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Output-feedback stabilization for descriptor Markovian jump systems with generally uncertain transition rates

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Abstract: This paper presents a dynamic output-feedback stabilization problem of descriptor Markovian jump systems with generally uncertain transition rates. First, a new necessary and sufficient condition to relax inequalities including generally uncertain transition rates is introduced. For the closed-loop systems with a dynamic output-feedback controller, the stabilization criterion is achieved as non-convex matrix inequalities. For the obtained criterion, this paper gives an improved necessary and sufficient condition in terms of linear matrix inequalities under completely known transition rates. Then, the proposed condition is extended for the descriptor Markovian jump systems with generally uncertain transition rates. To show the validity of the proposed control, a numerical example is given.

Keywords: Markovian jump system, descriptor system, generally uncertain transition rates, output-feedback control, linear matrix inequality.

1. INTRODUCTION

Markovian jump systems (MJSs) can be modeled by a set of linear systems in which the transitions among different modes are subject to a Markov chain. MJSs have gained an increasing attention due to the fact that MJSs are useful to describe real world systems subject to random abrupt variations in their structure or parameters. Thus, MJSs have been widely studied both in practical systems such as networked control systems (Kim and Park (2009); Zhu et al. (2010)), manufacturing systems (Martinelli (2007)), Random DoS attack (Ni et al. (2019)), energy system (Hu et al. (2019) and in theoretical researches (Qiu et al. (2015); Kwon et al. (2016); Park et al. (2018); Zhang et al. (2019a,b)). In practice, it is hard to precisely estimate the transition rates (TRs) in the jumping process. Thus, researches on the MJSs with incomplete knowledge of TRs attract attention: bounded uncertain TRs (Shi and Boukas (1997); Xiong et al. (2005)), partly unknown TRs (Zhang and Boukas (2009); Zhang et al. (2008); Wang et al. (2011); Kwon et al. (2016); Park et al. (2019); Shin and Park (2019); Lin et al. (2019)). The authors of Guo and Wang (2013) proposed a new description for the uncertain TRs, which is named as generally uncertain transition rates (GUTRs) in which each TR can be completely known or only its estimate is known. Such feature makes this model more flexible than that of both bounded TR and partly unknown TR.

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On the other hand, descriptor systems provide convenient representations in the description of many practical systems: (Boukas (2008); Xu and Lam (2006); Nguyen et al. (2018)). Recently, descriptor MJSs (DMJSs), which are descriptor systems with Markovian switching, have gained a substantial amount of attention due to the fact that DMJSs can better express the behavior of some physical systems than normal MJSs (Xia et al. (2009); Kim (2015); Kwon et al. (2017a); Zhang et al. (2017); Zheng et al. (2019)). Especially, the authors of Kao et al. (2014) proposed a state-feedback control for DMJSs with GUTRs. However, the relaxation condition in Kao et al. (2014) to deal with the problem of GUTRs requires additional constraint in Lyapunov function matrices.

In practice, the system states are usually difficult to be measured precisely, so output-feedback (OF) control problems for DMJSs were studied (Raouf and Boukas (2009); Zhang et al. (2012); Chen et al. (2016); Kwon et al. (2017b)). In Raouf and Boukas (2009), OF stabilization condition for DMJSs was studied in the form of bilinear matrix inequalities (BMIs). The authors of Zhang et al. (2012) considered finite-time static OF control of DMJSs. Moreover, the authors of Chen et al. (2016) studied the static OF stabilization problems for DMJS with GUTRs in terms of BMIs. Especially, the necessary and sufficient conditions for dynamic OF stabilization for DMJSs were first introduced in Kwon et al. (2017b). However, to obtain the controller gain matrices in Kwon et al. (2017b), the complete knowledge of TRs is needed. To overcome this difficulty, the authors of Park et al. (2019) give new necessary and sufficient conditions for dynamic OF stabilization for DMJSs with partly unknown TRs. However, the conditions in Park et al. (2019) can be more simplified. Additionally, to the best of our knowledge, there is no related article considering the dynamic OF control of DMJSs with GUTRs in the form of linear matrix inequalities (LMIs), which motivates us for this study.

