Preprints of the 21st IFAC World Congress (Virtual)
Berlin, Germany, July 12-17, 2020

Immersion based observer for the switched reluctance motor ⋆

Alejandra de la Guerra * **Alejandro Gutiérrez–Giles** ** **Gildas Besançon** ***

* *Metropolitan Autonomous University (UAM), Av. San Pablo Xalpa 180, Reynosa Tamaulipas, Azcapotzalco, CDMX, Mexico (e-mail: ale_delaguerra@comunidad.unam.mx).*
** *Center of Research and Advanced Studies of the National Polytechnic Institute (Cinvestav), Av. Instituto Politécnico Nacional 2508, CDMX, Mexico (e-mail: alejandro.giles@cinvestav.mx)*
*** *GIPSA-Lab, CNRS Univ. Grenoble Alpes, CNRS, Grenoble INP GIPSA-lab, 38000 Grenoble, France (e-mail: gildas.besancon@gipsa-lab.grenoble-inp.fr)*

Abstract: This article presents a high–gain Kalman–like observer for the switched reluctance motor that can reconstruct the angular position and velocity from the measured variables, *i. e.*, stator currents and voltages. This is possible after the results obtained from an observability analysis, which in turn permits the application of an immersion to transform the motor model into a suitable form for the observer implementation. Numerical simulations are incorporated to explain and validate the observer design. The procedure includes an active phase detection stage and a current–based speed controller that defines the commutation required for this kind of motors. The proposed observer is capable of reconstructing the mechanical variables (rotor position and speed) by employing only electrical ones (currents and voltages).

Keywords: switched reluctance motors, observers, observability, sensorless control

1. INTRODUCTION

This paper is motivated by the great qualities of the Switched Reluctance Motor (SR motor), such as its simple and endurable construction, no need of permanent magnets, and its torque-speed characteristics. In particular, the SR motor has the unique property that it can still operate during certain fault conditions (Saha and Choudhury, 2016), which makes this motor a reliable source of motion. Moreover, these properties make it a serious candidate for traction in Electric Vehicles (EV) and Hybrid Electric Vehicles (HEV), as documented in Rahman et al. (2000), Zeraoulia et al. (2006), and Ehsani et al. (2018). In the case of wind energy generation, there are cost problems associated to the employment of rare-earth elements to produce permanent magnets, such as neodymium (Nd). One of the solutions proposed by Jacobson and Delucchi (2011) is to avoid the use of this element by replacing the permanent magnet generators (PMG) with generators of similar performance and size such as the Switched Reluctance Generator (SRG). However, there are some disadvantages for using this motor. For example, the control of this machine is more complicated than for other motors. In particular, it cannot be open loop controlled. Another drawback is that most industrial controllers present speed ripple and noise. Additionally, it is not as commercially available as other motors.

Another control challenge is the *sensorless control*, *i.e.*, the implementation of a speed control by using only electrical input (voltages) and output (currents) measurements, without mechanical sensors. The main motivation behind the *sensorless control* is of economic nature, since it can diminish the cost of the entire drive while, from a technical point of view, the elimination of mechanical sensors decreases the complexity and maintenance of the system.

The sensorless problem for the switched reluctance motor has been extensively studied and partially solved using different methods, which can be classified as: 1) active-phase detection, 2) current gradient, 3) flux linkage reconstruction, and 4) state observers. In general, the first two methods are signal-based and the last two ones are model-based methods. However, a review of the state of the art shows several problems associated with the design of these estimators: most of the reported estimation techniques were made for a determined speed interval, e.g., low, medium or high speed. Also, few state observers have been reported, which includes the Luenberger-like observers proposed by Lumsdaine and Lang (1990) and Elmas and Zelaya-De La Parra (1996), and the sliding mode observer introduced by McCann et al. (2001) and lately modified by Brandstetter and Krna (2013). In more recent articles, such as Ouddah et al. (2015), Brandstetter et al. (2016), Xiao Wang et al. (2016), and Li et al. (2017), the authors work with very simplified models of the motor and the observers employed are based on the works already

⋆ Alejandra de la Guerra thanks **CONACYT** under scholarship **CVU: 332351**. Alejandro Gutiérrez–Giles thanks **CONACYT** under scolarship **CVU: 334785**

mentioned. On the other hand, the *starting hesitation* has been a constant problem for this motor, as explained by Bu and Xu (2001). As a consequence, some research effort has been done to detect the initial absolute rotor position as in Gao et al. (2001), Hossain et al. (2003), Bamba et al. (2007), and Komatsuzaki et al. (2008) to name a few. Moreover, Hossain et al. (2003) explicitly mentions, without a formal proof, that the motor is not observable at zero speed when the phase currents are zero.

