Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Disturbance Observer Based Repetitive Control System with Non-minimal State Space Realization and Anti-windup Mechanism

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Abstract: This paper develops a disturbance observer-based repetitive control system using a non-minimal state-space realization in which all state variables are chosen to correspond to the system's input and output variables and their past values. To enable the repetitive control system to follow a periodic reference signal or reject a disturbance signal of the same nature, a disturbance observer is used to estimate an input disturbance that contains the same frequency characteristics. This new approach differs from previously published design in repetitive control because it separates the design procedure into two simple tasks: first, stabilization by the design of a non-minimal state feedback control; and second, to independently incorporate the periodic modes via the estimation of the disturbance. Moreover, because this design ensures the stability of the disturbance observer, its implementation contains an anti-windup mechanism when the control signal reaches its maximum or minimum value. Without the complication of an observer for the state variables, the detection of a disturbance occurs earlier and the repetitive controller acts much faster than in the case of minimal state controller incorporating an observer. This leads to considerable performance improvement, with excellent disturbance rejection achieved with smaller control signal variations.

Keywords: repetitive control, non-minimal state-space realization, disturbance observer, disturbance rejection, anti-windup mechanism.

1. INTRODUCTION

Mathematical models play a fundamental role in control engineering applications. Two types of models are typically encountered. One is the class of physical models, such as those for electrical machines and power converters, which are based on the application of physical laws. Another is obtained through data analysis using the tools from system identification (for background see, e.g., Ljung [1999], Soderstrom [2018], Young [2012]). The second type of model is most commonly obtained in transfer-function form and is typically encountered in process control.

If the state variables cannot be measured in an application of state feedback control, it is a common practice to use an observer to estimate them. In the case of transfer-function models, the state variables are not known unless they are specifically chosen to correspond to the sets of input and output variables. This is indeed the framework of non-minimal state-space (NMSS) feedback control (Young et al. [1987], Wang and Young [1988], Taylor et al. [2013]). The non-minimal state-space realization has also been used in the design of model predictive controllers (Wang and Young [2006],Wang [2009]). The advantages of exploiting NMSS state feedback control include the avoidance of observer design and implementation and hence faster closed-loop response to disturbance rejection.

The main objective of this paper is to explore how the design of repetitive control systems can be enhanced by utilising an NMSS model for full state feedback control. In particular, it considers repetitive control systems that have the capability to track a multi-frequency periodic reference signal, or reject the same type of disturbance signal. To achieve these objectives, the 'internal model control' principle (Francis and Wonham [1976]), requires that the characteristics of the reference signal or the disturbance, as appropriate, have to be embedded into the control system design. In this context, there are two interesting related topics:

(1) First, the use of a disturbance observer to produce a repetitive control system that will naturally embed the periodic modes either identified from the reference signal or the disturbance signal;
(2) second, to avoid the estimation of the state variables using the NMSS model and hence a reduction of

the dimensions of the estimated variables, leading to simplicity in design and implementation.

In practical terms, deploying a disturbance observer in a repetitive control system design naturally provides an anti-windup mechanism in the event that the control signal reaches its operational limits. Next, the required background results are given.

2. DISTURBANCE OBSERVER BASED REPETITIVE CONTROL

2.1 Mathematical Model for Repetitive Controller Design

Assume that a discrete-time single-input single-output linear system is described by the difference equation:

$$\begin{aligned} y(k+1) = &-a_1 y(k) - a_2 y(k-1) \ldots - a_n y(k-n) \\ &+ b_1 u(k) + b_2 u(k-1) + \ldots + b_n u(k-n) \end{aligned} \quad (1)$$

where $u(k)$ and $y(k)$ are the input and output variables. The model coefficients a_1, a_2, …, a_n and b_1, b_2, …, b_n are obtained either from system identification or from mathematical modeling.

