Minimal Feedback Optimal Control of Linear-Quadratic-Gaussian Systems: No Communication is also a Communication

Dipankar Maity * **John S. Baras** **

* Georgia Institute of Technology, USA, (Email: dmaity@gatech.edu)
** University of Maryland, USA, (Email: baras@umd.edu)

Abstract: We consider a linear-quadratic-Gaussian optimal control problem where the sensor and the controller are remotely connected over a communication channel. The communication of the measurement from the sensor to the controller requires a certain cost which is augmented with the quadratic control cost. We formulate a control and communication co-design problem where we solve for the joint optimal pair of controller and transmitter. We emphasize on the fact that absence of measurement communication at any time instance also conveys certain information to the controller, and such implicit information should be taken into account while designing a controller. We decompose the problem into two subproblems to construct the optimal controller and the optimal transmitter. While the optimal controller can be constructed by solving a certain Riccati equation, the optimal transmitter can be found solving a certain dynamic programming problem. We first characterize a sub-optimal solution for this dynamic program and then design an iterative algorithm to further improve the sub-optimal solution.

Keywords: LQG systems, Optimal Control, Intermittent-feedback Control, co-design of Control and Communication

1. INTRODUCTION

Increasingly many control systems are becoming networked where the plant, the controller, and the sensors are not geographically collocated, rather connected by some communication network. Often times, many subsystems are connected through a shared communication network, and hence, the performance of the overall system is dictated by the limits on the available communication resources. Such resource limitations must explicitly be considered while designing a controller, and thus, leading to a co-design problem of control and communication.

Traditionally, the controls community has majorly focused on synthesizing *closed-loop* feedback controllers where it is assumed that sensors measurements are always available at the controller for computing the control input. On the other end of the spectrum, attention has also been given to synthesize controllers when no information is available, i.e., *open-loop* controllers. An intermittent-feedback is a scenario where the sensor measurements are transmitted to the controller in a sporadic manner in order to balance the trade-off between the control performance and the communication costs. A major challenge in the co-design of a controller and a transmitter (a decision-maker that decides the optimal instances to send sensory measurements to the controller) lies in the fact that the absence of the arrival of a measurement at the controller-site also implicitly communicates some information. Such implicit flow of information in absence of communication renders the problem difficult to solve, and often times intractable. A few approximate solutions to some related problems

have been recently discussed in Xu and Hespanha (2004); Molin and Hirche (2009, 2012).

In this work, we revisit the classical linear-quadratic-Gaussian (LQG) control problem with an added constraint that the availability of the measurements is decided by an active decision-maker. Transmission of each measurement requires a (communication) cost, and therefore, measurements must be transmitted sporadically. However, intermittent absence of measurements causes degradation in the control performance and increases the control performance cost. Therefore, a balance must be kept between the control cost and the communication cost by simultaneously designing a controller and a transmitter.

1.1 Related Work

Our work is along the lines of Molin and Hirche (2009, 2012); Soleymani et al. (2018) where the authors have considered a similar co-design problem. In Molin and Hirche (2009), the authors show that such a joint co-design problem can be decoupled for the LQG case, however, the the work does not provide any characterization on the transmitter's policy. In Soleymani et al. (2018), the authors make an assumption that the implicit information carried by observing an absence of information is discarded at the controller. Similar lines of work have been performed in Lafortune (1985); Aoki and Li (1969); Sawaragi et al. (1978); Olgac et al. (1985); Cooper and Hahi (1971) where they consider the cost for operating a sensor to obtain measurements.

Recent developments in control have led to a framework where sensor measurements or control commands are only transmitted when an *event* has occurred. In this

framework, Aström (2008); Heemels et al. (2008); Arzén (1999) and others have considered reducing the amount of communication in order to reduce the usage of the communication resources. These methods are primarily known as event-triggered control as discussed in Astrom and Bernhardsson (2002); Maity and Baras (2019, 2015), as self-triggered control in Velasco et al. (2003); Lemmon et al. (2007), and as periodic-time control in Bian and French (2005); Nešić et al. (1999). In the majority of these works, the authors propose an event-generating function a priori, and the measurements are transmitted according to the predefined event-generating functions. Thus, in essence, these works are not co-design problems, rather they (quantitatively) study how much reduction in communication happens by using event-triggered communication protocol. Applications of this framework in the context of LQG optimal control can be found in Demirel et al. (2016); Goldenshluger and Mirkin (2017) and the references therein.

Distantly related to our problem, Shi et al. (2012) and others also have studied LQG problems where the controller and the actuator are connected over a communication channel; and whether to send a control signal at the actuator at time t is a design variable. The problems in such scenarios boil down to jointly determining the role of the actuator in absence of a control input and determining the conditions for sending a control update to the actuator. Interested readers may confer Dolgov et al. (2013); Antunes and Heemels (2014); Quevedo et al. (2007) and the references therein. Another line of somewhat related works is the event-based remote estimation considered by Sijs and Noack (2017) and others, where the estimator takes into account the information encoded when a measurement is not sent. Contrary to this approach where the measurement transmission (event-triggering) condition is predefined, we design the optimal measurement transmission condition in this work.

1.2 Contributions

In this work we consider a minimal feedback LQG optimal control problem. The minimality is in the number of transmissions of the sensory data to the remote controller. We cast this problem as a co-design problem of control and communication, where,

i) We first show that under an innovation based transmission scheme, the controller and communication strategy can be decoupled into two problems, a result similar to Molin and Hirche (2009, 2012); Soleymani et al. (2018).

ii) Next, using the decoupled structure of the problem we construct a controller that is certainty-equivalence type, but the controller structure depends on the communication strategy from the sensor to the controller.

iii) We formulate a dynamic programming problem whose solution provides the optimal measurement transmission strategy from the sensor to the controller. While the dynamic programming is difficult to solve in general, we derive bounds on the value function which lets us construct a sub-optimal solution for the transmission strategy.

iv) Finally, we construct an algorithm that iteratively improves a sub-optimal solution to produce a solution which is at least a local optimal if not the global optimal solution for the transmission strategy.
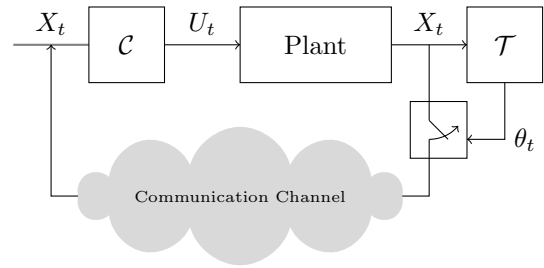

Fig. 1. Schematic of an intermittent-feedback control system. The transmitter $\mathcal{T}$ determines whether a measurement is sent to the controller $\mathcal{C}$.

1.3 Outline

We formally define the problem in Section 2, Section 3 solves the co-design problem, and describes the structure of the optimal controller and the optimal transmitter, and finally, the paper is concluded in Section 4.

