

Adaptive speed tracking controller for a brush-less DC motor using singular perturbation.^{*}

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Abstract: This work proposes a speed tracking controller for a brush-less DC (BLDC) motor in presence of plant parameters uncertainty and lack of current sensors. The designed controller is based on singular perturbation and adaptive control methods. Singular perturbation method is used to reduce the plant model order neglecting the current fast dynamics. Based on this reduced order model, an adaptive control law that has no dependency of the phase currents measurements, is formulated. The effectiveness of the proposed controller is demonstrated by numerical simulations.

Keywords: Brush-less motors; Adaptive control; Speed control; Singular perturbation method; Uncertain dynamic systems.

1. INTRODUCTION

The importance of the motor control technology has resurfaced recently because the high capabilities and better heat management of motors, since their efficiency is closely linked to the reduction of the petroleum dependency and greenhouse gases as mentioned in Nam (2018).

According to Xiaojuan and Jinglin (2010) in the last decades, permanent magnet synchronous motors (PMSM) started to replace other famous traditional kinds of motors such as Brushed DC motor or induction motor due to their mechanical friction and electric erosion. Therefore, the trend is to use high efficiency motors such as PMSM in appliances such as refrigerators, air conditioners, washing machines, etc., due to their low maintenance, long life and low noise as Bender and Orszag (2013) mentioned in their work.

A PMSM can be broadly classified into two categories according to the patterns of their EMFs. One category is characterized by having sinusoidal EMFs and the other, by having trapezoidal patterns. The former are known as permanent magnet brush-less DC (BLDC) motors.

BLDC motors are usually described as a three-phase circuit consisting of inductors, resistors and EMFs in each of its phases. However, unlike a conventional DC motor, the switching of a BLDC motor is achieved electronically by a controller. The importance of these controllers is well

known, as stated in the work of Hinch (1991), in order for BLDC motors to work correctly.

Controlling the BLDC motors speed is an important challenge since it allows to produce a desired torque to accomplish the specified task. Conventional control techniques such as PID and PI have been used to control the rotor speed of BLDC motors. Nevertheless, these controllers can achieve only local stability, become slower in the presence of a time-varying output reference and, moreover, uncertainty in the load torque affects the performance considerably.

To overcome these problems, modern nonlinear controllers were proposed in Xiaojuan and Jinglin (2010) and Hafez et al. (2019). Unfortunately, to implement these controllers, the measurements of all state variables are needed as well as the knowledge of the plant parameters that results in very complex controllers. However, in practice, the plant parameters are usually unknown and, in many cases, the only available sensors are Hall effect sensors from which it can be obtained only the position and speed.

In this paper, we propose a solution to the motor speed adaptive tracking problem under parametric uncertainty and unknown load torque. This solution will be accomplished leaving out the current measurements, relying only on position and speed measurements that can be obtained from Hall Effect sensors. For this purpose, the singular perturbation method is used to neglect the currents fast dynamics and simplify the BLDC motor model. This simplification leads to a reduced order system that has relative degree equal to one with respect to the tracking error. This

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permits to design a simple adaptive control law that has no dependency on the phase currents measurements. This controller is suitable for the real time implementation for a wide range of BLDC motors of various power without accurate knowledge of the parameters.

The work is organized as follows. Section 2 describes the BLDC motor model. Section 3 presents a basic insight on singular perturbation theory. In Section 4, the singular perturbation method is applied to obtain a BLDC motor reduced order model. Based on this model, an adaptive control law is then formulated. Section 5 shows the effectiveness of the proposed controller by numerical simulations. Last, Section 6 draws some conclusions.

2. BLDC MOTOR MATHEMATICAL MODEL

In this section, we present the dynamic model for a three-phase (a, b, c) BLDC motor with a concentrated full-pitch symmetric Y-connected winding equipped with three Hall sensors placed symmetrically at intervals of 120° electrical degrees under the following assumptions, Xia (2012) and Nam (2018):

Assumption 1. The inner rotor has a non-salient pole structure.

Assumption 2. The core saturation is ignored, as well as the eddy current losses and the hysteresis losses.

Assumption 3. The reaction of the armor is ignored, and the distribution of the air-gap magnetic field is a trapezoidal wave with a flat top width of 120° electrical degrees.

Assumption 4. Conductor distribution is continuous and even on the surface of the armature.