Denoting the periodic reference signal by $r(k)$, the error between the output variable and the reference signal is

$$e(k) = y(k) - r(k)$$

If the input periodic disturbance is denoted by $\mu(k)$, the intermediate control signal, with the disturbance, is written as

$$\tilde{u}(k) = u(k) + \mu(k) \quad (2)$$

Using these reference and the disturbance signals, the discrete-time model on which subsequent analysis is based has the form

$$\begin{aligned} e(k+1) = &-a_1 e(k) - a_2 e(k-1) \ldots - a_n e(k-n) \\ &+ b_1 \tilde{u}(k) + b_2 \tilde{u}(k-1) + \ldots + b_n \tilde{u}(k-n) \end{aligned} \quad (3)$$

To convert this model into a state-space form, the state variables are selected as the input and the output signals, including their relevant past values, i.e.,

$$x(k) = [e(k) \;\; \ldots \; e(k-n) \;\; \tilde{u}(k-1) \; \ldots \tilde{u}(k-n)]^T$$

The non-minimal state-space representation of (3) can then be written as:

$$x(k+1) = A_m x(k) + B_m \tilde{u}(k) \quad (4)$$

$$e(k) = C_m x(k) \quad (5)$$

where

$$A_m = \begin{bmatrix} -a_1 & -a_2 & \ldots & b_2 & \ldots & b_n \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ldots & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 & \ddots & 0 \\ 0 & 0 & \ldots & 0 & 1 & 0 \end{bmatrix}; \quad B_m = \begin{bmatrix} b_1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C_m = [1 \; 0 \; 0 \; \ldots \; 0 \; 0]$$

2.2 Repetitive Control Law

The repetitive control system is based on the design of a state feedback controller and the disturbance observer. The controller gain K is chosen such that the closed-loop system:

$$x(k+1) = (A_m - B_m K)x(k)$$

is stable with its eigenvalues strictly within the unit circle of the complex plane. With this design, the intermediate control signal is calculated as

$$\tilde{u}(k) = -Kx(k)$$

and the control signal is

$$u(k) = \tilde{u}(k) - \hat{\mu}(k)$$

where $\hat{\mu}(k)$ is the estimated disturbance signal.

2.3 Disturbance Observer

It is now necessary to consider how to estimate the periodic input disturbance $\hat{\mu}(k)$ for the repetitive control system design, given that the whole of the state vector $x(k)$ is measured.

As in previous work (Wang et al. [2013]), it is assumed that either the reference signal $r(k)$ or the input disturbance signal $\mu(k)$ has been analyzed to obtain the dominant frequency components, leading to a polynomial $D(q^{-1})$, where q^{-1} denotes the backward difference operator, as model of these components. Also the input disturbance signal $\mu(k)$ can be written as:

$$\mu(k) = \frac{\epsilon(k-1)}{D(q^{-1})} \quad (6)$$

where $\epsilon(k)$ is a zero-mean white noise sequence. The polynomial $D(q^{-1})$ has all zeros on the unit circle, as derived from the frequency analysis of either the reference signal or the disturbance signal. For example, if the reference signal is sinusoidal with N samples over a period T, the corresponding $D(q^{-1})$ is

$$D(q^{-1}) = 1 - 2\cos\frac{2\pi}{N} q^{-1} + q^{-2}$$

Also it is assumed that the $D(q^{-1})$ polynomial has order n_d and is written as

$$D(q^{-1}) = 1 + d_1 q^{-1} + d_2 q^{-2} + \ldots + d_{n_d} q^{-n_d} \quad (7)$$

where the coefficients d_1, d_2, …, d_{n_d} are known.

From the definition of the input disturbance (2) and the state-space model (4), it follows that the input disturbance $\mu(k)$ satisfies the following equation:

$$B_m \mu(k) = x(k+1) - A_m x(k) - B_m u(k) \quad (8)$$

Multiplying across this equation from the left by C_m gives

$$\begin{aligned} C_m B_m \mu(k) &= C_m x(k+1) - C_m A_m x(k) - C_m B_m u(k) \\ &= e(k+1) - C_m A_m x(k) - C_m B_m u(k) \end{aligned} \quad (9)$$

and one possible way to reconstruct the input periodic disturbance $\mu(k)$ based on this last equation. However, this would not be sufficiently accurate to generate the repetitive control signal because of the uncertainties in the mathematical model and the requirement to have available the feedback error $e(k+1)$ at the current time k. Consequently, the design of an observer to estimate $\mu(k)$ based on the disturbance model (6) is required.

