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New Results on Stabilization of Stochastic Switching Systems Subject to Partly Available Semi-Markov Kernel ⋆

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Abstract: This paper investigates the stabilization issue for a class of discrete-time stochastic switching systems. The switching behavior is dominated by a semi-Markov process with finite sojourn time. Allowing for the fact that it is often difficult to get complete semi-Markov kernel (SMK) in practice, the elements in SMK of the model under study are considered to be partly accessible, which is more general than both semi-Markov model with complete SMK and Markov model with unknown transition probabilities. Sufficient stability condition is derived for the underlying system without any a priori knowledge, based on which a stabilization criterion is presented such that the closed-loop stochastic switching systems can be mean-square stable. In the end, the validity of the theoretical results is testified by a numerical example.

Keywords: Partly available semi-Markov kernel, semi-Markov chain, stability and stabilization, stochastic switching systems.

1. INTRODUCTION

For the past several decades, as a typical class of stochastic switching systems, Markov jump systems (MJSs) have been extensively investigated with considerably launched results, see, e.g., Sun and Zhao [2018], Benjelloun and Boukas [1998], Geromel et al. [2016], Lin et al. [2020], Shi et al. [1999], Vargas et al. [2013], Shen et al. [2011], Wang and Liu [2018]. However, in practice, it may be quite difficult to access all transition probabilities (TPs) accurately Ghaoui and Rami [1996], Karan et al. [2006], and moreover, controller design under incorrect TPs may bring about degraded performance, and even instability Xiong et al. [2005], Kalyanasundaram et al. [2004]. Aiming at coping with such a problem, unavailable TPs are depicted in the form of polytopic-type uncertainties Ghaoui and Rami [1996], Costa et al. [1999], norm-bounded uncertainties Xiong et al. [2005], Karan et al. [2006], as well as in the time-varying form Bolzern et al. [2010], Wu et al. [2014], requiring a priori knowledge for these uncertain TPs. Afterwards, some techniques are proposed in Zhang and Boukas [2009], Zhang and Lam [2010] to tackle the stability and stabilization issues of MJSs with partially unknown TPs without any knowledge of unknown TPs. However, such memoryless TPs in the Markov systems may be inapplicable to those practical systems involving TPs of memory property.

Fortunately, semi-Markov jump systems (S-MJSs) have been introduced to control community. The TPs in S-MJSs rely on the information of all the elapsed jump sequences and thus have the memory capacity. Moreover, the sojourn time between consecutive jumps does not have to be confined to exponential distribution and geometric distribution, which is quite distinct from MJSs. In view of these, the semi-Markov model is more powerful in modeling stochastic switching systems than the Markov model, and can describe practical dynamics more accurately, such as population systems Kao [1973], multiple-bus systems Mudge and Al-Sadoun [1985], and reward systems Ciardo et al. [1990]. Due to the memory feature of TPs of S-MJSs, achievements used to progress more slowly in contrast with MJSs. After 2000, a growing research interest in semi-Markov jump linear systems (S-MJLSs) has been witnessed and quite a few results have been reported, see, e.g., Schwartz and Haddad [2003], Hou et al. [2006], Huang and Shi [2013], Schioler et al. [2014], Qi et al. [2017], Tian et al. [2020a,b]. Among them, Schwartz and Haddad [2003], Hou et al. [2006] consider some specific types of probability distributions for sojourn time, Huang and Shi [2013], Qi et al. [2017] require a priori information of upper and lower bounds for time-varying TPs, and Schioler et al. [2014], Yang et al. [2016] assume complete knowledge of semi-Markov kernel. However, these ideal assumptions limit the applicability of these theoretical results.

With the introduction of SMK approach, the stability problem of S-MJLSs can be addressed by completely utilizing probability distribution information of sojourn time, which gets rid of the assumptions of assigned types of probability distributions for sojourn time and a priori information of both upper and lower bounds for TPs, but requires all the elements in the SMK have to be exactly known.

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Unfortunately, it is often very costly and inconvenient to get adequate samples for the probabilities of jumps and the probability distribution functions of sojourn time among different modes. Thus, obtaining all the elements in SMK can be quite laborious in engineering practice, and thereby investigations into S-MJLSs with incomplete information of SMK is of significance from control perspective. Up to now, the problem of stabilization under incomplete SMK remains unsolved, which motivates us for this study.

This paper aims at addressing the stability analysis and stabilization problems for a family of discrete-time s-tochastic switching systems. The switching dynamics of the system under study are governed by a semi-Markov chain with partially unavailable elements of the SMK. The main contribution lies in the consideration of incomplete SMK brought by the difficulty in obtaining adequate samples for the probabilities of mode switchings and the distribution probabilities of sojourn time between different modes, such that the designed controller can be competent to stabilize a larger scope of practical stochastic systems. Compared with Ning et al. [2020], no a priori information of unknown elements in SMK is required, which is a main advantage of the proposed theoretical results in this paper.

Notations: $\mathbb{R}$ and $\mathbb{Z}$ represent the sets of real numbers and integers, respectively; $\mathbb{R}_{>0}$, $\mathbb{R}_{\geq 0}$ and $\mathbb{R}_{[N_1,N_2]}$ signify the sets of positive real numbers, non-negative real numbers, and $\{l \in \mathbb{R} | N_1 \leq l \leq N_2\}$, respectively, while $\mathbb{Z}_{\geq N}$ and $\mathbb{Z}_{[N_1,N_2]}$ the sets $\{l \in \mathbb{Z} | l \geq N\}$ and $\{l \in \mathbb{Z} | N_1 \leq l \leq N_2\}$, respectively. Then, $\mathbb{R}^n$ indicates the n-dimensional Euclidean space. $\|\cdot\|$ denotes the Euclidean vector norm. The superscripts "-1" and "T" describe the inverse and the transposition of a vector or a matrix, respectively. The symbol "$\otimes$" symbolizes the Kronecker product of matrices. Additionally, diag$\{\cdots\}$ stands for a block-diagonal matrix. To reduce clutter, any off-diagonal entries introduced by symmetry are replaced by "$*$". Moreover, $\mathfrak{E}\{\cdot\}|_x$ represents the expectation operator conditioned on x. In addition, "$Q \succ \mathbf{0}$" (or "$Q \prec \mathbf{0}$") means that Q is a symmetric and positive (or negative) definite matrix. $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ are respectively used to imply the maximal and minimal eigenvalues of matrix Q, respectively. $\mathbf{0}$ and I_n stand for zero matrix and n-dimensional identity matrix, respectively.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider a stochastic switching system governed by a semi-Markov chain on a complete probability space as

$$x(k+1) = A_{r(k)}x(k) + B_{r(k)}u(k) \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ and $u(k) \in \mathbb{R}^{n_u}$ are the vectors of system state and control input, respectively; $\{r(k)\}_{k\in\mathbb{Z}_{\geq 0}}$ is a discrete-time semi-Markov process taking values in a finite set $\mathbb{M} \triangleq \{1, 2, \ldots, M\}$, and governs the switching among the M modes. Both the coefficient matrices $A_{r(k)}$ and $B_{r(k)}$ are known real-valued matrix functions of $r(k)$ with appropriate dimensions.