Motivated from the aforementioned needs, this paper considers the dynamic OF stabilization problem of DMJSs with GUTRs. First, a new necessary and sufficient condition to relax inequalities including GUTRs is introduced. For the closed-loop systems with a dynamic OF controller, the stabilization criterion is achieved as non-convex matrix inequalities. For the obtained criterion, this paper gives an improved necessary and sufficient condition in terms of LMIs under completely known TRs based on the method proposed in Park et al. (2019). Then, the proposed condition is extended for the DMJSs with GUTRs. A numerical example shows the validity of the derived results. Further, the contributions of this paper can be pointed out as follows:

- A generalized relaxation method considering the problem of GUTRs is proposed as a necessary and sufficient condition. Then, it can be shown in this paper that the method can be easily applied to the inequalities containing the GUTRs. Consequently, differently from Chen et al. (2016) which is obtained from the relaxation method Kao et al. (2014), this paper successfully provides a dynamic OF stabilization condition of DMJS with GUTRs in terms of LMIs.
- By utilizing the set-equivalent conditions proposed in Feng and Shi (2017), this paper successfully gives a simplified necessary and sufficient conditions for the conditions in Park et al. (2019).

1.1 Notation

- For a vector v, v^T means the transpose of v.
- $\mathbf{Inte}(N) = \{1, 2, \ldots, N\}$
- For symmetric matrices A and B, $A \geq B$ stands for that $P - Q$ is positive semi-definite.
- For symmetric matrices A and B, $A > B$ stands for that $P - Q$ is positive definite.
- For square matrix M, $\mathbf{He}(M) = M + M^T$.
- For square matrix N, $diag(N, N) = N \oplus N$.
- I is an identity matrix.
- For matrices A_i, and set $\mathcal{S} = \{s_1, s_2, \ldots, s_k\}$, $[A_i]_{i \in \mathcal{S}} \triangleq [A_{s_1},\ A_{s_2}, \ldots,\ A_{s_k}]$, and

$$diag[A_i]_{i\in\mathcal{S}} \triangleq \begin{bmatrix} A_{s_1} & 0 & \cdots & 0 \\ 0 & A_{s_2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & A_{s_k} \end{bmatrix}.$$

1.2 Abbreviations

- DMJS: descriptor Markovian jump system
- GUTR: generally uncertain transition rate
- OF: output-feedback
- LMI (BMI): linear matrix inequality (bilinear matrix inequlaity)
- *iff*: if and only if

2. PROBLEM DESCRIPTION

Consider the following descriptor Markovian jump systems (DMJSs):

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t), \\ y(t) = C(r_t)x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^{n_u}$ and $y(t) \in \mathbb{R}^p$ is the state, control input, and output, respectively. The matrix $E \in \mathbb{R}^{n\times n}$ has $rank(E) = r < n$. $\{r(t), t \geq 0\}$ is a Markov chain, which takes values in $\mathbf{Inte}(N)$. The mode transition probabilities of the Markov chain $r(t)$ are as follows:

$$\begin{aligned} &\text{Prob}(r_{t+\delta t} = j | r_t = i) \\ &= \begin{cases} \lambda_{ij}\delta t + o(\delta t) & if\ \ j \neq i, \\ 1 + \lambda_{ii}\delta t + o(\delta t) & otherwise, \end{cases} \end{aligned} \tag{2}$$

where $\delta t > 0$, $\lim_{\delta t \to 0}(o(\delta t)/\delta t) = 0$, and $\lambda_{ij} \geq 0$, for $j \neq i$, is the transition rate from mode i at time t to mode j at time $t+\delta t$, which satisfies $\lambda_{ii} = -\sum_{j=1,j\neq i}^{N} \lambda_{ij}$. That is to say, the probability of jumping to the next state depends only on the present mode and no on the previous modes. To simplify the notation, A_i, B_i and C_i denote the $A(r(t) = i)$, $B(r(t) = i)$ and $C(r(t) = i)$, respectively.

