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Cooperative Adaptive Cruise Control of Vehicle Platoons with Fading Signals and Heterogeneous Communication Delays

Xiulan Song, Feng Xiao, Hong Peng, Defeng He

College of Information Engineering, Zhejiang University of Technology, Hangzhou 310023, P.R. China.
(Tel: 86057185290372; e-mail: songxl2008@zjut.edu.cn).

Abstract: This paper proposes a new cooperative adaptive cruise control (CACC) approach of vehicle platoons with fading signals and heterogeneous communication delays. The CACC model with variable input delays is established to describe the varying time-delays from transmitting acceleration of the front vehicle. The fading signal gains may be unknown and uncertain due to heterogeneous V2V wireless channels. Then a set of decentralized time-delay feedback CACC controllers is computed in such way that each vehicle evaluates its own control strategy using only neighborhood information. In order to establish string stability of the platoon with the decentralized controllers, some sufficient conditions are obtained in form of linear matrix inequalities. The scenarios, consisting of seven different cars with heterogeneous wireless channels, are used to demonstrate the effectiveness of the presented method.

Keywords: Cooperative adaptive cruise control, vehicle platoons, networked control system, robustness, string stability

1. INTRODUCTION

In recent years, cooperative adaptive cruise control (CACC) of vehicle platoons has attracted considerable attention in both industrial and academic communities. With information exchange between vehicles by V2X (vehicle-to-vehicle, infrastructure,…) wireless communication, CACC improves traffic flow stability, throughput, driving safety and ride comfort of vehicles (Dey, et al., 2016; Milanes, et al., 2014; Wang, et al., 2018; Al-Jhayyish and Schmidt, 2018; He, et al., 2019). Then CACC is shown to be one of the promising intelligent transportation systems technologies (Dey, et al., 2016; Wang, et al., 2018; Milanes and Shladover, 2014).

The main goal of CACC is to adjust multiple vehicles to form a platoon and then maintain an optimal, safe spacing (Dey, et al., 2016). String stability is the important aim of theoretical analysis of vehicle CACC systems. This property implies that in a platoon perturbation on the leading vehicle are not amplified downstream through the platoon (Dey, et al., 2016; Milanes, et al., 2014). Several CACC methods have been proposed to establish the string stability of vehicle platoons (e.g., see Li, et al. (2017), Kayacan (2017), Ploeg, et al. (2014), Guo and Yue (2014), Yu, et al. (2018)).

In CACC systems, the real-time behaviours of front vehicles are dispatched to the nearest following vehicles in the V2X wireless network. Hence, the V2X network has important effects on the string stability of a vehicle platoon due to uncertainties and time-varying communication delays (Dey, et al., 2016). Many works addressed these effects within the networked control systems framework (Oncu, et al., 2014; Firooznia, et al., 2017; Orosz, 2016; Abdessameuda, et al., 2015; Bernardo, et al., 2015; Izadi, et al., 2009; Song, et al., 2019). For instance, the effect of network-induced constant delays or sample-and-hold devices on string stability was studied within a networked control system perspective (e.g., Oncu, et al., 2014). The results on CACC were general derived by the assumption of constant time-delays of the V2X network. Fewer results have been obtained to deal with time-varying heterogeneous communication delays (Bernardo, et al., 2015; Song, et al., 2019).

Most of the existing results on CACC generally assume that the information transmitted by V2X are reliable all the times. However, signal fading might happen to wireless network in practical vehicles for reasons such as low battery power and interference of radar signals. For example, the signal-to-noise ratio of the information transmitted by V2X may noticeably decay at low battery power (Xu, et al., 2018; Su and Chesi, 2017). Vulnerability of V2X wireless network to battery power and cyber-attacks was studied by many researches (e.g., see Sharma and Kaushik (2019), Petit and Shladover (2015) and the references therein). Under signal fading, the vehicle can still have access to the state (e.g., acceleration) of the preceding car via V2X but the state is inaccurate, which may degrade the string stability of a vehicle platoon. How would signal fading affect the cooperative control of vehicle platoons is still an open and challenging problem.

The aim of this paper is to solve CACC problem in the networked control framework by taking into account fading signals and heterogeneous communication delays. The CACC model with variable input delays is used to describe the time varying heterogeneous communication. The CACC controller is defined on the state of the host vehicles and on fading signals transmitted from the preceding vehicles via V2X communication network. The platoon formation and its stability is guaranteed in the presence of fading signals and time-varying delays by some LMI conditions. Numerical results illustrate the effectiveness of the proposed CACC method for string stability of a seven-vehicle platoon in the varying speed scenario.

Notations: Throughout the paper, P>0 means that the matrix P is positive definite and diag{...} denotes a block-diagonal matrix. The symmetric terms in a symmetric matrix are denoted by '*', e.g., $\begin{bmatrix} M & N \\ * & L \end{bmatrix} = \begin{bmatrix} M & N \\ N^{\mathrm{T}} & L \end{bmatrix}$.