If (6) is written in the following difference equation form:

$$\mu(k+1) = -d_1\mu(k) - d_2\mu(k-1) - \ldots - d_{n_d}\mu(k-n_d) + \epsilon(k) \tag{10}$$

then the state vector $p(k)$ can be formulated as

$$p(k) = [\mu(k)\ \mu(k-1)\ \ldots\ \mu(k-n_d)]^T$$

and the associated state-space model describing the dynamics of the disturbance takes the form

$$p(k+1) = A_d p(k) + B_d \epsilon(k) \tag{11}$$

$$\mu(k) = C_d p(k) \tag{12}$$

where (in the case of $n_d = 3$ for ease of presentation)

$$A_d = \begin{bmatrix} -d_1 & -d_2 & -d_{n_d} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B_d = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_d = [1\ 0\ 0]$$

The measurement of disturbance is $C_m B_m \mu(k)$, based on the right-hand side of (9). With the assumption $C_m B_m \neq 0$, the pair $A_d,\ C_m B_m C_d$ is observable. Hence a disturbance observer is constructed for the estimation of $p(k)$, leading to:

$$\begin{aligned}\hat{p}(k+1) &= A_d\hat{p}(k) + K_{ob}(C_m B_m \mu(k) - C_m B_m C_d \hat{p}(k)) \\ &= A_d\hat{p}(k) + K_{ob}(e(k+1) - C_m A_m x(k) \\ &\quad - C_m B_m u(k) - C_m B_m C_d \hat{p}(k))\end{aligned} \tag{13}$$

where the observer gain K_{ob} is chosen based on the pair $\{A_d, C_m B_m C_d\}$ such that the observer error system is stable. Here, the observer error system is defined as

$$\tilde{p}(k+1) = (A_d - K_{ob} C_m B_m C_d)\tilde{p}(k) + B_d \epsilon(k) \tag{14}$$

where $\tilde{p}(k) = p(k) - \hat{p}(k)$. This error system is obtained by substituting (9) into (13), and subtracting (13) from (11).

The repetitive control system design procedure involves two tasks. The first is to design the state feedback controller K; and the second is to design the disturbance observer gain K_{ob}. Both of these tasks can be done in a straightforward manner. In particular, the disturbance dynamics are of relatively low order because the use of the non-minimal state-space model avoids the estimation of the state variables. This is especially important when the system model is of higher order.

The repetitive control system with the anti-windup mechanism is not implementable in the form given above since the disturbance observer (13) is not implementable in its current form since the right-hand side involves the feedback error at $k+1$. In order to avoid this problem, an intermediate variable is defined as $\hat{q}(k) = \hat{p}(k) - K_{ob}e(k)$ and then by routine manipulations

$$\begin{aligned}\hat{q}(k+1) &= (A_d - K_{ob} C_m B_m C_d)\hat{q}(k) \\ &\quad + (A_d - K_{ob} C_m B_m C_d) K_{ob} e(k) \\ &\quad - K_{ob}(C_m A_m x(k) + C_m B_m u(k))\end{aligned} \tag{15}$$

which is implementable. Given an initial state vector $\hat{q}(0)$ and the control signal $u(k)$, output signal $y(k)$ and the reference signal $r(k)$, (15) provides a real-time estimation of the disturbance signal $\hat{\mu}(k)$.

3. IMPLEMENTATION OF REPETITIVE CONTROL SYSTEM WITH ANTI-WINDUP MECHANISM

In addition to the simplicity in the design of the repetitive control system when using the disturbance observer, implementation of the control system has a naturally occurring anti-windup mechanism when the control signal reaches its operational limits. This is because the sinusoidal modes that have been embedded into the repetitive control system are introduced through the estimation, which is a stable realization (see (15)) in the disturbance model.

For the implementation of the repetitive control system with its anti-windup mechanism, we assume that the control signal is constrained such that

$$u^{min} \leq u(k) \leq u^{max}$$

At the initial stage, the current and past control signal and output signal are known so that the initial state vector $x(0)$ is given, and the initial state $\hat{q}(0)$ is chosen. The resulting computational algorithm is summarized as follows.