The mode TR matrix Λ is considered to be generally uncertain Guo and Wang (2013): for N operation modes,

$$\Lambda \triangleq \begin{bmatrix} \hat{\lambda}_{11} + \Delta_{11} & ? & \cdots & ? \\ ? & ? & \cdots & \hat{\lambda}_{2N} + \Delta_{2N} \\ \hat{\lambda}_{31} + \Delta_{31} & ? & \cdots & ? \\ \vdots & \vdots & \ddots & \vdots \\ ? & \hat{\lambda}_{N2} + \Delta_{N2} & \cdots & ? \end{bmatrix}, \tag{3}$$

where $\hat{\lambda}_{ij}$ and $\Delta_{ij} \in [-\delta_{ij}, \delta_{ij}](\delta_{ij} \geq 0)$ represent the known estimate value and estimate error of the uncertain TR λ_{ij}, respectively. '?' represents the completely unknown TR. For convenience, for all $i \in \mathbf{Inte}(N)$, the set U^i denotes that $U^i = U_k^i \cup U_{uk}^i$ with

$U_k^i \triangleq \{j :$ The estimate of λ_{ij} is known for $j \in \mathbf{Inte}(N)\}$,

$U_{uk}^i \triangleq \{j :$ The estimate of λ_{ij} is unknown for $j \in \mathbf{Inte}(N)\}$.

Additionally, if $i \in U_{uk}^i$, a lower bound λ_i^d must be given.

Remark 1. The GUTRs is more general than either bounded uncertain TRs or partly unknown TRs. Both the bounded uncertain TR and the partly unknown TR are the special cases of GUTR model: the GUTR matrix (3) with $U_{uk}^i = \emptyset$ can be represented as the bounded uncertain TR matrix, and the GUTR matrix with $\delta_{ij} = 0$, $\forall i \in \mathbb{N}_N^+$, $\forall j \in U_k^i$ are the partly unknown TR matrix. Therefore, it can be said that the GUTR model is used to be applied to more practical situations.

Throughout this paper, it is assumed that the system state $x(t)$ is not accessible for feedback, (A_i, B_i) is stabilizable and (A_i, C_i) is detectable for all $i \in \mathbf{Inte}(N)$. This paper consider the following dynamic OF controller:

$$\begin{cases} E\dot{x}_c(t) = A_c(r_t)x_c(t) + B_c(r_t)y(t), \\ u(t) = C_c(r_t)x_c(t) + D_c(r_t)y(t). \end{cases} \tag{4}$$

We present some assumptions about the above controller:

- $x_c(t)$ is the controller state that has same dimension to system state $x(t)$.

- The controller gain matrices have the same dimension as (1) and are dependent on the mode:
  $[A_{ci}\ B_{ci}\ C_{ci}\ D_{ci}]$
  $\triangleq [A_c(r(t)=i)\ B_c(r(t)=i)\ C_c(r(t)=i)\ D_c(r(t)=i)]$
- The matrix E is same to the singular matrix E in (1)

Now, we give some lemmas which will be used in the proof of the main results.

Lemma 1. Uezato and Ikeda (1999) Let P be symmetric such that $E_L^T P E_L > 0$ and Φ be nonsingular. Then, $PE + R^T \Phi S^T$ is nonsingular and its inverse is expressed as $(PE + R^T \Phi S^T)^{-1} = \bar{P}E^T + S\bar{\Phi}R$, where E_L and E_R are full column rank with $E = E_L E_R^T$, $R \in \mathbb{R}^{(n-r)\times n}$, and $S \in \mathbb{N}^{n\times(n-r)}$ satisfy $RE = 0$ and $ES = 0$, respectively. $\bar{P}$ is symmetric and $\bar{\Phi}$ is nonsingular such that $E_R^T \bar{P} E_R = (E_L^T P E_L)^{-1}$, and $\bar{\Phi} = (S^T S)^{-1}\Phi^{-1}(RR^T)^{-1}$.