In this article, an observability analysis, based on the *observability rank condition* from Hermann and Krener (1977) and the *local weak observability* property defined therein, is presented. Then, the results of this analysis are employed in the design of both, an active-phase detector and a local observer for each stator phase. The observability analysis shows the two main challenges to design an state observer for this system: 1) the model is non-uniformly observable, *i.e.*, the observability depends on the input, and 2) the observability map presents singularities, which are linked to the motor's commutation stage. It must be mentioned that these results are in accordance with the observability analysis based on the indistinguishable dynamics of the SR motor reported in de la Guerra et al. (2015).

The main contribution of this paper is: from the observability analysis results, an immersion of the mathematical model of the SR motor is developed, which in turns serves to implement a Kalman-like observer following the results of Besançon and Ticlea (2007).

This article is organized as follows: Section 2 presents the SR motor model and its commutation. In Section 3, an observability analysis is developed. Section 4 presents the proposed immersion and the corresponding observer. Section 5 includes simulation results to validate the proposed scheme. Finally, in Section 6 some conclusions and directions for future work are provided.

2. SWITCHED RELUCTANCE MOTOR MODEL

A mathematical model for a m–phases SR motor is given by (de la Guerra et al., 2016)

$$\frac{\mathrm{d}\boldsymbol{i}}{\mathrm{d}t} = \boldsymbol{L}^{-1}(\theta)(-\omega \boldsymbol{C}(\theta)\boldsymbol{i} - \boldsymbol{R}\boldsymbol{i} + \boldsymbol{u}) \tag{1a}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \omega \tag{1b}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{2\mathrm{J}}\boldsymbol{i}^{\mathrm{T}}\mathbf{C}(\theta)\boldsymbol{i} - \frac{\mathrm{d}}{\mathrm{J}}\omega\,, \tag{1c}$$

where $\boldsymbol{i} \in \mathbb{R}^m$ is the vector of stator currents, $\boldsymbol{u} \in \mathbb{R}^m$ is the vector of voltage inputs, $\omega \in \mathbb{R}$ is the angular velocity, $\boldsymbol{R} \in \mathbb{R}^{m\times m}$ is a diagonal matrix accounting for the winding resistances, $\mathrm{J} \in \mathbb{R}$ is the rotor inertia and $\mathrm{d} \in \mathbb{R}$ the viscous friction coefficient. In this work, in contrast with de la Guerra et al. (2016), non-saturated (triangular) winding inductances are considered, which are defined by

$$L_j(\theta) = \begin{cases} l_{\mathrm{u}} + c_j\left(\theta_j + \dfrac{\pi}{N_{\mathrm{r}}}\right) & \text{if } -\dfrac{\pi}{N_{\mathrm{r}}} < \theta_j \leq 0 \\ l_{\mathrm{a}} - c_j\theta_j & \text{if } 0 < \theta_j \leq \dfrac{\pi}{N_{\mathrm{r}}} \end{cases}, \tag{2}$$

where $j = 1, 2, 3, \ldots, m$, $c_j = (l_{\mathrm{a}} - l_{\mathrm{u}})N_{\mathrm{r}}/\pi$, $\phi_j = 2\pi\,(j-1)\,/\,(N_{\mathrm{r}}m)$ is the phase shift, $\theta \in \mathbb{R}$ is the angular rotor position, $\theta_j \triangleq \theta - \phi_j \pmod{\pi/N_{\mathrm{r}}}$, and $l_{\mathrm{a}} > l_{\mathrm{u}} > 0$ are the aligned and unaligned phase inductances, respectively. Thus, $\boldsymbol{L}(\theta) \in \mathbb{R}^{m\times m}$ is a diagonal matrix of winding inductances and $\boldsymbol{C}(\theta) \in \mathbb{R}^{m\times m}$ is given by

$$\boldsymbol{C}(\theta) = \frac{\partial \boldsymbol{L}(\theta)}{\partial\theta}\,.$$

Fact 1. The winding inductances matrix is symmetric and positive definite $\mathbf{L}(\theta) = \mathbf{L}^{\mathrm{T}}(\theta) > 0$, for every value of θ.

Define the vector $\boldsymbol{x} \triangleq [\boldsymbol{x}_1 \quad x_2 \quad x_3]^{\mathrm{T}} = [\boldsymbol{i} \quad \theta \quad \omega]^{\mathrm{T}}$. Then, the state–space form of model (1) is

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{L}^{-1}(x_2)(-x_3\boldsymbol{C}(x_2)\boldsymbol{x}_1 - \boldsymbol{R}\boldsymbol{x}_1 + \boldsymbol{u}) \tag{3a}$$

$$\dot{x}_2 = x_3 \tag{3b}$$

$$\dot{x}_3 = \frac{1}{2\mathrm{J}}\boldsymbol{x}_1{}^{\mathrm{T}}\mathbf{C}(x_2)\boldsymbol{x}_1 - \frac{\mathrm{d}}{\mathrm{J}}x_3\,. \tag{3c}$$

