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New Passivity Conditions for Linear Time Varying Output Feedback Systems

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Abstract: The paper presents new passivity conditions for square linear time varying (LTV) output feedback systems. The new conditions enable the formulation of a new simple test for almost strict passivity, which is necessary for the closed loop to be output strictly passive. The new test requires the solution of an algebraic Riccati equation in the linear time invariant (LTI) case and the solution of a forward differential Riccati equation in the LTV case. The proposed test simplifies the synthesis and design of output strictly passive systems. The examples discussed in the paper demonstrate the efficiency of the test.

Keywords: Passivity, Almost Strict Passivity test, Simple Adaptive Control

I. INTRODUCTION

Passive systems play a significant role in the design of adaptive systems [10], [11]. A closed loop output feedback system is output strictly passive (OSP) [1] if the controlled plant is almost strictly passive (ASP) [10]. Using simple adaptive control (SAC) methodology [3], it is shown in [11] that ASP conditions in nonstationary and nonlinear systems are sufficient to guarantee that the closed loop system with output feedback is stable, robust with disturbances, and has asymptotically perfect tracking in ideal conditions. This result motivates the derivation of a new simple ASP test for closed loop output feedback LTV systems.

In LTI systems, the strict passivity (SP) [1] of the system coincides with the strict positive realness (SPR) [1, 2] property. Strict positive realness is proved to be a very desirable and important feature of systems with applications in various fields, such as control, adaptive control and networks.

It has been shown that closed-loop stability with nonstationary control can be guaranteed in those special LTI systems that can become SPR via constant output feedback. Because only a constant output feedback separates such systems from being strictly positive real, they have been called 'almost strictly positive real' (ASPR) in the LTI case [3, 4], or 'almost strictly passive' (ASP) [10] in the more general case. The ASP feature has played a crucial role in systems with uncertainty [5–7], and in adaptive control [3, 8, and 9] of LTI systems.

The ASPR lemma [15] is the basis for design and synthesis of parallel feedforward compensator (PFC). The PFC renders the controlled plant to be ASPR and its design is the main design issue in the implementation of simple adaptive control. References [18], [19], [20], [21], and [22] deal with the design of parallel feedforward compensator in LTI systems. Reference [10] presents the non-stationary and nonlinear version of the ASPR lemma and implements it in the design of robotic manipulator. Reference [23] uses the non-stationary and nonlinear ASP lemma of [10] in the design of entry capsules.

The existing output strictly passive (OSP) conditions for linear time varying (LTV) system are presented in [12] and stated below.

OSP conditions for LTV systems

Assume that the system $\{F(t),\ G(t),\ H^T(t)\}$ is completely controllable and completely observable.

Then the system is OSP if there exist continuous, bounded

$P(t) = P^T(t) > 0$ and $Q(t) = Q^T(t) > 0$ such that

$$\dot{P}(t) + P(t)F(t) + F^T(t)P(t) = -Q \quad (1a)$$

$$P(t)G(t) = H(t) \quad (1b)$$

See Lemma 2, Corollary IV.2 of reference [12].

Observe that (1b) leads to

$G^T(t)P(t)G(t) = G^T(t)H(t) = H^T(t)G(t) > 0.$

These existing OSP conditions are composed of two parts: one is the differential Lyapunov equation and the other is the structural constraint $PG = H$. The new OSP conditions derived in this paper contain forward differential Riccati equation similar to the one used in filtering, and the structural constraint $P^{-1}G = H$.

As already mentioned, the application of SAC in LTI systems is based on the Almost Strictly Positive Real (ASPR) lemma. In the case of LTV systems the equivalent lemma is the ASP lemma. The two lemmas are stated below. These lemmas are the basis for the existing ASPR and ASP tests.

ASPR lemma for LTI systems

Any linear minimum-phase plant $\{F,\ G,\ H^T\}$ with H^TG Positive Definite Symmetric is ASPR.

See reference [15], 1st ed. pp.55, Lemma 1.

ASP lemma for LTV systems

The linear time varying system $\{F(t),\ G(t),\ H^T(t)\}$ with continuous and uniformly bounded matrices is ASP if:

(i) It is minimum phase, which means that it has "stable zero dynamics". (See the next subsection for details)
(ii) The product $H^T(t)G(t)$ is uniformly positive definite symmetric.