Thus, the dynamical behavior of the BLDC motor is given by

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \Psi \quad (1)$$

$$\Psi = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ab} & L_{bb} & L_{bc} \\ L_{ac} & L_{bc} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + N \begin{bmatrix} \phi_a(\theta) \\ \phi_b(\theta) \\ \phi_c(\theta) \end{bmatrix} \quad (2)$$

where v_a, v_b, v_c and i_a, i_b, i_c are the voltages and currents of phases a, b, c , respectively, Ψ corresponds to the magnetic flux linkages in the stator windings, L_{aa}, L_{bb}, L_{cc} and L_{ab}, L_{ac}, L_{bc} are the self and mutual inductances, respectively, and ϕ_a, ϕ_b, ϕ_c are the flux linkages of the permanent magnets induced by the rotor to each phase in relation to the electrical angle θ .

Substituting (2) into (1), yields

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \left\{ \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ab} & L_{bb} & L_{bc} \\ L_{ac} & L_{bc} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \right\} + N \frac{d}{dt} \begin{bmatrix} \phi_a(\theta) \\ \phi_b(\theta) \\ \phi_c(\theta) \end{bmatrix} \quad (3)$$

where $N \frac{d}{dt} [\phi_a(\theta) \ \phi_b(\theta) \ \phi_c(\theta)]^T$ represents the EMF in each phase according to the Faraday induction law (Serway et al. (2005)) and can be rewritten as

$$N \frac{d}{dt} \begin{bmatrix} \phi_a(\theta) \\ \phi_b(\theta) \\ \phi_c(\theta) \end{bmatrix} = N \frac{\partial}{\partial \theta} \begin{bmatrix} \phi_a(\theta) \\ \phi_b(\theta) \\ \phi_c(\theta) \end{bmatrix} \frac{d\theta}{dt}. \quad (4)$$

In addition, BLDC motors are synchronous motors according to Krishnan (2017), so the rotor speed ω_r is defined as

$$\omega_r := \frac{2}{P} \frac{d\theta}{dt} \quad (5)$$

where P is the number of rotor poles.

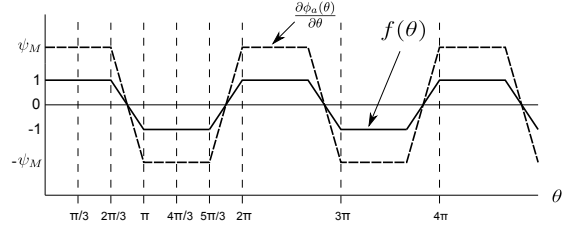


Fig. 1. Trapezoidal function $\frac{\partial \phi(\theta)}{\partial \theta}$ (dotted) and its normalization $f(\theta)$ (continuous) of one phase.

By Assumption 3, the vector $\frac{\partial}{\partial \theta} [\phi_a(\theta) \ \phi_b(\theta) \ \phi_c(\theta)]^T$ has trapezoidal wave form as shown in Fig. 1. Moreover, for symmetrical Y-connected BLDC motors each trapezoidal wave form can be represented as

$$\frac{\partial}{\partial \theta} \begin{bmatrix} \phi_a(\theta) \\ \phi_b(\theta) \\ \phi_c(\theta) \end{bmatrix} = \psi_M \begin{bmatrix} f(\theta) \\ f(\theta - \frac{2\pi}{3}) \\ f(\theta + \frac{2\pi}{3}) \end{bmatrix} \quad (6)$$

where ψ_M is the maximum magnitude of the trapezoidal wave form and $f(\theta)$ is described in Table 1.

Table 1. $f(\theta)$ with respect to the electrical angle θ

Electrical angle θ	$f(\theta)$
$\theta \in [0, \frac{\pi}{3})$	1
$\theta \in [\frac{\pi}{3}, \frac{2\pi}{3})$	1
$\theta \in [\frac{2\pi}{3}, \pi)$	$-\frac{6}{\pi}\theta + 5$
$\theta \in [\pi, \frac{4\pi}{3})$	-1
$\theta \in [\frac{4\pi}{3}, \frac{5\pi}{3})$	-1
$\theta \in [\frac{5\pi}{3}, 2\pi)$	$\frac{6}{\pi}\theta - 11$