2. PROBLEM FORMULATION

Consider a group string of n vehicles moving along a single lane and assume that they run in horizontal environment (see Fig. 1). In this platoon, each vehicle shares its acceleration with the following vehicles through a predecessor-following communication topology network that consists of N ($1<N<\infty$) heterogeneous wireless channels. The vehicles are also equipped with onboard sensors to measure the inter-vehicle distance and relative velocity with respect to its preceding vehicle. The first vehicle in the platoon presents the reference trajectory of the string according to some safety spacing policies. Let L_i, q_i, v_i and a_i be the length, position, velocity and acceleration of the ith vehicle in the platoon for i=0,1,…,N, where i=0 stands for the leading vehicle.

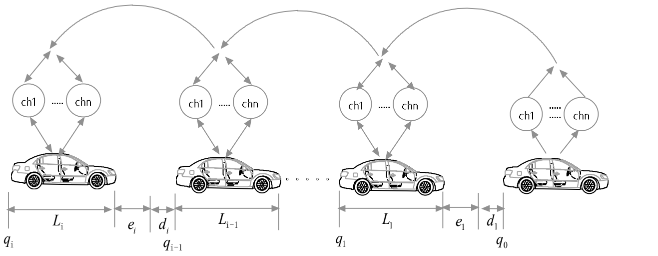


Fig. 1 A schematic CACC system of the vehicle platoon

Considering the ith vehicle in this platoon, the three-order linear model is adopted to describe the longitudinal dynamics of the vehicle (Guo, et. al., 2014; Oncu, et al., 2014)

$$\begin{aligned} \dot{q}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= a_i(t) \\ \dot{a}_i(t) &= -1/\varsigma_i\, a_i + 1/\varsigma_i\, u_i \end{aligned} \tag{1}$$

where q_i, v_i and a_i are the position, velocity and acceleration, respectively, control u_i is the acceleration command and constant ς_i represents the internal actuator dynamics. The model can be obtained by feedback linearization (Guo and Yue, 2014), which simplifies the complexity of modeling the longitudinal dynamics of vehicles. It has been shown in (Oncu, et al., 2014) that the model (1) adequately describes the longitudinal dynamics of the acceleration-controlled vehicles via the experimental validation and has been widely used to design CACC controllers of vehicle platoons.

The goal of CACC is generally to regulate the inter-vehicle distance d_i to a small desired spacing while guaranteeing string stability. In this study, the vehicle platoon moves with a reference constant velocity, namely, the leading vehicle satisfies that $\dot{q}_0(t) = v_0(t)$ and $\dot{v}_0(t) = 0$. Moreover, the constant time headway is adopted as the safe spacing policy of the proposed CACC system. The desired safe spacing is defined as

$$d_{r,i}(t) = D_i + h_i v_i(t) \tag{2}$$

where $d_{r,i}$ is the desired spacing between vehicles i and i–1, D_i is the desired distance at standstill and h_i is the time gap. The actual inter-vehicle distance is equal to $d_i=q_{i-1}-q_i-L_i$ where q_{i-1} is the position of predecessor. The difference between actual and desired inter-vehicle distances defines error $e_i=d_i-d_{r,i}=q_{i-1}-q_i-L_i-d_{r,i}$.

This paper selects the error state vector of the ith vehicle as $x_i=[e_i \;\; \Delta v_i \;\; a_i]^{\mathrm{T}}$. Then the dynamics of the error variables for the vehicle can be represented as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + G_i x_{i-1}(t) \tag{3}$$

with matrices

$$A_i = \begin{bmatrix} 0 & 1 & -h_i \\ 0 & 0 & -1 \\ 0 & 0 & -1/\varsigma_i \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ 1/\varsigma_i \end{bmatrix}, G_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

for i=1,...,N. Since the ith vehicle receives the signals transmitted by onboard sensors and V2V communication, respectively, the output of the vehicle in the platoon is defined as

$$y_i(t) = C_i \left[x_i^{\mathrm{T}}(t) \quad a_{i-1}(t - \tau_i(t)) \right]^{\mathrm{T}} \tag{4}$$

where τ_i is the unavoidable time-varying communication delay and C_i=diag{$\rho_{i,1}$, $\rho_{i,2}$, $\rho_{i,3}$, $\rho_{i,4}$} is the uncertain gain matrix of the state measurement. Typically, τ_i is updated over finite time intervals and can be assumed bounded by some a maximum value, i.e., $0 \le \tau_i \le h$ with the bound h>0. Moreover, ($\rho_{i,1}$, $\rho_{i,2}$, $\rho_{i,3}$) and $\rho_{i,4}$ denote the gains of onboard sensors and V2V channels, respectively, and also are assumed to be bounded between minimal and maximum values, i.e., $\rho_{i,\min} \le \rho_{i,j} \le \rho_{i,\max}$ with the bounds $\rho_{i,\min}$, $\rho_{i,\max}$>0 for j=1,...,4. Note that if onboard sensors and V2V channels are normal, then $\rho_{i,j}$=1 for j=1,...,4. In practice, however, these signal gains are generally not accurately known but uncertain due to battery power, heterogeneous V2V wireless channels, etc.