(1) Compute the estimated input disturbance:
$$\hat{p}(k) = \hat{q}(k) + K_{ob}e(k); \quad \hat{\mu}(k) = C_d\hat{p}(k).$$
(2) Compute the control signal by subtracting the estimated disturbance from the feedback control law:
$$u(k) = -Kx(k) - \hat{\mu}(k).$$
(3) Implement saturation limits on the control signal:
$$u(k)^{act} = \begin{cases} u^{min}, & if\ u(k) < u^{min} \\ u(k), & if u^{min} \leq u(k) \leq u^{max} \\ u^{max}, & if u(k) > u^{max}. \end{cases}$$
(4) Update the disturbance observer with the saturation information based on (15), with the control signal replaced by $u(k)^{act}$.
(5) Send the control signal $u(k)^{act}$ to the actuator and repeat from Step 1 when the next sampling period is available.

4. CASE STUDY 1

To highlight the application of the new design this section gives the results of application to the model of the dynamics of a gantry robot, shown in Figure 1, which replicates the 'pick and place' operation, commonly found in a variety of industrial applications. Moreover, the operations required must be performed in synchronization with a conveyor system.

4.1 Modelling and Control System Design

For modeling and control design purposes, this gantry robot can be treated as three single-input single-output systems (one for each axis) that can operate simultaneously to locate the end effector anywhere within a cuboid work envelope. In order to obtain a model for controller design, each axis of the robot was modeled independently by means of sinusoidal frequency response tests. Here, only the X-axis is considered and the following 7th-order transfer-function (with s denoting the Laplace transform variable) is used in the design as an approximation of the dynamics. Sampling at $\Delta t = 0.01$ secs with a zero-order

$$G(s) = \frac{(s + 500.19)(s + 4.90 \times 10^5)(s + 10.99 \pm j169.93)(s + 5.29 \pm j106.86)}{s(s + 69.74 \pm j459.75)(s + 10.69 \pm j141.62)(s + 12.00 \pm j79.10)} \quad (16)$$

Fig. 1. The gantry robot

hold gives

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} \quad (17)$$

where the coefficients of the numerator are $b_1 = 0.5174$, $b_2 = -0.0108$, $b_3 = 0.2863$, $b_4 = 0.1053$, $b_5 = -0.0816$, $b_6 = 0.0081$, $b_7 = -0.0006$ and those of the denominator are $a_1 = -1.5314$, $a_2 = 0.9717$, $a_3 = -0.3821$, $a_4 = -0.0056$, $a_5 = -0.0557$, $a_6 = 0.0036$, $a_7 = -0.0005$.

The non-minimal state-space model is formed by choosing the measured input and output variables as the state variables. In this case, although the dimension of the state vector is quite high at 13×1, the implementation of the state vector is performed by shifting the data vector in real-time in order to reduce the computational load.

4.2 Controller Design

The state feedback controller is designed based on a linear quadratic regulator with cost function:

$$J = \sum_{k=0}^{\infty} x(k)^T Q x(k) + \sum_{k=0}^{\infty} \tilde{u}(k)^T R \tilde{u}(k)$$

where Q is an identity matrix and $R = 1$. The MATLAB function dlqr was used to find the state feedback controller gain K.

4.3 Disturbance Observer Design

The polynomial $D(q^{-1})$ is

$$D(q^{-1}) = (1 - q^{-1})(1 - 2\cos\frac{2\pi}{N} q^{-1} + q^{-2})$$

where $N = 200$. The first term in $D(q^{-1})$ corresponds to the DC component and the second term to the dominant frequency in the reference signal (Wang et al. [2013]). For this design, the system matrix is

$$A_d = \begin{bmatrix} 2.999 & -2.999 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and the output matrix is

$$C_m B_m C_d = \begin{bmatrix} 0.5174 \times 10^{-3} & 0 & 0 \end{bmatrix}$$

Also the design of the disturbance observer is simplified because of the low dimension of the state variables. The MATLAB function dlqr was used again to find the observer gain K_{ob}, where the matrix Q is an identity matrix and $R = 0.1$. As measurement noise is one of the challenges for a repetitive control system, noise is added to the output to reflect this in the simulations.