Lemma 2. Feng and Shi (2017) The unforced DMJSs (1) is stochastically admissible *iff* there exist symmetric matrices $P_i \in \mathbb{R}^{n\times n}$ and nonsingular matrices $\Phi_i \in \mathbb{R}^{(n-r)\times(n-r)}$, such that the following coupled LMIs hold, for all $i \in \mathbf{Inte}(N)$:

$$0 < E_L^T P_i E_L \tag{5}$$

$$0 > \mathbf{He}\{(P_i E + R^T \Phi_i S^T)^T A_i\} + \sum_{j=1}^{N} \lambda_{ij} E^T P_j E. \tag{6}$$

Remark 2. The condition in Lemma 2 is not needed to satisfy the positivity of the matrix P_i for all $i \in \mathbf{Inte}(N)$, which is required in Lemma 2 of Kwon et al. (2017b). Consequently, it is obvious that the condition in Lemma 2 is less conservative than that in Lemma 2 of Kwon et al. (2017b). Additionally, for the problem of dynamic OF control of DMJSs, the condition in Lemma 2 yields more simple and less conservative condition than that in Lemma 2 of Kwon et al. (2017b), which will be discussed in main results.

To solve the problem of GUTRs, the following Lemma 3 is introduced:

Lemma 3. For the GUTR matrix (3), the inequality $0 > Q + \sum_{j\in U^i} \lambda_{ij} P_j$ holds *iff* the following inequalities hold:

(i) $(i \in U_k^i)$ For all $s \in U_k^i$, $j \in U_{uk}^i$, $r_s^i \in \{1, 2\}$

$$0 > Q + \sum_{q\in U_k^i} \tilde{\lambda}_{iq} P_q + (-\sum_{q\in U_k^i} \tilde{\lambda}_{iq}) P_j, \tag{7}$$

(ii) $(i \in U_{uk}^i)$ For all $s \in U_k^i$, $j \in U_{uk}^i$, $r_s^i \in \{1, 2\}$

$$0 > Q + \lambda_i^d P_i + \sum_{q\in U_k^i} \tilde{\lambda}_{iq} P_q - (\lambda_i^d + \sum_{q\in U_k^i} \tilde{\lambda}_{iq}) P_j, \tag{8}$$

where $\tilde{\lambda}_{iq} = \hat{\lambda}_{iq} + (-1)^{r_q^i} \delta_{iq}$.

Proof. The inequality $0 > Q + \sum_{j\in U^i} \lambda_{ij} P_j$ can be represented as

$$0 > Q + \sum_{j\in U_k^i} \lambda_{ij} P_j + \sum_{j\in U_{uk}^i} \lambda_{ij} P_j. \tag{9}$$

(i) $(i \in U_k^i)$ In this case, $\lambda_i^k < 0$. By the Theorem 1 of Zhang and Lam (2010), (9) is ensured *iff* the following condition holds:

$$0 > Q + \sum_{q\in U_k^i} \lambda_{iq} P_q - \lambda_i^k P_j,\ \forall j \in U_{uk}^i. \tag{10}$$

Now, the above condition can be represented as $\forall j \in U_{uk}^i$,

$$0 > Q + \sum_{q\in U_k^i} \hat{\lambda}_{iq} P_q + \sum_{q\in U_k^i} \Delta_{iq} P_q - \sum_{q\in U_k^i} (\hat{\lambda}_{iq} + \Delta_{iq}) P_j. \tag{11}$$

Since $-\delta_{ij} \le \Delta_{ij} \le \delta_{ij}$, we have $\Delta_{ij} = \alpha_{ij}\delta_{ij} + (1 - \alpha_{ij})(-\delta_{ij})$, where $0 \le \alpha_{ij} \le 1$. Thus, (11) holds *iff* (7) holds.

(ii) $(i \in U_{uk}^i)$ In this case, λ_{ii} is unknown, $\lambda_i^k \ge 0$ and $\lambda_{ii} \le -\lambda_i^k$. Also, we only consider $\lambda_{ii} < -\lambda_i^k$ here since if $\lambda_{ii} = \lambda_i^k$, then the ith row of the TR matrix is completely known. By the Theorem 1 of Zhang and Lam (2010), (9) holds *iff* the following condition holds:

$$0 > Q + \sum_{q\in U_k^i} \lambda_{iq} P_q + \lambda_i^d P_i - \lambda_i^d P_j - \lambda_i^k P_j, \forall j \in U_{uk}^i. \tag{12}$$

Then, with similar process in previous case, (12) holds *iff* (8) holds.