The control objective of the paper is to design an output controller for the ith vehicle in the platoon, i.e.

$$u_i(t) = k_{i,1} e_i + k_{i,2} \Delta v_i + k_{i,3} a_i + k_{i,4} a_{i-1} =: K_i y_i(t) \tag{5}$$

that regulates the error state x_i to zero while guaranteeing string stability of the vehicle platoon in (3) in the presence of signal gain-uncertainties and heterogeneous varying delays. The gain vector $K_i=[k_{i,1} \; k_{i,2} \; k_{i,3} \; k_{i,4}]$ of controller (5) will be determined in the next section. In what follows, some well-known results are recalled.

Definition 1(Stability): (Guo, et al. 2014) For a step change of desired speed v at time instant t=0, the error state of each vehicle in the platoon described by (1) is asymptotically stabilized to the origin, i.e., vehicles in the platoon reach the reference speed.

Definition 2(String stability): (Oncu, et al., 2014) The vehicle platoon system (4) is string stable if the inequality $\|G(jw)\|\leq 1$ holds for any w, where $G(s)=a_i(s)/a_{i-1}(s)$ with $a_i(s)$ and $a_{i-1}(s)$ are the Laplace transforms of $a_i(t)$ and $a_{i-1}(t)$, respectively.

String stability of a platoon implies that the oscillations do not amplify with the platoon caused by any manoeuvre of leading vehicle. Hence, string stability can be seen as a measurement on the amplification of perturbations along the vehicle string. When the onboard sensors and wireless V2V work in a normal situation, the system (4) is string stable under some well accepted conditions, e.g., Oncu, et al., (2014). However, due to the uncertain and unknown gains of output signals and varying delays, the system is getting to an uncertain system subject to varying delays and uncertain gains of output signals. This makes the CACC problem rather challenging.

3. CACC OF VEHICLE STRINGS

The general form of the closed-loop CACC vehicle system is obtained by combining the dynamics of error states in (3) with the CACC law (5). To this end, substituting (4) to the CACC law (5) yields

$$\begin{aligned} u_i(t) &= K_i C_i \left[x_i^{\mathrm{T}}(t) \quad a_{i-1}(t-\tau_i(t)) \right]^{\mathrm{T}} \\ &= [k_{i,1} \quad k_{i,2} \quad k_{i,3}]\mathrm{diag}\{\rho_{i,1},\rho_{i,2},\rho_{i,3}\}x_i(t) \\ &\quad + k_{i,4}[0 \quad 0 \quad \rho_{i,4}]x_{i-1}(t-\tau_i(t)) \\ &\triangleq K_{i,1}Y_{i,1}x_i(t) + K_{i,2}Y_{i,2}x_{i-1}(t-\tau_i(t)) \end{aligned} \tag{6}$$

where $K_{i,1}=[k_{i,1}\ k_{i,2}\ k_{i,3}]$, $K_{i,2}=k_{i,4}$, $Y_{i,1}=\mathrm{diag}\{\rho_{i,1},\rho_{i,2},\rho_{i,3}\}$ and $Y_{i,2}=[0\ 0\ \rho_{i,4}]$. Here the signal gains $Y_{i,1}$ and $Y_{i,2}$ are generally uncertain and unknown. The gain K_i will be computed elaborately for any signal gains $Y_{i,1}$ and $Y_{i,2}$ in Subsection 3.2.

Applying the control law (6) into the system (3), the closed-loop CACC system of the ith vehicle is derived as

$$\begin{aligned} \dot{x}_i(t) &= (A_i + B_i K_{i,1} Y_{i,1}) x_i(t) \\ &\quad + B_i K_{i,2} Y_{i,2} x_{i-1}(t-\tau_i(t)) + G_i x_{i-1}(t) \end{aligned} \tag{7}$$

Let $x=[x_1^{\mathrm{T}}\ x_2^{\mathrm{T}} \cdots x_n^{\mathrm{T}}]^{\mathrm{T}}$. The closed-loop CACC system of the vehicle platoon can be re-written in a compact form of

$$\dot{x}(t) = (A + BK_1Y_1)x(t) + \bar{B}K_2Y_2x(t-\tau(t)) \tag{8}$$

with $B=\mathrm{diag}\{B_1,B_2,...,B_n\}$, $K_1=\mathrm{diag}\{K_{1,1},K_{1,2},...,K_{1,n}\}$, $Y_1=\mathrm{diag}\{Y_{1,1},Y_{1,2},...,Y_{1,n}\}$, $K_2=\mathrm{diag}\{K_{2,2},K_{2,3},...,K_{2,n},0\}$, $Y_2=\mathrm{diag}\{Y_{2,2},Y_{2,3},...,Y_{2,n},0\}$

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ G_2 & A_2 & \cdots & 0 \\ \cdots & \ddots & \ddots & \cdots \\ 0 & \cdots & G_n & A_n \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ B_2 & 0 & \cdots & 0 \\ \cdots & \ddots & \ddots & \cdots \\ 0 & \cdots & B_n & 0 \end{bmatrix}$$

and $\tau=\{\tau_1,\tau_2,..,\tau_n\}$. In what follows, the gains K_i will be computed to guarantee the closed-loop CACC system (7) is vehicle stable and the closed-loop string system (8) is string stable in the presence of heterogeneous communication delays and uncertain gains of output signals.