To stabilize system (1) with full adaptation to the mode switchings, a mode-dependent controller is adopted as

$$u(k) = K_{r(k)}x(k) \tag{2}$$

where $K_{r(k)}$ is the controller gain awaiting determination. After substituting (2) into (1), it gives the following resulting closed-loop system:

$$x(k+1) = \bar{A}_{r(k)}x(k) \tag{3}$$

where $\bar{A}_{r(k)} \triangleq A_{r(k)} + B_{r(k)}K_{r(k)}$.

In the sequel, R_n, k_n and S_n are used to represent the index of $r(k)$ at the nth jump, the time at the nth jump with $k_0 = 0$ and the sojourn time between the nth jump and the $(n+1)$th jump, respectively. We suppose that the sojourn time is upper-bounded by T_p for each $p \in \mathbb{M}$. Then, we invoke the following definitions for later use.

Definition 1. $\{(R_n, k_n)\}_{n\in\mathbb{Z}_{\geq 0}}$ is a discrete-time homogeneous Markov renewal chain (HMRC) if $\forall p \neq q \in \mathbb{M}$, $\forall s \in \mathbb{Z}_{[1,T_p]}$ and $\forall n \in \mathbb{Z}_{\geq 0}$, $\Pr(R_{n+1} = q, S_n = s | R_0, k_0, R_1, k_1, \ldots, R_n = p, k_n) = \Pr(R_{n+1} = q, S_n = s | R_n = p) = \Pr(R_1 = q, S_0 = s | R_0 = p)$.

Definition 2. The HMRC $\{(R_n, k_n)\}_{n\in\mathbb{Z}_{\geq 0}}$ is given. Then $\{R_n\}_{n\in\mathbb{Z}_{\geq 0}}$ is the embedded Markov chain (EMC) of the HMRC. $\{r(k)\}_{k\in\mathbb{Z}_{\geq 0}}$ is a semi-Markov chain (SMC) associated with the HMRC, if $r(k) = R_{n_{\max}(k)}$, $\forall k \in \mathbb{Z}_{\geq 0}$ where $n_{\max}(k) \triangleq \max\{n \in \mathbb{Z}_{\geq 0} | k \geq k_n\}$.

Since the mode switchings of system (1) are governed by an SMC, which unavoidably concerns mode transitions and probability distributions of sojourn time, the following notions and properties are crucial for later derivations.

Definition 3. Consider an SMC $\{r(k)\}_{k\in\mathbb{Z}_{\geq 0}}$ associated with the HMRC $\{(R_n, k_n)\}_{n\in\mathbb{Z}_{\geq 0}}$, the transition probabilities (TPs) of the EMC are defined by $\pi_{pq} \triangleq \Pr(R_{n+1} = q | R_n = p)$ and $\pi_{pp} \triangleq 0$ with $0 \leq \pi_{pq} \leq 1$, $\forall p \neq q \in \mathbb{M}$; the semi-Markov kernel (SMK) of the SMC is defined by $\Theta(s) \triangleq [\theta_{pq}(s)]_{p,q\in\mathbb{M}}$ where $\theta_{pq}(s) \triangleq \Pr(R_{n+1} = q, S_n = s | R_n = p)$ and $\theta_{pp}(s) \triangleq 0$ with $0 \leq \theta_{pq}(s) \leq 1$, $\forall p \neq q \in \mathbb{M}$, $\forall s \in \mathbb{Z}_{[1,T_p]}$; the sojourn-time probability mass functions (ST-PMFs) depending on both the current mode p and the target mode q are defined by $f_{pq}(s) \triangleq \Pr(S_n = s | R_{n+1} = q, R_n = p)$, $\forall p \neq q \in \mathbb{M}$, $\forall s \in \mathbb{Z}_{[1,T_p]}$, and particularly $f_{pp}(s) \triangleq 0$, $\forall p \in \mathbb{M}$, $\forall s \in \mathbb{Z}_{[1,T_p]}$.

Remark 1. In fact, TPs of EMC are actually TPs for the horizon of jump instants. Thus, we can deduce $\sum_{q\in\mathbb{M},q\neq 0} \pi_{pq} = 1$ and $\pi_{pp} = 0$, $\forall p \in \mathbb{M}$. Moreover, according to Definition 3, we can infer that $\sum_{s=1}^{T_p} \sum_{q\in\mathbb{M}} \theta_{pq}(s) = 1$ with $\theta_{pq}(s) = \pi_{pq} f_{pq}(s)$, $\forall p, q \in \mathbb{M}, \forall s \in \mathbb{Z}_{[1,T_p]}$.

In this paper, we consider a more general and practical scenario that not all the elements in the SMK $\Theta(s)$ can be accessed, which includes both the case of completely known SMK and the case of arbitrary switching. Further, we define that $\forall p \in \mathbb{M}$,

$$\begin{cases} \bar{\mathbb{M}}_p \triangleq \left\{q \in \mathbb{M} | \text{if } \theta_{pq}(s) \text{ is known}, \forall s \in \mathbb{Z}_{[1,T_p]}\right\} \\ \tilde{\mathbb{M}}_p \triangleq \left\{q \in \mathbb{M} | \text{if } \theta_{pq}(s) \text{ is unknown}, \exists s \in \mathbb{Z}_{[1,T_p]}\right\} \end{cases} \tag{4}$$

Further, if $\bar{\mathbb{M}}_p \neq \emptyset$, it can be described as $\bar{\mathbb{M}}_p \triangleq \{\bar{m}_p(1), \bar{m}_p(2), \ldots, \bar{m}_p(\bar{M}_p)\}$, where $1 \leq \bar{m}_p(1) < \bar{m}_p(2) < \ldots < \bar{m}_p(\bar{M}_p) \leq M$, in which $\bar{m}_p(n)$ represents the nth known element of $\bar{\mathbb{M}}_p$, which signifies the $\bar{m}_p(n)$th element of $\mathbb{M}$ as well; $\bar{M}_p \in \mathbb{Z}_{[1,M]}$ is the number of elements of $\bar{\mathbb{M}}_p$.

