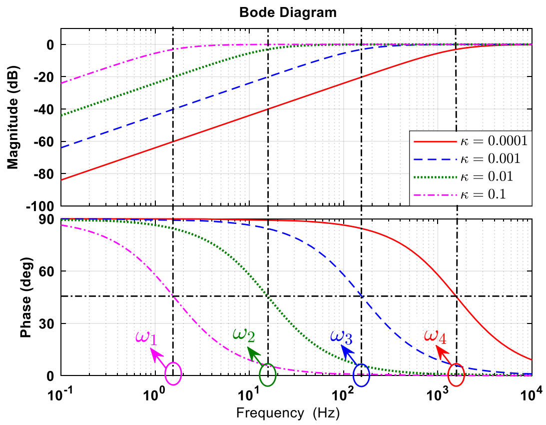

Fig. 1. Bode diagram of estimation error dynamics.

From Fig.1, we can clearly see that the bandwidth can be increased when the filter parameter is set sufficiently small, i.e., $\omega_1<\omega_2<\omega_3<\omega_4$. Therefore, a larger bandwidth can be used to eliminate the uncertain dynamics by setting a smaller filter parameter. Hence, a small filter parameter can be used to rapidly diminish the estimation error.

2.3 Robustness Analysis

This subsection will analyse the robustness of this proposed USDE against to bounded noise, which is crucial for practical applications. Denote ε as the measurement noise perturbing the state x (note the control signal u in (1) is calculated by the controller, and thus is free of measurement noise). Then the measured variable $\bar{x}$ used for the estimator design is

$$\bar{x}=x+\varepsilon, \varepsilon\in\mathbb{R}^n. \tag{15}$$

It is assumed that the noise signal is bounded by $\|\varepsilon\|\le \lambda_1$, $\|\dot{\varepsilon}\|\le\lambda_2$ for constants $\lambda_1,\lambda_2>0$. Owing to the existence of noise signal ε, the filter operations given in (3) can be modified as

$$\begin{aligned}&\kappa\dot{\bar{x}}_f+\bar{x}_f=\bar{x},\ \ \bar{x}_f(0)=0,\\ &\kappa\dot{u}_f+u_f=u,\ \ u_f(0)=0,\end{aligned} \tag{16}$$

such that the designed estimator (8) can be presented as

$$\hat{F}=\frac{\bar{x}-\bar{x}_f}{\kappa}-A\bar{x}_f-Bu_f. \tag{17}$$

Consider (8) and (15), then (17) can be further written as

$$\hat{F}=\dot{x}_f-Ax_f-Bu_f+\dot{\varepsilon}_f-A\varepsilon_f, \tag{18}$$

where $\dot{\varepsilon}_f$ and ε_f are the filtered version of $\dot{\varepsilon}$ and ε in terms of $1/(\kappa s+1)$. Then the estimation error $\tilde{F}$ subject to measurement noise is given in the frequency-domain as

$$\tilde{F}(s)=\frac{\kappa s}{\kappa s+1}[F]+\frac{1}{\kappa s+1}[A\varepsilon]-\frac{s}{\kappa s+1}[\varepsilon], \tag{19}$$

which can be rewritten in the time-domain as

$$\dot{\tilde{F}}=-\frac{1}{\kappa}\tilde{F}+\dot{F}+\frac{1}{\kappa}(A\varepsilon-\dot{\varepsilon}). \tag{20}$$

In this case, to simplify the analysis and show the robustness of this estimator, we will carry out further analysis on (20). For the ease of analysis in terms of Bode diagram, we assume $\dot{F}=0$ here (Since we focus on the effect of noise in this analysis, the term $\dot{F}$ does not change the conclusions). Moreover, the system matrix A is set as a constant $-\alpha,\alpha>0$ to make system (1) stable. Then, the transfer function from the noise ε to the estimation error $\tilde{F}$ can be given as

$$G(s)=\frac{\tilde{F}(s)}{\varepsilon(s)}=-\frac{s+\alpha}{\kappa s+1}, \tag{21}$$

where the estimation error $\tilde{F}$ is the output and the noise signal ε is the input, such that we can use Bode diagram to show the effects induced by the measurement noise ε on the estimation under different filter parameter κ. The results can be shown in Fig.2. From Fig.2, we can see that the amplitude of estimation error will be significantly increased as the filter parameter is set very small; this means that the designed estimator (17) is sensitive to the measurement noise when we choose a small filter parameter. Hence, we need to make a trade-off between the robustness and the convergence performance when setting the filter parameter κ.