4.4 Simulation results

The results were obtained white noise with zero mean and standard deviation 0.01

Figure 2 (a) compares the output response $y(k)$ with the reference signal $r(k)$, however, because of the measurement noise, it is not visible. The error signal $r(k)-y(k)$ is shown separately in Figure 2 (b). It is clear that the repetitive control system has tracked the reference signal with small errors despite of the measurement noise even though repetitive controllers are sensitive to measurement noise due to fundamental limitations in feedback control (see Wang et al. [2013]). The measurement noise was assumed to be white with zero mean and standard deviation 0.01

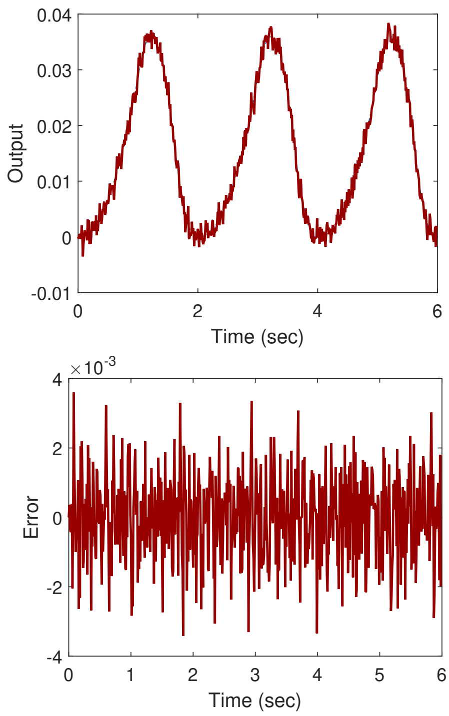

Fig. 2. Trajectory tracking of the X-axis with measurement noise.

5. CASE STUDY 2

This section compares the new design in this paper with a minimal state-space model state feedback control law. The system under study has the transfer-function:

$$G(z^{-1}) = \frac{(1 - 0.1z^{-1})(1 - 0.3z^{-1})}{(1 - 0.5z^{-1})^2(1 - 0.9z^{-1})}$$

Also the disturbance signal is a series of step changes with amplitude ± 1. The control objective is to maintain steady-state operation in the presence of step disturbances and hence the disturbance model $D(q^{-1})$ is selected as $1 - q^{-1}$.

5.1 Minimal State-space Realization

When the system model is given by a transfer-function, an observer is usually required to estimate the state variable. Suppose, therefore that the system considered has an input disturbance $\mu(k)$ and hence the state-space model is:

$$x_p(k+1) = A_p x_p(k) + B_p(u(k) + \mu(k)) \tag{18}$$

$$y(k) = C_p x_p(k) \tag{19}$$

where:

$$A_p = \begin{bmatrix} 1.9 & -1.15 & 0.225 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad B_p = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_p = [1 \ -0.4 \ 0.03]$$

The function of the observer is to estimate $x_p(k)$ together with the constant disturbance $\mu(k)$. To this end, augmented state variables are defined as follows

$$x_F(k) = \left[x_p(k)^T \ \mu(k)\right]^T$$

and the associated system matrices are

$$A_F = \begin{bmatrix} A_p & B_p \\ O & 1 \end{bmatrix}; B_F = \begin{bmatrix} B_p \\ 0 \end{bmatrix}$$

$$C_F = [C_p \ 0]$$

The state feedback controller is designed using the pair (A_p, B_p), and the MATLAB function dlqr produces the following controller gain vector

$$K_p = [1.3819 \ -0.9361 \ 0.1921]$$

when Q is the identity matrix and $R = 1$. The associated closed-loop eigenvalues are $0.1479 \pm j0.3553$, 0.2224.

An observer was designed using the pair (A_F, C_F) with the MATLAB dlqr to give

$$K_{ob} = [2.39 \ 0.9411 \ -0.0846 \ 0.5177]^T$$

where Q is the identity matrix and $R = 0.1$. The closed-loop error system has eigenvalues at 0.499, 0.3386, $0.0257 \pm j0.1872$.

