Feedback Control in the Presence of Input and Output Disturbances [*]

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Abstract: A novel control law is proposed to attenuate the influence of input and output disturbances for systems with vector output and sector bounded nonlinearities. The control law is based on estimation of the disturbance in the output. Differently from the existing results, the ultimate bound of the closed-loop system depends on only one component of the output disturbance vector (as well as, on the input disturbance). The results are formulated in terms of LMIs. The efficiency and advantages of the results over the existing methods are demonstrated by numerical examples.

Keywords: Disturbance attenuation, LMI.

1. INTRODUCTION

For practical implmentation of control methods it is important to take into account input and output disturbances Tao and Kokotovic (1996); Fradkov et al. (1999); Popescu et al. (2017). The following control methods are efficient in the presence of disturbances: H_∞-control Chen et al. (2015); Sanchez-Pena and Sznaier (1998), invariant ellipsoid method Schweppe (1973); Polyak and Topunov (2008), and method of rejection of sinusoidal disturbances Bodson and Douglas (1997); Fedele and Ferrise (2013); Pyrkin and Bobtsov (2016). However, the accuracy in the steady state of Chen et al. (2015); Sanchez-Pena and Sznaier (1998); Schweppe (1973); Polyak and Topunov (2008); Bodson and Douglas (1997); Fedele and Ferrise (2013); Pyrkin and Bobtsov (2016) depends on magnitudes of all system disturbances.

Recently a new control method has been suggested in Furtat (2017, 2018). This method decreases the resulting ultimate bound of the closed-loop system. Differently from Chen et al. (2015); Sanchez-Pena and Sznaier (1998); Schweppe (1973); Polyak and Topunov (2008); Bodson and Douglas (1997); Fedele and Ferrise (2013); Pyrkin and Bobtsov (2016), the above bound depends only on one component of the output disturbance vector (as well as on the input disturbance). Thus, the control law of Furtat (2017, 2018) may reject output disturbances with

large magnitudes. However, the disturbances considered in Furtat (2017, 2018) are assumed to be differentiable.

The objective of the present paper is to design a novel method that rejects nonsmooth input/output disturbances.

The paper is organized as follows. Problem formulation is presented in Section 2. Section 3 describes the control law design under input and output disturbances. Section 4 contains the main results. Section 5 illustrates an efficiency of the proposed method and its advantages compared with the existing methods. Section 6 collects some conclusions.

Notations. Throughout the paper the superscript T stands for matrix transposition; $\mathbb{R}^n$ denotes the n dimensional Euclidean space with vector norm $|\cdot|$; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices; the notation $P > 0$ for $P \in \mathbb{R}^{n \times n}$ means that P is symmetric and positive definite; $\lambda_{\min}(P)$ stands for the minimum eigenvalue of the matrix P; $E_j = [0, ..., 0, 1, 0, ..., 0]^T$ is a vector, where the jth component is equal to 1 and other components are equal to 0; $\tilde{E}$ is the $(n-1) \times n$ matrix obtained from the identity matrix of order n by eliminating the ith row, i.e. $\tilde{E}^T = [E_1, ..., E_{i-1}, E_{i+1}, ..., E_n]$; I is the identity matrix of corresponding order; the notation $\Theta(\chi)$ for $\chi \in \mathbb{R}$ means that $\lim_{\chi \to 0} \frac{\Theta(\chi)}{\chi} = C$, where C is a constant, diag$\{\cdot\}$ denotes a block diagonal matrix.

2. PROBLEM FORMULATION

Let a plant model be described by the following equations

$$\dot{x}(t) = Ax(t) + Bu(t) + D\phi(x,t) + Gf(t), \quad (1)$$
$$y(t) = x(t) + \xi(t), \quad (2)$$

where $t \geq 0$, $x(t) \in \mathbb{R}^n$ is the unmeasured state vector, $n \geq 2$, $u(t) \in \mathbb{R}^m$ is the control signal, $y(t) \in \mathbb{R}^n$ is the measured signal, $f(t) \in \mathbb{R}^v$ is the input disturbance and $|f(t)| \leq \kappa_1$, $\kappa_1 > 0$, $\phi(x,t) \in \mathbb{R}^l$ is the unknown nonlinear function satisfying the condition $|\phi(x,t)| \leq \chi|x|$, $\chi > 0$, $\xi(t) = [\xi_1(t), ..., \xi_n(t)]^T$ is the output disturbance and $\kappa_2^j = \lim\limits_{t \to \infty} \sup\limits_{t \geq 0} |\xi_j(t)|$, $\kappa_2^j > 0$, $j = 1, .., n$. The matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $D \in \mathbb{R}^{n \times l}$, $G \in \mathbb{R}^{n \times v}$ and the constants κ_1, κ_2^j, $j = 1, ..., n$ and χ are known.