2. PROBLEM FORMULATION

Let us consider the following Linear-Quadratic-Gaussian optimal control problem where equation (1) represents the dynamics of the state,

$$X_{t+1} = AX_t + BU_t + W_t, \qquad (1)$$

where $X_t \in \mathbb{R}^n$, $U_t \in \mathbb{R}^m$ and $\{W_t\}_{t \in \mathbb{N}_0}$ is an i.i.d. sequence of Gaussian noise with $W_0 \sim \mathcal{N}(0, \mathcal{W})$, and $X_0 \sim \mathcal{N}(\mu_0, \Sigma_{X_0})$ is the initial state which is independent of the noise sequence $\{W_t\}_{t \in \mathbb{N}_0}$. For notational compactness, we will denote $X_0 = \mu_0 + W_{-1}$ where $W_{-1} \sim \mathcal{N}(0, \Sigma_{X_0})$.

The control objective function that needs to be minimized is the following quadratic cost

$$J = \mathbb{E}\left[\sum_{t=0}^{T-1}\left(X_t^{\mathsf{T}}QX_t + U_t^{\mathsf{T}}RU_t\right) + X_T^{\mathsf{T}}QX_T\right]. \qquad (2)$$

The observation available at the controller $\mathcal{C}$ is determined by a decision-maker $\mathcal{T}$ located at the sensor's side as shown in Figure 1. We assume that the sensor has the perfect state measurement, and at each time instance t, the decision-maker decides whether to send the sensed state value to the controller. In the event when the decision-maker decides not to send the state value, the controller receives a null symbol $\varnothing$; otherwise, the controller receives the perfect state measurement X_t without any delay or distortion. The action of this decision-maker at time t is a binary variable representing the decision to send or not to send the state measurement to the controller. Let us denote the action of this decision maker by $\theta_t \in \{0, 1\}$ such that

$$\theta_t = \begin{cases} 0, & \text{measurement is not sent,} \\ 1, & \text{measurement is sent.} \end{cases}$$

Thus, the observation arriving at the controller follows the pattern

$$Y_t = \begin{cases} \varnothing, & \text{if } \theta_t = 0, \\ X_t, & \text{if } \theta_t = 1. \end{cases}$$

For each state-measurement X_t that is sent to the controller, a cost $C(X_t)$ is incurred that accounts for the transmission cost. Thus, the expected transmission cost over the horizon is $\mathbb{E}[\sum_{t=0}^{T-1} \theta_t C(X_t)]$. In a network control

system with limited communication budget, the decision-maker at the sensor side aims to minimize the communication related cost over the horizon $[0, T]$. In this work we assume that the transmission cost does not depend on the state value, and hence, $C(x) = c$ for all $x \in \mathbb{R}^n$. The controller ($\mathcal{C}$) and the decision-maker ($\mathcal{T}$) at the sensor side jointly try to optimize the weighted cost function J_β that combines the control and the communication cost

$$J_\beta = \mathbb{E}\Big[\sum_{t=0}^{T-1} \big(X_t^\mathsf{T} Q X_t + U_t^\mathsf{T} R U_t\big) + X_T^\mathsf{T} Q X_T \Big] + \beta \mathbb{E}\Big[\sum_{t=0}^{T-1} \theta_t c \Big], \tag{3}$$

where $\beta \geq 0$ is the weight, and $c > 0$ is the given transmission cost. By defining $\lambda = \beta c$, we rewrite J_β as

$$J_\lambda = \mathbb{E}\Big[\sum_{t=0}^{T-1} \big(X_t^\mathsf{T} Q X_t + U_t^\mathsf{T} R U_t + \lambda \theta_t\big) + X_T^\mathsf{T} Q X_T \Big]. \tag{4}$$

This is a two-agent decision making problem where the agents collaboratively try to find the joint optimal strategies for each of them. Such a formulation is atypical in optimal control where only the controller directly affects the state. In this case, the decision-maker at the sensor's side does not directly affect the state evolution, but it impacts the controller's performance and hence is eventually affecting the state. In most of the previous works in the field of event-based control, a rigid decision-making rule is preassigned, and according to that rule, the measurements are transmitted. In those works, when the controller does not receive any measurement, the controller does not take into consideration the fact that the state value must belong to some particular set (in $\mathbb{R}^n$) which led to the absence of transmission. Rather, the absence of a measurement is merely treated as an absence of new information. However, in such a two agent scenario, an absence of transmission can also convey certain information; and this is one of the crucial aspects to be discussed in this paper.

2.1 Information Structures of the Decision Makers

Let us denote the state, control, observation, and θ histories at time t as $\mathcal{X}_t = \{X_0, \ldots, X_t\}$, $\mathcal{U}_t = \{U_0, \ldots, U_t\}$, $\mathcal{Y}_t = \{Y_0, \ldots, Y_t\}$, and $\Theta_t = \{\theta_0, \ldots, \theta_t\}$. The information available at the controller at time t is denoted as $\mathfrak{I}_t^c = \{\mathcal{Y}_t, \Theta_t, \mathcal{U}_{t-1}\}$, and the information available at the decision-maker is denoted as $\mathfrak{I}_t^d = \{\mathcal{X}_t, \Theta_t, \mathcal{U}_t\}$.

It is natural to ask that measurements must only be sent if such measurements contain new *information* which cannot be inferred otherwise. Consequently, the decision θ_t will be taken based on the *new information* present in $\mathfrak{I}_t^d \setminus \mathfrak{I}_{t-1}^d$. Thus, the decision of whether to transmit the measurement or not at time t, is made based on a signal $\xi_t = \mathcal{I}(\mathfrak{I}_t^d)$ that captures this new information. Therefore, this entails a sequential structure of the decision-making process. Precisely, the transmitter will be divided into two parts, namely, the one that will generate the signal ξ_t based on the information $\mathfrak{I}_t^d$ and, the other that will generate θ_t from the signal $\xi_t \in \mathbb{R}^n$. Let us denote $\gamma_t^\theta : \mathbb{R}^n \to \{0, 1\}$ to be the strategy for deciding whether to transmit or not. Thus, the strategy at the decision-maker ($\mathcal{T}$) is given by the combination of the mappings $\gamma_t^\theta : \mathbb{R}^n \to \{0, 1\}$ and $\mathcal{I} : \mathfrak{I}_t^d \to \mathbb{R}^n$ as follows

$$\xi_t = \mathcal{I}(\mathfrak{I}_t^d), \quad \theta_t = \gamma_t^\theta(\xi_t). \tag{5}$$

Therefore, the overall strategy at the decision-maker is given by the composition mapping $\gamma_t^\theta \circ \mathcal{I} : \mathfrak{I}_t^d \to \{0, 1\}$.