Taking into account equations (4)-(6) the EMF vector $[e_a \ e_b \ e_c]$ is defined as follows

$$\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} := \frac{P}{2} K_e \begin{bmatrix} f(\theta) \\ f(\theta - \frac{2\pi}{3}) \\ f(\theta + \frac{2\pi}{3}) \end{bmatrix} \omega_r, \quad K_e = N \psi_M. \quad (7)$$

Using now (7), the equations (3) become

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \left\{ \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ab} & L_{bb} & L_{bc} \\ L_{ac} & L_{bc} & L_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \right\} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}. \quad (8)$$

In Xia (2012), it was mentioned that, under Assumption 1, the flow is isotropic in all directions and therefore, self and mutual inductances will not vary over time. In addition, since the three-phase stator windings are symmetrical, the self-inductances will be equal to each other, as well as the mutual-inductances, i.e.

$$\begin{aligned} L_{aa} &= L_{bb} = L_{cc} = L \\ L_{ab} &= L_{ac} = L_{bc} = M. \end{aligned}$$

Therefore, the equations (8) are changed to

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}. \quad (9)$$

Using Kirchhoff's law ($Mi_a + Mi_b = -Mi_c$), the equations (9) can be reduced to

$$\underbrace{\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}}_V = R \underbrace{\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}}_I + (L - M) \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}. \quad (10)$$

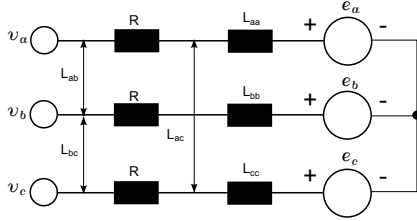


Fig. 2. Equivalent circuit for a 3-phase Y-connected BLDC motor.

To complete the mathematical model of the BLDC motor, Newton's second law is used to describe the rotational motion as

$$T_e - T_l = J \frac{d}{dt} \omega_r + \beta \omega_r \quad (11)$$

where T_l , J and β are the load torque, the moment of inertia and the coefficient of viscous friction, respectively. The electrical torque T_e and its equation can be deduced from an energy balance perspective, where the power transferred to the rotor (electromagnetic power) is equal to the sum of the product of the currents and the EMFs of the three phases, i.e.

$$P_e = [e_a \ e_b \ e_c] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}. \quad (12)$$

Ignoring mechanical losses, all electromagnetic energy becomes kinetic energy, then $P_e = T_e \omega_r$ using the equations (7) and (12) finally, T_e is described by

$$T_e = \frac{P}{2} K_e \underbrace{\left[f(\theta) \ f(\theta - \frac{2\pi}{3}) \ f(\theta + \frac{2\pi}{3}) \right]}_{F^T} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}. \quad (13)$$

3. SINGULAR PERTURBATION

The aim of this section is to provide a basic insight on singular perturbation theory. For deeper and formal definitions on perturbation methods please refer to Hinch (1991), Nayfeh (2008), Bender and Orszag (2013).

Singular perturbation methodology is an analytic approximation strategy to find solutions for systems of the form

$$\frac{d}{dt} x = f(t, x, z), \quad x(t_0) = x_0 \quad (14)$$

$$\epsilon \frac{d}{dt} z = g(t, x, z), \quad z(t_0) = z_0 \quad (15)$$

where $x \in \mathbb{R}^n$, $z \in \mathbb{R}^m$, the functions $f(\cdot)$ and $g(\cdot)$ are continuously differentiable in all its arguments; ϵ is a small

parameter such that when $\epsilon = 0$ causes a structural change in the dynamic properties of the system given that the differential equation (15) degenerates into an algebraic equation

$$0 = g(t, x, z). \quad (16)$$

Suppose that the equation (16) has an isolated solution

$$z = h(t, x). \quad (17)$$

Substituting (17) in (14) results in the following reduced n-dimensional system called slow motion system:

$$\frac{d}{dt} x = f(t, x, h(t, x)). \quad (18)$$

Now, define the following change of variable to ensure the origin to be an equilibrium point:

$$y := z - h(t, x).$$

System (14) - (15) in new variables (x, y) is now given by

$$\frac{d}{dt} x = f(t, x, y + h(t, x)) \quad (19)$$

$$\epsilon \frac{d}{dt} y = g(t, x, y + h(t, x)) - \epsilon \frac{\partial h}{\partial t} \quad (20)$$

$$- \epsilon \frac{\partial h}{\partial x} f(t, x, y + h(t, x)).$$

Defining now a new time

$$\tau := t\epsilon^{-1} \implies \frac{d\tau}{dt} = \epsilon^{-1} \quad (21)$$

and setting $\epsilon = 0$ in (20) results in the following fast dynamics:

$$\frac{d}{d\tau} y = g(t, x, y + h(t, x)) \quad (22)$$

which is called also the boundary layer system where x, t are treated as fixed parameters.