The minimal state feedback control law based on the above computations is:

$$u(k) = -K_p \hat{x}_p(k) - \hat{\mu}(k)$$

where $\hat{x}_p(k)$ and $\hat{\mu}(k)$ are estimated with the observer:

$$\hat{x}_F(k+1) = A_F \hat{x}_F(k) + B_F u(k) + K_{ob}(y(k) - r(k) - C_F \hat{x}_F(k))$$

in which $r(k)$ is the reference signal.

5.2 Non-minimal State-space Realization

The non-minimal state-space realization has system matrices:

$$A_m = \begin{bmatrix} 1.9 & -1.15 & 0.225 & -0.4 & 0.03 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}; \quad B_m = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$C_m = [1 \ 0 \ 0 \ 0 \ 0]$$

and application of the MATAB dlqr function gives

$$K = [1.2087 \ -0.8638 \ 0.1819 \ -0.3146 \ 0.0243]$$

The closed-loop eigenvalues for the state feedback control system are then $0.3230 \pm j0.2965$, 0.3599, with the remaining two at zero.

The disturbance observer is a first order system but, to be consistent, the MATLAB dlqr function is again used to find the observer gain $K_{ob} = 0.9161$, which corresponds to a closed-loop eigenvalue of the observer error system at 0.0839. For this design, the matrices Q and R are scalar and selected as $Q = 1$ and $R = 0.1$.

5.3 Simulation Results

To evaluate the two repetitive control systems, measurement noise with zero mean and standard deviation of 0.01 is added to the output measurement, with a step reference signal applied at the beginning of the simulation. As an input disturbance, a square wave signal with amplitude of ± 1 is added after 150 samples and the control signal amplitude is constrained to ± 2.5.

5.4 Disturbance rejection

Figure 3 gives comparative data the closed-loop responses under identical simulation conditions where a step disturbance with amplitude of 2 is introduced at sampling instant $k = 300$. From the top plot in this figure it is seen that both control systems maintain steady-state operation in the presence of a disturbance amplitude of 2. The repetitive controller using the non-minimal state-space realization, quickly detects the disturbance and responds very quickly (see red Line (1)), so that the maximum deviation from the steady-state operation occurs at sampling instant 301 and is equal to the amplitude of the disturbance (2). In contrast, the control system with minimal state-space realization detects the disturbance slower and responds more slowly (see the black dash-dot Line (2)). A consequence of this is that the maximum deviation from the steady-state operation occurs at sampling instant 302 and has value 5, i.e. the maximum deviation is 2.5 times that of the original disturbance amplitude.

The behavior of the control signals are also entirely different for the two implementations, as shown in Figure 3 (b), where the control signal in the non-minimal state-space realization requires a smaller amplitude to achieve a better disturbance rejection. Conversely, the later detection of the disturbance by the minimal state-space realization controller means that the feedback error becomes much larger and it produces a large amplitude input to compensate for the effect of disturbance. Note that both control signals have triggered the saturation limit and the anti-windup mechanisms has become active in both cases.

5.5 Reference Input following

Figures 4 (a) and (b) show the closed-loop output response and the control signal response to a step reference signal. It is clear that the early detection of the reference change by the repetitive controller results in better transient performance and reduces the required control signal amplitude.

One question that arises is: can the closed-loop performance be improve if a deadbeat observer is used for the minimal state-space realization, where all the closed-loop

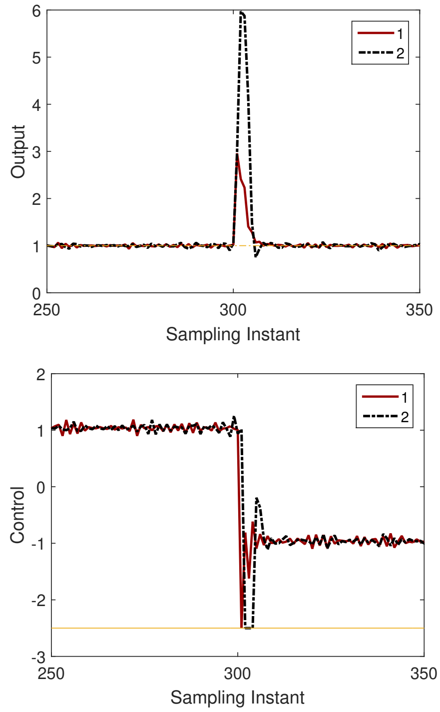

Fig. 3. Comparative studies: disturbance rejection. Key: line (1) Non-minimal realization repetitive control system; line (2) Minimal realization repetitive control system.