The ASP lemma is a direct corollary of Theorem 2 in [10]. Under these assumptions the system can be made ASP via static output feedback.

Zero Dynamics in LTV Systems

Consider the LTV system $\{F(t), G(t), H^T(t)\}$ where $F(t)$ is $n \times n$, $G(t)$ is $n \times m$, $H^T(t)$ is $m \times n$, and all the system matrices are uniformly bounded. The system $\{F(t), G(t), H^T(t)\}$ is defined to be minimum-phase if its zero dynamics is uniformly asymptotically stable [10].

Following [16] and [17], the 'zero dynamics' of this system is defined by [10] as

$$\dot{z} = [\dot{N}(t)M(t) + N(t)F(t)M(t)]z(t) \tag{2}$$

where $M_{n,n-m}(t)$, $N_{n-m,n}(t)$ assumed to exist, are the solution of

$$H^T(t)\, M(t) = 0_{m,n-m} \tag{3}$$

$$N(t)G(t) = 0_{n-m,m} \tag{4}$$

$$N(t)M(t) = I_{n-m} \tag{5}$$

The 'zero state' $z(t)$ is defined via

$$x(t) = M(t)z(t) \tag{6}$$

and satisfies

$$z(t) = N(t)M(t)z(t) = N(t)x(t) \tag{7}$$

Remarks

1. In LTI systems, the SP property coincides with the SPR property.
2. Observe that any SISO transfer function (not necessarily stable) with relative degree 1, which is minimum phase, is ASPR.
3. The calculation of M and N for LTI systems is well defined in [16]. In the LTI case the system zeros are the eigenvalues of the matrix NFM. The system is minimum phase if these zeros lie in the left half plane.
4. Lemma 2 of [10] states that if a system cannot become SP via static output feedback, no dynamic feedback can render it to be SP.
5. In the LTI case the ASPR test is clearly defined via the eigenvalues of the matrix NFM and the matrix product $H^T G$. In the LTV case the existing ASP test requires the calculation of the matrices $M(t)$ and $N(t)$ and a proof of stability for the zeros differential equation.

The present paper derives a new ASP test for LTV systems. The new test is simple and can also be used as an alternative ASPR test.

The contribution of the paper is the derivation of a new ASP test for LTV systems which requires the solution of a forward differential Riccati equation in the LTV case, and an algebraic Riccati equation in the linear time invariant (LTI) case.

The paper proceeds as follows: Statement of the problem is given in Section II. Section III is devoted to preliminaries. Section IV.A defines new closed loop OSP conditions for LTV systems in terms of a forward differential Riccati equation. Section IV.B deals with the synthesis of OSP systems and presents a new ASP test. Two examples which demonstrate the efficiency of the test are discussed in Section V. The conclusions are summarized in Section VI. Appendix A presents the proof of Theorem 1 appeared in section IV.A

II. STATEMENT OF THE PROBLEM

Given the square linear time varying (LTV) system

$$\dot{x} = F(t)x + G(t)u_c\,, \quad x(t_0) = x_0 \tag{8a}$$

$$y = H^T(t)x \tag{8b}$$

where $x \in \mathbb{R}^n$, $u_c \in \mathbb{R}^m$, $y \in \mathbb{R}^m$, $u \in \mathbb{R}^m$ and $F(t), G(t)$, $H(t)$ are continuous and uniformly bounded matrices with appropriate dimensions over the time interval of interest. It is also given that the pair $F(t), G(t)$ is completely controllable and the pair $F(t)$, $H^T(t)$ is completely observable.

Find sufficient conditions for system (8) to be ASP such that the closed loop output feedback system presented in Fig. 1 and described by

$$\dot{x} = Fx + GKe \tag{9a}$$

$$y = H^T x \tag{9b}$$

$$e = u - y \tag{10}$$

$$u_c = Ke \tag{11}$$

is Output Strictly Passive (OSP) [1].

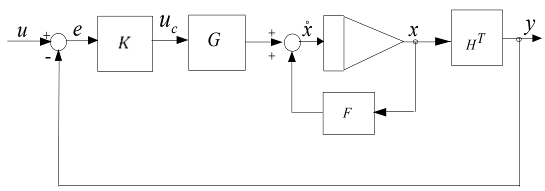

Figure 1: Output feedback system under consideration

III. PRELIMINARIES

The preliminary material includes two lemmas. Lemma 1 deals with a condition, in terms of energy function, that a system should satisfy if it is OSP. Lemma 2 deals with the parameterization of P that satisfies $PG = H$.