2.1 Commutation

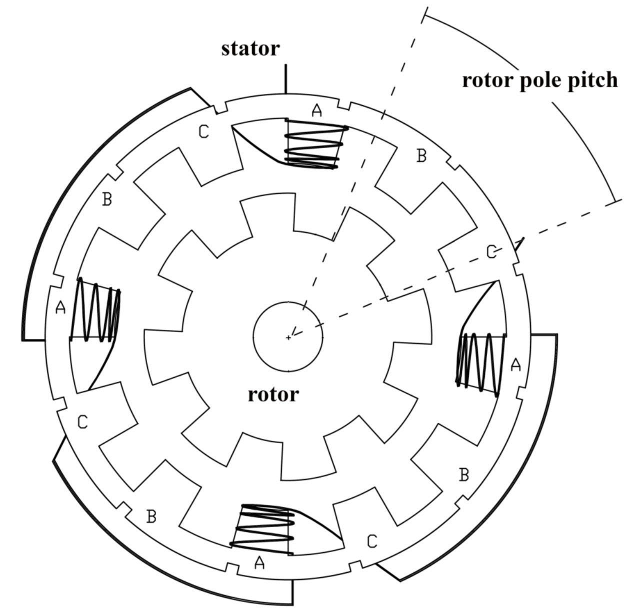

Fig. 1. Rotor and stator of a three phases SR motor showing the pole pitch section.

Consider the diagram of the SR motor shown in Figure 1. A current must circulate in a given winding to attract the poles of the rotor close to the stator poles of the corresponding phase. The position shown in Figure 1 corresponds to the maximum (aligned) inductance value, l_{a}, for the phase B. When the rotor poles are completely unaligned with the corresponding phase, the value of the inductance reaches its minimum, denoted by l_{u}. During each fundamental switching period, labeled as rotor pole-pitch in Figure 1, all phases are excited once and the interval between the excitation of two consecutive phases is called the *stroke angle* (Miller, 2001). One stroke is equal to $S = mN_r$, so the stroke angle is

$$\theta_{\mathrm{s}} = \frac{2\pi}{mNr}\,. \tag{4}$$

The total produced torque is the sum of the torque produced for each phase. There must be one stroke per rotor pole–pitch for each phase, and the current in each phase flows in a fraction of this cycle.

3. OBSERVABILITY ANALYSIS

In this section, an observability analysis of the SR motor model by using the so-called *observability rank condition* (Hermann and Krener, 1977) is presented. The state space model of the SR motor (3) can be rewritten in the form

$$\begin{aligned}\dot{\boldsymbol{x}} &= f_0(\boldsymbol{x}) + f_1(\boldsymbol{x})\bar{\boldsymbol{u}}\\ \boldsymbol{y} &= h_j(\boldsymbol{x})\,,\end{aligned} \tag{5}$$

where $\bar{\boldsymbol{u}} = -\boldsymbol{R}\boldsymbol{x}_1 + \boldsymbol{u}$. For the sake of exposition, the foregoing analysis will be presented only for phase 1, although the same procedure can be straightforwardly applied for the remaining ones. For phase 1, the vector fields $f_0, f_1 \in \mathbb{R}^3$ and the output function $h_j \in \mathbb{R}$ are defined as

$$\begin{aligned} f_0 &= \begin{bmatrix} -\dfrac{C_1(x_2)x_{11}x_3}{L_1(x_2)} & x_3 & T_\mathrm{e} - \dfrac{\mathrm{d}}{\mathrm{J}}x_3 \end{bmatrix}^\mathrm{T}\\ f_1 &= \begin{bmatrix} \dfrac{1}{L_1(x_2)} & 0 & 0 \end{bmatrix}^\mathrm{T}\\ h_1 &= x_{11}\,, \end{aligned} \tag{6}$$

where x_{11} is the first component of the vector x_1, and

$$T_\mathrm{e} = \frac{1}{2\mathrm{J}}C_1(x_2)x_{11}^2.$$

For the system (6), the observation space $\mathcal{O}(h)$ is formed with h_1 and its Lie derivatives along the vector fields f_0 and f_1 as

$$\mathcal{O}(h) = \{h_1, L_{f_0}h_1, L_{f_1}h_1, L_{f_0}^2 h_1, L_{f_1 f_0}h_1, \dots\}\,.$$

For a distribution based on these fields to be nonsingular at some point x_0, the observability matrix, $\mathbf{O}$, constructed with the differentials of the elements of the observation space, has to be nonsingular at x_0. In this case, the output is $h_1 = x_{11}$ and the Lie derivatives are given by