Then, if we can find a solution $\hat{x}(t, \epsilon)$ for the reduced system (18) and the origin of the boundary layer system (22) is exponentially stable equilibrium point, uniformly in the fixed parameters, then the solutions for the full order system (14) - (15) $x(t, \epsilon)$ and $z(t, \epsilon)$ meet that

$$x(t, \epsilon) - \hat{x}(t) = O(\epsilon)$$

$$z(t, \epsilon) - h(t, \hat{x}(t)) - \hat{y}(\tau) = O(\epsilon)$$

for some small value ϵ , where $\hat{y}(\tau)$ is the solution of the boundary layer system (22).

These results are presented formally in Khalil (2002).

4. CONTROL DESIGN PROCEDURE

In this section, the aim is to analyze the dynamic model that describes the behavior of a BLDC motor with the purpose of finding an adaptive control law which forces the rotor speed ω_r to track a given smooth reference signal ω_{ref} . This analysis will be accomplished under the following assumptions:

Assumption 5. None of the plant parameters are known.

Assumption 6. Phase currents i_a, i_b, i_c are not available for measurement.

Assumption 7. Load torque T_l is unknown and varies slow, thus $\frac{d}{dt} T_l = 0$.

Assumption 8. The reference signal ω_{ref} and its derivative $\frac{d}{dt} \omega_{ref}$ are bounded continuous known functions.

For this purpose, the singular perturbation analysis will be used to find a reduced order system (slow dynamics) eliminating the phase currents i_a, i_b, i_c fast dynamics.

In general, BLDC motors are constructed so that L and M are small, that leads to the intuition that the motor model can be simplified, approximating it by a minor order system, as it was shown in Section 3. This implies that is possible to analyze only the speed differential equation, since the phase currents differential equations will become algebraic equations by setting $L - M = 0$. Such simplification will make possible to find an adaptive control law that depends only on the speed measurement. This kind of simplification is not always feasible, so it is necessary to verify that the boundary layer system (fast dynamics) is exponentially stable before designing the controller using the reduced order system (slow dynamics).

We will start by defining the speed error e_ω as the difference between the actual rotor speed ω_r and the smooth reference signal ω_{ref} as follows

$$e_\omega := \omega_r - \omega_{ref}. \quad (23)$$

Differentiating (23) with respect to time and using (11), the dynamics of the error are given by

$$\frac{d}{dt}e_\omega = \frac{PK_e}{2J}F^T I - \frac{T_l}{J} - \frac{\beta}{J}(e_\omega + \omega_{ref}) - \frac{d}{dt}\omega_{ref} \quad (24)$$

where F^T denotes $\left[f(\theta) \ f(\theta - \frac{2\pi}{3}) \ f(\theta + \frac{2\pi}{3}) \right]$ and I denotes $[i_a \ i_b \ i_c]^T$.

To simplify the notation, define a new variable ζ as follows

$$\zeta := \frac{PK_e}{2J}I. \quad (25)$$

Using the new variable (25), equation (24) can be written as

$$\frac{d}{dt}e_\omega = F^T \zeta - \frac{T_l}{J} - \frac{\beta}{J}(e_\omega + \omega_{ref}) - \frac{d}{dt}\omega_{ref}, \quad (26)$$

whereas the dynamic equation for (25) can be obtained using (10), thus

$$\frac{(L - M)}{R} \frac{d}{dt}\zeta = -\zeta - \frac{P^2 K_e^2}{4RJ} F(e_\omega + \omega_{ref}) + \frac{PK_e}{2RJ} V \quad (27)$$

where V denotes $[v_a \ v_b \ v_c]^T$.