Consider the closed-loop string system (8) and let $A_k=A+BK_1Y_1$ and $B_k=\bar{B}K_2Y_2$ for simplicity. The closed-loop string system is equal to

$$\dot{x}(t) = A_k x(t) + B_k x(t-\tau(t)) \tag{9}$$

This paper introduces a known lemma that will be instrumental for the proof of vehicle stability of the CACC system presented later in this section.

Lemma 1: (Zhang, et al., 2005) Let P_i, Q_i and R_i be any positive-definite symmetric matrices with appropriate dimension for i=1 and 2. Then there exists a symmetric matrix M such that

$$\begin{bmatrix} P_1 - M & Q_1 \\ Q_1^{\mathrm{T}} & R_1 \end{bmatrix} > 0, \quad \begin{bmatrix} P_2 + M & Q_2 \\ Q_2^{\mathrm{T}} & R_2 \end{bmatrix} > 0 \tag{10}$$

if and only if

$$\begin{bmatrix} P_2 + P_2 & Q_1 & LQ_2 \\ * & R_1 & 0 \\ * & * & R_2 \end{bmatrix} > 0 \cdot \tag{11}$$

Now the vehicle stability of the vehicle platoon in the presence of fading signals and heterogeneous time-varying delays can be guaranteed under the hypothesis of the following theorem.

Theorem 1: Consider the closed-loop string system (9). If there exist some positive-definite symmetric matrices $P\in\Re^{3n\times 3n}$, $Q\in\Re^{3n\times 3n}$ and $R\in\Re^{3n\times 3n}$ such that the following nonlinear matrix inequality holds:

$$\Phi = \begin{bmatrix} M_{11} & M_{12}^{\mathrm{T}} & M_{13}^{\mathrm{T}} \\ * & -hR^{-1} & 0 \\ * & * & -hR \end{bmatrix} < 0 \tag{12}$$

where matrices $M_{11} = \begin{bmatrix} \vartheta & PB_k - N_1^{\mathrm{T}} + N_2 \\ * & -\sigma Q - N_2^{\mathrm{T}} - N_2 \end{bmatrix}$, $M_{12}=[A_k \quad B_k]h$ and $M_{13}=[N_1\ N_2]h$ with $\vartheta=PA_k+A_k^{\mathrm{T}}P+Q+N_1^{\mathrm{T}}+N_1$, $\sigma=1-\mu$ and some free matrices $N_i\in\Re^{3n\times 3n}$ for i=1 and 2, then the system is asymptotically stable.

Proof: Consider the closed-loop string system in (9). From the Leibniz-Newton formula, it is known that $\varphi(t)$=0 with

$$\varphi(t) = x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds \cdot$$

Then for the matrices N_1 and N_2, it is derived that $[x(t)^{\mathrm{T}}N_1^{\mathrm{T}}+x(t-\tau(t))^{\mathrm{T}}N_2^{\mathrm{T}}]\varphi(t)=0$. Moreover, let $\eta_1(t)=[x(t)^{\mathrm{T}}\ x(t-\tau(t))^{\mathrm{T}}]^{\mathrm{T}}$ and $\Lambda_{ij}=h(S_{ij}-S_{ij})$ with any matrices $S_{ij}\in\Re^{3n\times 3n}$ for i, j=1, 2. Then Λ_{ij}=0 and $\eta_1(t)^{\mathrm{T}}\Lambda\eta_1(t)$=0 with $\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{bmatrix}$.

Consider the following Lyapunov-Krasovskii candidate function of the string system:

$$\begin{aligned} V(t) &= x(t)^{\mathrm{T}}Px(t) + \int_{t-\tau(t)}^{t} x(s)^{\mathrm{T}}Qx(s)ds \\ &\quad + \int_{-h}^{0}\int_{t-\theta}^{t} \dot{x}(s)^{\mathrm{T}}R\dot{x}(s)dsd\theta \end{aligned} \tag{13}$$

Taking the derivative of $V(t)$ along (9) gives

$$\begin{aligned}\dot{V}(t) &= x(t)^{\mathrm{T}}(PA_k + A_k^{\mathrm{T}}P + Q)x(t) \\ &\quad + 2x(t)^{\mathrm{T}}PB_k x(t-\tau(t)) + \dot{x}(t)^{\mathrm{T}} hR\dot{x}(t) \\ &\quad - \int_{t-h}^{t} \dot{x}(s)^{\mathrm{T}} R\dot{x}(s)ds \\ &\quad - x(t-\tau(t))^{\mathrm{T}}(1-\dot{\tau}(t))Qx(t-\tau(t))\end{aligned} \tag{14}$$