$$L_{f_0}h_1 = -\frac{C_1(x_2)x_{11}x_3}{L_1(x_2)} \tag{7}$$

$$L_{f_1}h_1 = \frac{1}{L_1(x_2)}\,. \tag{8}$$

Thus, the observability matrix is

$$\mathbf{O} = \begin{bmatrix} 1 & 0 & 0\\ \dfrac{\partial L_{f_0}h_1}{\partial x_{11}} & \dfrac{\partial L_{f_0}h_1}{\partial x_2} & \dfrac{\partial L_{f_0}h_1}{\partial x_3}\\ 0 & \dfrac{\partial L_{f_1}h_1}{\partial x_2} & 0 \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} 1 & 0 & 0\\ -\dfrac{C_1(x_2)x_3}{L_1(x_2)} & \dfrac{C_1^2(x_2)x_{11}x_3}{L_1^2(x_2)} & -\dfrac{C_1(x_2)x_{11}}{L_1(x_2)}\\ 0 & \dfrac{C_1(x_2)}{L_1^2(x_2)} & 0 \end{bmatrix},$$

where it was substituted $\partial C_1(x_2)/\partial x_2 = 0$, given the triangular inductances definition (2). After some calculations, it can be shown that the rank condition is fulfilled if the determinant of matrix $\mathbf{O}$ satisfies

$$\det(\mathbf{O}) = -\frac{C_1^2(x_2)x_{11}}{L_1^3(x_2)} \neq 0\,. \tag{10}$$

In other words, from Fact 1, the expression (10) is satisfied if $C_1(x_2)$ and x_{11} are different from zero simultaneously. Therefore, the SR motor model (3) is locally and weakly observable (Hermann and Krener, 1977) in a region defined, for the j–th phase, where the following conditions are satisfied

$$i_j \neq 0\,, -\frac{\pi}{2N_\mathrm{r}} < \theta_j < 0\,. \tag{11}$$

Remark 1. The conditions (11) are directly related with the way in which the phases of the SR motor are switched to generate continuous movement of the rotor shaft, because $C_j(x_2) = 0$ determines the switching instants of the j–th phase. This means that the mechanical variables can be reconstructed with the sum of the contributions of each phase in a power generation cycle if the current for the corresponding phase is different from zero.

Remark 2. When the input of a given phase is identically zero, that is, $\bar{u}_j \equiv 0$, it can be seen from (3a) that the corresponding phase current tends exponentially to zero. Thus, eventually $\mathbf{O}$ will be singular. This implies (11) will be fulfilled only if $\bar{u}_j \neq 0$ which means that the observability depends on the input: model (1) is non-uniformly observable. This result is in accordance with the one obtained in de la Guerra et al. (2015).

4. MAIN RESULT

In this section the main result of this article is presented. *i.e.*, the design of an observer for the SR motor based on a transformed model derived from an immersion procedure. This immersion transformation is based on the construction of an observability-like space and is explained in the next subsection.

4.1 Immersion-based observer

As shown in the last section, model (3) is non-uniformly observable. To surmount this challenge, it may be possible to use a transformation to put this model in a suitable form for the observer design. Accordingly, in this section the immersion procedure taken from Besançon and Ticlea (2007) is recalled to *locally immerse* the control-affine system (5) satisfying the observability rank condition at some point x_0, into the form

$$\begin{aligned}\dot{\boldsymbol{z}} &= \boldsymbol{A}(u,y) + \boldsymbol{B}(u,z)\\ \boldsymbol{y} &= \boldsymbol{C}(u)z + \boldsymbol{D}(u)\,.\end{aligned} \tag{12}$$

Immersion procedure:

- Build a vector $z_1(x)$ of all state–dependent functions $h_j(x)$ of (5), $j = 0, \dots, m$.
- At step $k+1$, assume the vectors z_1 to z_k have been constructed in the previous steps, and choose among the differentials of the elements in $z_1, \dots, z_k$ a basis $\{d\phi_1, \dots, d\phi_k\}$ regular around x_0 for the co-distribution spanned by these differentials. Let ν_k be the dimension of such basis.
  - · If $\nu_k = n$ with n the system order, end the procedure.
  - · If not, construct a vector $z_{k+1}(x)$ by taking all functions $L_{f_i}z_k^l$, $i = 0, \dots, m$, $l = 1, \dots, N_k$ that do not satisfy $d\phi_1\wedge, \dots, d\phi_{\nu_k}\wedge dL_{f_i}z_k^l = 0$ around x_0, with
  $$z_k = \begin{bmatrix} z_k^1 & \cdots & z_k^{N_k}\end{bmatrix}^\mathrm{T}.$$
- Notice that by construction, the elements in the vectors $z_1, z_2, \dots$, belong to the observation space of

the system, $\mathcal{O}$, which means that their differentials are elements of $\mathbf{O}$.