3. MAIN RESULTS

The goal of this paper is to derive a dynamic OF control for DMJSs with GUTRs. By utilizing Lemma 3, the Theorem 1 in Park et al. (2019) is extended to the case of GUTRs. By (1) and (4), the augmented closed-loop system is able to be written as

$$\bar{E}\dot{\bar{x}}(t) = A_{cl,i}\bar{x}(t) \tag{13}$$

where $\bar{E} \triangleq diag(E, E)$, $\bar{x}(t) \triangleq [x^T(t)\ x_c^T(t)]^T$, and

$$A_{cl,i} \triangleq \begin{bmatrix} A_i + B_i D_{ci} C_i & B_i C_{ci} \\ B_{ci} C_i & A_{ci} \end{bmatrix}. \tag{14}$$

Applying the closed-loop system (13) to Lemma 2 provides the stochastically admissibility criterion as follows:

$$0 > \Omega_i + U_i \Pi_i V_i^T + V_i \Pi_i^T U_i^T,\ \forall i \in \mathbf{Inte}(N), \tag{15}$$

$$0 < E_L^T P_i E_L,\ \forall i \in \mathbf{Inte}(N), \tag{16}$$

where $\Omega_i = \Psi_i^T \bar{A}_i + \bar{A}_i^T \Psi_i + \sum_{j=1}^N \lambda_{ij} \bar{E}^T P_j \bar{E}$, $U_i = \Psi_i^T \bar{B}_i$, $V_i = \bar{C}_i^T$, $\Psi_i = P_i \bar{E} + \bar{R}\Phi_i \bar{S}^T$ and

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix},\ \bar{B}_i = \begin{bmatrix} 0 & B_i \\ I & 0 \end{bmatrix},$$

$$\bar{C}_i = \begin{bmatrix} 0 & I \\ C_i & 0 \end{bmatrix},\ \Pi_i = \begin{bmatrix} A_{ci} & B_{ci} \\ C_{ci} & D_{ci} \end{bmatrix}.$$

By applying the elimination lemma Boyd et al. (1994) to the free variable Π_i, it can be clearly known that (15) holds *iff* $0 > U_i^{\perp T} \Omega_i U_i^{\perp}$ and $0 > V_i^{\perp T} \Omega_i V_i^{\perp}$. This results in the follows:

$$0 > \bar{B}_i^{\perp T} \left(\mathbf{He}(\bar{A}_i \bar{\Psi}_i) + \sum_{j=1}^{N} \lambda_{ij} \bar{\Psi}_i^T \bar{E}^T P_j \bar{E} \bar{\Psi}_i \right) \bar{B}_i^{\perp}, \tag{17}$$

$$0 > (\bar{C}_i^T)^{\perp T} \left(\mathbf{He}(\Psi_i^T \bar{A}_i) + \sum_{j=1}^{N} \lambda_{ij} \bar{E}^T P_j \bar{E} \right) (\bar{C}_i^T)^{\perp}, \tag{18}$$

where $\bar{\Psi}_i \triangleq \Psi_i^{-1} = \bar{P}_i \bar{E} + \bar{R}\bar{\Phi}_i \bar{S}^T$.

In the following theorem, the stabilization problem of DMJSs with completely known transition rates is derived based on the Theorem 1 of Park et al. (2019).

