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Non-fragile Exponential Consensus of Nonlinear Multi-agent Systems via Sampled-data Control ⋆

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Abstract: In this article, non-fragile exponential consensus problem is investigated for nonlinear multi-agent systems (MASs) through the use of sampled-data controllers. The sampled-data system is translated to a continuous system with time-varying delay through input delay approach via control input. With the introduction of a sampled-data approach, the information is sent only to the network at each sampling instant and is inevitably subject to a transmission delay. By using the tools from algebraic graph theory and Lyapunov–Krasovskii functional (LKF) technique, it is proved that the concerned non-fragile consensus problem is solvable if the resultant consensus error system can be exponentially stabilized. Numerical example is given to illustrate the merits of the results obtained.

Keywords: Nonlinear multi-agent systems, Non-fragile consensus control, Sampled-data control

1. INTRODUCTION

In the past decade, consensus control for multi-agent systems (MASs) has received extensive research attention due to its potential applications in various science and engineering areas, such as formation control, multi-robot teams, unmanned aircraft cooperative control, sensor networks and coordinated defence systems to name a few. In general, the main objective in consensus is to achieve an agreement among the agents on a parameter of interest. The parameter of interest is application-specific. For example, in vehicular MASs, the parameter of interest can be the motion of the agents (positions and velocities). In estimation applications, it can be the temperature measured by different sensors placed at different locations Yu et al. (2018); Amini et al. (2016); Dong et al. (2018). Studies of consensus problems for MASs have been explored by various scholars from different perspectives Wen et al. (2013); Wu et al. (2019); Guo et al. (2015); Liu et al. (2016). For instance, the problem of consensus in directed networks of nonlinear MASs with sampled-data control was studied in Wen et al. (2013). The non-fragile consensus control of discrete time-varying nonlinear MASs with uniform quantization and randomly occurring deception attacks have been investigated in Wu et al. (2019). The distributed non-fragile H_∞ consensus problem have been investigated for MASs with external disturbances and unknown initial disturbances under switching weighted balanced directed topologies in Guo et al. (2015). The problem of consensus for linear discrete-time networked MASs with directed topologies and communication delays was investigated in Liu et al. (2016).

Most existing implementations for consensus in nonlinear MASs, are based on continuous-time (real-time) information exchanges among the neighbouring agents, thus not suitable under networked environment. In many practical applications, however, the exchange of information between agents can only occur in unique model cases due to the use of digital sensors and limited bandwidth communication channels. To handle this problem, using sampled-data control rather than continuous-time control would be a viable approach. Sampled-data systems contain continuous-time plants under discrete-time control updates. Sampled-data control approach has a long-standing record in various research fields among the control community Jiang et al. (2020); Fu et al. (2018); Du and Yu (2019); Wu et al. (2015). In Jiang et al. (2020), the sampled-data non-fragile H_∞ consensus tracking problem for nonlinear MASs with switching topologies and exogenous disturbances was investigated. In Fu et al. (2018), the exponential consensus problem for MASs with Lipschitz nonlinear dynamics using sampled-data information have been investigated. In Du and Yu (2019), the leader-following and leaderless consensus problem of high-order MASs have been discussed via sampled-data control approach. Consensus problem of MASs have been investigated by using aperiodic sampled-data control Wu et al. (2015). However, non-fragile exponential consensus problem for

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nonlinear MASs by using sampled-data control has not been considered adequately, which motivates this study.

Inspired by the above observations, we have explored the problem of non-fragile exponential consensus control for nonlinear MASs via sampled-data approach. Using algebraic graph theory and the Lyapunov functional technique, sufficient conditions for the solvability of consensus is obtained based on linear matrix inequalities (LMIs). The contributions of this paper mainly include the following:

(1) The corresponding non-fragile consensus protocols are designed for nonlinear MASs using sampled-data control. Regarding the sampled-data scheme, we assume that the sampling intervals are time-varying.
(2) In this work, an efficient criteria for non-fragile exponential consensus are established while most of the existing related results are concerned only with asymptotic consensus.
(3) In Fu et al. (2018), exponential sampled-data consensus of linear MASs is investigated. Compared with Fu et al. (2018), non-fragile exponential consensus for nonlinear MAS is considered using sampled-data mechanism and the non-fragile consensus protocol is designed with the norm-bounded parametric uncertainty. Thus, the MASs under consideration is more general than Fu et al. (2018).