Notice that, one may aim for directly designing a function $\tilde{\gamma}_t^\theta$ such that $\theta_t = \tilde{\gamma}_t^\theta(\mathfrak{I}_t^d)$. Such an approach may not necessarily reveal the structure of the *new information* based on which θ_t is designed. Thus, we explicitly represent $\tilde{\gamma}_t^\theta(\mathfrak{I}_t^d)$ as $\gamma_t^\theta \circ \mathcal{I}$. Although, both γ_t^θ and $\mathcal{I}$ are the decision variables for the decision-maker, we will adhere to a specific structure for the mapping $\mathcal{I}$ that extracts the new information. Particularly, in this work we will consider that, for all $t \in \mathbb{N}$,

$$\mathcal{I}(\mathfrak{I}_t^d) = X_t - A X_{t-1} - B U_{t-1} = W_{t-1}, \tag{6}$$

and $\mathcal{I}(\mathfrak{I}_0^d) = X_0 - \mu_0 = W_{-1}$. The motivation behind this particular structure for $\mathcal{I}$ is due to the fact that the decision regarding the transmission of the measurement is based on the amount of innovations, captured by W_{t-1}, present in the measurement. If the realization of W_{t-1} has a small value then X_t does not differ much from $A X_{t-1} + B U_{t-1}$ and hence the transmitter may wish not to send this measurement [1]. One may adopt a different structure for $\mathcal{I}$ based on mutual-information or other information-theoretic metrics, however, as an initial attempt to this problem, we will consider this approach. Since the structure of $\mathcal{I}(\mathfrak{I}_t^d)$ is given in (6), therefore the optimization over the strategies of the decision-maker is equivalent to the optimization over $\gamma_t^\theta(\cdot)$.

The admissible control strategies at any time t is a measurable function from the Borel σ-algebra generated by $\mathfrak{I}_t^c$ to $\mathbb{R}^m$. Let us denote such strategies by $\gamma_t^u(\cdot)$ and the space they belong to by Γ^u. Similarly, an admissible strategy for the transmitter is a measurable function from the Borel σ-algebra generated by W_{t-1} to $\{0, 1\}$. Let us denote such strategies by $\gamma_t^\theta(\cdot)$ and the space they belong to by Γ_t^θ. Since $\gamma_t^\theta(\cdot)$ maps $\mathbb{R}^n$ to $\{0, 1\}$, then $\gamma_t^\theta(\cdot)$ is of the form [2]

$$\gamma_t^\theta(x) = 1_{\mathcal{S}_t}(x) \tag{7}$$

for some $\mathcal{S}_t \in \mathcal{B}(\mathbb{R}^n)$ where $\mathcal{B}(\mathbb{R}^n)$ is the space of Borel sets of $\mathbb{R}^n$, and $1_\mathcal{S}(\cdot)$ is the indicator function of the set $\mathcal{S}$. Therefore, if $W_{t-1} \in \mathcal{S}_t$, then the state measurement is transmitted, otherwise, no communication occurs. We seek to find the optimal sequence of the sets $\{\mathcal{S}_0, \cdots, \mathcal{S}_{T-1}\}$ that minimizes the cost (4) where each $\mathcal{S}_t$ is a Borel measurable set of $\mathbb{R}^n$.

Let γ^Θ denote the entire sequence $\{\gamma_0^\theta(\cdot), \gamma_1^\theta(\cdot), \cdots, \gamma_{T-1}^\theta(\cdot)\}$ and let Γ^Θ denote the space where γ^Θ belongs. Similarly, $\gamma^\mathcal{U}$ and $\Gamma^\mathcal{U}$ are defined as well. Therefore, for a given strategy pair $(\gamma^\mathcal{U}, \gamma^\Theta)$, the cost (4) is re-written as

---

[1] Although probabilistically unlikely, but it may happen that the realizations of $W_{t-k}, W_{t-k+1}, \ldots, W_{t-1}$ all have small values and hence the transmitter did not send measurements in the whole interval $[t-k, t]$. However, the cumulative noise $\sum_{i=1}^{k} A^{i-1} W_{t-i}$ will have a large value for high k. Therefore, instead of looking at the quantity W_{t-1}, it is generally desired to look into the quantity $\sum_{i=1}^{k} A^{i-1} W_{t-i}$ (where $t-k$ is the last time instance before t when a measurement was sent) to make a decision on the transmission.

[2] In this case we consider deterministic strategies for the ease of this exposition; one can consider probabilistic strategies as well using similar steps.

$$J_\lambda(\gamma^{\mathcal{U}}, \gamma^\Theta) = \mathbb{E}\Big[\sum_{t=0}^{T-1} \big(X_t^\mathsf{T} Q X_t + U_t^\mathsf{T} R U_t + \lambda\theta_t \big) \tag{8}$$

$$+ X_T^\mathsf{T} Q X_T | U_t = \gamma_t^u(\mathfrak{I}_t^c), \theta_t = \gamma_t^\theta(\mathcal{I}(\mathfrak{I}_t^d)) \Big].$$

3. DECISION MAKING AT CONTROLLER AND TRANSMITTER

In this section we find the optimal $\gamma^{\mathcal{U}*}$ and $\gamma^{\Theta*}$ that minimizes the cost (8) amongst all admissible strategies, i.e.,

$$(\gamma^{\mathcal{U}*}, \gamma^{\Theta*}) = \underset{\gamma^{\mathcal{U}} \in \Gamma^{\mathcal{U}}, \gamma^\Theta \in \Gamma^\Theta}{\arg\min} J_\lambda(\gamma^{\mathcal{U}}, \gamma^\Theta). \tag{9}$$

Before starting the analysis for finding the optimal strategies, let us define certain filtering processes which are fundamental elements in our subsequent analysis.

3.1 Estimation under Intermittent Feedback

Let us define the estimate of the state at the controller side by $\hat{X}_t = \mathbb{E}[X_t | \mathfrak{I}_t^c]$. If, at time t, $\theta_t = 1$, then $\hat{X}_t = X_t$, otherwise $\hat{X}_t = \mathbb{E}[X_t | \mathfrak{I}_{t-1}^c, U_{t-1}, \theta_t = 0]$. In a compact form,

$$\hat{X}_t = \theta_t X_t + (1 - \theta_t)\mathbb{E}[X_t | \mathfrak{I}_{t-1}^c, U_{t-1}, \theta_t = 0]$$

The term $\mathbb{E}[X_t | \mathfrak{I}_{t-1}^c, U_{t-1}, \theta_t = 0]$ can be expanded as:

$$\mathbb{E}[X_t | \mathfrak{I}_{t-1}^c, U_{t-1}, \theta_t = 0]$$
$$= \mathbb{E}[A X_{t-1} + B U_{t-1} + W_{t-1} | \mathfrak{I}_{t-1}^c, U_{t-1}, \theta_t = 0]$$
$$\overset{(a)}{=} A\mathbb{E}[X_{t-1} | \mathfrak{I}_{t-1}^c] + B U_{t-1} + \mathbb{E}[W_{t-1} | \theta_t = 0]$$
$$= A\hat{X}_{t-1} + B U_{t-1} + \mathbb{E}[W_{t-1} | \theta_t = 0] \tag{10}$$

where (a) holds due to the fact that θ_t is a measurable function of the random variable W_{t-1} which is independent of $\{\mathfrak{I}_{t-1}^c, U_{t-1}\}$, and hence, the event $\{\theta_t = 0\}$ is independent of X_{t-1}.