Our objective is to design the controller that guarantees the input-to-state stability (ISS) of (1) leading to ultimate bound

$$\lim_{t \to \infty} \sup_{t \geq 0} |x(t)| < \delta, \qquad (3)$$

where $\delta = \Theta(\kappa_1^2 + (\kappa_2^i)^2)$, the ith component due to the designer choice. This is different from the existing results Chen et al. (2015); Sanchez-Pena and Sznaier (1998); Schweppe (1973); Polyak and Topunov (2008); Bodson and Douglas (1997); Fedele and Ferrise (2013); Selivanov et al. (2015); Pyrkin and Bobtsov (2016), where $\delta = \Theta(\kappa_1^2 + \sum_{j=1}^n (\kappa_2^j)^2)$. Moreover, unlike Furtat (2017, 2018) the signals f and ξ may be not differentiable. Sufficient condition for our objective is given below in Theorems 1.

Let us briefly describe the design method. Since plant (1), (2) contains input and output disturbances, at least two measurement channels are required for getting information about these signals (thus, $n \geq 2$). Let us consider the ith equation in (1), (2) for getting information about disturbance f. The other equations are used for getting information about output disturbance. The output signal y contains output disturbances, therefore, we design an algorithm that allows to estimate the part of output disturbance vector without ith component, i.e. $\tilde{\xi} = [\xi_1, ..., \xi_{i-1}, \xi_{i+1}, ..., \xi_n]^T$ (see "Estimator of $\tilde{\xi}$" in Fig. 1, where $\hat{\tilde{\xi}}$ is the estimate of $\tilde{\xi}$). Thus, having information about $\tilde{\xi}$, the state vector estimate $\hat{x}$ is constructed and used for design the control law (Fig. 1), reducing the influence of f.
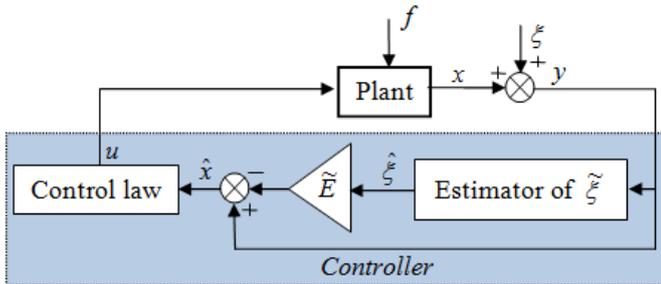

Fig. 1. The control scheme structure.

Remark 1. Let we know *a priori*, that there exists the lth component of the vector ξ such that $\lim\limits_{t \to \infty} \sup\limits_{t \geq 0} |\xi_l(t)| < \lim\limits_{t \to \infty} \sup\limits_{t \geq 0} |\xi_k(t)|$ for $l \in \{1, ..., n\}$ and $k \in \{1, ..., l-1, l+1, ..., n\}$. In this case, we will use the lth equation in (1), (2) instead of ith equation, because with this choice

the ultimate bound δ in (3) will take the smallest value. Otherwise, $\xi_i(t)$ is chosen arbitrary.

3. CONTROL LAW DESIGN

Introduce the control law (see Fig. 1) in the form

$$u(t) = K\hat{x}(t), \qquad (4)$$

where the matrix $K^T \in \mathbb{R}^n$ is chosen such that the closed-loop system is ISS, the signal $\hat{x}(t)$ is the estimate of the state vector $x(t)$ obtained by

$$\hat{x}(t) = y(t) - \tilde{E}^T \hat{\tilde{\xi}}(t). \qquad (5)$$

Here $\hat{\tilde{\xi}}(t)$ is the estimate of $\tilde{\xi}(t) = [\xi_1(t), ..., \xi_{i-1}(t), \xi_{i+1}(t), ..., \xi_n(t)]^T$.