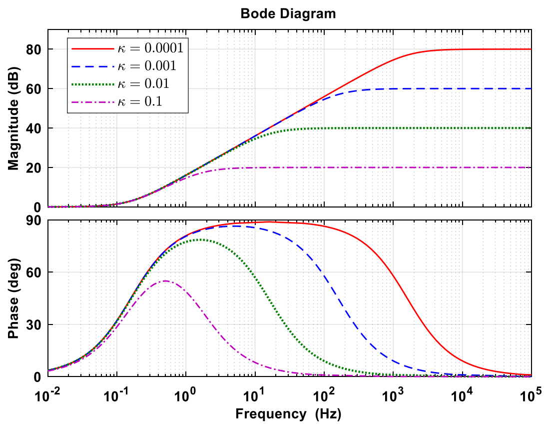

Fig. 2. Bode plots of transfer function (21) with different κ.

3. ENHANCED UNKNOWN SYSTEM DYNAMICS ESTIMATOR

In this section, we will present an EUSDE based on two-layer low-pass filters to enhance the robustness performance.

3.1 Preliminaries and inspiration

Although the low-pass filter $1/(\kappa s+1)$ leads to a phase lag, it can store the history data of the input signal. According to (Han, 2009), the following fact holds

$$\dot{x} \approx (x(t)-x(t-\kappa))/\kappa, \tag{22}$$

However, we cannot directly use the above formulation to calculate the derivative $\dot{x}$ to design the estimator (Youcef-Toumi & Reddy, 1992), since the following issues should be considered:

i) The state $x(t)$ is usually measured by using sensors, and the measurement noise is unavoidable, thus $x(t-\kappa)$ may not represent the actual signal $x(t)$ at $t-\kappa$ moment;

ii) For a sufficiently small coefficient κ, the measurement noise perturbing the measurement $x(t)$ can be amplified significantly as shown in (20), so as to deteriorate the estimation performance;

iii)Low-pass filter can eliminate high-frequency measurement noise. However, it will lead to a small phase lag.

According to (Han, 2009), we can define two low-pass filters $\kappa\dot{x}_f + x_f = x, x_f(0)=0$, and $\kappa_1\dot{x}_{f1}+x_{f1}=x, x_{f1}(0)=0$ with two different filter parameters κ and κ_1, and then have

$$x_f(t) \approx x(t-\kappa),\ x_{f1}(t) \approx x(t-\kappa_1), \tag{23}$$

In this respect, we can use $x_f(t), x_{f1}(t)$ to replace $x(t-\kappa)$, $x(t-\kappa_1)$ to calculate the derivative and design the estimator. To clearly show the above fact, a schematic diagram is used to describe the mechanism of low-pass filter, which is given in Fig.3. Considering the above analysis, we use the filtered version of the measured signal $x(t)$ to design the estimator instead of using $x(t-\kappa)$, such that the above problems related to the measurement noise can be addressed.

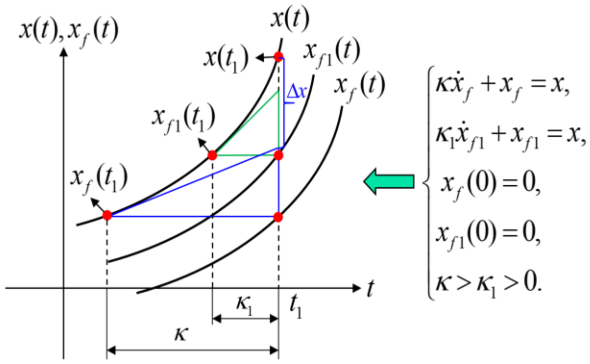

Fig. 3. Schematic diagram of low-pass filter.