An interesting observation lies in the term $\mathbb{E}[W_{t-1} | \theta_t = 0]$ which reflects the fact 'communication in absence of communication'. $\theta_t = 0$ implies that there will be no communication of the state value from the decision-maker to the controller, or in other words, the amount of information in W_{t-1} is not large enough to trigger a communication. However, this absence of communication reveals some information to the controller about the realization of W_{t-1}. Precisely, since $\theta_t = \gamma_t^\theta(W_{t-1})$, then from (7), it is evident that $W_{t-1} \in \mathcal{S}_t^c$ where $\mathcal{S}_t^c$ is the complement of the set $\mathcal{S}_t \subseteq \mathbb{R}^n$. Since the events $\{\theta = 0\}$ and $\{W_{t-1} \in \mathcal{S}_t^c\}$ are equivalent, then

$$\mathbb{E}[W_{t-1} | \theta_t = 0] = \mathbb{E}[W_{t-1} | W_{t-1} \in \mathcal{S}_t^c]$$
$$= \frac{1}{\mathbb{P}_t(\mathcal{S}_t^c)} \int_{\mathcal{S}_t^c} w \mathbb{P}_t(dw). \tag{11}$$

Under the assumption that $W_{t-1} \sim \mathcal{N}(0, \mathcal{W})$, (for all $t = 1, \ldots, T$) we have

$$\mathbb{P}_t(dw) = \frac{1}{\sqrt{(2\pi)^n \det(\mathcal{W})}} e^{-\frac{w^\mathsf{T} \mathcal{W}^{-1} w}{2}} dw,$$

and since, $W_{-1} \sim \mathcal{N}(0, \Sigma_{X_0})$, we have

$$\mathbb{P}_0(dw) = \frac{1}{\sqrt{(2\pi)^n \det(\Sigma_{X_0})}} e^{-\frac{w^\mathsf{T} \Sigma_{X_0}^{-1} w}{2}} dw.$$

In the subsequent analysis, we will suppress the subscript t in $\mathbb{P}_t$ and simply write it as $\mathbb{P}$.

Although, $W_{t-1} \sim \mathcal{N}(0, \mathcal{W})$, the event $\{\theta_t = 0\}$ updates the distribution of W_{t-1} at the controller side. The *posterior distribution*, $\mathbb{P}(dw | \theta_t = 0)$, of W_{t-1} has as support $\mathcal{S}_t^c$ and is given by $\mathbb{P}(dw | \theta_t = 0) = \frac{\mathbb{P}(dw)}{\mathbb{P}(\mathcal{S}_t^c)}$. One may further notice that, for certain choices of $\mathcal{S}_t^c$ (e.g., $\mathcal{S}_t^c = \{\|x\| \le 1\}$), $\mathbb{E}[W_{t-1} | \theta_t = 0] = \frac{1}{\mathbb{P}(\mathcal{S}_t^c)} \int_{\mathcal{S}_t^c} w \mathbb{P}(dw) = 0 = \mathbb{E}[W_{t-1}]$. Thus, for these choices of $\mathcal{S}_t$ (or equivalently $\mathcal{S}_t^c$), although the posterior distribution of W_{t-1} changes, some statistics are unchanged, such as the posterior mean.

In absence of communication, let us denote the posterior noise mean by $\hat{W}_{t-1} \triangleq \mathbb{E}[W_{t-1} | \theta_t = 0]$. Furthermore, let us denote the estimation error by $e_t \triangleq X_t - \hat{X}_t$ which satisfies the dynamics

$$e_{t+1} = X_{t+1} - \big(\theta_{t+1} X_{t+1} + (1 - \theta_{t+1})(A\hat{X}_t + B U_t + \hat{W}_t)\big)$$
$$= (1 - \theta_{t+1})\big(A e_t + W_t - \hat{W}_t\big)$$
$$= (1 - \theta_{t+1})\big(A e_t + \tilde{W}_t\big) \tag{12}$$

where $\tilde{W}_t \triangleq W_t - \hat{W}_t$. Note that, although not expressed explicitly, $\tilde{W}_t$ depends on the choice of the set $\mathcal{S}_t$. The initial condition for (12) is given by $e_0 = (1 - \theta_0)(X_0 - \mathbb{E}[X_0 | \theta_0 = 0]) = (1 - \theta_0)(W_{-1} - \hat{W}_{-1}) = (1 - \theta_0)\tilde{W}_{-1}$.

Let us denote the posterior noise covariances of $\tilde{W}_t$ at time $t+1$ by $M_{t|t+1} = \mathbb{E}[\tilde{W}_t \tilde{W}_t^\mathsf{T} | \theta_{t+1} = 0]$. Therefore,

$$M_{t|t+1} = \mathbb{E}[\tilde{W}_t \tilde{W}_t^\mathsf{T} | \theta_{t+1} = 0]$$
$$= \mathbb{E}[W_t W_t^\mathsf{T} | \theta_{t+1} = 0] - \hat{W}_t \hat{W}_t^\mathsf{T}$$
$$= \mathbb{E}[W_t W_t^\mathsf{T} | W_t \in \mathcal{S}_{t+1}^c] - \hat{W}_t \hat{W}_t^\mathsf{T}$$
$$= \frac{1}{\mathbb{P}(\mathcal{S}_{t+1}^c)} \int_{\mathcal{S}_{t+1}^c} w w^\mathsf{T} \mathbb{P}(dw) - \hat{W}_t \hat{W}_t^\mathsf{T}. \tag{13}$$

Since $M_{t|t+1}$ can be computed by knowing $\mathcal{S}_{t+1}$ and the distribution of W_t, this computation can be performed offline if $\mathcal{S}_{t+1}$ and $\mathbb{P}(dw)$ are known a priori. We also define

$$M_{-1|0} = \mathbb{P}_0(\mathcal{S}_0^c)\left(\frac{1}{\mathbb{P}_0(\mathcal{S}_0^c)} \int_{\mathcal{S}_0^c} w w^\mathsf{T} \mathbb{P}_0(dw) - \hat{W}_{-1}\hat{W}_{-1}^\mathsf{T} \right)$$

$$= \int_{\mathcal{S}_0^c} w w^\mathsf{T} \mathbb{P}_0(dw)$$

$$- \frac{1}{\mathbb{P}_0(\mathcal{S}_0^c)} \left(\int_{\mathcal{S}_0^c} w \mathbb{P}_0(dw) \right)\left(\int_{\mathcal{S}_0^c} w \mathbb{P}_0(dw) \right)^\mathsf{T}.$$