Further we design the algorithm for estimation of $\tilde{\xi}(t)$. Using relation

$$\xi(t) = \sum_{j=1}^n E_j \xi_j(t) = \tilde{E}^T \tilde{\xi}(t) + E_i \xi_i(t),$$

rewrite $y(t)$ given by (2) as follows

$$y(t) = x(t) + \tilde{E}^T \tilde{\xi}(t) + E_i \xi_i(t). \qquad (6)$$

Eliminate the ith equation in (6) and rewrite result w.r.t. $\tilde{\xi}(t)$. To this end, pre-multiplying (6) by $\tilde{E}$ and setting $\tilde{y}(t) = \tilde{E} y(t)$, we have

$$\tilde{\xi}(t) = \tilde{y}(t) - \tilde{E} x(t). \qquad (7)$$

Integrating (1) in t and employing (7), we get

$$\tilde{\xi}(t) = \tilde{y}(t) - \tilde{E} \int_0^t \big[Ax(s) + Bu(s) \\ + D\phi(x(s), s) + Gf(s) \big] ds. \qquad (8)$$

Denoting

$$\tilde{A} = \tilde{E} A \tilde{E}^T, \quad \tilde{A}_1 = \tilde{E} A, \quad \tilde{A}_2 = \tilde{E} A E_i, \\ \tilde{B} = \tilde{E} B, \quad \tilde{D} = \tilde{E} D, \quad \tilde{G} = \tilde{E} G \qquad (9)$$

and substituting $x(t)$ from (6) into (8), we have

$$\tilde{\xi}(t) = \int_0^t [\tilde{A}\tilde{\xi}(s) - \tilde{A}_1 y(s)]ds + \tilde{y}(t) \\ - \int_0^t [\tilde{B}u(s) + \tilde{D}\phi(x, s) + \tilde{G}f(s) - \tilde{A}_2\xi_i(s)]ds. \qquad (10)$$

The second row in (10) contains unknown functions $\phi(x,t)$, $f(t)$ and $\xi_i(t)$, while the first row in (10) can be used for design the estimate of $\tilde{\xi}(t)$. Therefore, introduce the estimate of $\tilde{\xi}(t)$ (see "Estimator of $\tilde{\xi}$" in Fig. 1) in the form

$$\hat{\tilde{\xi}}(t) = \int_0^t \left[\tilde{A}\hat{\tilde{\xi}}(s) - \tilde{A}_1 y(s) \right] ds + \tilde{y}(t). \qquad (11)$$

As a result, the proposed algorithm is presented by control law (4), (5) and output disturbance estimator (11). In the next section we derive the closed-loop system and formulate the sufficient condition for ISS.

4. MAIN RESULT

Consider the output disturbance estimation error

$$e(t) = \tilde{\xi}(t) - \hat{\xi}(t) \tag{12}$$

and, taking into account (5), rewrite control law (4) as follows

$$u(t) = K[x(t) + \tilde{E}^{\mathrm{T}}e(t) + E_i\xi_i(t)]. \tag{13}$$

Substituting (13) into (1), we get

$$\dot{x}(t) = (A + BK)x(t) + BK\tilde{E}^{\mathrm{T}}e(t) \\ + BKE_i\xi_i(t) + D\phi(x,t) + Gf(t). \tag{14}$$

Since equation (14) contains the variable e, it is necessary to obtain dynamics of e. Employing (10), (11) and (13), differentiate (12) in t and rewrite result in the form

$$\dot{e}(t) = (\tilde{A} - \tilde{B}K\tilde{E}^{\mathrm{T}})e(t) - \tilde{B}Kx(t) - \tilde{D}\phi(x(t),t) \\ - \tilde{G}f(t) + (\tilde{A}_2 - \tilde{B}KE_i)\xi_i(t). \tag{15}$$

Combine equations (14) and (15). To this end, introduce the following vectors and matrices

$$\begin{aligned}
x_a(t) &= col\{x(t), e(t)\}, \\
\psi(t) &= col\{\xi_i(t), f(t)\}, \\
A_a &= \begin{bmatrix} A + BK & BK\tilde{E}^{\mathrm{T}} \\ -\tilde{B}K & \tilde{A} - \tilde{B}K\tilde{E}^{\mathrm{T}} \end{bmatrix}, \\
G_a &= \begin{bmatrix} D \\ -\tilde{D} \end{bmatrix}, \\
F_a &= \begin{bmatrix} BKE_i & G \\ \tilde{A}_2 - \tilde{B}KE_i & -\tilde{G} \end{bmatrix}.
\end{aligned} \tag{16}$$