For the SR motor model, in accordance with the observability analysis, the system is observable if the output, *i.e.*, the stator current for a given phase, is different from zero in the corresponding rotor-pole pitch interval. To fulfill this condition the procedure will be made per-phase. Thus, for phase 1 the first variable is

$$h_1 = z_1^1 = x_{11}\,.$$

Recalling the definition of fields f_0 and f_1 used in Section 3, the Lie derivatives along the vector fields f_0 and f_1 are given by (7) and (8). Since these Lie derivatives cannot be expressed only in terms of z_1^1, both become new variables, z_2^1 and z_2^2. As the differentials of these three variables are locally linearly independent, they already define a basis for the observation space, thus the procedure stops here. The new state vector is defined as $\boldsymbol{z} = [z_1^1 \quad z_2^1 \quad z_2^2]^{\mathrm{T}}$ and the input is $u = \bar{u}_1$. Then, the system has the form

$$\dot{\boldsymbol{z}} = \boldsymbol{A}(u,y)\boldsymbol{z} + \boldsymbol{b}(u,\boldsymbol{z})\,, \quad y = \mathbf{C}\boldsymbol{z} \tag{13}$$

with

$$\mathbf{A}(\boldsymbol{u},y) = \begin{bmatrix} 0 & 1 & u \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \mathbf{C} = [1 \ \ 0 \ \ 0] \ , \text{ and}$$

$$\boldsymbol{b}(\boldsymbol{z},\boldsymbol{u}) = \begin{bmatrix} 0 \\ \dfrac{2(z_2^1)^2}{z_1^1} - \dfrac{c_1^2}{2\mathrm{J}} z_2^2 (z_1^1)^3 - \dfrac{\mathrm{d}}{\mathrm{J}} z_2^1 + \dfrac{z_2^1 z_2^2}{z_1^1} u \\ \dfrac{z_2^1 z_2^2}{z_1^1} \end{bmatrix},$$

where $c_1 = (l_{\mathrm{a}} - l_{\mathrm{u}})N_{\mathrm{r}}/\pi$ is a positive constant. As a consequence, the conditions (11) are satisfied in the region of interest. Notice also that these conditions imply $z_1^1 \neq 0$. Therefore, m systems like model (13) must be defined to estimate the angular rotor position in one operation cycle following the torque generation cycle defined in the Section 2.

4.2 Observer

Once the model (3) has been transformed into to the form (12), a high–gain Kalman–like observer, proposed by Besançon and Ticlea (2007), can be designed as

$$\begin{aligned} \dot{\hat{\boldsymbol{z}}} &= \mathbf{A}(u,y) + \boldsymbol{b}(u,\hat{\boldsymbol{z}}) - \boldsymbol{\Gamma}(\lambda)\boldsymbol{S}^{-1}\boldsymbol{C}^T(u)(\hat{y} - y) \\ \dot{\boldsymbol{S}} &= -\lambda(-\gamma\boldsymbol{S} - \mathbf{A}(u,y)^T\boldsymbol{S} - \boldsymbol{S}\mathbf{A}(u,y) + \boldsymbol{C}^T\boldsymbol{C}) \\ \hat{y} &= \boldsymbol{C}\hat{\boldsymbol{z}} \end{aligned} \tag{14}$$

with

$$\boldsymbol{\Gamma}(\lambda) = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda^2 & 0 \\ O & 0 & \lambda^2 \end{bmatrix}, \tag{15}$$

where λ is associated to z_1^l variables while λ^2 is associated to z_2^l variables. Asymptotic estimation of the states is guaranteed if there exists a sufficient persistent input for system (13) and if $\boldsymbol{b}(u,\boldsymbol{z})$ is Lipschitz uniformly in u and $\boldsymbol{z}$. The estimates of the original variables are then obtained by applying the inverse transformation given in Subsection 5.3.

5. NUMERICAL SIMULATION

5.1 Current-driven speed controller

As stated in Remark 2, the system under consideration is non-uniformly observable. For this system in particular, the observability depends strongly on the input. This input should be capable of driving the machine at a constant speed, while guaranteeing that the condition $i_j \neq 0$ is satisfied for each phase. Finally, the input must not rely on the measurement of mechanical variables, *i.e.*, position and velocity. Taking into account these considerations, the designed velocity controller is given by the desired current profiles for the three stator phases shown in Figure 2. These trapezoidal profiles are designed to have a period

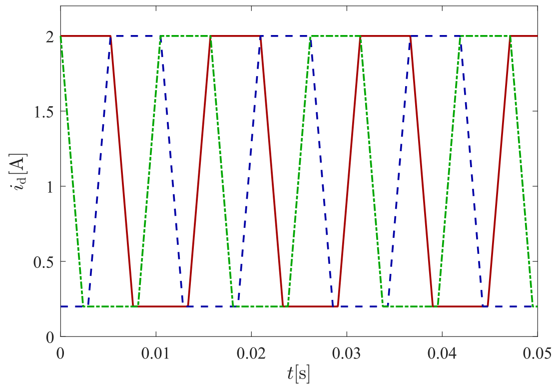

Fig. 2. Desired currents: phase 1 (—), phase 2 (- - -), and phase 3 (- . -).

of $T_{\mathrm{id}} = \theta_{\mathrm{s}}/\omega_{\mathrm{d}}$, where ω_{d} is the desired rotor speed and θ_{s} is the stroke angle defined in (4). The minimum and maximum amplitude for the desired currents are set to 0.2[A] and 2[A], respectively, to satisfy the condition $i_j \neq 0$. Finally, the following PD controller is employed to satisfy $\boldsymbol{i} \to \boldsymbol{i}_{\mathrm{d}}$

$$\boldsymbol{u} = -k_{\mathrm{p}}\boldsymbol{e} - k_{\mathrm{d}}\dot{\boldsymbol{e}}\,, \tag{16}$$

where $\boldsymbol{e} = \boldsymbol{i} - \boldsymbol{i}_{\mathrm{d}}$ and $\mathrm{d}\boldsymbol{i}/\mathrm{d}t$ is obtained through a first-order derivative filter.