where $\mathbb{P}_0(dw) \sim \mathcal{N}(0, \Sigma_{X_0})$. The conditional error covariance $\Sigma_t \triangleq \mathbb{E}[e_t e_t^\mathsf{T} | \mathfrak{I}_t^c]$ has the following dynamics

$$\Sigma_{t+1} = (1 - \theta_{t+1})(A \Sigma_t A^\mathsf{T} + M_{t|t+1}), \tag{14}$$
$$\Sigma_0 = (1 - \theta_0) M_{0|-1}$$
$$= (1 - \theta_0)\left(\frac{1}{\mathbb{P}(\mathcal{S}_0^c)} \int_{\mathcal{S}_0^c} w w^\mathsf{T} \mathbb{P}(dw) - \hat{W}_{-1}\hat{W}_{-1}^\mathsf{T} \right).$$

where $\hat{W}_{-1} = \mathbb{E}[W_{-1} | \theta_0 = 0]$. Note that Σ_t is a random variable, and the randomness is induced by $\{\theta_0, \theta_1, \ldots, \theta_t\}$. Let us define

$$\bar{\Sigma}_t \triangleq \mathbb{E}[\Sigma_t] = \mathbb{E}[e_t e_t^\mathsf{T}] \tag{15}$$

After defining $\bar{\Sigma}_{-1} = 0$, the dynamics of $\bar{\Sigma}_t$ can compactly be written as,

$$\bar{\Sigma}_{t+1} = \mathbb{P}(\mathcal{S}_{t+1}^c)(A \bar{\Sigma}_t A^\mathsf{T} + M_{t|t+1}), \tag{16}$$
$$\bar{\Sigma}_{-1} = 0; \tag{17}$$

and therefore, $\bar{\Sigma}_t$ is a function of all the sets $\mathcal{S}_0, \ldots, \mathcal{S}_t$. By solving the dynamics (16), we can write the solution as

$$\bar{\Sigma}_t = \sum_{k=0}^{t} \left(\prod_{\ell=k}^{t} \mathbb{P}(\mathcal{S}_\ell^c) \right) A^{t-k} M_{k-1|k} A^{t-k^\mathsf{T}}. \quad (18)$$

3.2 Optimal Strategies for Controller-Transmitter Pair

In order to proceed with the optimization of $J_\lambda(\gamma^{\mathcal{U}}, \gamma^\Theta)$ in (8), let us define the value function

$$V_k(x) = \min_{\{\gamma_t^u\}_{t=k}^{T-1}, \{\gamma_t^\theta\}_{t=k}^{T-1}} \mathbb{E}\Big[\sum_{t=k}^{T-1} (X_t^\mathsf{T} Q_1 X_t + U_t^\mathsf{T} R U_t + \lambda \theta_t)$$
$$+ X_T^\mathsf{T} Q_2 X_T \mid U_t = \gamma_t^u(\mathfrak{I}_t^c), \theta_t = \gamma_t^\theta(\mathcal{I}(\mathfrak{I}_t^d)),$$
$$X_k = x, t = k, \cdots, T-1 \Big]. \quad (19)$$

By the dynamic programming principle,

$$V_k(x) = \min_{\gamma_k^u \in \Gamma_k^u, \gamma_k^\theta \in \Gamma_k^\theta} \mathbb{E}\Big[(X_k^\mathsf{T} Q_1 X_k + U_k^\mathsf{T} R U_k + \lambda \theta_k)$$
$$+ V_{k+1}(X_{k+1}) \mid U_k = \gamma_k^u(\mathfrak{I}_k^c), \theta_k = \gamma_k^\theta(\mathcal{I}(\mathfrak{I}_k^d)),$$
$$X_k = x \Big]. \quad (20)$$

If γ_k^{u*} and $\gamma_k^{\theta*}$ minimize the right-hand-side of (20), then the optimal decision on the transmission of the state at time k is given by $\theta_k^* = \gamma_k^{\theta*}(\mathcal{I}(\mathfrak{I}_k^d))$ and the optimal control is $U_k^* = \gamma_k^{u*}(\mathfrak{I}_k^c)$. From the definition of the value function in (19), we have

$$\min_{\gamma^{\mathcal{U}} \in \Gamma^{\mathcal{U}}, \gamma^\Theta \in \Gamma^\Theta} J(\gamma^{\mathcal{U}}, \gamma^\Theta) = \mathbb{E}[V_0(X_0)], \quad (21)$$

where the expectation is over the random variable $X_0 \sim \mathcal{N}(\mu_0, \Sigma_{X_0})$. The main results are given in the following Theorem.

Theorem 1. (Certainty Equivalence Controller). Given the information $\mathfrak{I}_t^c$ to the controller, the optimal control strategy $\{\gamma_k^{u*}\}_{k=0}^{T-1}$ that minimizes (8) is given by

$$U_k^* = \gamma_k^{u*}(\mathfrak{I}_k^c) = -L_k \mathbb{E}[X_k | \mathfrak{I}_k^c], \quad (22)$$

where for all $k = 0, 1, \ldots, T-1$, L_k and P_k are obtained by

$$L_k = (R + B^\mathsf{T} P_{k+1} B)^{-1} B^\mathsf{T} P_{k+1} A, \quad (23a)$$
$$N_k = L_k^\mathsf{T} (R + B^\mathsf{T} P_{k+1} B) L_k, \quad (23b)$$
$$P_k = Q_1 + A^\mathsf{T} P_{k+1} A - N_k, \quad (23c)$$
$$P_T = Q_2. \quad (23d)$$

Proof. The full proof has been omitted due to space constraints. We provide below some key steps.

The proof of this theorem is based on the dynamic programming principle on the value function defined in (19). Let us hypothesize that the value function has the form

$$V_k(x) = x^\mathsf{T} P_k x + r_k + C_k \quad (24)$$

where P_k is defined in (23c), and for all $k = 0, 1, \ldots, T-1$,

$$C_k = \min_{\{\gamma_t^\theta\}_{t=k}^{T-1}} \mathbb{E}\left[\sum_{t=k}^{T-1} (\|e_t\|_{N_t}^2 + \lambda \theta_t) \Big| \ \theta_t = \gamma_t^\theta(\mathcal{I}(\mathfrak{I}_t^d)), \forall t \right], \quad (25)$$

and $r_k \in \mathbb{R}$ is given by

$$r_k = \sum_{t=k+1}^{T} \mathrm{tr}(P_t \mathcal{W}). \quad (26)$$

Using dynamic program principle, C_k can be written as

$$C_k = \min_{\gamma_k^\theta \in \Gamma_k^\theta} \mathbb{E}[\|e_k\|_{N_k}^2 + \lambda \theta_k + C_{k+1} \mid \theta_k = \gamma_k^\theta(\mathcal{I}(\mathfrak{I}_k^d))]. \quad (27)$$