Employing (16), rewrite (14) and (16) in the form

$$\dot{x}_a(t) = A_a x_a(t) + G_a\phi(x,t) + F_a\psi(t). \tag{17}$$

As a result, the closed-loop system (17) depends on ξ_i and f only, while the closed-loop systems in Chen et al. (2015); Sanchez-Pena and Sznaier (1998); Schweppe (1973); Polyak and Topunov (2008); Bodson and Douglas (1997); Fedele and Ferrise (2013); Pyrkin and Bobtsov (2016) depend on whole vector ξ and disturbance f. The following result is thus in order.

Theorem 1. Given a matrix K and a scalar α, let there exist constants $\beta > 0$, $\tau > 0$ and matrix $P > 0$ that satisfy the following LMI

$$\Psi_a = \begin{bmatrix} \Psi_{11} & PG_a & PF_a \\ * & -I & 0 \\ * & * & -\beta I \end{bmatrix} < 0, \tag{18}$$

where $\Psi_{11} = A_a^{\mathrm{T}}P + PA_a + 2\alpha P + \tau\chi^2 C^{\mathrm{T}}C$, "$*$" denotes a symmetrical block of a symmetric matrix, $C = [I \ 0]$. Consider (1) under control law (4), where $\hat{x}$ is given by (5) and (11). The solutions of this system are ultimately bounded and (3) holds with $\delta = \sqrt{\frac{\beta[\kappa_1^2 + (\kappa_2^i)^2]}{2\alpha\lambda_{\min}(P)}}$.

Proof 1. For the ISS analysis of (17) introduce Lyapunov function in the form

$$V_1 = x_a^{\mathrm{T}}Px_a. \tag{19}$$

Employing (17) and (19), consider the following relation

$$\dot{V}_1 + 2\alpha V_1 - \beta\psi^{\mathrm{T}}\psi = x_a^{\mathrm{T}}(A_a^{\mathrm{T}}P + PA_a + 2\alpha P)x_a \\ + 2x_a^{\mathrm{T}}PG_a\phi(x,t) + 2x_a^{\mathrm{T}}PF_a\psi - \beta\psi^{\mathrm{T}}\psi. \tag{20}$$

Denoting

$$\begin{aligned}
z(t) &= col\{x_a(t), \phi(x,t), \psi(t)\}, \\
\Psi &= \Psi_a - \mathrm{diag}\{\tau\chi^2 C^{\mathrm{T}}C, -I, 0\},
\end{aligned}$$

represent (20) as follows

$$\dot{V}_1 + 2\alpha V_1 - \beta\psi^{\mathrm{T}}\psi = z^{\mathrm{T}}\Psi z. \tag{21}$$

Taking into account $x(t) = Cx_a(t)$ and $|\phi(x,t)| \leq \chi|x|$, consider the following estimate $\phi^{\mathrm{T}}(x,t)\phi(x,t) \leq \chi^2 x_a^{\mathrm{T}}(t)C^{\mathrm{T}}Cx_a(t)$ in the form

$$z^{\mathrm{T}}(t)\mathrm{diag}\{\chi^2 C^{\mathrm{T}}C, -I, 0\}z(t) \geq 0. \tag{22}$$

According to S-procedure, inequalities (21) and (22) simultaneously hold, if LMI (18) holds. Thus, $z(t)$ is ultimately bounded. Therefore, $x(t)$ and $e(t)$ are ultimately bounded. It follows from comparison principle and $\limsup_{t\to\infty}\sup_{t\geq 0}|\psi(t)|^2 \leq \kappa_1^2 + (\kappa_2^i)^2$, that $\lambda_{\min}(P)\limsup_{t\to\infty}\sup_{t\geq 0}|x(t)|^2 \leq \limsup_{t\to\infty}\sup_{t\geq 0}(x_a^{\mathrm{T}}(t)\times Px_a(t)) \leq 0.5\alpha^{-1}\beta[\kappa_1^2 + (\kappa_2^i)^2]$. Thus, Theorem 1 is proven.