Let

$$\alpha_1 := \frac{T_l}{J}, \quad \alpha_3 := \frac{(L - M)}{R}, \quad \alpha_5 := \frac{P^2 K_e^2}{4RJ},$$

$$\alpha_2 := \frac{\beta}{J}, \quad \alpha_4 := \frac{PK_e}{2RJ},$$

then the system (26) - (27) can be written as

$$\frac{d}{dt}e_\omega = F^T \zeta - \alpha_1 - \alpha_2(e_\omega + \omega_{ref}) - \frac{d}{dt}\omega_{ref} \quad (29)$$

$$\alpha_3 \frac{d}{dt}\zeta = -\zeta - \alpha_5 F(e_\omega + \omega_{ref}) + \alpha_4 V. \quad (30)$$

It is worth to note that the system (29) - (30) has the form of (14) - (15), so using α_3 as a perturbation parameter, we now proceed with the boundary layer analysis shown in Section 3.

4.1 Boundary layer analysis

By setting $\alpha_3 = 0$ equation (30) becomes

$$0 = -\zeta - \alpha_5 F(e_\omega + \omega_{ref}) + \alpha_4 V. \quad (31)$$

Then, it can be found that (31) has a unique real root given by

$$\zeta^* = -\alpha_5 F(e_\omega + \omega_{ref}) + \alpha_4 V.$$

Now, proceeding with the analysis shown in Section 3, let Γ be

$$\Gamma := \zeta - \zeta^*. \quad (32)$$

Then, the system (29)-(30) in terms of the new variable (32) is stated as

$$\frac{d}{dt}e_\omega = F^T \Gamma - \alpha_5 \|F\|^2 (e_\omega + \omega_{ref}) - \alpha_1 \quad (33)$$

$$- \alpha_2 (e_\omega + \omega_{ref}) - \frac{d}{dt}\omega_{ref} + \alpha_4 F^T V$$

$$\alpha_3 \frac{d}{dt}\Gamma = -\Gamma - \alpha_3 \left\{ -\alpha_5 \frac{P}{2} (e_\omega + \omega_{ref})^2 \frac{\partial}{\partial \theta} F \right. \quad (34)$$

$$\left. - \alpha_5 F \left(\frac{d}{dt}e_\omega + \frac{d}{dt}\omega_{ref} \right) + \alpha_4 \frac{d}{dt}V \right\}.$$

Defining a new time variable τ to analyze the boundary layer

$$\tau = t\alpha_3^{-1} \implies \frac{d\tau}{dt} = \alpha_3^{-1},$$

and setting $\alpha_3 = 0$, the equation (34) can be rewritten as

$$\frac{d}{d\tau}\Gamma = -\Gamma. \quad (35)$$

Thus, the fast dynamics are described by the linear system (35) that is exponentially stable. Therefore, system (29) - (30) can be approximated by the reduced order slow motion model (36).

4.2 Adaptive control design with reduced model

In this subsection, an adaptive control law will be designed based on the reduced slow dynamics described by (33) when $\Gamma = 0$, i.e.

$$\frac{d}{dt}\tilde{e}_\omega = -\alpha_5 \|F\|^2 (\tilde{e}_\omega + \omega_{ref}) - \alpha_1 \quad (36)$$

$$- \alpha_2 (\tilde{e}_\omega + \omega_{ref}) - \frac{d}{dt}\omega_{ref} + \alpha_4 F^T V.$$

To stabilize the tracking error system (36), the control law $V = V(\tilde{e}_\omega, \hat{\theta}, t)$ is formulated of the form

$$V(\tilde{e}_\omega, \hat{\theta}, t) = \frac{F}{\|F\|^2} \eta(\tilde{e}_\omega, \hat{\theta}, t),$$

$$\eta(\tilde{e}_\omega, \hat{\theta}, t) = \hat{\theta}_1 \|F\|^2 (\tilde{e}_\omega + \omega_{ref}) + \hat{\theta}_2 + \hat{\theta}_3 \omega_{ref} \quad (37)$$

$$+ \hat{\theta}_4 \frac{d}{dt}\omega_{ref} - \lambda \tilde{e}_\omega$$

where $\lambda > 0$ is a control design gain and $\hat{\theta}_i, i = 1, \dots, 4$, corresponds to the estimation of the parameters $\theta_1 = \alpha_4^{-1} \alpha_5$, $\theta_2 = \alpha_4^{-1} \alpha_1$, $\theta_3 = \alpha_4^{-1} \alpha_2$ and $\theta_4 = \alpha_4^{-1}$ respectively, with adaptation laws given by