From the changes of the time-delay, it is obtained that

$$\begin{aligned}\dot{V}(t) &\le x(t)^{\mathrm{T}}(PA_k + A_k^{\mathrm{T}}P + Q)x(t) + 2x(t)^{\mathrm{T}}PB_k x(t-\tau(t)) \\ &\quad + \dot{x}(t)^{\mathrm{T}} hR\dot{x}(t) + \eta_1(t)^{\mathrm{T}}[N_1^{\mathrm{T}} \quad N_2^{\mathrm{T}}]\varphi(t) + \eta_1(t)^{\mathrm{T}}\Lambda\eta_1(t) \\ &\quad - \int_{t-\tau(t)}^{t} \dot{x}(s)^{\mathrm{T}} R\dot{x}(s)ds - x(t-\tau(t))^{\mathrm{T}}\sigma Qx(t-\tau(t))\end{aligned} \tag{15}$$

which yields

$$\begin{aligned}\dot{V}(t) &\le x(t)^{\mathrm{T}}(PA_k + A_k^{\mathrm{T}}P + Q + N_1^{\mathrm{T}} + h(S_{11} - S_{11}))x(t) \\ &\quad + x(t)^{\mathrm{T}}(PB_k - N_1^{\mathrm{T}} + h(S_{12} - S_{12}))x(t-\tau(t)) \\ &\quad + x(t-\tau(t))^{\mathrm{T}}(-\sigma Q - N_2^{\mathrm{T}} + h(S_{22} - S_{22}))x(t-\tau(t)) \\ &\quad + x(t-\tau(t))^{\mathrm{T}}(B_k^{\mathrm{T}}P + N_2^{\mathrm{T}} + h(S_{12}^{\mathrm{T}} - S_{12}^{\mathrm{T}}))x(t) \\ &\quad + \eta_1(t)^{\mathrm{T}}[A_k^{\mathrm{T}} \quad B_k^{\mathrm{T}}]^{\mathrm{T}} hR[A_k \quad B_k]\eta_1(t) \\ &\quad - \int_{t-\tau(t)}^{t} (\dot{x}(s)^{\mathrm{T}} R\dot{x}(s) + x(t)^{\mathrm{T}} N_1^{\mathrm{T}}\dot{x}(s)ds \\ &\quad + \int_{t-\tau(t)}^{t} x(t-\tau(t))^{\mathrm{T}} N_2^{\mathrm{T}}\dot{x}(s))ds\end{aligned} \tag{16}$$

Let $\eta_2(t,s) = [\eta_1(t)^{\mathrm{T}} \ \dot{x}(s)^{\mathrm{T}}]^{\mathrm{T}}$. Then function $V(t)$ satisfies that

$$\begin{aligned}\dot{V}(t) &\le x(t)^{\mathrm{T}}(PA_k + A_k^{\mathrm{T}}P + Q + N_1^{\mathrm{T}} + N_1 + hS_{11})x(t) \\ &\quad + x(t)^{\mathrm{T}}(PB_k - N_1^{\mathrm{T}} + N_2 + hS_{12})x(t-\tau(t)) \\ &\quad + x(t-\tau(t))^{\mathrm{T}}(-\sigma Q - N_2^{\mathrm{T}} - N_2 + hS_{22})x(t-\tau(t)) \\ &\quad + x(t-\tau(t))^{\mathrm{T}}(B_k^{\mathrm{T}}P + N_2^{\mathrm{T}} - N_1 + hS_{12}^{\mathrm{T}})x(t) \\ &\quad + \eta_1(t)^{\mathrm{T}} h\Gamma_1^{\mathrm{T}}R\Gamma_1\eta_1(t) - \int_{t-\tau(t)}^{t} \eta_2(t,s)^{\mathrm{T}}\psi\eta_2(t,s)ds\end{aligned} \tag{17}$$

where $\Gamma_1=[A_k \ B_k]=h^{-1}M_{12}$ and $\psi = \begin{bmatrix} S_{11} & S_{12} & N_1^{\mathrm{T}} \\ * & S_{22} & N_2^{\mathrm{T}} \\ * & * & R \end{bmatrix}$. Moreover, the inequality (17) can be re-written as

$$\dot{V}(t) \le \eta_1(t)^{\mathrm{T}}(H + h\Gamma_1^{\mathrm{T}}R\Gamma_1)\eta_1(t) - \int_{t-\tau(t)}^{t} \eta_2(t,s)^{\mathrm{T}}\psi\eta_2(t,s)ds$$

where $H = \begin{bmatrix} H_{11} & H_{12} \\ * & H_{22} \end{bmatrix}$ with $H_{11}=\vartheta+hS_{11}$, $H_{12}=PB_k-N_1^{\mathrm{T}}+N_2+hS_{12}$ and $H_{22}=-\sigma Q-N_2^{\mathrm{T}}-N_2+hS_{22}$. Clearly, $V(t)$ monotonically decreases along the trajectories of (9) if $H+h\Gamma_1^{\mathrm{T}}R\Gamma_1<0$ and $\psi>0$. Using Shur's complement, it is easy to see that

$$\begin{aligned}H + h\Gamma_1^{\mathrm{T}}R\Gamma_1 < 0 &\Leftrightarrow \begin{bmatrix} H & h\Gamma^{\mathrm{T}} \\ * & -hR^{-1} \end{bmatrix} < 0 \\ &\Leftrightarrow \begin{bmatrix} -M_{11} - hS & -M_{12}^{\mathrm{T}} \\ * & hR^{-1} \end{bmatrix} > 0\end{aligned} \tag{18}$$