Remark 2. Let us show the boundedness of all signals in the closed-loop system. Since the signals x and e are ultimately bounded, then the signal u is ultimately bounded from (13). The ultimate boundedness of $\hat{\xi}$ follows from (12). Therefore, the ultimate boundedness of $\tilde{y}$ follows from (7). It follows from (11) that $\int_0^t \left[\tilde{A}\hat{\xi}(s) - \tilde{A}_1 z(s)\right] ds$ is bounded. As a result, all signals are bounded in the closed-loop system.

5. EXAMPLES

Example 1. Consider plant (1), (2). The known matrices are presented as follows

$$A = \begin{bmatrix} -3 & 1 & 0 \\ -3 & 0 & 1 \\ 0.1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.1 \\ 1.5 \\ 3 \end{bmatrix}, \quad G = \begin{bmatrix} -0.01 \\ 1.03 \\ 1.97 \end{bmatrix}.$$

The unknown parameters and signals in (1), (2) are given in the forms

$$\phi(x,t) = \sin(t)[\sin(x_1) + \sin(2x_2) + \sin(3x_3)] \quad (\chi = \sqrt{14}),$$
$$x(0) = [1 \ 1 \ 1]^{\mathrm{T}},$$
$$f(t) = 1 + 2\sin(0.7t),$$
$$\xi_1(t) = 1 + 10\sin(3t),$$
$$\xi_2(t) = -2 + 7\cos(3t),$$
$$\xi_3(t) = 0.01\sin(0.8t).$$

We will choose further the parameters of the proposed control law. Let $i = 3$ in (7). Then

$$\tilde{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} -3 & 1 \\ -1 & 0 \end{bmatrix}, \quad \tilde{A}_1 = \begin{bmatrix} -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

in (11). By using pole placement, we choose $K = 20[1\ 1\ 3]^{\mathrm{T}}$ in control law (4) such that $A + BK$ is Hurwitz. LMI (18) is feasible for setting parameters.

We will demonstrate the transients for the proposed control law and compare results with the state-feedback $u = Ky$ (where disturbances are not compensated), invariant ellipsoid based algorithm Khlebnikov et al. (2011), high gain robust algorithm Furtat et al. (2015), and speed-gradient algorithm Orlov et al. (2017).

The high gain robust algorithm is presented by $u = -100[1\ 2\ 1]y$. The speed-gradient algorithm is given by $u = -10B^{\mathrm{T}}y = -10[0\ 1\ 2]y$. The invariant ellipsoid based algorithm is presented in the form

$$\dot{g} = Ag + Bu + L(g - y),$$
$$u = Kg,$$

where the matrices $L = \begin{bmatrix} -0.27 & -0.66 & -0.28 \\ -0.69 & -2.53 & -2.99 \\ -0.24 & -2.95 & -6.27 \end{bmatrix}$ and $K = -[0.09\ 0.62\ 1.13]$ are calculated such that the ellipsoid $x^{\mathrm{T}}Px = 1$, $P > 0$ has the smallest semiaxes.

The transients in $x_1(t)$, $x_2(t)$ and $x_3(t)$ are presented for the controller $u = Ky$ in Fig. 2, for the high gain robust algorithm in Fig. 3, for the speed-gradient algorithm in Fig. 4 for the invariant ellipsoid based algorithm in Fig. 5, and for the proposed algorithm in Fig. 6. The transients in Figs. 2–5 depend on ξ_1, ξ_2, ξ_3, f, while the transients from Fig. 6 depends on ξ_3 and f only. The advantage of the proposed control law is clearly seen: the ultimate bounds under the control law $u = Ky$ and the control laws from Khlebnikov et al. (2011); Furtat et al. (2015); Orlov et al. (2017) are at least 5 time larger (approximately 6.7, 7.8, 3.1 and 1.5) than the one under the proposed control law (approximately 0.3).
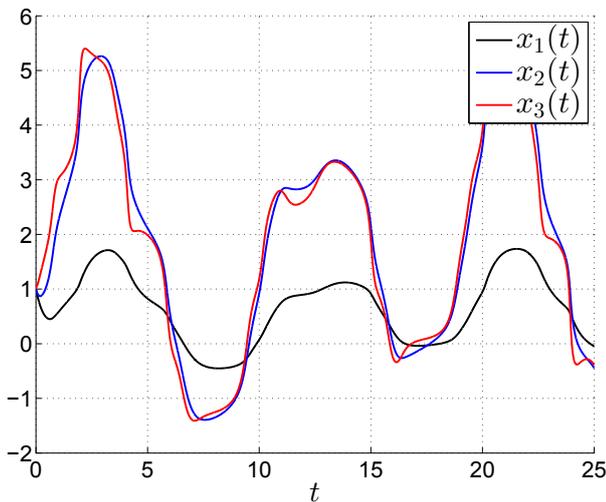

Fig. 2. The transients in $x_1(t)$, $x_2(t)$ and $x_3(t)$ are obtained for the control law $u = Ky$.