Remark 2. From the definitions in (4), it can be seen that $\mathbb{M}=\bar{\mathbb{M}}_p\cup\tilde{\mathbb{M}}_p$ with $\bar{\mathbb{M}}_p\cap\tilde{\mathbb{M}}_p=\emptyset$. If $\tilde{\mathbb{M}}_p=\emptyset$, then $\bar{\mathbb{M}}_p=\mathbb{M}$ in which $\bar{m}_p(n)=n,\forall n\in\mathbb{Z}_{[1,\bar{M}_p]}$ and $\bar{M}_p=M$.

Now, we give the definition of mean-square stability for the class of systems under study.

Definition 4. System (1) with $u(k)\equiv\mathbf{0}$ is mean-square (MS) stable if

$$\lim_{k\to\infty}\mathfrak{E}\left\{\|x(k)\|^2\right\}\Big|_{x(0),r(0)}=0 \tag{5}$$

holds for any initial conditions $x(0)\in\mathbb{R}^{n_x},r(0)\in\mathbb{M}$.

Remark 3. As the information about SMK $\Theta(s)$ is incomplete, we cannot figure out the σ in Yang et al. [2016] exactly. Thus, we choose MS stability instead of σ-error MS stability.

Our purposes are to establish an MS stability criterion for the system (1) in unforced form, and to design a state-feedback controller of form (2) such that the resulting closed-loop system (3) is MS stable.

3. MAIN RESULTS

For the purpose of establishing stability and stabilization criteria for the concerned system, we give the criterion of MS stability for system (1).

Theorem 1. Consider stochastic switching system (1) of unforced form with incomplete SMK. Given finite constants $\rho_p\in\mathbb{R}_{>0}$, $T_p\in\mathbb{Z}_{\geq1},p\in\mathbb{M}$, if there exists a set of matrices $P_p(h)\succ\mathbf{0},p\in\mathbb{M},h\in\mathbb{Z}_{[0,T_p-1]}$, such that $\forall p\in\mathbb{M},\forall h\in\mathbb{Z}_{[1,T_p-1]}$ $(T_p\in\mathbb{Z}_{\geq2})$,

$$\left(A_p^{\mathrm{T}}\right)^hP_p(h)A_p^h-\rho_pP_p(0)\prec\mathbf{0} \tag{6}$$

$\forall p\in\mathbb{M}$,

$$\sum_{s=1}^{T_p}\sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s)\left[\left(A_p^{\mathrm{T}}\right)^sP_q(0)A_p^s-P_p(0)\right]\prec\mathbf{0} \tag{7}$$

and $\forall p\in\mathbb{M},\forall q\in\tilde{\mathbb{M}}_p,\forall s\in\mathbb{Z}_{[1,T_p]}$,

$$\left(A_p^{\mathrm{T}}\right)^sP_q(0)A_p^s-P_p(0)\prec\mathbf{0} \tag{8}$$

then the unforced system is MS stable.

Proof. We construct a Lyapunov function as

$$\mathcal{F}(x(k),r(k),h)\triangleq x^{\mathrm{T}}(k)P_{r(k)}(h)x(k) \tag{9}$$

where $h\triangleq k-k_n$ denotes the time interval since the last mode switching. Letting $R_{\tilde{n}(k)}=p,\forall k\in\mathbb{Z}_{[k_n,k_{n+1}-1]}$ and $\forall p\in\mathbb{M}$, it holds that

$$\lambda_1\|x(k)\|^2\leq\mathcal{F}(x(k),p,h)\leq\lambda_2\|x(k)\|^2 \tag{10}$$

where $\lambda_1\triangleq\inf_{p\in\mathbb{M},h\in\mathbb{Z}_{[0,T_p-1]}}\{\lambda_{\min}(P_p(h))\}$ and $\lambda_2\triangleq\sup_{p\in\mathbb{M},h\in\mathbb{Z}_{[0,T_p-1]}}\{\lambda_{\max}(P_p(h))\}$ with $\lambda_{\min}(P_p(h))$ and $\lambda_{\max}(P_p(h))$ denoting the minimal and maximal eigenvalues of $P_p(h)$, respectively. From (9), $\mathcal{K}_1(\|x(k)\|)\leq\mathcal{F}(x(k),r(k),k-k_n)\leq\mathcal{K}_2(\|x(k)\|)$ can be satisfied where $\mathcal{K}_1(\cdot)=\lambda_1(\cdot)^2$ and $\mathcal{K}_2(\cdot)=\lambda_2(\cdot)^2$ are class $\mathcal{K}_\infty$ functions.

For $R_{\tilde{n}(k)}=p,\forall k\in\mathbb{Z}_{[k_n,k_{n+1}-1]}$, from (6), one has that $\forall S_n\in\mathbb{Z}_{\geq2},\forall k\in\mathbb{Z}_{[k_n+1,k_{n+1}-1]}$,

$$\begin{aligned}&\mathcal{F}(x(k_n+h),p,h)-\rho_p\mathcal{F}(x(k_n),p,0)\\&=x^{\mathrm{T}}(k)\left[\left(\bar{A}_p^{\mathrm{T}}\right)^hP_p(h)\bar{A}_p^h-\rho_pP_p(0)\right]x(k)<0\end{aligned} \tag{11}$$

which further implies that $\forall k\in\mathbb{Z}_{[k_n+1,k_{n+1}-1]}$, we have $\mathcal{F}(x(k),r(k),k-k_n)\leq\rho_{r(k_n)}\mathcal{F}(x(k_n),r(k_n),0)$, $S_n\in\mathbb{Z}_{\geq2}$.