$$\psi > 0 \Leftrightarrow \begin{bmatrix} hS & M_{13}^{\mathrm{T}} \\ * & hR \end{bmatrix} > 0 \tag{19}$$

where $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$. Then applying Lemma 1, it has that

$$\begin{cases} H + h\Gamma_1^{\mathrm{T}}R\Gamma_1 < 0 \\ \psi > 0 \end{cases} \Leftrightarrow \begin{bmatrix} -M_{11} & -M_{12}^{\mathrm{T}} & M_{13}^{\mathrm{T}} \\ * & hR^{-1} & 0 \\ * & * & hR \end{bmatrix} > 0 \tag{20}$$

which is equal to the inequality (12). Hence, the condition in (12) yields that $V(t)$ monotonically decreases along the trajectories of (9) and it is a Lyapunov function of the closed-loop string system (9). This establishes the asymptotic stability of the system from the Lyapunov-Krasovskii's argument.

Remark 1: Theorem 1 presents a sufficient condition to ensure stability of the closed-loop string system (9). However, the condition (12) is nonlinear matrix inequality, e.g., R^{-1}, PA_k. Hence, it is not easy to compute the CACC controllers in (5) by the known linear matrix inequality (LMI) tool. One method is to introduce the linearization method (Zhang, et al., 2005) to compute the CACC controllers via the available LMI tool. Namely, consider the closed-loop string system (9) and given some numbers λ and γ. If there exist some matrices $0 < \overline{P} \in \Re^{3n\times 3n}$, $0 < \overline{Q} \in \Re^{3n\times 3n}$, $0 < \overline{R} \in \Re^{3n\times 3n}$, $W_1 \in \Re^{1\times 3n}$ and $W_2 \in \Re^{1\times 3n}$ such that the following LMI holds:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & \overline{P} \\ * & \Xi_{22} & \Xi_{23} & h\overline{R} & 0 \\ * & * & -h\overline{R} & 0 & 0 \\ * & * & * & h\overline{R} & 0 \\ * & * & * & * & -\overline{Q} \end{bmatrix} < 0 \tag{21}$$

where
$\Xi_{11} = A\overline{P} + \overline{P}A^{\mathrm{T}} + BW_1 + W_1^{\mathrm{T}}B^{\mathrm{T}} - \lambda\gamma^{-1}(\overline{B}W_2 + W_2^{\mathrm{T}}\overline{B}^{\mathrm{T}}) - \sigma\lambda^2\gamma^{-2}\overline{Q}$, $\Xi_{12} = \gamma^{-1}\overline{B}W_2 + \overline{P} + \lambda\gamma^{-1}\overline{Q} + \sigma\lambda\gamma^{-2}\overline{Q}$, $\Xi_{23} = h\gamma^{-1}W_2^{\mathrm{T}}\overline{B}^{\mathrm{T}}$, $\Xi_{13} = h(\overline{P}A^{\mathrm{T}} + W_1^{\mathrm{T}}B^{\mathrm{T}} - \lambda\gamma^{-1}W_2^{\mathrm{T}}\overline{B}^{\mathrm{T}}$, and $\Xi_{22} = -(2+\sigma)\gamma^{-2}\overline{Q}$, then the closed-loop string system is asymptotically stable with the CACC gains $K_1 = W_1\overline{P}^{-1}Y_1^{-1}$ and $K_2 = W_2\overline{Q}^{-1}Y_2^{-1}$. Then the matrix inequality (21) is linear with respect to the matrices $\overline{P}$, $\overline{Q}$, $\overline{R}$, W_1 and W_2. Hence, these matrices can be obtained by solving the feasibility problem with the solver 'feasp' in the LMI toolbox (Boyd, et al., 1994).

4. STRING STABILITY ANALYSIS

Although the closed-loop system (7) guarantees the zero steady-state spacing error for each vehicle in the platoon, it gives no specific restrictions on the transient spacing errors and string stability. To meet the string stability requirement, i.e., the transient spacing errors are not amplified downstream along the platoon, the controller need to be supplemented with the conditions on transient spacing errors.

Considering the desired safe spacing (2), it is obtained that the Laplace transform of the spacing

$$\delta_i(s) = \frac{a_{i-1}(s) - a_i(s)}{s^2} - \frac{ha_i(s)}{s},$$
$$\Delta v_i(s) = \frac{a_{i-1}(s) - a_i(s)}{s} \tag{22}$$

Substituting (5) and (22) into (3), and making some algebra operations, it is obtained that

$$G_i(s) = \frac{a_i(s)}{a_{i-1}(s)} = \frac{k_{i,1}\rho_{i,1} + k_{i,2}\rho_{i,2}s + k_{i,4}\rho_{i,4}s^2 e^{-\tau_i s}}{s^3\zeta_i + (1 - k_{i,3}\rho_{i,3})s^2 + (hk_{i,1}\rho_{i,1} + k_{i,2}\rho_{i,2})s + k_{i,1}\rho_{i,1}} \tag{23}$$