Example 2. Consider the control of an amplidyne (see Fig. 7). An amplidyne Kwakernaak and Sivan (1972); Deshpande (2001); Macmillan (2016) is an electric machine used to control a large dc power through a small dc voltage. Amplidynes are used for electric elevators, point naval guns, antiaircraft artillery radar, control processes
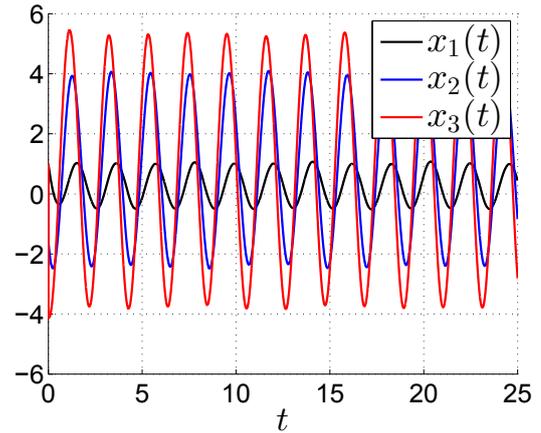

Fig. 3. The transients in $x_1(t)$, $x_2(t)$ and $x_3(t)$ are obtained for high gain robust algorithm Furtat et al. (2015).
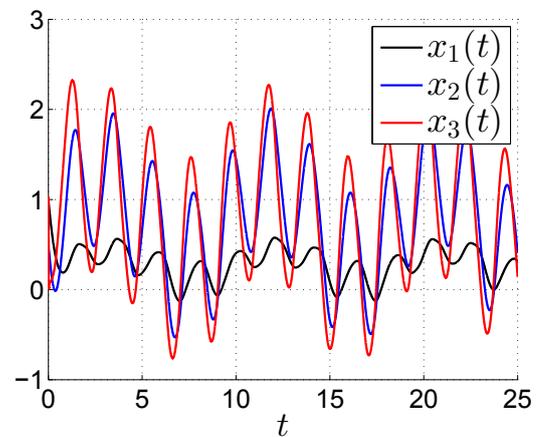

Fig. 4. The transients in $x_1(t)$, $x_2(t)$ and $x_3(t)$ are obtained for speed-gradient algorithm Orlov et al. (2017).
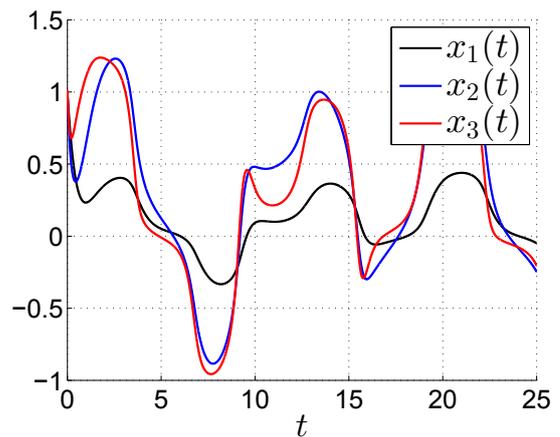

Fig. 5. The transients in $x_1(t)$, $x_2(t)$ and $x_3(t)$ are obtained for invariant ellipsoid based algorithm Khlebnikov et al. (2011).