Afterwards, we consider $R_{\tilde{n}(k_n)}=p,R_{\tilde{n}(k_{n+1})}=q$, $\forall p\neq q\in\mathbb{M}$, and assign $S_n=s$, $\forall s\in\mathbb{Z}_{[1,T_p]}$. Combining (7) and (8), together with the facts that $\sum_{s=1}^{T_p}\sum_{q\in\tilde{\mathbb{M}}_p}\theta_{pq}(s)=\varepsilon_p-\sum_{s=1}^{T_p}\sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s),\forall p\in\mathbb{M}$, one infers that

$$\begin{aligned}&\mathfrak{E}\{\mathcal{F}(x(k_{n+1}),q,0)\}|_{x(k_n),R_n=p\ (S_n\in\mathbb{Z}_{[1,T_p]})}-\mathcal{F}(x(k_n),p,0)\\
&=x^{\mathrm{T}}(k_n)\left[\sum_{s=1}^{T_p}\sum_{q\in\mathbb{M}}\frac{\theta_{pq}(s)}{\varepsilon_p}(A_p^{\mathrm{T}})^sP_q(0)A_p^s\right]x(k_n)\\
&\quad-x^{\mathrm{T}}(k_n)P_p(0)x(k_n)\\
&=\frac{x^{\mathrm{T}}(k_n)}{\varepsilon_p}\left\{\sum_{s=1}^{T_p}\sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s)\left[(A_p^{\mathrm{T}})^sP_q(0)A_p^s-P_p(0)\right]\right\}x(k_n)\\
&\quad+\sum_{s=1}^{T_p}\sum_{q\in\tilde{\mathbb{M}}_p}\frac{\theta_{pq}(s)}{\varepsilon_p}x^{\mathrm{T}}(k_n)\left[\left(A_p^{\mathrm{T}}\right)^sP_q(0)A_p^s-P_p(0)\right]x(k_n)\\
&\leq-\frac{1}{\varepsilon_p}\lambda_{\min}\left(-\sum_{s=1}^{T_p}\sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s)\left[(A_p^{\mathrm{T}})^sP_q(0)A_p^s-P_p(0)\right]\right)\\
&\quad\times\|x(k_n)\|^2-\sum_{s=1}^{T_p}\sum_{q\in\tilde{\mathbb{M}}_p}\frac{\theta_{pq}(s)}{\varepsilon_p}\\
&\quad\times\lambda_{\min}\left(-\left[(A_p^{\mathrm{T}})^sP_q(0)A_p^s-P_p(0)\right]\right)\|x(k_n)\|^2\\
&\leq\left(-\frac{1}{\varepsilon_p}\lambda_3-\sum_{s=1}^{T_p}\sum_{q\in\tilde{\mathbb{M}}_p}\frac{\theta_{pq}(s)}{\varepsilon_p}\lambda_4\right)\|x(k_n)\|^2\\
&\leq-\lambda_3\|x(k_n)\|^2\end{aligned}$$

where $\lambda_3\triangleq\inf_{p\in\mathbb{M}}\{\lambda_{\min}(-\sum_{s=1}^{T_p}\sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s)[(A_p^{\mathrm{T}})^s\times P_q(0)A_p^s-P_p(0)])\}$ and $\lambda_4\triangleq\inf_{p\in\mathbb{M},q\in\tilde{\mathbb{M}}_p,s\in\mathbb{Z}_{[1,T_p]}}\{\lambda_{\min}((A_p^{\mathrm{T}})^sP_q(0)A_p^s-P_p(0))\}$. Then, the following can be guaranteed

$$\begin{aligned}&\mathfrak{E}\{\mathcal{F}(x(k_{n+1}),r(k_{n+1}),0)\}|_{x(k_n),R_n}\\&\qquad-\mathcal{F}(x(k_n),r(k_n),0)\leq-\mathcal{K}_3(\|x(k_n)\|)\end{aligned}$$

where $\mathcal{K}_3(\cdot)\triangleq\lambda_3(\cdot)^2$ is a class $\mathcal{K}_\infty$ function.

By employing Lemma 1 in Yang et al. [2016], the MS stability of S-MJLS (1) has been proved. □

In the following, the condition for stabilization is given.

Theorem 2. Consider stochastic switching system (1) with incomplete SMK. Given finite constants $\rho_p\in\mathbb{R}_{>0}$, $T_p\in\mathbb{Z}_{\geq1},p\in\mathbb{M}$, the closed-loop system (3) is MS stable, if there exist sets of positive definite matrices $J_p(h,a)$, $\mathcal{J}_p(s,b)$, $\bar{\mathcal{J}}_p(T_p)$ and $J_p(0,c)$, $p\in\mathbb{M}$, $h\in\mathbb{Z}_{[0,T_p-1]}$, $a\in\mathbb{Z}_{[0,h]}$, $s\in\mathbb{Z}_{[1,T_p]}$, $b,c\in\mathbb{Z}_{[0,s]}$ and sets of matrices U_p and V_p, $p\in\mathbb{M}$, such that $\forall p\in\mathbb{M},\forall h\in\mathbb{Z}_{[1,T_p-1]}$ $(T_p\in\mathbb{Z}_{\geq2}),\forall a\in\mathbb{Z}_{[0,h-1]}$,

$$\begin{bmatrix}-J_p(h,a) & *\\ A_pV_p+B_pU_p & J_p(h,a+1)-V_p-V_p^{\mathrm{T}}\end{bmatrix}\prec\mathbf{0} \tag{12}$$

$$J_p(h,0)-\rho_pJ_p(0,0)\prec\mathbf{0} \tag{13}$$

$\forall p\in\mathbb{M},\forall b\in\mathbb{Z}_{[0,T_p-1]}$,

$$\begin{bmatrix} -\bar{\mathcal{J}}_p(b) & * & * \\ (\mathbf{A}_p\mathbf{V}_p+\mathbf{B}_p\mathbf{U}_p)\,\boldsymbol{\theta}_p(b+1) & \mathbf{L}_p & * \\ (A_pV_p+B_pU_p)\,\boldsymbol{\imath}_p(b+1) & \mathbf{0} & \bar{\mathcal{L}}_p(b+1) \end{bmatrix} \prec \mathbf{0} \quad (14)$$

$$\bar{\mathcal{J}}_p(0) - \sum_{s=1}^{T_p}\sum_{q\in\bar{\mathbb{M}}_p} \theta_{pq}(s) J_p(0,0) \prec \mathbf{0} \quad (15)$$

and $\forall p\in\mathbb{M}, \forall q\in\tilde{\mathbb{M}}_p, \forall c\in\mathbb{Z}_{[0,s-1]}, \forall s\in\mathbb{Z}_{[1,T_p]}$,

$$\begin{bmatrix} -J_p(0,c) & * \\ A_pV_p+B_pU_p & J_p(0,c+1)-V_p-V_p^{\mathrm{T}} \end{bmatrix} \prec \mathbf{0} \quad (16)$$