$$\frac{d}{dt}\hat{\theta}_1 = -\gamma_1 \|F\|^2 \tilde{e}_\omega (\tilde{e}_\omega + \omega_{ref}), \quad \frac{d}{dt}\hat{\theta}_3 = -\gamma_3 \tilde{e}_\omega \omega_{ref},$$

$$\frac{d}{dt}\hat{\theta}_2 = -\gamma_2 \tilde{e}_\omega, \quad \frac{d}{dt}\hat{\theta}_4 = -\gamma_4 \tilde{e}_\omega \frac{d}{dt}\omega_{ref} \quad (38)$$

where $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are positive constants.

It is important to note that there is no singularity in (37) since $2 \leq \|F\|^2 \leq 3$ as it can be seen in Table 1.

Proposition. A solution of the closed-loop system (36)-(38) is bounded and the tracking error $\tilde{e}_\omega(t)$ asymptotically tends to zero. i.e.

$$\lim_{t \rightarrow \infty} |\tilde{e}_\omega(t)| = 0.$$

Proof. Define parameter estimation errors as

$$\begin{aligned} \tilde{\theta}_1 &= \theta_1 - \hat{\theta}_1, & \tilde{\theta}_3 &= \theta_3 - \hat{\theta}_3, \\ \tilde{\theta}_2 &= \theta_2 - \hat{\theta}_2, & \tilde{\theta}_4 &= \theta_4 - \hat{\theta}_4. \end{aligned} \quad (39)$$

Then, using (39), the closed-loop system (36) - (38) is represented as

$$\begin{aligned} \frac{d}{dt} \tilde{e}_\omega &= -\alpha_4 \{ (\theta_3 + \lambda) \tilde{e}_\omega + \tilde{\theta}_1 \|F\|^2 (\tilde{e}_\omega + \omega_{ref}) \\ &+ \tilde{\theta}_2 + \tilde{\theta}_3 \omega_{ref} + \tilde{\theta}_4 \frac{d}{dt} \omega_{ref} \}, \end{aligned} \quad (40)$$

$$\frac{d}{dt} \tilde{\theta}_1 = \gamma_1 \|F\|^2 \tilde{e}_\omega (\tilde{e}_\omega + \omega_{ref}), \quad (41)$$

$$\frac{d}{dt} \tilde{\theta}_2 = \gamma_2 \tilde{e}_\omega, \quad (42)$$

$$\frac{d}{dt} \tilde{\theta}_3 = \gamma_3 \tilde{e}_\omega \omega_{ref}, \quad (43)$$

$$\frac{d}{dt} \tilde{\theta}_4 = \gamma_4 \tilde{e}_\omega \frac{d}{dt} \omega_{ref}. \quad (44)$$

Recalling that all $\theta_i, i = 1, \dots, 4$ are positive combinations of constant parameters, and following adaptive backstepping methodology (Krstic et al. (1995)), the function

$$v = \frac{1}{2} (\theta_4 \tilde{e}_\omega^2 + \gamma_1^{-1} \tilde{\theta}_1^2 + \gamma_2^{-1} \tilde{\theta}_2^2 + \gamma_3^{-1} \tilde{\theta}_3^2 + \gamma_4^{-1} \tilde{\theta}_4^2) \quad (45)$$

is proposed as a Lyapunov function candidate. Calculating the time derivative of (45) and using (40), yields

$$\begin{aligned} \frac{d}{dt} v &= \tilde{e}_\omega \{ -(\theta_3 + \lambda) \tilde{e}_\omega - \tilde{\theta}_1 \|F\|^2 (\tilde{e}_\omega + \omega_{ref}) \\ &- \tilde{\theta}_2 - \tilde{\theta}_3 \omega_{ref} - \tilde{\theta}_4 \frac{d}{dt} \omega_{ref} \} + \gamma_1^{-1} \tilde{\theta}_1 \frac{d}{dt} \tilde{\theta}_1 \\ &+ \gamma_2^{-1} \tilde{\theta}_2 \frac{d}{dt} \tilde{\theta}_2 + \gamma_3^{-1} \tilde{\theta}_3 \frac{d}{dt} \tilde{\theta}_3 + \gamma_4^{-1} \tilde{\theta}_4 \frac{d}{dt} \tilde{\theta}_4. \end{aligned} \quad (46)$$