in steelworks, remote control rods in nuclear submarine designs, diesel-electric locomotive control systems. The amplidyne electrical dynamics is described by
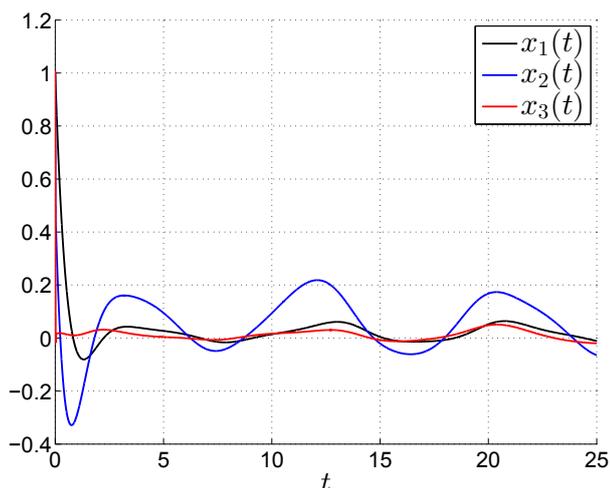
Fig. 6. The transients of $x_1(t)$, $x_2(t)$ and $x_3(t)$ are obtained for the proposed algorithm.

$$\dot{x} = \begin{bmatrix} -\dfrac{R_2}{L_2} & \dfrac{k_1}{L_2} \\ -\dfrac{R_1}{L_1} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \dfrac{1}{L_1} \end{bmatrix} (u(t) + f(t)), \qquad (23)$$

where $x = [i_1, i_2]^{\mathrm{T}}$, i_1 and i_2 are currents in first and second windings accordingly, $u = e_0$ is the input voltage, e_1 and e_2 are the induced voltages given by $e_1 = k_1 i_1$ and $e_2 = k_2 i_2$, L_1 and R_1 denote the inductance and resistance of the first field windings, L_2 and R_2 are those of the first armature windings together with the second field windings. According to Kwakernaak and Sivan (1972), the following numerical values are used: $R_1 = 5$ Ω, $L_1 = 0.5$ s^{-1}, $k_1 = 20$ V/A, $k_2 = 50$ V/A, $R_2 = 10$ Ω, $L_2 = 10$ s^{-1}.
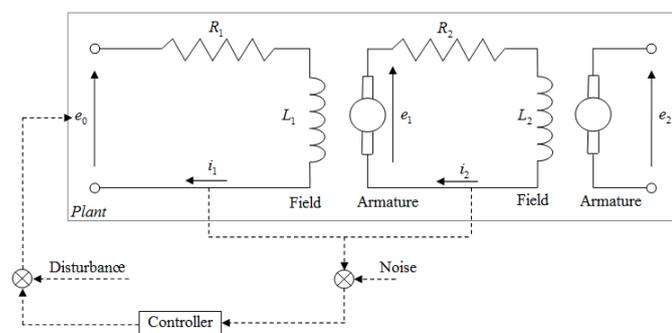

Fig. 7. Simplified representation of an amplidyne control scheme.

Choosing $i = 2$ in (7), we have $\tilde{E} = [1\ 0]$, $\tilde{A} = -3$ and $\tilde{A}_1 = [-3\ 1]$ in (11). Let $K = [10\ 1]^{\mathrm{T}}$. Compare the proposed algorithm with the static control laws $u = Ky$ and $u = [0\ 1]y$ from Kwakernaak and Sivan (1972); Macmillan (2016). LMI (18) is feasible for setting parameters.

Let $f = 0.1\sin(0.7t) + d_1(t)$, $z_1(t) = q_1(x_1) + 100\sin(1.7t) + d_2(t)$, $z_2(t) = q_2(x_2) + 10^{-3}\sin(0.5t) + d_3(t)$ in the simulations, where q_1 and q_2 are the quantization functions with the quantization intervals are 0.5 and 0.05 respectively, the signals $d_1(t)$, $d_2(t)$ and $d_3(t)$ are obtained by the band-limited white noise blocks in Matlab Simulink with the following parameters: noise power $1, 3, 10^{-4}$ and sample time 0.1 (s), 0.01 (s), 0.03 (s) accordingly. The plots of