Let s=jw. From the Euler formula on $e^{-\tau i s}$, this paper obtains that

$$\begin{aligned} &|G_i(\mathrm{j}w)| \\ &= \frac{|k_{i,1}\rho_{i,1} + \mathrm{j}wk_{i,2}\rho_{i,2}s - k_{i,4}\rho_{i,4}w^2(\cos(w\tau) - \mathrm{j}\sin(w\tau))|}{|-\mathrm{j}w^3\zeta_i - (1 - k_{i,3}\rho_{i,3})w^2 + \mathrm{j}(hk_{i,1}\rho_{i,1} + k_{i,2}\rho_{i,2})w + k_{i,1}\rho_{i,1}|} \\ &=: \frac{\sqrt{a}}{\sqrt{a+b}} \end{aligned} \tag{24}$$

In order to guarantee $|G_i(\mathrm{j}w)|=|a_i(\mathrm{j}w)/a_{i-1}(\mathrm{j}w)|\leq 1$ for any w and i=1,...,n, it is derived that

$$\begin{aligned} &w^4\zeta^2 + [1 + (k_{i,3}\rho_{i,3})^2 - (k_{i,4}\rho_{i,4})^2 - 2k_{i,3}\rho_{i,3} - 2\zeta hk_{i,1}\rho_{i,1} \\ &-2k_{i,2}\rho_{i,2}]w^2 - 2k_{i,2}\rho_{i,2}k_{i,4}\rho_{i,4}\sin(\tau_i w)w \\ &+2k_{i,4}\rho_{i,4}k_{i,1}\rho_{i,1}\cos(\tau_i w) + 2k_{i,1}\rho_{i,1}k_{i,3}\rho_{i,3} \\ &+h^2k_{i,1}^2\rho_{i,1}^2 + 2hk_{i,1}\rho_{i,1}k_{i,2}\rho_{i,2} - 2k_{i,1}\rho_{i,1} \geq 0 \end{aligned}$$

Using Taylor series, it is obtained that

$$\sin(\tau_i w) \approx \tau_i w - (\tau_i w)^3/6, \quad \cos(\tau_i w) \approx 1 - (\tau_i w)^2/2$$

and

$$\begin{aligned} &(\zeta_i^2 + \frac{k_{i,2}\rho_{i,2}k_{i,4}\rho_{i,4}\tau_i^3}{3})w^4 + [1 + (k_{i,3}\rho_{i,3})^2 - (k_{i,4}\rho_{i,4})^2 - 2k_{i,3}\rho_{i,3} \\ &-2\zeta_i hk_{i,1}\rho_{i,1} - 2k_{i,2}\rho_{i,2} - 2k_{i,4}\rho_{i,4}k_{i,1}\rho_{i,1}\tau_i - k_{i,4}\rho_{i,4}k_{i,1}\rho_{i,1}\tau_i^2]w^2 \\ &+2k_{i,1}\rho_{i,1}k_{i,3}\rho_{i,3} + h^2k_{i,1}^2\rho_{i,1}^2 + 2hk_{i,1}\rho_{i,1}k_{i,2}\rho_{i,2} \\ &-2k_{i,1}\rho_{i,1} + 2k_{i,4}\rho_{i,4}k_{i,1}\rho_{i,1} \geq 0 \end{aligned}$$

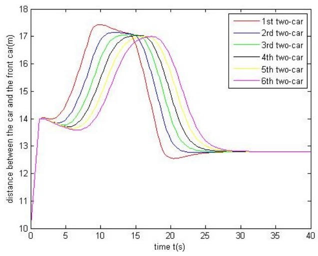
Since $\zeta_i > 0, \rho_{i,1}, \rho_{i,2}, \rho_{i,3}, \rho_{i,4} > 0, k_{i,1}, k_{i,2}, k_{i,4} > 0$, it is obtained that

$$\begin{aligned} &1 + (k_{i,3}\rho_{i,3})^2 - (k_{i,4}\rho_{i,4})^2 - 2k_{i,3}\rho_{i,3} - 2\zeta_i hk_{i,1}\rho_{i,1} - 2k_{i,2}\rho_{i,2} \\ &-2k_{i,4}\rho_{i,4}k_{i,1}\rho_{i,1}\tau_i - k_{i,4}\rho_{i,4}k_{i,1}\rho_{i,1}\tau_i^2 \geq 0 \\ &2k_{i,1}\rho_{i,1}k_{i,3}\rho_{i,3} + h^2k_{i,1}^2\rho_{i,1}^2 + 2hk_{i,1}\rho_{i,1}k_{i,2}\rho_{i,2} \\ &-2k_{i,1}\rho_{i,1} + 2k_{i,4}\rho_{i,4}k_{i,1}\rho_{i,1} \geq 0 \end{aligned}$$

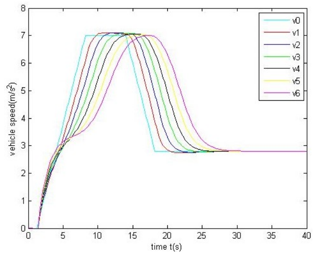
Then it is true that $|G_i(\mathrm{j}w)|=|a_i(\mathrm{j}w)/a_{i-1}(\mathrm{j}w)|\leq 1$ for any w and i=1,...,n. The string stability property of the vehicle platoon is ensured.