Grouping similar terms, equation (46) can be rewritten as

$$\begin{aligned} \frac{d}{dt} v &= -(\theta_3 + \lambda) \tilde{e}_\omega^2 \\ &+ \tilde{\theta}_1 (-\|F\|^2 \tilde{e}_\omega (\tilde{e}_\omega + \omega_{ref}) + \gamma_1^{-1} \frac{d}{dt} \tilde{\theta}_1) \\ &+ \tilde{\theta}_2 (-\tilde{e}_\omega + \gamma_2^{-1} \frac{d}{dt} \tilde{\theta}_2) + \tilde{\theta}_3 (-\tilde{e}_\omega \omega_{ref} + \gamma_3^{-1} \frac{d}{dt} \tilde{\theta}_3) \\ &+ \tilde{\theta}_4 (-\tilde{e}_\omega \frac{d}{dt} \omega_{ref} + \gamma_4^{-1} \frac{d}{dt} \tilde{\theta}_4). \end{aligned} \quad (47)$$

Substituting (41) - (44) in (47), results in

$$\frac{d}{dt} v = -(\theta_3 + \lambda) \tilde{e}_\omega^2 \leq 0. \quad (48)$$

This implies that

$$v(t) \leq v(t_0), \quad \forall t \geq t_0.$$

Therefore, the solutions $\tilde{e}_\omega(t)$ and $\tilde{\theta}_i(t)$ with $i = 1, \dots, 4$ of (40) - (44) are bounded for all $t > t_0$.

Moreover, taking the time derivative of (48) yields

$$\begin{aligned} \frac{d^2}{dt^2} v &= 2\alpha_4 (\theta_3 + \lambda) \tilde{e}_\omega \{ (\theta_3 + \lambda) \tilde{e}_\omega \\ &+ \tilde{\theta}_1 \|F\|^2 (\tilde{e}_\omega + \omega_{ref}) + \tilde{\theta}_2 \\ &+ \tilde{\theta}_3 \omega_{ref} + \tilde{\theta}_4 \frac{d}{dt} \omega_{ref} \}. \end{aligned} \quad (49)$$

Recalling that $\|F\|^2 \leq 3$, and $\tilde{e}_\omega(t), \tilde{\theta}_i(t)$ with $i = 1, \dots, 4$ are bounded; then, under Assumption 8, it can be

conclude that (49) is also bounded. Hence, the Lyapunov function derivative (48) is uniformly continuous. Thus, all conditions from Barbalat's lemma (Marino and Tomei (1996)) are satisfied, resulting in $\lim_{t \rightarrow \infty} |\tilde{e}_\omega(t)| = 0$. \square

5. NUMERICAL SIMULATIONS

This section shows the behavior of the complete mathematical model, obtained for the BLDC motor in Section 2, closed by the adaptive control law (37) proposed in Section 4 from the reduced model (36). In addition, a sharp change in the load torque T_l from $0Nm$ to $20Nm$ at $t = 1.5s$ has been added as a disturbance in order to see how the controller rejects it and observe how non-modeled dynamics affect the performance of this controller. In order to have a significant feedback that it is able to track an output reference rejecting T_l disturbances, the proposed adaptive control (37) is compared with the one published in Bayardo and Loukianov (2018) that uses the complete BLDC model. The parameters used for the simulation are presented in Table 2.

Table 2. Parameters used for simulation

Motor Parameters	Value
Number of poles P	12
Moment of inertia J	$0.18 \text{ Kg}m^2$
Damping coefficient β	0.05
Self inductance L	$10.63e-3 \text{ H}$
Mutual inductance M	$5.13e-3 \text{ H}$
Resistance R	$2.02 \text{ } \Omega$
Back EMF constant K_e	0.06 V/Rad/s

Fig. 3 shows the reference and the response of both controllers, the *Complete Model Control* (CMC) and the *Reduced Model Control* (RMC). We can appreciate that both, CMC and RMC, track the reference and successfully reject the load torque disturbance.