Since e_t does not depend on the control input, C_k does not depend on the controller strategy $\gamma^{\mathcal{U}}$. One may verify that V_{T-1} is indeed of the form (24). By assuming that the hypothesis (24) holds for some $k \le T-1$, we can show

$$V_{k-1}(x)$$
$$= \min_{\gamma_{k-1}^u \in \Gamma_{k-1}^u, \gamma_{k-1}^\theta \in \Gamma_{k-1}^\theta} \mathbb{E}\Big[(X_{k-1}^\mathsf{T} Q_1 X_{k-1} + U_{k-1}^\mathsf{T} R U_{k-1}$$
$$+ \lambda \theta_{k-1}) + X_k^\mathsf{T} P_k X_k + C_k + r_k \mid U_{k-1} = \gamma_{k-1}^u(\mathfrak{I}_{k-1}^c),$$
$$\theta_{k-1} = \gamma_{k-1}^\theta(\mathcal{I}(\mathfrak{I}_{k-1}^d)), X_{k-1} = x \Big].$$
$$= \min_{\gamma_{k-1}^u \in \Gamma_{k-1}^u, \gamma_{k-1}^\theta \in \Gamma_{k-1}^\theta} \mathbb{E}\Big[X_{k-1}^\mathsf{T} P_{k-1} X_{k-1}$$
$$+ \|U_{k-1} + L_{k-1} X_{k-1}\|_{R+B^\mathsf{T} P_k B}^2 + \lambda \theta_{k-1} + C_k$$
$$+ r_k + \mathrm{tr}(P_k \mathcal{W}) \mid U_{k-1} = \gamma_{k-1}^u(\mathfrak{I}_{k-1}^c),$$
$$\theta_{k-1} = \gamma_{k-1}^\theta(\mathcal{I}(\mathfrak{I}_{k-1}^d)), X_{k-1} = x \Big].$$

Since $\mathbb{E}[\|U_{k-1} + L_{k-1} X_{k-1}\|_{R+B^\mathsf{T} P_k B}^2]$ is the only term in the last equation that depends on U_{k-1}, the $\mathfrak{I}_{k-1}^c$ measurable function minimizing $\mathbb{E}[\|U_{k-1} + L_{k-1} X_{k-1}\|_{R+B^\mathsf{T} P_k B}^2]$ is $-L_{k-1}\mathbb{E}[X_{k-1} \mid \mathfrak{I}_{k-1}^c]$. Therefore, we have

$$U_{k-1}^* = \gamma_{k-1}^{u*} = -L_{k-1}\mathbb{E}[X_{k-1} | \mathfrak{I}_{k-1}^c].$$

After substituting the optimal control in the expression for $V_{k-1}(x)$, we obtain

$$V_{k-1}(x) = x^\mathsf{T} P_{k-1} x + r_{k-1} + C_{k-1}.$$

Thus, using induction, the hypothesis (24) is true for all k. This completes the sketch of the proof. $\square$

From Theorem 1 we have

$$V_0(x) = x^\mathsf{T} P_0 x + r_0$$
$$+ \min_{\{\gamma_t^\theta\}_{t=0}^{T-1}} \mathbb{E}\left[\sum_{t=0}^{T-1} (\|e_t\|_{N_t}^2 + \lambda \theta_t) \mid \theta_k = \gamma_k^\theta(\mathcal{I}(\mathfrak{I}_k^d)), \forall k \right]$$

The optimal strategy for θ can be found by performing the optimization

$$\min_{\{\gamma_t^\theta\}_{t=0}^{T-1}} \mathbb{E}\left[\sum_{t=0}^{T-1} (\|e_t\|_{N_t}^2 + \lambda \theta_t) \mid \theta_k = \gamma_k^\theta(\mathcal{I}(\mathfrak{I}_k^d)), \forall k \right].$$

Thus, the new optimization problem to find the optimal transmission strategy is

$$\min_{\{\gamma_t^\theta\}_{t=0}^{T-1}} \mathbb{E}\left[\sum_{t=0}^{T-1} (\|e_t\|_{N_t}^2 + \lambda \theta_t) \mid \theta_t = \gamma_t^\theta(\mathcal{I}(\mathfrak{I}_t^d)), \ \forall t \right]$$
$$\text{s.t.} \quad e_{t+1} = (1 - \theta_{t+1})(Ae_t + \tilde{W}_t). \quad (28)$$

Problem (28) is reminiscent of a Markov-Decision-Process (MDP) with state e and action θ, along with deterministic initial state $e_{-1} = 0$. Even though the cost function of this MDP is quadratic, however, due to the bi-linear dynamics of e_t and the binary nature of θ_t, obtaining a closed form solution of this MDP is a distant reality. From the theory of MDP, it is sufficient to look for Markovian policies of the form $\gamma_t^{\theta, \mathrm{Markov}}(e_t)$. Although (28) is difficult to solve, but when solved, it produces the optimal transmission strategy $\gamma_t^{\theta*}(\cdot) = 1_{\mathcal{S}_t^*}(\cdot)$. Using the definition (15), an equivalent formulation of (28) is given in the following Theorem.

Theorem 2. (Optimal Transmission Strategy). The optimal transmission strategy $\gamma_t^{\theta*}(\cdot) = 1_{\mathcal{S}_t^*}(\cdot)$ can be found by optimizing the following problem

$$\min_{\{\mathcal{S}_t\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} \mathrm{tr}(\bar{\Sigma}_t N_t) + \lambda \mathbb{P}(\mathcal{S}_t) \qquad (29)$$

$$\text{s.t.} \quad \bar{\Sigma}_{t+1} = \mathbb{P}(\mathcal{S}_{t+1}^c)(A\bar{\Sigma}_t A^\mathsf{T} + M_{t|t+1}),$$

$$M_{t|t+1} = \frac{1}{\mathbb{P}(\mathcal{S}_{t+1}^c)} \int_{\mathcal{S}_{t+1}^c} ww^\mathsf{T} \mathbb{P}(dw) - \hat{W}_t \hat{W}_t^\mathsf{T}.$$

$$\hat{W}_t = \mathbb{E}[W_t | W_t \in \mathcal{S}_{t+1}^c]$$

$$\bar{\Sigma}_{-1} = 0, \qquad t = -1, \ldots, T-2.$$

Proof. From the previous discussion we have

$$\min_{\gamma^{\mathcal{U}} \in \Gamma^{\mathcal{U}}, \gamma^{\Theta} \in \Gamma^{\Theta}} J(\gamma^{\mathcal{U}}, \gamma^{\Theta}) = \mathbb{E}[V_0(X_0)]$$

$$= r_0 + \mathrm{tr}(P_0 \mathbb{E}[X_0 X_0^\mathsf{T}])$$

$$+ \min_{\{\gamma_t^{\theta}\}_{t=0}^{T-1}} \mathbb{E}\left[\sum_{t=0}^{T-1} (\|e_t\|_{N_t}^2 + \lambda \theta_t) \mid \theta_k = \gamma_k^{\theta}(\mathcal{I}(\mathfrak{I}_k^d)), \forall k \right].$$

At this point, one may verify that the dynamic program (29) is equivalent to the one in (28), and this completes the proof. □

Solving the dynamic program (29) is equally computationally expensive as solving (28), and therefore, we resort to find an approximate solution(s). First, we will provide an upper and lower bound on the value function C_k in Lemma 3. Then, based on these bounds, we construct an approximate solution $\{\mathcal{S}_0^{\circ}, \ldots, \mathcal{S}_{T-1}^{\circ}\}$ which will be shown to perform better than both continuous feedback and open-loop scenarios. Finally we will provide an algorithm to further improve the initial solution $\{\mathcal{S}_0^{\circ}, \ldots, \mathcal{S}_{T-1}^{\circ}\}$ and construct a solution $\{\mathcal{S}_0^*, \ldots, \mathcal{S}_{T-1}^*\}$ which is at least locally optimal if not globally optimal.