5. SIMULATIONS

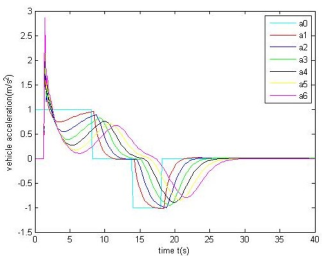
In this section, an example of a seven-vehicle platoon is used to show the implementation of the proposed control method. The platoon runs in a virtual environment established using System Build software package in MATLAB. The vehicles' parameters used in the simulation experiments are selected as minimum vehicle distance d_i = 8 m, length of vehicle L_i = 2m, time constants ς_1=ς_6=ς_7=ς_4=0.3 s, ς_2=ς_5=0.25 s, ς_3=0.2 and time gap h_i =1 for i=1,…,7. The upper bound of varying delays of the ACC system considered here is picked as τ^u=1.05 s. As the traffic messages of the seven vehicles and the acceleration of the front vehicle are transmitted by heterogeneous wireless channels, the gain channels for the platoon are chosen as C_1=[1.0, 0.95, 0.97, 0.97], C_2=[0.95, 0.93, 1.0, 0.96], C_3=[0.98, 1.0, 0.95, 0.95], C_4=[0.99, 0.97, 0.96, 0.98], C_5=[0.94, 0.96, 0.93, 0.94], and C_6=[0.93, 0.97, 0.98, 0.98]. In order to compute the CACC gains k_i, let λ=–0.257, ρ=0.5, and μ_1=0. By solving the LMI (21), the gains are computed as K_1=[0.7247, 1.8876, -1.2673, 0.0032], K_2=[0.7048, 1.8793, -0.8303, 0.0046], K_3=[0.5843, 1.4580, -0.4828, 0.0099], K_4= [0.9294, 2.2878, -1.2498, 0.0128], K_5=[0.8762, 1.9249, -0.9120, 0.0255] and K_6=[1.1532, 2.1864, -1.2495, 0]. In simulation, the varying time-delay of the ACC system is produced by a stochastic signal satisfying the bound τ^u. The scenario is initialized that all vehicles stop and the inter-vehicle distance errors are set as 4 m.



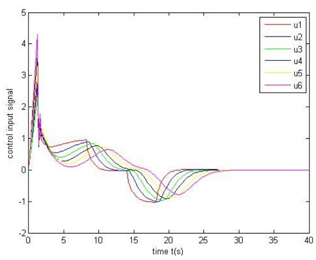
(a) Actual spacing



(b) Velocity



(c) Vehicle accelerations



(d) Control signals

Fig. 2 Profiles of states and control of the vehicle platoon

Fig. 2(c)-(d) show the trajectories of all vehicles when using the proposed method, where subplot (a) pictures the actual relative distance between the adjacent vehicles and subplots (b)-(d) picture the time evolutions of the vehicle speed, acceleration and control inputs of the host vehicles, respectively. Moreover, Fig. 3 shows that the frequency response of the acceleration transfer function on the adjacent vehicles in the platoon. From Figs. 2 and 3, it is seen that the trajectories of all vehicles can quickly converge in the face of fading signals and varying delays induced by the V2X channel transmission. In other words, each vehicle has the ability to track its immediately preceding one and maintain the inter-vehicle distance at the desired spacing as well as ensuring a consensus speed of all vehicles in the platoon. These results demonstrate the effectiveness of the presented method.

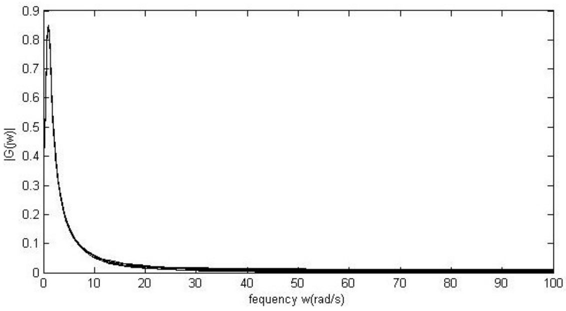


Fig. 3 Frequency response for any ω>0

6. CONCLUSION

This paper has presented a new cooperative adaptive cruise control (CACC) method for vehicle platoons connected by time-delay heterogeneous channel transmission and fading signals. Some linear matrix inequalities (LMIs) are solved to design the CACC controller of the vehicle platoon. Moreover, the LMI conditions are established to guarantee the asymptotic stability and string stability properties of the vehicle platoon in the presence of fading signals and varying time-delays induced by the time-varying heterogeneous wireless communication delays. The simulation results demonstrated the effectiveness of the method presented here and certified string stability of the connected vehicles with fading signals and varying time-delays induced by the time-varying heterogeneous wireless communication delays.

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