$$J_q(0,0) - J_p(0,s) \prec \mathbf{0} \quad (17)$$

where $\mathbf{A}_p \triangleq I_{\bar{M}_p}\otimes A_p$; $\mathbf{B}_p \triangleq I_{\bar{M}_p}\otimes B_p$; $\mathbf{U}_p \triangleq I_{\bar{M}_p}\otimes U_p$; $\mathbf{V}_p \triangleq I_{\bar{M}_p}\otimes V_p$; $\mathbf{L}_p \triangleq \mathrm{diag}\{L_{\bar{m}_p(1)}, L_{\bar{m}_p(2)},\ldots,L_{\bar{m}_p(\bar{M}_p)}\}$ with $L_c \triangleq J_c(0,0)-V_c-V_c^{\mathrm{T}}$, $c\in\bar{\mathbb{M}}_p, p\in\mathbb{M}$; and $\bar{\mathcal{L}}_p(b) \triangleq \bar{\mathcal{J}}_p(b)-V_p-V_p^{\mathrm{T}}$, $b\in\mathbb{Z}_{[1,T_p]}$ with $\bar{\mathcal{J}}_p(b) \triangleq \sum_{s=b+1}^{T_p}\mathcal{J}_p(s,b)$, $b\in\mathbb{Z}_{[0,T_p-1]}$; $\boldsymbol{\theta}_p(b) \triangleq \left[\sqrt{\theta_{a\bar{m}_p(1)}(b)}I_{n_x} \;\; \sqrt{\theta_{a\bar{m}_p(2)}(b)}I_{n_x} \;\cdots\; \sqrt{\theta_{a\bar{m}_p(\bar{M}_p)}(b)}I_{n_x}\right]^{\mathrm{T}}$, $b\in\mathbb{Z}_{[1,T_p]}$; $\boldsymbol{\imath}_p(b) \triangleq I_{n_x}, b\in\mathbb{Z}_{[1,T_p-1]}$, $\boldsymbol{\imath}_p(T_p)\triangleq\mathbf{0}$.

Proof. We notice the variables $J_p(h,a)\succ\mathbf{0}$, $\bar{\mathcal{J}}_p(b)\succ\mathbf{0}$ and $J_p(0,c)\succ\mathbf{0}$, $\forall p\in\mathbb{M}$, $\forall q\in\tilde{\mathbb{M}}_p$, $\forall h\in\mathbb{Z}_{[0,T_p-1]}$, $\forall a\in\mathbb{Z}_{[0,h]}$, $\forall b\in\mathbb{Z}_{[1,h]}$, $\forall c\in\mathbb{Z}_{[0,s]}$, $\forall s\in\mathbb{Z}_{[1,T_p]}$. As we can prove $J_p(h,a)-V_p-V_p^{\mathrm{T}} \succ -V_pJ_p^{-1}(h,a)V_p^{\mathrm{T}}$, $\bar{\mathcal{J}}_p(b)-V_p-V_p^{\mathrm{T}} \succ -V_p\bar{\mathcal{J}}_p^{-1}(b)V_p^{\mathrm{T}}$, and $(J_p(0,c)-V_p)\,J_p^{-1}(0,c)(J_p(0,c)-V_p)^{\mathrm{T}}\succ\mathbf{0}$, defining

$$J_p(h,a)\triangleq V_p^{\mathrm{T}}Q_p(h,a)V_p,\;\; \bar{\mathcal{J}}_p(b)\triangleq V_p^{\mathrm{T}}\bar{\mathcal{Q}}_p(b)V_p, \quad J_p(0,c)\triangleq V_p^{\mathrm{T}}Q_p(0,c)V_p \quad (18)$$

one can get

$$\begin{aligned} -Q_p^{-1}(h,a) &\prec J_p(h,a)-V_p-V_p^{\mathrm{T}}, \\ -\bar{\mathcal{Q}}_p^{-1}(b) &\prec \bar{\mathcal{J}}_p(b)-V_p-V_p^{\mathrm{T}}, \\ -Q_p^{-1}(0,c) &\prec J_p(0,c)-V_p-V_p^{\mathrm{T}} \end{aligned} \quad (19)$$

Combining (12)–(19), as well as replacing U_p with K_pV_p, $\forall p\in\mathbb{M}$, we obtain that $\forall p\in\mathbb{M}, \forall h\in\mathbb{Z}_{[1,T_p-1]}$ $(T_p\in\mathbb{Z}_{\geq 2}), \forall a\in\mathbb{Z}_{[0,h-1]}$,

$$\begin{bmatrix} -V_p^{\mathrm{T}}Q_p(h,a)V_p & * \\ (A_p+B_pK_p)V_p & -Q_p^{-1}(h,a+1) \end{bmatrix} \prec \mathbf{0} \quad (20)$$

$$V_p^{\mathrm{T}}Q_p(h,0)V_p - \rho_pV_p^{\mathrm{T}}Q_p(0,0)V_p \prec \mathbf{0} \quad (21)$$

$\forall p\in\mathbb{M}, \forall b\in\mathbb{Z}_{[0,T_p-1]}$,

$$\begin{bmatrix} -V_p^{\mathrm{T}}\bar{\mathcal{Q}}_p(b)V_p & * & * \\ (\mathbf{A}_p+\mathbf{B}_p\mathbf{K}_p)\,\mathbf{V}_p\boldsymbol{\theta}_p(b+1) & -\mathbf{Q}_p^{-1} & * \\ (A_p+B_pK_p)V_p\boldsymbol{\imath}_p(b+1) & \mathbf{0} & -\bar{\mathcal{Q}}_p^{-1}(b+1) \end{bmatrix} \prec \mathbf{0} \quad (22)$$

$$V_p^{\mathrm{T}}\bar{\mathcal{Q}}_p(0)V_p - \sum_{s=1}^{T_p}\sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s)V_p^{\mathrm{T}}Q_p(0,0)V_p \prec \mathbf{0} \quad (23)$$

and $\forall p\in\mathbb{M}, \forall q\in\tilde{\mathbb{M}}_p, \forall c\in\mathbb{Z}_{[0,s-1]}, \forall s\in\mathbb{Z}_{[1,T_p]}$,

$$\begin{bmatrix} -V_p^{\mathrm{T}}Q_p(0,c)V_p & * \\ (A_p+B_pK_p)V_p & -Q_p^{-1}(0,c+1) \end{bmatrix} \prec \mathbf{0} \quad (24)$$

$$V_p^{\mathrm{T}}Q_q(0,0)V_p - V_p^{\mathrm{T}}Q_p(0,s)V_p \prec \mathbf{0} \quad (25)$$

where $\bar{\mathcal{Q}}_p(b)\triangleq\sum_{s=b+1}^{T_p}\mathcal{Q}_p(s,b)$, $b\in\mathbb{Z}_{[0,T_p-1]}$, $\mathbf{K}_p\triangleq I_{\bar{M}_p}\otimes K_p$ and $\mathbf{Q}_p\triangleq\mathrm{diag}\{Q_{\bar{m}_p(1)},Q_{\bar{m}_p(2)},\ldots,Q_{\bar{m}_p(\bar{M}_p)}\}$.