Lemma1

Lemma 1 is Corollary 2.3 of Theorem 2.2 in [1].

Assume that there exist a continuously differentiable function $V(\cdot) \geq 0$ and a measurable function $d(\cdot)$ such that $\int_0^t d(s)ds \geq 0$ for all $t \geq 0$. Then a system with input $u(t)$ and output y(t) is output strictly passive (OSP) if there exists $\varepsilon > 0$ such that $\dot{V} \leq y^T(t)u(t) - \epsilon y^T(t)y(t) - d(t)$ for all $t \geq 0$ and all functions $u(\cdot)$.

Lemma 2

Lemma 2 is Lemma 3 of [13].

Suppose that G and H are full rank. Then there exists a matrix $P = P^T > 0$ that satisfies (1b) if and only if

$$G^T H = H^T G > 0 \tag{12}$$

Furthermore, when (12) holds, all solution of (1b) are given by

$$P = H(G^T H)^{-1} H^T + G_\perp X G_\perp^T \tag{13}$$

where X is an arbitrary positive definite matrix with proper dimensions and $G_\perp$ is the orthogonal null space of G.

Note that $G_\perp^T G = 0$, $[G, G_\perp]$ is invertible and $G_\perp^T G_\perp = I$.

Also note that $PG = H(G^T H)^{-1} H^T G + G_\perp X G_\perp^T G = H$

IV. MAIN RESULTS

A CLOSED LOOP OSP CONDITIONS

The theorem below defines new closed loop OSP conditions for LTV systems in terms of a forward differential Riccati.

Theorem 1

If:

(i) F, Q and R^{-1} are uniformly bounded and $R^{-1} > 0$, $Q \geq 0$ are symmetric matrices

(ii) $\left[F, HR^{-1/2}\right]$ is uniformly completely observable

(iii) There exists a continuous, uniformly bounded symmetric positive definite $P(t)$ such that

$$\dot{P}(t) = P(t)F^T(t) + F(t)P(t) - P(t)H(t)R^{-1}(t)H^T(t)P(t) + Q(t) \ ; \ P(t_0) = P_0 \tag{14a}$$

$$P^{-1}(t)G(t) = H(t) \tag{14b}$$

(iv) The closed loop gain K is determined by $K = \alpha R^{-1}$ where R^{-1} is a parameter and $\alpha < 1/2$

Then the closed loop system of Fig. 1 is OSP.

Proof

The proof uses Corollary 2.3 of Theorem 2.2 in [1] (see Lemma 1of section III) together with the energy function $V(x,t) = \frac{1}{2}x^T P^{-1} x$ where P is uniformly bounded symmetric positive definite matrix that simultaneously satisfies equations (14a) and (14b). The details of the proof are presented in Appendix A.

Remarks:

1. Equation (14a) is identical to the covariance matrix equation associated with Kalman filter state estimation error. This equation is a forward differential Riccati equation in opposite to the backward differential Riccati equation usually used in optimal control.
2. Conditions (i) and (ii) imply that the solution $P(t)$ of the forward differential Riccati equation is continuous and uniformly bounded symmetric positive definite matrix.
3. The OSP conditions are given in terms of the solution $P(t)$ of a differential Riccati equation. The simultaneous solution of equations (14a) and (14b) is derived via the parametrization of $P^{-1}(t)$ that satisfies equation (14b). By using the parameterization of Lemma 2 of section III, the simultaneous solution of (14a) and (14b) can be obtained as a solution of an additional differential Riccati equation. See Theorem 2 in the next section.
4. In the case of linear time invariant system the new OSP conditions are defined by the steady state value of $P(t)$.

B. SYNTHESIS OF OSP LTV SYSTEMS

The synthesis of OSP linear time varying output feedback systems is based on Theorem 1 of the previous section. It is required to find a continuous, uniformly bounded symmetric positive definite solution $P(t)$ that simultaneously satisfies equations (14a) and (14b). The simultaneous solution of equations (14a) and (14b) is derived via the parametrization of $P^{-1}(t)$ that satisfies equation (14b).