Theorem 1. For all $i \in \mathbf{Inte}(N)$, there exist matrices P_i and Φ_i such that (16)-(18) hold *iff* there exist matrices X_i, $\bar{X}_i$, Φ_{1i} and $\bar{\Phi}_{1i}$ such that

$$0 < \begin{bmatrix} E_L^T X_i E_L & I \\ I & E_R^T \bar{X}_i E_R \end{bmatrix}, \quad (19)$$

$$0 > \left[\begin{array}{c|c} B_i^{\perp T}\mathcal{L}_{1i}B_i^{\perp} & \left[\sqrt{\lambda_{ij}} B_i^{\perp T} E \bar{X}_i E_R\right]_{j\in \mathbf{Inte}(N)/\{i\}} \\ \hline (*) & -diag\left[E_R^T \bar{X}_j E_R\right]_{j\in \mathbf{Inte}(N)/\{i\}} \end{array}\right], \quad (20)$$

$$0 > (C_i^T)^{\perp T}\left[\mathcal{L}_{2i} + \sum_{j=1}^{N}\lambda_{ij}E^T X_j E\right](C_i^T)^{\perp}, \quad (21)$$

where $\mathcal{L}_{1i} \triangleq \mathbf{He}(A_i(\bar{X}_i E^T + S\bar{\Phi}_{1i}R)) + \lambda_{ii}E\bar{X}_i E^T$ and $\mathcal{L}_{2i} \triangleq \mathbf{He}(A_i^T(X_i E + R^T\Phi_{1i}S^T))$.

Proof. The proof is omitted because of the space limit.

Remark 3. The Theorem 1 of Park et al. (2019) requires the positivity of X_i, which is not needed in Theorem 1. From this point of view, it can be said that this paper successfully gives a simplified necessary and sufficient conditions for the conditions in Park et al. (2019).

Now, the Theorem 1 is extended to the case of generally uncertain transition rates with a help of Lemma 3.

Theorem 2. For all $i \in \mathbf{Inte}(N)$, there exist matrices P_i and Φ_i such that (16)-(18) hold *iff* there exist matrices X_i, $\bar{X}_i$, Φ_{1i} and $\bar{\Phi}_{1i}$ such that

$$0 < \begin{bmatrix} E_L^T X_i E_L & I \\ I & E_R^T \bar{X}_i E_R \end{bmatrix}, \quad (22)$$

(i) ($i \in U_k^i$) $\forall s \in U_k^i$, $j \in U_{uk}^i$, $r_s^i \in \{1,2\}$, (23) and (24),

(ii) ($i \in U_{uk}^i$) $\forall s \in U_k^i$, $j \in U_{uk}^i$, $r_s^i \in \{1,2\}$, (25) and (26),

where $\tilde{\lambda}_{iq} = \hat{\lambda}_{iq} + (-1)^{r_q^i}\delta_{iq}$, $\mathcal{L}_{1i} \triangleq \mathbf{He}(A_i(\bar{X}_i E^T + S\bar{\Phi}_{1i}R)) + \lambda_{ii}E\bar{X}_i E^T$, $\mathcal{L}_{2i} \triangleq \mathbf{He}(A_i^T(X_i E + R^T\Phi_{1i}S^T))$, and $Q_i = B_i^{\perp T}\mathcal{L}_{1i}B_i^{\perp} + \lambda_i^d B_i^{\perp T}E^T\bar{X}_i E B_i^{\perp}$.

Proof. The proof is omitted because of the space limit.

Remark 4. Differently from the result in Chen et al. (2016) which considers the static OF stabilization condition for DMJS with GUTRs in terms of *BMIs*, the Theorem 2 successfully provides the dynamic OF stabilization condition for DMJS with GUTRs in terms of *LMIs*.

4. ILLUSTRATIVE EXAMPLE

Let us consider DMJSs (1) that have the following parameters with three modes:

$$A_1 = \begin{bmatrix} 0.3 & -0.4 \\ -0.8 & -0.4 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.8 & -0.2 \\ -0.8 & -1.2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.2 & -0.1 \\ -0.8 & -1.0 \end{bmatrix},\ B_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C_1 = C_2 = C_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T,$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ E_L = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ E_R = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$R = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T,\ S = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad (27)$$

and the TR matrices are as follows:

$$\Lambda_1 = \begin{bmatrix} -3.1 & 2.5 & 0.6 \\ 0.5 & -2.4 & 1.9 \\ 0.9 & 2.9 & -3.8 \end{bmatrix},$$

$$\Lambda_2 = \begin{bmatrix} -3.2+\Delta_{11} & ? & ? \\ ? & ? & 2+\Delta_{23} \\ ? & ? & ? \end{bmatrix},$$