Furthermore, perform congruence transformations to (20) and (24) by $\mathrm{diag}\{V_p^{-1},I_{n_x}\}$, to (21), (23) and (25) by V_p^{-1}, and to (22) by $\mathrm{diag}\{V_p^{-1}, I_{(\bar{M}_p+1)n_x}\}$. Then, applying Schur complement yields that $\forall p\in\mathbb{M}$, $\forall h\in\mathbb{Z}_{[1,T_p-1]}$ $(T_p\in\mathbb{Z}_{\geq 2})$, $\forall a\in\mathbb{Z}_{[0,h-1]}$,

$$\bar{A}_p^{\mathrm{T}}Q_p(h,a+1)\bar{A}_p - Q_p(h,a) \prec \mathbf{0} \quad (26)$$

$$Q_p(h,0) - \rho_pQ_p(0,0) \prec \mathbf{0} \quad (27)$$

$\forall p\in\mathbb{M}$, $\forall b\in\mathbb{Z}_{[0,T_p-1]}$,

$$\sum_{s=b+1}^{T_p}\left(\bar{A}_p^{\mathrm{T}}\mathcal{Q}_p(s,b+1)\bar{A}_p - \mathcal{Q}_p(s,b)\right) \prec \mathbf{0} \quad (28)$$

$$\sum_{s=1}^{T_p}\left(\mathcal{Q}_p(s,0) - \sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s)Q_p(0,0)\right) \prec \mathbf{0} \quad (29)$$

and $\forall p\in\mathbb{M}, \forall q\in\tilde{\mathbb{M}}_p, \forall c\in\mathbb{Z}_{[0,s-1]}, \forall s\in\mathbb{Z}_{[1,T_p]}$,

$$\bar{A}_p^{\mathrm{T}}Q_p(0,c+1)\bar{A}_p - Q_p(0,c) \prec \mathbf{0} \quad (30)$$

$$Q_q(0,0) - Q_p(0,s) \prec \mathbf{0} \quad (31)$$

where $\mathcal{Q}_p(s,s)\triangleq\sum_{q\in\bar{\mathbb{M}}_p}\theta_{pq}(s)Q_q(0,0)$.

From (26), we can get

$$\begin{aligned} &\sum_{a=0}^{h-1}\left(\bar{A}_p^{\mathrm{T}}\right)^a\left(\bar{A}_p^{\mathrm{T}}Q_p(h,a+1)\bar{A}_p - Q_p(h,a)\right)\bar{A}_p^a \\ &= \left(\bar{A}_p^{\mathrm{T}}\right)^hQ_p(h,h)\bar{A}_p^h - Q_p(h,0) \\ &\prec \mathbf{0} \end{aligned} \quad (32)$$

Combining (27) and (32), together with $Q_p(h,h)\triangleq P_p(h)$, $p\in\mathbb{M}, \forall h\in\mathbb{Z}_{[1,T_p]}$, it can give rise to (6) after replacing $\bar{A}_p$ with A_p. Moreover, according to (28), we can deduce

$$\begin{aligned} &\sum_{b=0}^{T_p-1}\left(\bar{A}_p^{\mathrm{T}}\right)^b\left[\sum_{s=b+1}^{T_p}\left(\bar{A}_p^{\mathrm{T}}\mathcal{Q}_p(s,b+1)\bar{A}_p - \mathcal{Q}_p(s,b)\right)\right]\bar{A}_p^b \\ &= \sum_{s=1}^{T_p}\left(\bar{A}_p^{\mathrm{T}}\right)^s\mathcal{Q}_p(s,s)\bar{A}_p^s - \sum_{s=1}^{T_p}\mathcal{Q}_p(s,0) \\ &\prec \mathbf{0} \end{aligned} \quad (33)$$

Also, it can be obtained from (30) that

$$\begin{aligned} &\sum_{c=0}^{s-1}\left(\bar{A}_p^{\mathrm{T}}\right)^c\left(\bar{A}_p^{\mathrm{T}}Q_p(0,c+1)\bar{A}_p - Q_p(0,c)\right)\bar{A}_p^c \\ &= \left(\bar{A}_p^{\mathrm{T}}\right)^sQ_p(0,s)\bar{A}_p^s - Q_p(0,0) \\ &\prec \mathbf{0} \end{aligned} \quad (34)$$

Then, pro- and post-multiplying $(\bar{A}_p^{\mathrm{T}})^s$ and $(\bar{A}_p)^s$ on (31), respectively, the following inequality can be obtained:

$$\left(\bar{A}_p^{\mathrm{T}}\right)^sQ_q(0,0)\left(\bar{A}_p\right)^s - \left(\bar{A}_p^{\mathrm{T}}\right)^sQ_p(0,s)\left(\bar{A}_p\right)^s \prec \mathbf{0} \quad (35)$$

By setting $Q_p(0,0)\triangleq P_p(0), Q_q(0,0)\triangleq P_q(0)$, $p\in\mathbb{M}, q\in\tilde{\mathbb{M}}_p$, and replacing $\bar{A}_p$ by A_p for all $p\in\mathbb{M}$, the combination of (29) and (33) implies (7) while the combination of (34) and (35) gives (8).

The MS stability of closed-loop system (3) can be guaranteed by following the proof in Theorem 1. □

Remark 4. It should be noted that if all the elements of the SMK are accessible, i.e., $\bar{\mathbb{M}}_p=\mathbb{M}$ and $\tilde{\mathbb{M}}_p=\emptyset, \forall p\in\mathbb{M}$, then Theorem 2 will reduce to Theorem 4 in Yang et al.