5.2 Active phase detection

To guarantee that the second condition of (11) is satisfied, an algorithm is implemented to detect whenever a given phase is in the motor-mode zone, *i.e.,* when the slope of the corresponding inductance is positive or, equivalently, when $C_j(x_2) = \partial L_j(x_2)/\partial x_2$ is positive. From (3a), for each phase one has

$$x_3 C_j(x_2) x_{1j} = u_j - L_j(x_2)\dot{x}_{1j} - r_j x_{1j}\,, \tag{17}$$

where r_j is the corresponding phase winding resistance. Assuming that the sign of the angular speed x_3 does not change (in the present case is assumed positive), the sign of $C_j(x_2)$ can be determined from (17). Since the electrical variables can be measured, and after Fact 1, such approximation can be carried out as

$$\operatorname{sign}\left(C_j(x_2)\right) \approx \operatorname{sign}\left(\left(u_j - l_{\mathrm{u}}\dot{x}_{1j} - r_j x_{1j}\right)/x_{ij}\right)\,, \tag{18}$$

where $L_j(x_2)$ has been substituted with the constant l_{u}, *i.e.,* the value of the minimum inductance, since x_2 is not available for measurement. Furthermore, a positive threshold for the right quantity on the right of (18) can be implemented to deal with measurement noise and to stay away from the singularity in the observability distribution given by $C_j(x_2) = 0$.

5.3 Observer implementation

After detecting the active phase, as explained above, an observer of the form (14) is implemented for the corresponding active phase. The initial conditions for this observer must be reset each time a phase becomes active. Given the definitions of z_1^1, z_2^1, and z_2^2, such initial conditions are chosen as

$$\hat{z}_j(t_{0j}) = [i_j(t_{0j}) \;\; -c_j\omega_{\mathrm{d}} i_j(t_{0j})/l_{\mathrm{u}} \;\; 1/l_{\mathrm{u}}]^{\mathrm{T}} \;, \qquad (19)$$

where t_{0j} is the time when the phase is switched from inactive to active.

To recover the mechanical signals, an inversion of the estimated variables $\hat{\boldsymbol{z}}$ must be carried out. From the definition of z_2^2, an estimation of each phase inductance is simply $\hat{L}_j = 1/\hat{z}_2^2$. Since the algorithm works only in the positive slope of each inductance, the map is invertible and the position is recovered by applying the inverse map to the largest estimated inductance, i.e.,

$$\hat{x}_2 = \left(\hat{l}_{\mathrm{M}} - l_{\mathrm{u}}\right)/c_j + 3\pi/\left(N_{\mathrm{r}}m\right) + \phi_j \;, \qquad (20)$$

where $\hat{l}_{\mathrm{M}} = \max_j \hat{L}_j$. On the other hand, to estimate the velocity, the following inversion is proposed

$$\hat{x_3} = \mathrm{abs}\left(\hat{z}_2^1/\left(c_j\hat{z}_1^1\hat{z}_2^2\right)\right) \;. \qquad (21)$$

This estimated variable is then filtered by an unity-gain low-pass first-order filter.

The results of a numerical simulation presented to validate the proposed approach are shown in Figures 3–6. In Figure 3, the desired, real, and estimated currents for phase 1 in steady state are shown, for the interval of time $t = [10, 10.05]$ s. In this figure, it can be seen that the proposed PD controller delivers the real currents close to the desired ones, although no perfect tracking is achieved. The estimated currents are however very close to the real ones. The real speed of the rotor shaft and the estimated one by the observer are shown in Figure 4. The speed control is not smooth, since it relies only on the current driven PD controller and makes no use of mechanical measurements. Nevertheless, the estimated speed is very close to the real one in steady state. The estimation of the phase inductances is displayed in Figure 5, where it can be appreciated that the estimation is carried out only for the half of the signals, *i.e.*, only when the phase is detected to be active by employing (18). Finally, the estimation of the rotor position, module $\pi/4$, is shown in Figure 6 for the time interval of $t = [10, 10.05]$ s. In this case, no filter is employed, but only the algoritm given by (20).