where $\Delta_{11} \in [-0.1, 0.1]$ and $\Delta_{23} \in [-0.2, 0.2]$. Λ_1 and Λ_2 indicate the completely known and the generally uncertain transition rate matrix, respectively. The lower bounds of the unknown diagonal elements are set as $\lambda_2^d = -10$, $\lambda_3^d = -10$. Then, for the completely known transition rate matrix Λ_1, the controller gain matrices can be obtained from Theorem 1 but the details are omitted because of the space limit. Fig. 1 shows that the open-loop system is stochastically unstable. Then, applying the obtained controller gain matrices to the system results in Fig. 2, which presents that the state trajectories of the closed-loop system go to zero. Therefore, the dynamic OF controller obtained from the Theorem 1 is stochastically stabilize the systems with the completely known transition rate matrix Λ_1.

Next, for the generally uncertain transition rate matrix Λ_2, the Theorem 2 provides the controller gain matrices as follows:

$$\Pi_1 = \left[\begin{array}{cc|c} -44.0747 & -4.8413 & -6.4304 \\ 4.6851 & -2.7034 & -31.7004 \\ \hline 4.6894 & -2.3024 & -30.8952 \end{array}\right],$$

$$\Pi_2 = \left[\begin{array}{cc|c} -14.5445 & -20.5170 & 37.5833 \\ -24.7162 & 19.7741 & 0.2099 \\ \hline 24.8119 & -20.9982 & -1.0930 \end{array}\right],$$

$$\Pi_3 = \left[\begin{array}{cc|c} -26.8372 & 4.3020 & 39.4622 \\ 27.2046 & 8.1728 & 16.9934 \\ \hline 27.3398 & 9.1953 & 17.7548 \end{array}\right].$$

Fig. 3 shows the state responses $x_1(t)$ and $x_2(t)$ of the closed-loop system with the above controller gain matrices, which yields that the Theorem 2 ensures the stochastic stabilizability of the SMJSs with generally uncertain transition rates.

5. CONCLUSIONS AND FUTURE WORKS

This paper dealt with the dynamic OF stabilization problem of DMJSs with GUTRs. A new necessary and sufficient condition to relax the inequalities including GUTRs was first introduced. For the systems with the dynamic OF controller, the stabilization criterion was achieved as nonconvex matrix inequalities. For the obtained criterion, this paper provided an improved necessary and sufficient condition in the form of linear matrix inequalities under completely known transition rates based on the method proposed in Park et al. (2019). Then, the proposed condition was extended for the DMJSs with GUTRs. A numerical example illustrated the feasibility of the derived control. Moreover, this paper considered the mode-independent singular matrix E. Therefore, it can be a meaningful future work to deal with the DMJSs that have mode-dependent singular matrix E_i. Also, we focus on applying the proposed control scheme to the real time implementation which can be modeled by DMJSs with GUTRs.

$$0 > \left[\begin{array}{c|c|c} B_i^{\perp T}\mathcal{L}_{1i}B_i^{\perp} & \left[\sqrt{\tilde{\lambda}_{iq}}B_i^{\perp T}E\bar{X}_iE_R\right]_{q\in U_k^i/\{i\}} & \sqrt{-\sum_{q\in U_k^i}\tilde{\lambda}_{iq}}B_i^{\perp T}E\bar{X}_iE_R \\ \hline (*) & -diag\left[E_R^T\bar{X}_qE_R\right]_{q\in U_k^i/\{i\}} & 0 \\ \hline (*) & (*) & -E_R^T\bar{X}_jE_R \end{array}\right], \tag{23}$$

$$0 > (C_i^T)^{\perp T}\left[\mathcal{L}_{2i} + \sum_{q\in U_k^i}\tilde{\lambda}_{iq}EX_qE^T + (-\sum_{q\in U_k^i}\tilde{\lambda}_{iq})EX_jE^T\right](C_i^T)^{\perp}. \tag{24}$$