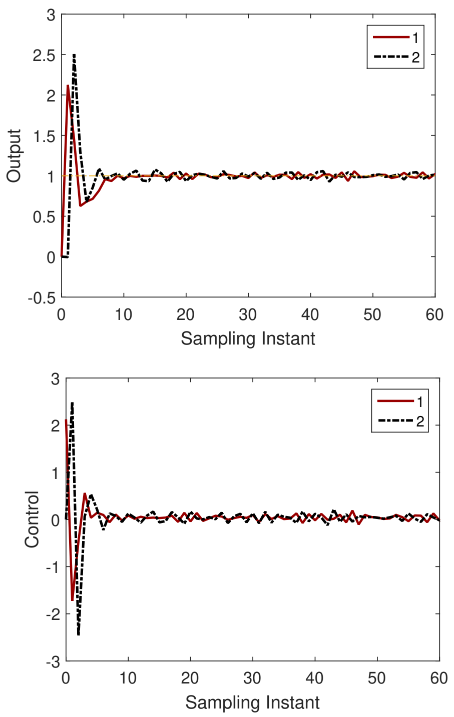

Fig. 4. Comparative studies: reference following. Key: line (1) Non-minimal realization repetitive control system; line (2) Minimal realization repetitive control system.

poles of the observer error system are positioned at zero? In which context, simulation studies have shown that this does not provide early detection of the disturbance and does not reduce the maximum deviation of the response. Moreover, due to the higher gain of the observer, the measurement noise is more amplified in the control signal.

6. CONCLUSIONS

This paper has developed a disturbance observer-based repetitive control system using a non-minimal state-space realization. By choosing the state variables based directly on the plant input and output variables, there is no need to use an observer to estimate the state variables. The task of repetitive control system design then becomes two simple, yet independent, tasks: first, the design of a state feedback control law and secondly the design of a disturbance observer that embeds the characteristics of the disturbance signal or the reference signal into the control algorithm. The associated simulation studies demonstrate the efficacy of the resulting robotic arm control. Furthermore, the related comparative studies illustrate that the repetitive control system using a non-minimal state-space realization can have considerable advantages over a repetitive control system design using a minimal state-space realization.

Ongoing research includes experimental validation on the gantry robot used in this paper.

REFERENCES

Wang, C., and Young, P. (1988). Direct digital control by input-output, state variable feedback: theoretical background. *International Journal of Control*, Vol.747, 97–109.

Francis, B. and Wonham, W. (1976). The internal model principle of control theory. *Automatica*, 12, 457–465.

Ljung, L. (1999). *System Identification: Theory for the User*. Prentice-Hall, Englewood Cliffs, New Jersey.

Soderstrom, T. (2018). *Errors-in-Variables Methods in System Identification*. Springer.

Taylor, C.J., Young, P.C., and Chotai, A. (2013). *True Digital Control: Statistical Modelling and Non-minimal State Space Design*. John Wiley & Sons.

Young, P.C., Behzadi, M.A., Wang, C., and Chotai, A. (1987) Direct digital and adaptive control by input-output, state variable feedback. *International Journal of Control*, 46, 1861–1881.

Wang, L., Freeman, C., and Rogers, E. (2013). Repetitive-predictive control system with constraints: design and implementation. *Journal of Process Control*, 23, 956–967.

Wang, L. and Young, P.C. (2006). An improved structure for model predictive control using non-minimal state space realisation. *Journal of Process Control*, 16, 355–371.

Young, P.C. (2012). *Recursive Estimation and Time-series Analysis: an Introduction*. Springer Science & Business Media.

Wang L. (2009). *Model Predictive Control System Design and Implementation using MATLAB*. Springer.