6. CONCLUSION

The observability analysis carried out in this work, establishes some well-known observability problems of the SR motor, including the non-uniformly observability nature of this system. The same analysis permitted to define a region where an immersion procedure can be applied to transform the motor model into a form suitable for implementation of a previously reported observer for non-uniformly observable systems. An *ad-hoc* controller, independent of the mechanical variables, was designed to simultaneously drive the rotor speed to a constant desired velocity, and to satisfy the required observability conditions. The simulation results show that it is possible to reconstruct in a continuous form the mechanical variables.

As future work, the analysis of robustness with respect to noises or uncertainty will be carried out along with

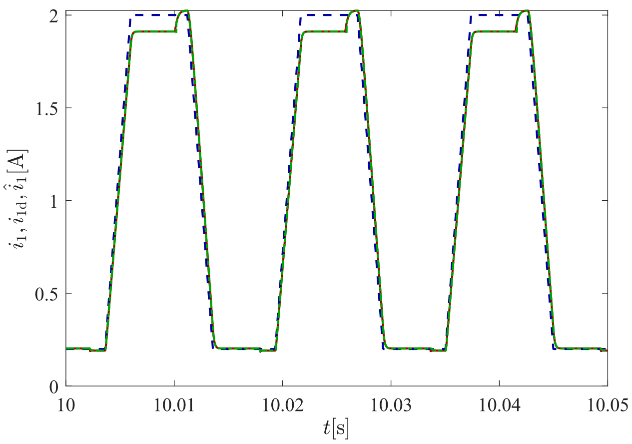

Fig. 3. Phase 1 current: desired (- - -), real (—), estimated (- . -).

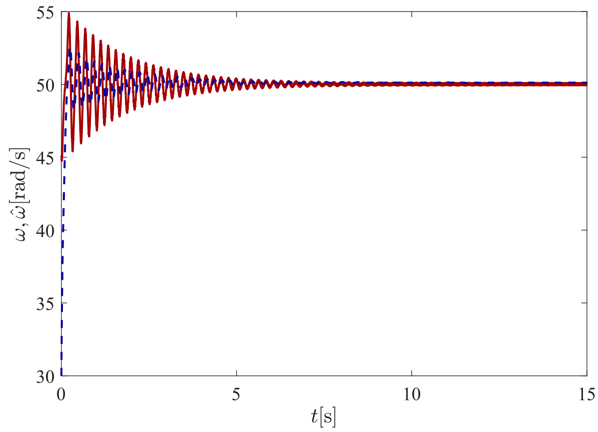

Fig. 4. Rotor speed: real (—), estimated (- - -).

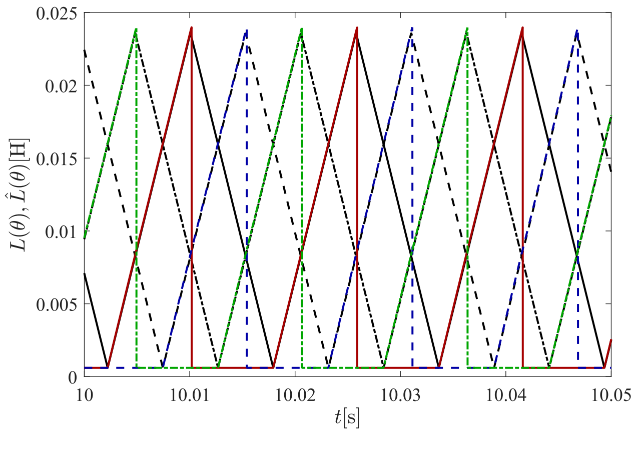

Fig. 5. Phase inductances. Phase 1: real (—), estimated (—). Phase 2: real (- - -), estimated (- - -). Phase 3: real (- . -), estimated (- . -).

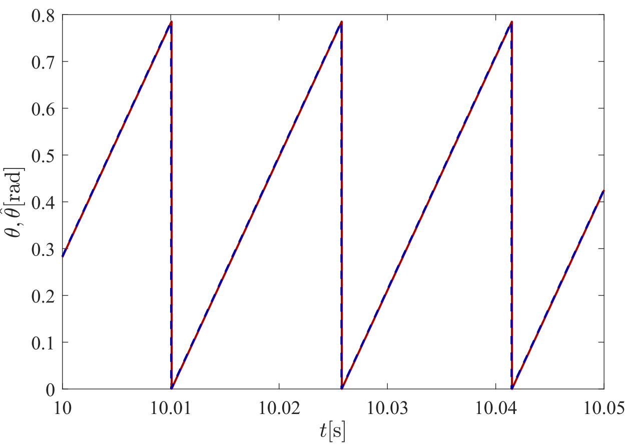

Fig. 6. Rotor position: real (—), estimated (- - -).

experimental validation of the proposed scheme. Its application for non triangular inductances (*e.g.*, sinusoidal and trapezoidal) will be studied as well.

REFERENCES

Bamba, T., Komatsuzaki, A., and Miki, I. (2007). Estimation of rotor position for switched reluctance motor at standstill. In *Power Conversion Conference, Nagoya, Japan*, 259–263. IEEE.