3.2 Enhanced Unknown System Dynamics Estimator

In this subsection, we will propose an enhanced unknown systems dynamics estimator (EUSDE) by using two-layer low-pass filters. The first-layer low-pass filter is used to obtain the derivatives of measurable states, and the second-layer low-pass filter is utilized to design the estimator. Similar to the design of (8), we define the following filtered variables x_f, x_{f1} of x as

$$\begin{cases} \kappa\dot{x}_f + x_f = x, & x_f(0)=0, \\ \kappa_1\dot{x}_{f1}+x_{f1}=x, & x_{f1}(0)=0, \end{cases} \tag{24}$$

where $\kappa>\kappa_1>0$ are filter parameters. Since $\kappa>\kappa_1$, $x_{f1}(t)$ will be close to $x_f(t)$ in comparison to $x(t)$ from Fig.3. Thus, the measured signal $x(t)$ in the first equation of (24) can be replaced by $x_{f1}(t)$ to calculate the derivative of $x_{f1}(t)$. Then, we have

$$\dot{x}_f(t) = \frac{x_{f1}(t)-x_f(t)}{\kappa-\kappa_1}, \tag{25}$$

which can be true for a sufficiently small $\kappa-\kappa_1$. In (25), since only the filtered variables of $x(t)$ are used to calculate the derivative of $x_f(t)$, the effect of the measurement noise ε can be diminished.

Moreover, to compensate the phase lag induced by the low-pass filter $\kappa\dot{x}_f + x_f = x$, we adopt the following operation

$$\hat{x}(t) = x_f(t) + \lambda\kappa\frac{x_{f1}(t)-x_f(t)}{\kappa-\kappa_1}, \tag{26}$$

where $\lambda>0$ is a constant parameter and $\hat{x}(t)$ denotes the reconstructed state variable. As shown in Fig.3, we have the fact $\Delta x \to 0$ for $\kappa \to 0$, which implies that we can use the reconstructed state $\hat{x}$ to arbitrarily approximate x at t moment. Moreover, for a sufficiently small $\kappa-\kappa_1$, we have $\dot{x}_f(t) = (x_{f1}(t)-x_f(t))/(\kappa-\kappa_1)$, such that we can write (26) in the Laplace-domain as

$$\hat{X}(s) = \frac{1}{\kappa s+1} X(s) + \frac{\lambda \kappa s}{\kappa s+1} X(s), \tag{27}$$

which implies that $\hat{X}(s) = X(s)$ for $\lambda = 1$. Hence, the problem of the phase lag can also be solved.

Then we can use the reconstructed state $\hat{x}(t)$ to replace the measured state $\bar{x}(t)$ to design the EUSDE. For this purpose, the second-layer low-pass filter as used in the design of USDE is given as

$$\begin{cases} \kappa \dot{\hat{x}}_f + \hat{x}_f = \hat{x}, & \hat{x}_f(0) = 0, \\ \kappa_1 \dot{\hat{x}}_{f1} + \hat{x}_{f1} = x, & \hat{x}_{f1}(0) = 0, \\ \kappa \dot{u}_f + u_f = u, & u_f(0) = 0. \end{cases} \tag{28}$$

According to the invariant manifold shown in Lemma 1, the EUSDE is given as

$$\hat{F} = \frac{\hat{x}_{f1} - \hat{x}_f}{\kappa - \kappa_1} - A\hat{x}_f - Bu_f. \tag{29}$$

Consider (9), (27) and (28), then (29) can be presented as

$$\hat{F}(s) = s\hat{X}_f(s) - A\hat{X}_f(s) - BU_f(s) = \frac{1}{\kappa s+1} F(s), \tag{30}$$

which also implies that the estimated dynamics $\hat{F}$ represent the filtered version of the unknown system dynamics F, i.e., $\hat{F} = F_f$. Therefore, the convergence performance of this EUSDE is similar to that given in Theorem 1, whose proof is omitted here. Therefore, the objective of the second-layer low pass filter has been achieved as well.