The first step of the parameterization is to rewrite (14a) in terms of $P^{-1}(t)$.

$$\begin{aligned} -\frac{d(P^{-1})}{dt} &= P^{-1}\dot{P}P^{-1} \\ &= P^{-1}(PF^T + FP - PHR^{-1}H^TP + Q)P^{-1} \\ &= F^TP^{-1} + P^{-1}F - HR^{-1}H^T + P^{-1}QP^{-1} \end{aligned}$$

Using the notation

$$\overline{P} = P^{-1} \ ; \overline{P}_0 = P_0^{-1} \tag{15}$$

then

$$-\dot{\overline{P}} = F^T\overline{P} + \overline{P}F - HR^{-1}H^T + \overline{P}Q\overline{P} \quad ; \overline{P}(t_0) = \overline{P}_0 \tag{16a}$$

$$\overline{P}G = H \ ; \overline{P}_0 = P_0^{-1} \tag{16b}$$

The second step of the parameterization is to apply Lemma 3 of [13] (see Lemma 2 of section III) to equation (16b). Assume that G and H are full rank. Then there exists a matrix $\overline{P} = \overline{P}^T > 0$ that satisfies (16b) if and only if

$$G^TH = H^TG > 0 \tag{17}$$

Furthermore, when (17) holds, all solutions of (16b) satisfy

$$\overline{P} = H(G^TH)^{-1}H^T + G_\perp X G_\perp^T \tag{18}$$

where X is an arbitrary positive definite matrix with proper dimensions and $G_\perp$ is the orthogonal null space of G.

By using the notation

$$\overline{P}_i = H(G^TH)^{-1}H^T \tag{19}$$

equation (18) can be expressed as

$$\overline{P} = \overline{P}_i + G_\perp X G_\perp^T \tag{20}$$

Observe that under the assumption $G^TH = H^TG > 0$ the matrix $\overline{P}_i$ is symmetric.

Substitution of (20) in (16a) leads to an additional forward differential Riccati equation as described in the following theorem.

Theorem 2

Assume that each of the equations (14a) and (14b) has positive definite symmetric solution. Then the simultaneous solution $\overline{X}$ of (14a) and (14b), if exists, is the solution of the following forward differential Riccati equation

$$\dot{\overline{X}} = \overline{X}\,\overline{F}^T + \overline{F}\,\overline{X} - \overline{X}\,\overline{H}\,\overline{R}^{-1}\overline{H}^T\overline{X} + \bar{Q} \ ; \overline{X}(t_0) = X^{-1}(t_0) \tag{21}$$

where

$$-\dot{X} = \overline{F}^T X + X\overline{F} - \overline{H}\ \overline{R}^{-1}\ \overline{H}^T + X\overline{Q}X \tag{22a}$$

$$X(t_0) = G_\perp^T(t_0)\left[\overline{P}(t_0) - \overline{P}_i(t_0)\right]G_\perp(t_0) \tag{22b}$$

$$\overline{X} = X^{-1} \tag{23}$$

$$\overline{F} = G_\perp^T\left(F + Q\overline{P}_i\right)G_\perp + \dot{G}_\perp^T G_\perp \tag{24a}$$

Assuming that S below is a symmetric matrix, then

$$\begin{aligned} S &= HR^{-1}H^T - \left(\overline{P}_iF + F^T\overline{P}_i\right) - \overline{P}_iQ\overline{P}_i - \dot{\overline{P}}_i \\ &= LDL^T \end{aligned} \tag{24b}$$

$$\overline{P}_i = H(G^TH)^{-1}H^T \tag{24c}$$

$$\overline{H} = G_\perp^T L D^{1/2} \tag{24d}$$

$$\overline{R} = I \tag{24e}$$

$$\overline{Q} = G_\perp^T Q G_\perp \tag{24f}$$

Proof

Substitution of (20) in (16a), multiplication by $G_\perp^T$ from the left and by $G_\perp$ from the right and using the fact that $G_\perp^T G_\perp = I$ lead to equations (21) to (24) where the initial value of X is determined via equation (20).

Remarks:

1. Any bounded, square and symmetric S can be decomposed as LDL^T where D is diagonal, the diagonal entries are the eigenvalues of S, and the columns of L are the corresponding eigenvectors. The matrix L is orthogonal and satisfies $L^T = L^{-1}$ i.e. $L^T L = LL^T = I$. Therefore square and symmetric matrix S can be decomposed as $S = (LD^{1/2})(LD^{1/2})^T$.