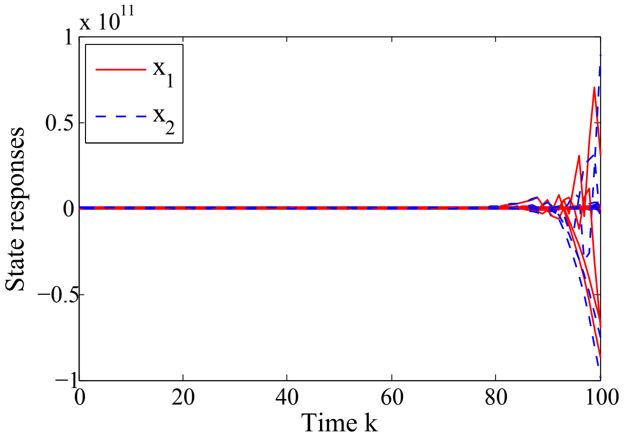

Fig. 1. State responses $x(k)$ of the unforced stochastic switching system for 100 realizations.

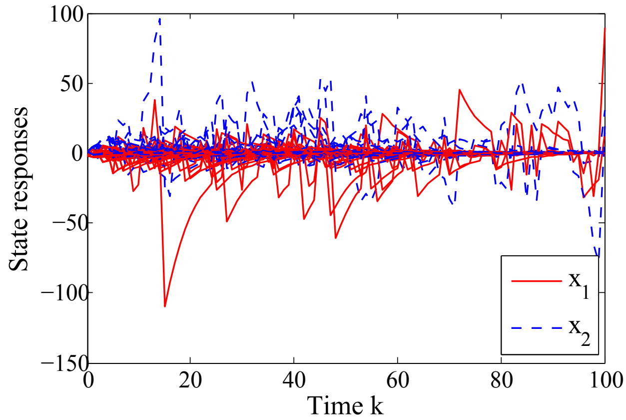

Fig. 2. State responses $x(k)$ of the closed-loop stochastic switching system for 100 realizations subject to erroneously presumptive SMK.

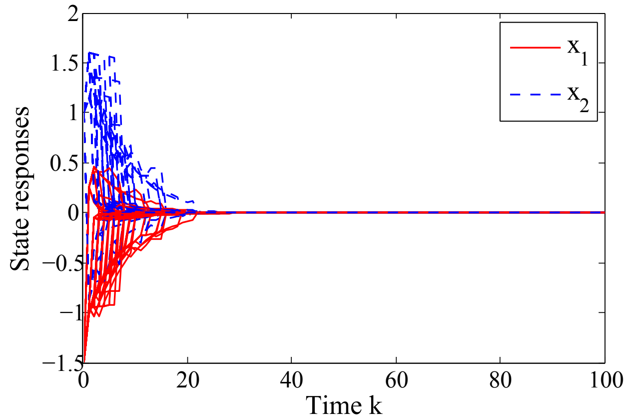

Fig. 3. State responses $x(k)$ of the closed-loop stochastic switching system for 100 realizations by Theorem 2.

[2016]. Moreover, if $\tilde{\mathbb{M}}_p = \mathbb{M}$ and $\bar{\mathbb{M}}_p = \emptyset$, $\forall p \in \mathbb{M}$, and $T_p \to \infty$ for all $p \in \mathbb{M}$, then the system we consider will become an arbitrary switching system.

4. NUMERICAL EXAMPLE

A stochastic switching system of form (1) with 3 modes is presented by the following parameter matrices:

$$A_1 = \begin{bmatrix} 0.4058 & 0.7402 \\ -0.4416 & 1.6236 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.8721 & -1.3955 \\ 1.0465 & 0.6975 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.2661 & 0.5100 \\ -1.3380 & 1.4563 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

It is considered that the mode switching is governed by an SMC with finite sojourn time among these modes. The upper bounds of sojourn time are set to be $T_1 = 10$, $T_2 = 8$, $T_3 = 6$. The accessible TPs of the associated EMC are assigned as $\pi_{21} = 0.7$, $\pi_{23} = 0.3$, $\pi_{31} = 0.2$ and $\pi_{32} = 0.8$, while π_{12} and π_{13} are unknown TPs. Meanwhile, we suppose that the known ST-PMFs are $f_{12}(s) = 0.6^s \cdot 0.4^{10-s} \cdot 10!/((10-s)!s!)$, $f_{13}(s) = 0.8^{s-1} \cdot 0.2$, $f_{23}(s) = 0.5^8 \cdot 8!/((8-s)!s!)$, $f_{31}(s) = 0.6^{s-1} \cdot 0.4$ and $f_{32}(s) = 0.3^{(s-1)^{0.8}} - 0.3^{s^{0.8}}$, and the inaccessible ST-PMF is $f_{21}(s)$. $\theta_{23}(s)$, $\theta_{31}(s)$ and $\theta_{32}(s)$ can be calculated out while $\theta_{12}(s)$, $\theta_{13}(s)$ and $\theta_{21}(s)$ are unavailable.

The actual expression of the inaccessible ST-PMF is $f_{21}(s) = 0.2^{(s-1)^2} - 0.2^{s^2}$ and the actual values of the unknown TPs are $\pi_{12} = 0.1$ and $\pi_{13} = 0.9$. As shown in Fig. 1, the system is unstable without control inputs. Then, let $\rho_1 = 0.5$, $\rho_2 = 1.2$ and $\rho_3 = 1.5$ in this example. With the employment of the state-feedback controller implemented based on the complete description of SMK by presuming the inaccessible ST-PMF to be $f_{21}(s) = 0.1^{(s-1)^2} - 0.1^{s^2}$ and the unknown TPs to be $\pi_{12} = 0.8$ and $\pi_{13} = 0.2$, Fig. 2 depicts the state responses of the resulting closed-loop S-MJLS with 100 realizations of different random jumping sequences. Then, turning to Theorem 2 in our paper under the actual $f_{21}(s)$, π_{12} and π_{13} the same as those in Fig. 2. The corresponding state responses of the closed-loop S-MJLS are plotted in Fig. 3 with 100 realizations of different random jumping sequences. The difference between Fig. 2 and Fig. 3 manifests that the controller designed under an incorrect SMK can deteriorate the stabilization performance of the system, and our proposed stabilization method can effectively resolve the problem against unavailable elements in SMK.

5. CONCLUSIONS

The paper has proposed a stabilization technique for a class of discrete-time stochastic switching systems in which the mode switchings are depicted by a semi-Markov chain with incomplete information of SMK. By virtue of Lyapunov stability theory, sufficient conditions for the MS stability of the unforced stochastic switching system have been exploited without prerequisite knowledge of unknown elements in SMK, based on which the desired mode-dependent controller has been carried out such that the resulting closed-loop system can be MS stable. Finally, the validity of the controller design strategy has been demonstrated by a numerical example.