4. NUMERICAL VALIDATION

To illustrate the efficiency of the proposed USDE and EUSDE, we consider the following system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0.2 & 1 \\ -1.2 & -0.5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ f(x,t) \end{bmatrix} \tag{31}$$

where x_1 and x_2 are the system states; u is the control signal; $f(x,t) = -0.2(x_1 + x_2) + 0.5x_1^2 x_2 - 1.3\sin(x_1)$ is a nonlinear function, which is used to simulate the lumped unknown system dynamics. In this case, we set the filter coefficient κ used in the USDE as 0.01, and the filter coefficients κ, κ_1 used in the EUSDE as 0.12 and 0.1. Moreover, a white noise ε with power 0.0001 and sample time 0.01 is inserted to both the position and velocity measurements as (15), which can be achieved by using *Band-Limited White Noise* in Matlab/Simulink. This indicates that the Signal to Noise Rations (SNR) of x_1 and x_2 are 27.73dB and 35.21dB, respectively.

Fig.4 shows the desired trajectories of state x_1, x_2 and the contaminated measurement of state $\bar{x}_1, \bar{x}_2$. From Fig.5, it is shown that both of the proposed USDE (8) and EUSDE (29) obtain satisfactory estimation performance when there is no measurement noise. We further test the robustness of the proposed USDE (8) against the added noise. Simulation results given in Fig.6 show that the USDE cannot guarantee satisfactory estimation convergence when there exists a fairly large noise ε, and a smaller filter parameter $\kappa = 0.001$ may further deteriorate the estimation. This result clarifies the statements discussed in subsection 2.3, that is the amplitude of estimation error can be increased by using a small filter parameter.

To eliminate the effect of sensor noise, we cannot directly use the contaminated measurement of state $\bar{x}_1, \bar{x}_2$ to implement the estimator. Instead, the reconstructed state variables based on the measured state $\bar{x}_1, \bar{x}_2$ are calculated and then used in the EUSDE (26), which can be seen from Fig.7. It is shown that there are a slight phase lag and oscillations in the reconstructed states $\hat{x}_1, \hat{x}_2$. With these reconstructed states, the performance of EUSDE is tested and given in Fig.8. From Fig.8, it is found that compared to the estimation result of USDE shown in Fig.6, the EUSDE shows better estimation performance even when the measurements are subjected to noise.

From these above simulations, it is evident that both the proposed USDE and EUSDE can effectively estimate the unknown system dynamics, and the EUSDE can maintain satisfactory estimation performance even in the presence of fair larger measurement noise.

5. CONCLUSIONS

In this paper, we provide an unknown system dynamics estimator (USDE) to *online* estimate the unknown system nonlinearities, external disturbances, or other modelling uncertainties in the control systems. To address the effects of measurement noise, we further design an enhanced USDE (EUSDE) by introducing two-layer low-pass filters. The convergence performance and robustness of the USDE and EUSDE are rigorously analysed in both the time-domain and frequency-domain. Numerical simulation results show their attractive estimation responses, where the EUSDE can maintain better estimation response than USDE in the presence of measurement noise. The proposed estimators can serve as an alternative solution to DOB, ESO or EID schemes, while having a simpler parameter tuning procedure. Hence, these estimators can be easily incorporated into feedback control schemes to enhance the performance of nonlinear uncertain control systems.