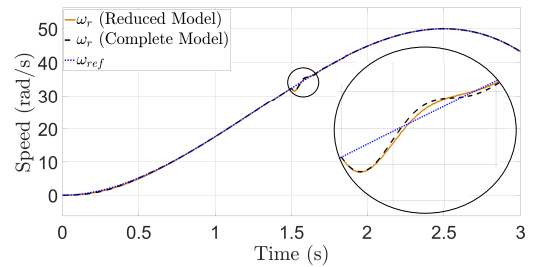


Fig. 3. Rotor speed response of the BLDC motor with both controllers exposed to a load torque change at $t=1.5s$

Even though RMC lacks the information of the current phases, Fig. 4 shows that the square error, from equation (23), has almost the same behaviour as CMC. On the other hand, the electrical torque T_e in RMC is affected by the non-modeled dynamics, as shown in Fig. 5. However, in traditional PID and PI controls the torque ripple (T_{ripple}) can be up to 50% of the average torque (T_{avg}), as addressed in Lin and Lai (2011). RMC improves this indicator, given that T_{ripple} computed for this controller never exceeds the 25%.

$$T_{ripple} = \frac{T_{max} - T_{min}}{T_{avg}} \quad (50)$$

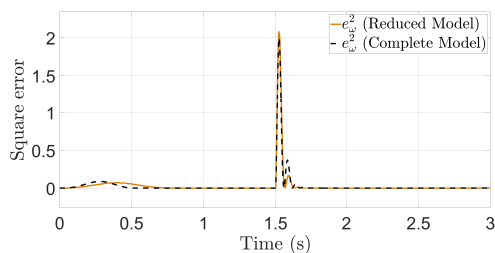


Fig. 4. Square error comparison between both controllers.

where T_{min} , T_{max} are the minimum and maximum values of the torque waveform.

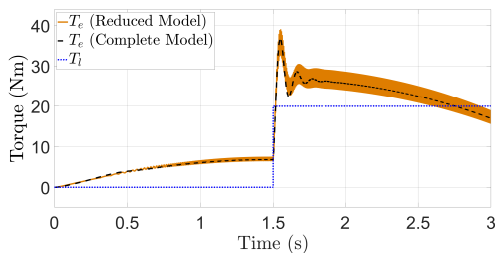


Fig. 5. Torque behaviour comparison

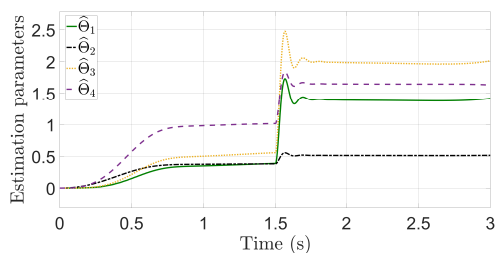


Fig. 6. Reduced order controller estimation parameters

The performance of the adaptive laws can be seen in Fig. 6, where the adapted parameters reach a steady state value, approximately, at time $t = 0.75s$. Even in the presence of disturbances the adapted parameters take less than $0.5s$ to reach the steady state again. Finally, Fig. 7 presents voltage and current phases of the RMC.

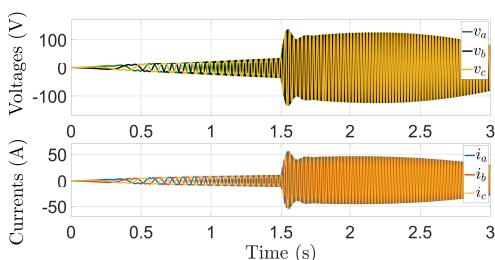


Fig. 7. Reduced order controller voltage and current in each phase

6. CONCLUSION

An adaptive tracking control law for the BLDC motor was designed without the dependency on the phase currents measurements or any parameter of the model. This control law was accomplished by a simplification of the BLDC

motor model that was justified by the analysis of the plant fast dynamics using singular perturbation method.

As shown in Section 5, the simplifications on the mathematical model had no negative effects on the tracking error, as a matter of fact, the behavior of the proposed adaptive controller, when compared with a previous work that uses the complete model, demonstrated similar results and showed great performance in the tracking of the reference signal ω_{ref} , even when the load torque T_l presented sudden changes. But this reduction brought an undesirable behavior in the electrical torque T_e , presented as ripple T_{ripple} . Nevertheless, this ripple remains less than the incurred by conventional PID controllers. Therefore, this controller is a good option when the situation requires to track a reference with parametric uncertainty, unknown load torque and no current measurements available. The implementation of the RMC controller is considered as future work.

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