2. Assuming square and symmetric S, then $S = LDL^T$. The condition $D \geq 0$ guarantees that $S \geq 0$ and that the solution $\overline{X}$, if exists, is positive definite. However, a positive definite solution may exists for S which is not semi-positive definite.

3. The matrix S is symmetric in each of the following cases. However, there are more special cases in which $\dot{\overline{P}}_i$ is symmetric and as a result S is also symmetric.

Case 1: The matrices F, G and H are time invariant. In this case, since $\dot{G} = \dot{H} = 0$ then $\dot{\overline{P}}_i = 0$ and S is symmetric.

Case 2: The matrices G and H are time invariant and the matrix $F(t)$ is time varying. In this case S is symmetric since $\dot{\overline{P}}_i = 0$.

Case 3: The case of SISO systems where $G^T H = H^T G$ is a scalar. In this case $\dot{\overline{P}}_i$ is symmetric if $\dot{H} = 0$. In this case S is symmetric since $\dot{\overline{P}}_i$ is symmetric and the matrices $F(t)$ and $G(t)$ remain time varying.

Theorem 3

If

(i) G and H are full rank and satisfy $G^T H = H^T G > 0$

(ii) The square matrix S is symmetric and can be decomposed as LDL^T with $D \geq 0$

(iii) $\overline{F}, \overline{Q}$ and $\overline{R}^{-1}$ are bounded and $\left[\overline{F},\ \overline{H}\,\overline{R}^{-1/2}\right]$ is uniformly completely observable.

Then the solution $X(t)$ of (22) and $\overline{X}(t)$ of (21) are positive definite bounded matrices and the system of Fig. 1 is OSP with $K = \alpha R^{-1}$ and $\alpha < 1/2$.

Proof

Following Lemma 3.2 of [14], if $\overline{F}, \overline{Q}$ and $\overline{R}^{-1}$ are bounded, and if $\left[\overline{F},\ \overline{H}\,\overline{R}^{-1/2}\right]$ is uniformly completely observable, then $\overline{X}(t)$ and $X(t)$ are positive definite bounded matrix. Following Theorem 1 and Lemma 2 of section III, if $P(t)$ is a common positive definite bounded solution of equations (14a) and (14b), and if

$X = G_\perp^T(\overline{P} - \overline{P}_i)G_\perp = G_\perp^T[P^{-1} - H(G^T H)^{-1}H^T]G_\perp$

is positive definite bounded matrix then the system of Fig. 1 with $K = \alpha R^{-1}$ and $\alpha < \frac{1}{2}$ is OSP.

Remarks:

1. If for specific values of R^{-1} and Q the matrix X is positive definite bounded matrix then the closed loop system is OSP.
2. The range of R^{-1} indicates the range of the gain K for which the closed loop system is OSP.
3. The matrix Q affects the matrix $\overline{F}$ of (24a) and hence the observability of $\left[\overline{F},\ \overline{H}\,\overline{R}^{-1/2}\right]$.
4. In linear time invariant system the differential Riccati equation of (21) is reduced to the algebraic Riccati equation .

Theorem 4 and Theorem 3 lead to Theorem 4 below which constitutes the new ASP test for LTV systems

Theorem 4

If

(i) $H^T G$ is positive definite symmetric matrix i.e., $G^T H = H^T G > 0$

(ii) F, Q and R^{-1} are uniformly bounded and $R^{-1} > 0$, $Q \geq 0$ are symmetric matrices

(iii) $[F, HR^{-1/2}]$ is uniformly completely observable

(iv) The solution $\bar{X}$ of equation (21) is positive definite symmetric matrix.

Then system (8) is ASP

Proof

This is a direct consequence of Theorem 2, Theorem 3 and the definition of ASP system [10], [11].

V. EXAMPLES

Example 1:

The system under test is the LTI system described by

$$G(s) = \frac{s^2 + s + 1}{s^3 + 1.1s^2 + 1.1s + 1}$$

This transfer function is minimum phase with zeros at $-0.5 \pm j\sqrt{3}/2$ and relative degree 1.