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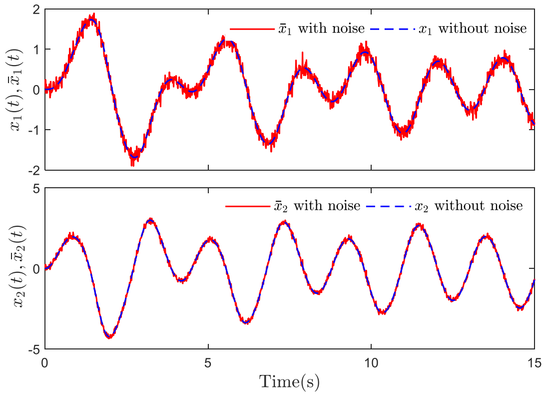

Fig. 4. Profiles of state trajectories with/without noise ε .

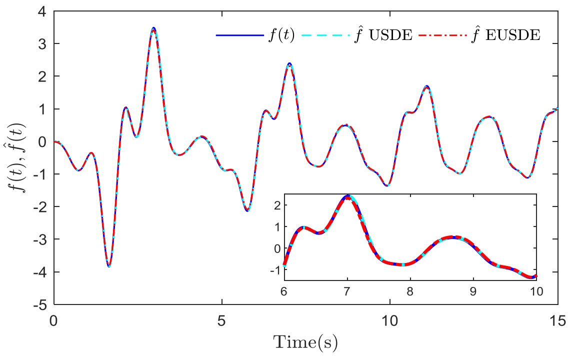

Fig. 5. Estimation of uncertain dynamics $f(t)$ by using USDE (8) and EUSDE (29) without noise.

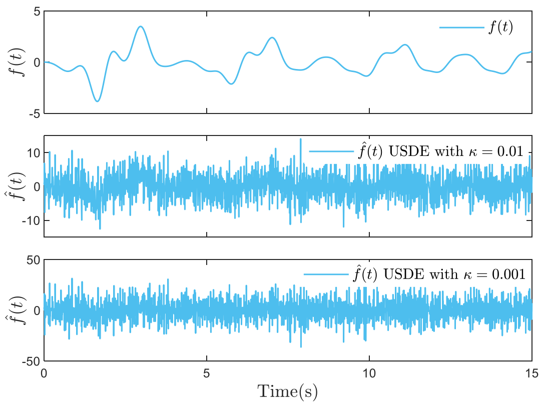

Fig. 6. Estimation of uncertain dynamics $f(t)$ by using USDE (8) under noise ε .

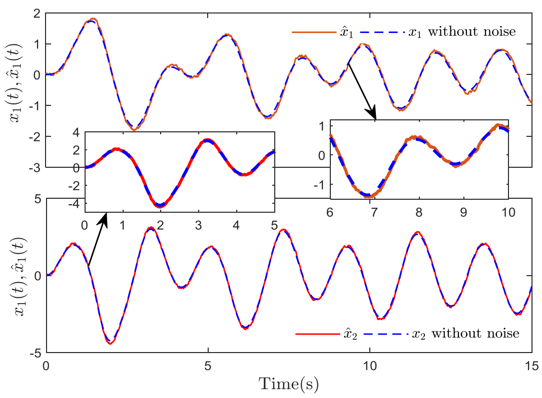

Fig. 7. Reconstructed state $\hat{x}_1, \hat{x}_2$ via (26) under noise ε .

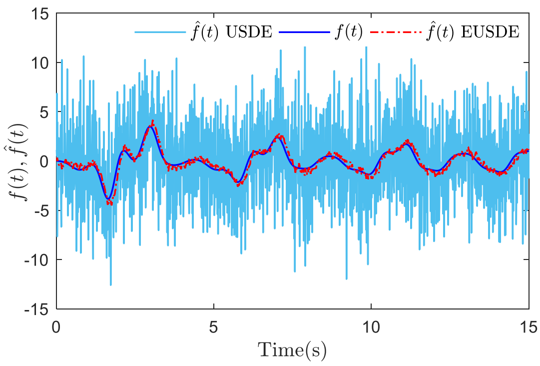

Fig. 8. Estimated uncertain dynamics $f(t)$ by using EUSDE under ε .

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