$$0 > \left[\begin{array}{c|c|c} Q_i & \left[\sqrt{\tilde{\lambda}_{iq}}B_i^{\perp T}E\bar{X}_iE_R\right]_{q\in U_k^i/\{i\}} & \sqrt{-\lambda_i^d-\sum_{q\in U_k^i}\tilde{\lambda}_{iq}}B_i^{\perp T}E\bar{X}_iE_R \\ \hline (*) & -diag\left[E_R^T\bar{X}_qE_R\right]_{q\in U_k^i/\{i\}} & 0 \\ \hline (*) & (*) & -E_R^T\bar{X}_jE_R \end{array}\right], \tag{25}$$

$$0 > (C_i^T)^{\perp T}\left[\mathcal{L}_{2i} + \lambda_i^dEX_iE^T + \sum_{q\in U_k^i}\tilde{\lambda}_{iq}EX_qE^T + (-\lambda_i^d - \sum_{q\in U_k^i}\tilde{\lambda}_{iq})EX_jE^T\right](C_i^T)^{\perp}, \tag{26}$$

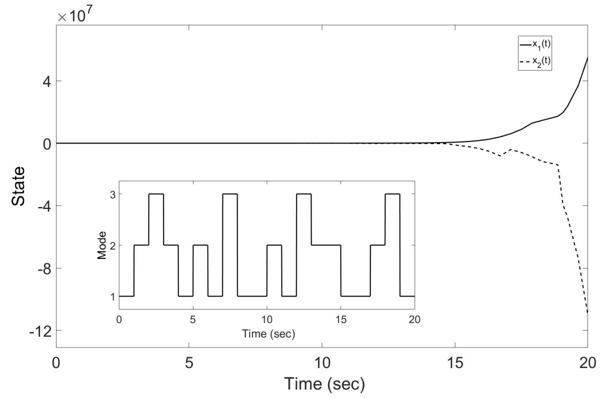

Fig. 1. Open-loop system simulation results: initial condition $x_0 = [1.2\ \ -0.6]^T$;

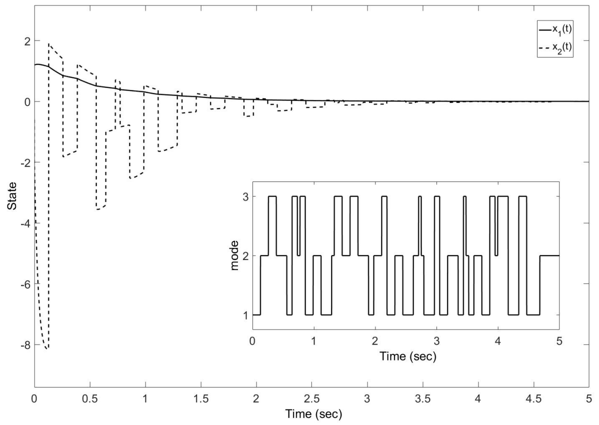

Fig. 2. Simulation results for the closed-loop system with completely known transition rate matrix Λ_1 simulation results: initial condition $x_0 = [1.2\ \ -0.6]^T$

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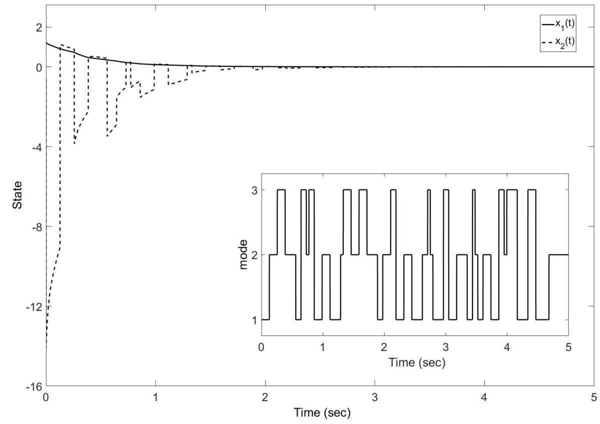

Fig. 3. Simulation results for the closed-loop system with generally uncertain transition rate matrix Λ_2 simulation results: initial condition $x_0 = [1.2\ \ -0.6]^T$

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