Lemma 3. For all $k = 0, \ldots, T$, $\underline{C}_k \leq C_k \leq \overline{C}_k$ where

$$\overline{C}_k = (T-k)\lambda + \mathrm{tr}(\bar{\Sigma}_{k-1} \Upsilon_k)$$
$$+ \sum_{t=k}^{T-1} \int_{\|w\|_{\Upsilon_{t+1}+N_t}^2 \leq \lambda} (\|w\|_{\Upsilon_{t+1}+N_t}^2 - \lambda) \mathbb{P}(dw) \quad (30a)$$

$$\underline{C}_k = (T-k)\lambda + \sum_{t=k}^{T-1} \int_{\|w\|_{N_t}^2 \leq \lambda} (\|w\|_{N_t}^2 - \lambda) \mathbb{P}(dw) \quad (30b)$$

and

$$\Upsilon_t = A^\mathsf{T}(\Upsilon_{t+1} + N_t)A, \qquad (31)$$
$$\Upsilon_T = 0.$$

Proof. The proof has been omitted due to limited space, but one may verify it through mathematical induction. □

Given the bounds on the value function C_k, we will construct an approximate solution for the strategies γ_t^{θ}. Precisely, in the proof of Lemma 3 one may notice that $\mathcal{S}_t = \{s \in \mathbb{R}^n \mid s^\mathsf{T}(N_t + \Upsilon_{t+1})s \geq \lambda\}$ minimizes the upper bound of the value function. In the following Lemma we will show that this strategy is indeed better than having perfect open-loop or perfect closed-loop operation.

Lemma 4. For all k,

$$\gamma_k^{\theta}(\cdot) = 1_{\bar{\mathcal{S}}_k}(\cdot),$$
$$\bar{\mathcal{S}}_k = \{s \in \mathbb{R}^n \mid s^\mathsf{T}(\Upsilon_{k+1} + N_k)s \geq \lambda\}$$

is a sub-optimal strategy that performs better than perfect closed-loop or perfect open-loop strategy.

Proof. The proof is straightforward and hence omitted due to page limitation. □

3.3 Algorithmic Solution for the Transmission Strategy

In this section, we will provide an algorithm which will take any $\bar{\mathcal{S}}$ (we will take the one as defined in Lemma 4) and improves $\bar{\mathcal{S}}$ until it converges to a better solution $\mathcal{S}^*$. Before presenting the algorithm, let us revisit the cost expression and illustrate some salient features which will help developing the algorithm. With a slight abuse of notation, let us denote $C(\mathcal{S})$ to be the cost incurred by an transmission strategy $\mathcal{S} = \{\mathcal{S}_0, \ldots, \mathcal{S}_{T-1}\}$, i.e,

$$C(\mathcal{S}) = \sum_{t=0}^{T-1} \mathrm{tr}(\bar{\Sigma}_t N_t) + \lambda \mathbb{P}(\mathcal{S}_t) \qquad (32)$$

After some simplifications, (32) can be written as

$$C(\mathcal{S}) = \sum_{t=0}^{T-1} \int_{\mathcal{S}_t^c} \left[\|w - \mathbb{E}[W_{t-1}|\mathcal{S}_t^c]\|_{F_t}^2 - \lambda \right] \mathbb{P}(dw) + \lambda T \qquad (33)$$

where F_t satisfies the dynamics

$$F_{t-1} = N_{t-1} + \mathbb{P}(\mathcal{S}_t^c)A^\mathsf{T} F_t A, \quad F_{T-1} = N_{T-1}.$$

Using standard inductive argument, it is trivial to verify that $F_t \preceq N_t + \Upsilon_{t+1}$ for all $t = 0, \ldots, T-1$ where Υ_t is defined in (31). Let us notice from (33) that the cost function is of the form $C(\mathcal{S}) = \lambda T + \sum_{t=0}^{T-1} G(t, \mathcal{S})$ where $G(t, \mathcal{S}) = \int_{\mathcal{S}_t^c} \left[\|w - \mathbb{E}[W_{t-1}|\mathcal{S}_t^c]\|_{F_t}^2 - \lambda \right] \mathbb{P}(dw)$ and $G(t, \mathcal{S})$ only depends on $\mathcal{S}_t, \mathcal{S}_{t+1}, \ldots, \mathcal{S}_{T-1}$. Thus, $\mathcal{S}_0$ only affects the term $G(0, \mathcal{S})$, and $\mathcal{S}_1$ affects two terms $G(0, \mathcal{S})$ and $G(1, S)$, and so on. This observation leads to a construction of an iterative algorithm in the following way: Let at the end of iteration i, the obtained solution be $\mathcal{S}^i = \{\mathcal{S}_t^i\}_{t=0}^{T-1}$. For iteration $i+1$, we first find $\mathcal{S}_0^{i+1}$ as

$$S_0^{i+1} = \underset{\mathcal{S}_0}{\arg\min}\, C(\mathcal{S}_0, \{\mathcal{S}_t^i\}_{t=1}^{T-1})$$

$$= \underset{\mathcal{S}_0}{\arg\min}\, G(0, (\mathcal{S}_0, \{\mathcal{S}_t^i\}_{t=1}^{T-1})).$$

After finding $\mathcal{S}_0^{i+1}$, $\mathcal{S}_1^{i+1}$ is found by performing

$$S_1^{i+1} = \underset{\mathcal{S}_1}{\arg\min}\, C(\mathcal{S}_0^{i+1}, \mathcal{S}_1, \{\mathcal{S}_t^i\}_{t=2}^{T-1})$$

$$= \underset{\mathcal{S}_1}{\arg\min}\, G(0, (\mathcal{S}_0^{i+1}, \mathcal{S}_1, \{\mathcal{S}_t^i\}_{t=2}^{T-1})) + G(1, (\mathcal{S}_1, \{\mathcal{S}_t^i\}_{t=2}^{T-1})).$$

In this way, having computed $\mathcal{S}_{j-1}^{i+1}$, $\mathcal{S}_j^{i+1}$ is computed by performing

$$\mathcal{S}_j^{i+1} = \arg\min_{S_j} \sum_{t=0}^{j} G(t, \{\mathcal{S}_k^{i+1}\}_{k=t}^{j-1}, \mathcal{S}_j, \{\mathcal{S}_k^i\}_{k=j+1}^{T-1}).$$

From the construction of the Algorithm, $C(\mathcal{S}^{i+1}) \leq C(\mathcal{S}^i)$ and thus, each iteration produces a solution which is at least as good as the previous one. Since from Lemma 3 we have that the value function is lower bounded, we are guaranteed that the optimal solution $\mathcal{S}^*$ will have $C(\mathcal{S}^*) \geq \underline{C}_0$ and hence $C(\mathcal{S}^i) \geq C(\mathcal{S}^*)$ for all i. Thus, convergence of the algorithm is guaranteed. In general, one may use a termination condition such as improvement is less than ϵ over an iteration, i.e., $C(\mathcal{S}^i) - C(\mathcal{S}^{i+1}) \leq \epsilon$.