A minimal realization of $G(s)$ is

$$F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1.1 & -1.1 \end{bmatrix} \ ; \ G = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ ;$$

$H^T = [1 \quad 1 \quad 1]$

Hence

$G^T H = H^T G = 1$

$$\overline{P}_i = H(G^T H)^{-1} H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$G_\perp = \begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

The steady state solution X of equation (22a) is obtained via equation (21) where $X = \overline{X}^{-1}$

Let $R^{-1} = 1$ and $Q = 0.01\,I_3$. Then the ARE solution X is

$$X = \begin{bmatrix} 52.6125 & 14.6246 \\ 14.6246 & 10.7986 \end{bmatrix} > 0$$

With $R^{-1} = 1000$ and $Q = 0.01\,\mathrm{I}_3$ the solution X is still positive definite.

$$X = \begin{bmatrix} 228.5620 & -138.8339 \\ -138.8339 & 282.9816 \end{bmatrix} > 0$$

The closed loop system with $K = \alpha R^{-1}$ where $\alpha < 1/2$ is OSP. Observe that for the tested range of R^{-1} the range of K is $0.5 < K < 500$.

Remarks:

1. Following the ASP theorem [10], [15, 1st ed. pp.55, Lemma 1], any linear minimum-phase plant $\{F, G, H^T\}$ with $H^T G$ Positive Definite Symmetric is ASPR. The transfer function of this example is minimum phase with $H^T G$ Positive Definite Symmetric and therefore it is expected to be ASPR. As a result that the closed loop is OSP.
2. Example 1 deals with LTI system. In this case $\dot{X} = 0$ since the ARE solution is applied. By using equation (18) we find that $\dot{X} = G_\perp^T \dot{\overline{P}} G_\perp$. Observe that $\dot{X} = 0$ implies that $G_\perp^T \dot{\overline{P}} G_\perp = 0$ and hence $\dot{\overline{P}}$, calculated from equations (18) and (16a), doesn't have to be zero.

Example 2:

The system under test is the LTV system described by

$$F = \begin{bmatrix} 2/(t_f - t) & 0 & 0 & 2666.667/(t_f - t) & 0 \\ 0 & -41.667 & -466.667 & -666.667 & 0 \\ 0 & 1.000 & 0 & 0 & 0 \\ 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5/6 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ 3.5 \\ 0 \\ 0 \\ 1/6 \end{bmatrix} \quad ; \quad H^T = [1 \quad 0 \quad 0 \quad 0 \quad 1]$$

$$G^T H = H^T G = \frac{1}{6} > 0$$

The system is tested in the time interval $t \in [0\,,\, t_f - 0.02]$ where $t_f = 5\,sec$. Note that since $\dot{G} = \dot{H} = 0$ then $\dot{\overline{P}}_\iota = 0$ and S of (24b) is symmetric.

Let $Q = 0.01\,\mathrm{I}_5$ and $\overline{X}(\mathrm{t}_0) = G_{a\perp}^T \left[I_5 + \overline{P}_i\right]^{-1} \mathrm{G}_{a\perp}$. Then the solution X of equation (22) is obtained via equation (21) where $X = \overline{X}^{-1}$. The minimal value of the eigenvalues of X as a function of time, for $R^{-1} = 1$ and $R^{-1} = 10^5$, are presented in Fig. 2. For both values of R^{-1} the matrix X has real positive eigenvalues and is symmetric positive definite. Observe that for the tested range of R^{-1} the range of K is $0.5 < K < 50{,}000$.

VI. CONCLUSIONS

The paper presents new passivity conditions for square LTV output feedback systems which enable the formulation of a simple test for almost strict passivity (ASP). The existing ASP test requires the calculation of the associated zero dynamics matrices *M(t)* and *N(t)* and a proof of stability for the zeros differential equation. The new test requires the solution of a forward differential Riccati equation in the LTV case, and the solution of an algebraic Riccati equation in the linear time invariant (LTI) case. The proposed test simplifies the synthesis and design of output strictly passive systems. The examples discussed in the paper demonstrate the efficiency of the test.