REFERENCES

Benjelloun, K. and Boukas, E.K. (1998). Mean square stochastic stability of linear time-delay system with Markovian jumping parameters. *IEEE Transactions on Automatic Control*, 43(10), 1456–1460.

Bolzern, P., Colaneri, P., and Nicolao, G.D. (2010). Markov jump linear systems with switching transition rates: Mean square stability with dwell-time. *Automatica*, 46(6), 1081–1088.

Ciardo, G., Marie, R.A., Sericola, B., and Trivedi, K.S. (1990). Performability analysis using semi-Markov reward processes. *IEEE Transactions on Computers*, 39(10), 1251–1264.

Costa, O.L.V., do Val, J.B.R., and Geromel, J.C. (1999). Continuous-time state-feedback H_2-control of Markovian jump linear systems via convex analysis. *Automatica*, 35(2), 259–268.

Geromel, J.C., Deaecto, G.S., and Colaneri, P. (2016). Minimax control of Markov jump linear systems. *International Journal of Adaptive Control & Signal Processing*, 30(8-10), 1152–1162.

Ghaoui, L.E. and Rami, M.A. (1996). Robust state-feedback stabilization of jump linear systems via LMIs. *International Journal of Robust & Nonlinear Control*, 6(9-10), 1015–1022.

Hou, Z., Luo, J., Shi, P., and Nguang, S.K. (2006). Stochastic stability of Ito differential equations with semi-Markovian jump parameters. *IEEE Transactions on Automatic Control*, 51(8), 1383–1387.

Huang, J. and Shi, Y. (2013). Stochastic stability and robust stabilization of semi-Markov jump linear systems. *International Journal of Robust & Nonlinear Control*, 23(18), 2028–2043.

Kalyanasundaram, S., Chong, E.K.P., and Shroff, N.B. (2004). Markov decision processes with uncertain transition rates: Sensitivity and max hyphen min control. *Asian Journal of Control*, 6(2), 253–269.

Kao, E.P.C. (1973). A semi-Markovian population model with application to hospital planning. *IEEE Transactions on Systems, Man and Cybernetics*, SMC-3(4), 327–336.

Karan, M., Shi, P., and Kaya, C.Y. (2006). Transition probability bounds for the stochastic stability robustness of continuous- and discrete-time Markovian jump linear systems. *Automatica*, 42(12), 2159–2168.

Lin, W., Li, X., Yang, Z., Xiong, S., Luo, Y., and Zhao, X. (2020). Modeling of 3D rock porous media by combining X-ray CT and Markov chain Monte Carlo. *Journal of Energy Resources Technology-Transactions of the ASME*, 142(1), 013001.

Mudge, T.N. and Al-Sadoun, H.B. (1985). A semi-Markov model for the performance of multiple-bus systems. *IEEE Transactions on Computers*, C-34(10), 934–942.

Ning, Z., Zhang, L., and Colaneri, P. (2020). Semi-Markov jump linear systems with incomplete sojourn and transition information: Analysis and synthesis. *IEEE Transactions on Automatic Control*, 65(1), 159–174.

Qi, W., Park, J.H., Cheng, J., and Kao, Y. (2017). Robust stabilization for nonlinear time-delay semi-Markovian jump systems via sliding mode control. *IET Control Theory & Applications*, 11(10), 1504–1513.

Schioler, H., Simonsen, M., and Leth, J. (2014). Stochastic stability of systems with semi-Markovian switching. *Automatica*, 50(11), 2961–2964.

Schwartz, C. and Haddad, A.H. (2003). Control of jump linear systems having semi-Markov sojourn times. In *Proceedings of 42nd IEEE Conference on Decision and Control*, 2804–2805. Mad, Hawaii, USA.

Shen, H., Chu, Y., Xu, S., and Zhang, Z. (2011). Delay-dependent H_∞ control for jumping delayed systems with two Markov processes. *International Journal of Control, Automation and Systems*, 9(3), 437–441.

Shi, P., Boukas, E.K., and Agarwal, R.K. (1999). Control of Markovian jump discrete-time systems with norm bounded uncertainty and unknown delay. *IEEE Transactions on Automatic Control*, 44(11), 2139–2144.

Sun, Y. and Zhao, J. (2018). Passivity-based stabilization for switched stochastic nonlinear systems. *IEEE/CAA Journal of Automatica Sinica*, DOI: 10.1109/JAS.2018.7511108.

Tian, Y., Yan, H., Dai, W., Chen, S., and Zhan, X. (2020a). Observed-based asynchronous control of linear semi-Markov jump systems with time-varying mode emission probabilities. *IEEE Transactions on Circuits and Systems II: Express Briefs*, DOI: 10.1109/TCSI-I.2020.2973242.

Tian, Y., Yan, H., Zhang, H., Yang, S.X., and Li, Z. (2020b). Observed-based finite-time control of nonlinear semi-Markovian jump systems with saturation constraint. *IEEE Transactions on Systems, Man and Cybernetics: Systems*, DOI: 10.1109/TSMC.2019.2962318.

Vargas, A.N., Costa, E.F., and do Val, J.B.R. (2013). On the control of Markov jump linear systems with no mode observation: application to a DC motor device. *International Journal of Robust & Nonlinear Control*, 23(10), 1136–1150.

Wang, J. and Liu, C. (2018). Stabilization of uncertain systems with Markovian modes of time delay and quantization density. *IEEE/CAA Journal of Automatica Sinica*, 5(2), 463–470.

Wu, Z.G., Shi, P., Su, H., and Chu, J. (2014). Asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities. *Automatica*, 50(1), 180–186.

Xiong, J., Lam, J., Gao, H., and Ho, D.W.C. (2005). On robust stabilization of Markovian jump systems with uncertain switching probabilities. *Automatica*, 41(5), 897–903.

Yang, T., Zhang, L., and Yin, X. (2016). Time-varying gain-scheduling σ-error mean square stabilisation of semi-Markov jump linear systems. *IET Control Theory & Applications*, 10(11), 1215–1223.

Zhang, L. and Boukas, E.K. (2009). Stability and stabilization of Markovian jump linear systems with partly unknown transition probabilities. *Automatica*, 45(2), 463–468.

Zhang, L. and Lam, J. (2010). Necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions. *IEEE Transactions on Automatic Control*, 55(7), 1695–1701.