Algorithm 1 Algorithm for Transmission Strategy

$\mathcal{S}^0 \leftarrow \overline{\mathcal{S}}$ (from Lemma 4),
$i \leftarrow 1$,
while termination condition is not met **do**
    for all $j = 0, \dots, T-1$ **do**

$$\mathcal{S}_j^{i+1} \leftarrow \underset{S_j}{\arg\min} \sum_{t=0}^{j} G(t, \{\mathcal{S}_k^{i+1}\}_{k=t}^{j-1}, \mathcal{S}_j, \{\mathcal{S}_k^i\}_{k=j+1}^{T-1}),$$

    end for
    $i \leftarrow i + 1$,
end while

---

4. CONCLUSION

In this work, we considered a co-design problem of optimal control and optimal measurement transmission strategy between a remote sensor and a controller. We propose an innovation based structure of the measurement transmission. Under this framework, we show that the joint design problem can be performed sequentially where a controller is designed first which implicitly depends on the measurement transmission strategy. Such a controller is shown to be linear where the gains of the controller can be computed through Riccati equations, and the estimation of the state depends on the measurement transmission strategy. The measurement transmission strategy can be found by solving a certain dynamic programming problem whose approximate solution is constructed as well. Finally, we also provide an iterative algorithm that can improve any approximate solution until that iterative process leads to a local/global optimal solution.

REFERENCES

Antunes, D. and Heemels, W. (2014). Rollout event-triggered control: Beyond periodic control performance. *IEEE Transactions on Automatic Control*, 59(12), 3296–3311.

Aoki, M. and Li, M.T. (1969). Optimal discrete-time control system with cost for observation. *IEEE Transactions on Automatic Control*, 14(2), 165–175.

Arzén, K.E. (1999). A simple event-based PID controller. In *Proc. 14th IFAC World Congress*, volume 18, 423–428.

Aström, K.J. (2008). Event based control. In *Analysis and design of nonlinear control systems*, 127–147. Springer.

Astrom, K.J. and Bernhardsson, B.M. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of the 41st IEEE Conference on Decision and Control, 2002.*, volume 2, 2011–2016. IEEE.

Bian, W. and French, M. (2005). General fast sampling theorems for nonlinear systems. *Systems & control letters*, 54(11), 1037–1050.

Cooper, C. and Hahi, N. (1971). An optimal stochastic control problem with observation cost. *IEEE Transactions on Automatic Control*, 16(2), 185–189.

Demirel, B., Gupta, V., Quevedo, D.E., and Johansson, M. (2016). On the trade-off between communication and control cost in event-triggered dead-beat control. *IEEE Transactions on Automatic Control*, 62(6), 2973–2980.

Dolgov, M., Fischer, J., and Hanebeck, U.D. (2013). Event-based LQG control over networks with random transmission delays and packet losses. *IFAC Proceedings Volumes*, 46(27), 23–30.

Goldenshluger, A. and Mirkin, L. (2017). On minimum-variance event-triggered control. *IEEE control systems letters*, 1(1), 32–37.

Heemels, W., Sandee, J., and Van Den Bosch, P. (2008). Analysis of event-driven controllers for linear systems. *International journal of control*, 81(4), 571–590.

Lafortune, S. (1985). On stochastic optimal control problems with selection among different costly observations. Technical report. URL https://www2.eecs.berkeley.edu/Pubs/TechRpts/1985/ERL-85-99.pdf.

Lemmon, M., Chantem, T., Hu, X.S., and Zyskowski, M. (2007). On self-triggered full-information H-infinity controllers. In *International Workshop on Hybrid Systems: Computation and Control*, 371–384. Springer.

Maity, D. and Baras, J.S. (2015). Event based control of stochastic linear systems. In *2015 International Conference on Event-based Control, Communication, and Signal Processing (EBCCSP)*, 1–8. IEEE.

Maity, D. and Baras, J.S. (2019). Optimal event-triggered control of nondeterministic linear systems. *IEEE Transactions on Automatic Control*, 65(2), 604–619.

Molin, A. and Hirche, S. (2009). On LQG joint optimal scheduling and control under communication constraints. In *Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28th Chinese Control Conference*, 5832–5838. IEEE.

Molin, A. and Hirche, S. (2012). On the optimality of certainty equivalence for event-triggered control systems. *IEEE Transactions on Automatic Control*, 58(2), 470–474.

Nešić, D., Teel, A.R., and Kokotović, P.V. (1999). Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximations. *Systems & Control Letters*, 38(4-5), 259–270.

Olgac, N.M., Anthony Cooper, C., and Longman, R.W. (1985). The impact of costly observations and observation delay in stochastic optimal control problems. *International Journal of Control*, 41(3), 769–785.

Quevedo, D.E., Silva, E.I., and Goodwin, G.C. (2007). Packetized predictive control over erasure channels. In *2007 American Control Conference*, 1003–1008. IEEE.

Sawaragi, Y., Soeda, T., Tomita, Y., and Imai, I. (1978). Optimal timing of the observation for the state estimation and control of the stochastic discrete linear system. *International Journal of Control*, 27(4), 621–637.

Shi, L., Yuan, Y., and Chen, J. (2012). Finite horizon LQR control with limited controller-system communication. *IEEE Transactions on Automatic Control*, 58(7), 1835–1841.

Sijs, J. and Noack, B. (2017). Event-based estimation in a feedback loop anticipating on imperfect communication. *IFAC-PapersOnLine*, 50(1), 2583–2588.

Soleymani, T., Baras, J.S., and Hirche, S. (2018). Value of information in feedback control. *arXiv preprint arXiv:1812.07534*.

Velasco, M., Fuertes, J., and Marti, P. (2003). The self triggered task model for real-time control systems. In *Work-in-Progress Session of the 24th IEEE Real-Time Systems Symposium (RTSS03)*, volume 384.

Xu, Y. and Hespanha, J.P. (2004). Optimal communication logics in networked control systems. In *2004 43rd IEEE Conference on Decision and Control (CDC)*, 3527–3532. IEEE.