Besançon, G. and Ticlea, A. (2007). An immersion-based observer design for rank-observable nonlinear systems. *IEEE Transactions on Automatic Control*, 52(1), 83–88.

Brandstetter, P., Petrtyl, O., and Hajovsky, J. (2016). Luenberger observer application in control of switched reluctance motor. In *17th International Scientific Conference on Electric Power Engineering, Prague Czech Republic*, 1–5.

Brandstetter, P. and Krna, P. (2013). Sensorless control of switched reluctance motor using sliding mode observer. In *International Conference on Applied Electronics, Pilsen, Czech Republic*, 1–4. IEEE.

Bu, J. and Xu, L. (2001). Eliminating starting hesitation for reliable sensorless control of switched reluctance motors. *IEEE Transactions on Industry Applications*, 37(1), 59–66.

de la Guerra, A., Arteaga-Pérez, M.A., Gutiérrez-Giles, A., and Maya-Ortiz, P. (2016). Speed-sensorless control of SR motors based on GPI observers. *Control Engineering Practice*, 46, 115 – 128.

de la Guerra, A., Maya-Ortíz, P., and Espinosa-Pérez, G. (2015). Global observability analysis of the SR motor under sensorless operation. In *54th IEEE Conference on Decision and Control, Osaka, Japan*, 1799–1804. IEEE.

Ehsani, M., Gao, Y., Longo, S., and Ebrahimi, K. (2018). *Modern electric, hybrid electric, and fuel cell vehicles.* CRC press.

Elmas, C. and Zelaya-De La Parra, H. (1996). Application of a full-order extended luenberger observer for a position sensorless operation of a switched reluctance motor drive. *IEE Proceedings-Control Theory and Applications*, 143(5), 401–408.

Gao, H., Salmasi, F., and Ehsani, M. (2001). Sensorless control of SRM at standstill. In *Sixteenth Annual IEEE Applied Power Electronics Conference and Exposition, Anaheim, CA, USA*, volume 2, 850–856. IEEE.

Hermann, R. and Krener, A. (1977). Nonlinear controllability and observability. *IEEE Transactions on Automatic Control*, 22(5), 728–740.

Hossain, S.A., Husain, I., Klode, H., Lequesne, B., Omekanda, A.M., and Gopalakrishnan, S. (2003). Four-quadrant and zero-speed sensorless control of a switched reluctance motor. *IEEE Transactions on Industry Applications*, 39(5), 1343–1349.

Jacobson, M.Z. and Delucchi, M.A. (2011). Providing all global energy with wind, water, and solar power, part I: Technologies, energy resources, quantities and areas of infrastructure, and materials. *Energy Policy*, 39(3), 1154–1169.

Komatsuzaki, A., Bamba, T., and Miki, I. (2008). A position estimation for switched reluctance motor at standstill. In *International Symposium on Power Electronics, Electrical Drives, Automation and Motion, Ischia, Italy*, 290–295. IEEE.

Li, P., Zhang, L., and Yu, Y. (2017). A novel sensorless for switched reluctance motor based on sliding mode observer. In *IEEE 2nd Advanced Information Technology, Electronic and Automation Control Conference, Chongqing, China*, 1560–1564.

Lumsdaine, A. and Lang, J.H. (1990). State observers for variable-reluctance motors. *IEEE Transactions on Industrial Electronics*, 37(2), 133–142.

McCann, R.A., Islam, M.S., and Husain, I. (2001). Application of a sliding-mode observer for position and speed estimation in switched reluctance motor drives. *IEEE Transactions on Industry Applications*, 37(1), 51–58.

Miller, T.J.E. (2001). *Electronic control of switched reluctance machines.* Elsevier.

Ouddah, N., Loukkas, N., Chaibet, A., Boukhnifer, M., and Monmasson, E. (2015). Experimental evaluation of second sliding modes observer and Extended Kalman Filter in a sensorless robust control of Switched Reluctance motor for EV application. In *23rd Mediterranean Conference on Control and Automation, Torremolinos, Spain*, 986–992.

Rahman, Z., Ehsani, M., and Butler, K. (2000). An investigation of electric motor drive characteristics for EV and HEV propulsion systems. Technical report, SAE Technical Paper.

Saha, N. and Choudhury, D. (2016). Analysis of different types of faults exhibited in switched reluctance motor drives. In *2016 10th International Conference on Intelligent Systems and Control, Coimbatore, Tamilnadu, India*, 1–5. IEEE.

Xiao Wang, Fei Peng, and Emadi, A. (2016). A position sensorless control of switched reluctance motors based on sliding-mode observer. In *IEEE Transportation Electrification Conference and Expo, Dearborn, Michigan, USA*, 1–6.

Zeraoulia, M., Benbouzid, M.E.H., and Diallo, D. (2006). Electric motor drive selection issues for HEV propulsion systems: A comparative study. *IEEE Transactions on Vehicular technology*, 55(6), 1756–1764.