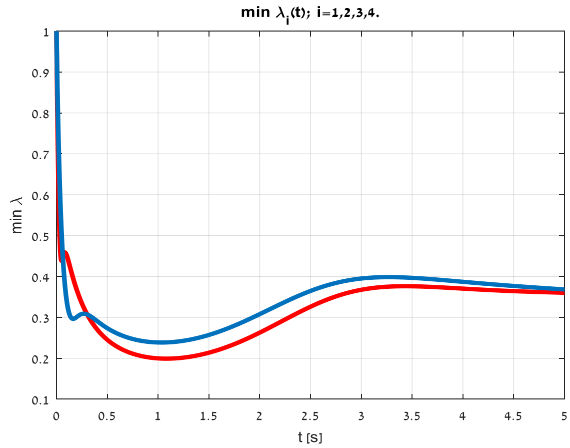

Figure 2: The minimal value of the eigenvalues of X as a function of time, for $R^{-1} = 1$ and $R^{-1} = 10^5$

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APPENDIX A

Proof of Theorem 1

The conditions for which the system is OSP are derived by using Lemma 1of section III and the energy function

$$V(x,t)=\tfrac{1}{2}x^TP^{-1}x \tag{A1}$$

where P is a continuous, uniformly bounded symmetric positive definite matrix that satisfies equation (14a) and (14b). Following Lemma 3.2 of [14] if F, Q and R^{-1} are bounded, and if $\left[F, HR^{-1/2}\right]$ is uniformly completely observable, then $P(t)$ of (14a) is bounded.

Differentiation of $V(x,t)$ yields

$$\begin{aligned}\dot V &= \tfrac{1}{2}\left(\dot x^TP^{-1}x + x^T\frac{d}{dt}(P^{-1})x + +x^TP^{-1}\dot x\right)\\ &= \tfrac{1}{2}(2x^TP^{-1}\text{GKe} + x^THR^{-1}H^Tx - x^TP^{-1}QP^{-1}x)\end{aligned}$$

Since P satisfies

$$P^{-1}G = H \tag{A2}$$

then

$$\begin{aligned}\dot V &= y^TKe + \tfrac{1}{2}y^TR^{-1}y - \frac{1}{2}x^TP^{-1}QP^{-1}x =\\ &= y^TK(u-y) + \tfrac{1}{2}y^TR^{-1}y - \tfrac{1}{2}x^TP^{-1}QP^{-1}x\\ &= y^TKu + y^T\left(K - \frac{1}{2}R^{-1}\right)y - \tfrac{1}{2}x^TP^{-1}QP^{-1}x\end{aligned}$$

If $K = \propto R^{-1}$ and $\alpha < 1/2$ then

$$\dot V = \alpha y^TR^{-1}u - y^T\left(\tfrac{1}{2} - \alpha\right)R^{-1}y - \tfrac{1}{2}x^TP^{-1}QP^{-1}x$$

Redefining the energy function V as

$$\overline{V} = \frac{V}{\alpha} = \frac{1}{2\alpha}x^TP^{-1}x \tag{A3}$$

leads to

$$\dot{\overline{V}} = y^TR^{-1}u - y^T\left(\tfrac{1}{2\alpha} - 1\right)R^{-1}y - \tfrac{1}{2\alpha}x^TP^{-1}QP^{-1}x \tag{A4}$$

Given that R^{-1} is uniformly bounded symmetric positive definite matrix then

$$0 < \beta_1 I \le R^{-1} \le \beta_2 I < \infty$$

where β_1 and β_2 are constant positive scalars. Thus

$$y^TR^{-1}u = u^TR^{-1}y \le \beta_2\, u^Ty$$

$$y^TR^{-1}y \ge \beta_1\, y^Ty$$

and therefore

$$\dot{\overline{V}} \le \beta_2\, u^Ty - \left(\frac{1}{2\alpha} - 1\right)\beta_1\, y^Ty - \frac{1}{2\alpha}x^TP^{-1}QP^{-1}x$$

Let $\tilde V$ be defined as

$$\tilde V = \frac{\overline{V}}{\beta_2} = \frac{1}{2\alpha\beta_2}x^TP^{-1}x \tag{A5}$$

Then

$$\dot{\tilde V} \le u^Ty - \left(\tfrac{1}{2\alpha} - 1\right)\frac{\beta_1}{\beta_2}\, y^Ty - \frac{1}{2\alpha\beta_2}x^TP^{-1}QP^{-1}x \tag{A6}$$

Equations (A5), (A6) satisfy the conditions of Lemma 1 of section III where

$$\tilde V = \frac{1}{2\alpha\beta_2}x^TP^{-1}x \ge 0\ ;\ \text{d(t)} = \frac{1}{2\alpha\beta_2}x^TP^{-1}QP^{-1}x \ge 0\ ;$$

$$\epsilon = \left(\frac{1}{2\alpha} - 1\right)\frac{\beta_1}{\beta_2} > 0$$

Q.E.D