$i_1(t)$ and $i_2(t)$ are depicted in Fig. 8–10 for control laws from Kwakernaak and Sivan (1972); Macmillan (2016) and the proposed algorithm. The transients in Fig. 8 depends on disturbances ξ_1, ξ_2 and f. In Fig. 9 the transients do not depend on ξ_1, but influence of the disturbance f is not attenuated. The transients in Fig. 10 do not depend on ξ_1 also, but the influence of disturbance f is attenuated. The advantage of the proposed control law is clearly seen: the ultimate bounds under the control laws $u = Ky$ and $u = [0\ 1]y$ (approximately 800 and 1.2) are at least 6 time larger than the one under the proposed control law (approximately 0.2). The additional simulations show that the proposed results are robust under small input and output disturbances. Also, the simulations illustrate that the proposed control law is efficient under unknown input and output time-varying delays.
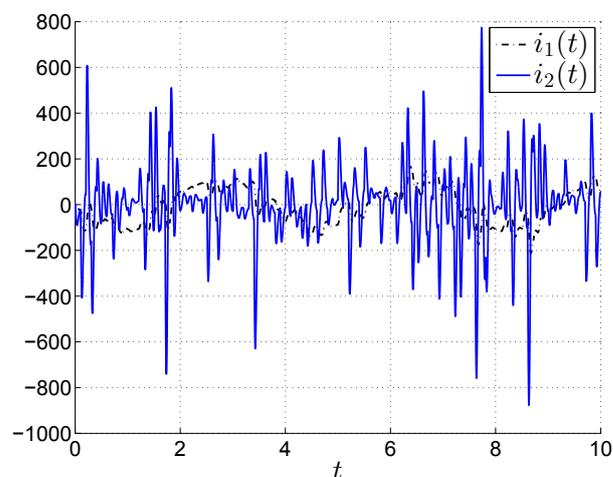

Fig. 8. The transients in $i_1(t)$ and $i_2(t)$ are obtained for the control law $u = Ky$.
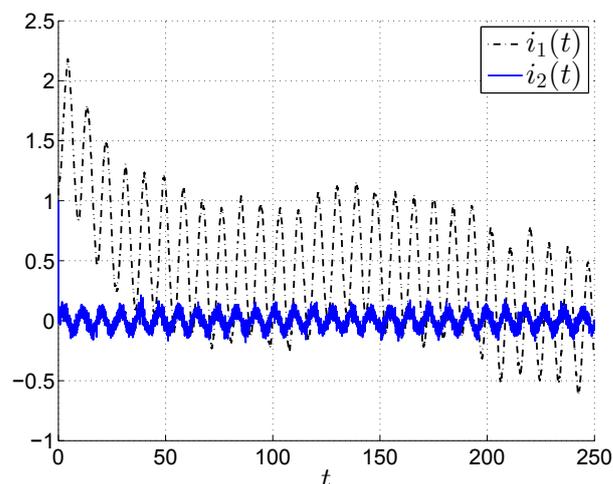

Fig. 9. The transients in $i_1(t)$ and $i_2(t)$ are obtained for the control law $u = [0\ 1]y$.

6. CONCLUSION

We have considered vector systems, where the full state is measured with the disturbances. A novel method has been proposed for attenuating the influence of input and output disturbances. Differently from recent results Furtat
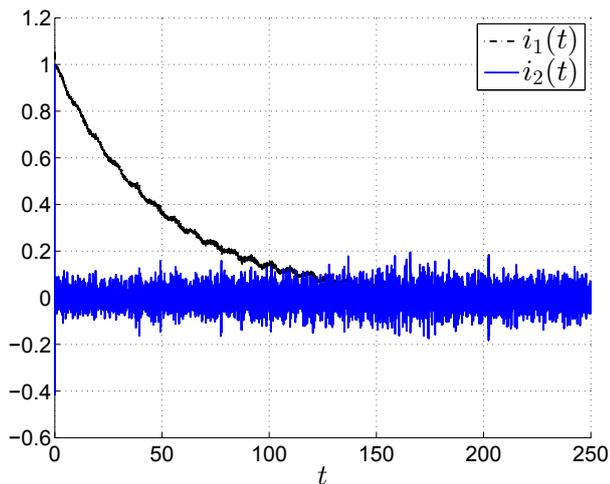
Fig. 10. The transients in $i_1(t)$ and $i_2(t)$ are obtained for the proposed algorithm.

(2017, 2018), the proposed design method does not require the smoothness of disturbances. The proposed method provides a better accuracy after transients, because the the closed-loop system depends on only one component of output disturbance vector and on input disturbance. The simulations in numerical examples illustrate the efficiency of the presented method and its advantages over alternative methods without disturbance compensation all disturbances. Also, the simulations illustrate that the proposed control law is efficient under unknown input and output time-varying delays.

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