The rest of this paper is organized as follows. Some useful definitions in graph theory, problem formulation and some basic notations are given in Section 2. The non-fragile exponential consensus problem of nonlinear MASs is investigated in Section 3. Simulation results are given in Section 4. Section 5 concludes the paper.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times n$ real matrices. A^T and A^{-1} denote the matrix transpose and inverse of A respectively. We say $X > 0$ implies that matrix X is real symmetric positive definite with appropriate dimensions. I denotes the identity matrix with appropriate dimensions. The Kronecker product of matrices $M \in \mathbb{R}^{m\times n}$ and $N \in \mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $M \otimes N$. Notation $*$ used in symmetric matrices represents the transpose of the corresponding block from the upper triangle. Notation $(.)^\dagger$ is the pseudo-inverse of the argument.

2. MODEL DESCRIPTION AND PRELIMINARIES

2.1 Graph Theory

Graphs are commonly used to model the interaction among the agents in MASs. In this paper, the communication topology among the agents is modeled by a weighted directed graph $\mathcal{G}(V, E, \mathcal{A})$ with a set of N agents $V = \{1, \dots, N\}$, in which the ith vertex indicates the ith agent while the edge set is $E \subseteq V \times V$. If the information of the jth agent is available for the ith one, then the pair (j, i) is an element of E and is depicted by $j \to i$ in graph representation. The set of neighbors of agent i is denoted by $\mathcal{N}_i = \{j \in V : (i, j) \in E\}$ and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix for $\mathcal{G}$, where $a_{ii} = 0$ and $a_{ij} = 1$ if $(i, j) \in V$ and $a_{ij} = 0$ otherwise. The Laplacian of $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$, where:

$$\mathcal{D} = diag\{\deg_1, \dots, \deg_N\},\ \deg_i = \sum_{j=1}^{N} a_{ij}.$$

Then, for any $i \neq j; i, j = 1, 2, \dots, N$, the Laplacian matrix $L = [l_{ij}]_{N\times N}$ of $\mathcal{G}$ is defined as

$$l_{ij} = -a_{ij},\ l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}. \tag{1}$$

2.2 Model Description

Consider the ith agent in the MASs, whose dynamics take the form of

$$\dot{z}_i(t) = \mathbb{A}z_i(t) + \mathbb{B}u_i(t) + \mathbb{C}f(t, z_i(t)), i = 1, 2, \dots, N. \tag{2}$$

where $z_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^p$ are the state and control input of the ith agent, respectively. The constant real known matrices $\mathbb{A}, \mathbb{B}$ and $\mathbb{C}$ are considered to preserve compatible dimensions. The vector-valued continuous activation function $f(t, z_i(t)) = [f(t, z_{i1}(t)), f(t, z_{i2}(t)), \dots, f(t, z_{in}(t))]^T$ denotes the nonlinear dynamics of ith agent.

Assumption 1. There exists a known real constant matrix $\mathbb{G}$ with appropriate dimensions such that the nonlinear vector function $f(t, z_i(t))$ in MAS (2) satisfies the following condition:

$$||f(t, z_i(t))|| \le ||\mathbb{G}z_i(t)||, \tag{3}$$

for any $z_i(t) \in \mathbb{R}^n$.

Then, we introduce a sampled-data consensus protocol with respect to uncertain perturbation to achieve non-fragile consensus as

$$u_i(t) = \delta(K + \Delta K) \sum_{j=1, j\neq i}^{N} a_{ij}(z_j(t_k) - z_i(t_k)), \tag{4}$$
$$t_k \le t < t_{k+1},$$

where $\delta > 0$ denotes the coupling strength and $K \in \mathbb{R}^{p\times n}$ is the controller gain to be designed. For appropriately dimensional matrices M, W and a time-varying matrix $\Delta(t)$ satisfying $\Delta^T(t)\Delta(t) \le I$, the norm-bounded parametric uncertainty ΔK is characterized as

$$\Delta K = M\Delta(t)W. \tag{5}$$

Sampled data information is utilized at sampling instants t_k by using a zero-order hold (ZOH) circuit. The control inputs are generated on the basis of this ZOH with a sequence of times $0 = t_0 < t_1 < \dots < t_k < \dots < \lim_{k\to\infty} t_k = +\infty$. The sampling intervals are periodic with an upper bound ϑ, such that $\vartheta_k \triangleq t_{k+1} - t_k \le \vartheta, \forall k \ge 0$.

Define

$$z(t) = \left[\, z_1(t)^T\ z_2(t)^T\ \dots\ z_N(t)^T \,\right]^T,$$
$$F(t, z(t)) = \left[\, f(t, z_1(t))^T\ f(t, z_2(t))^T\ \dots\ f(t, z_N(t))^T \,\right]^T.$$

Substituting (4) into (2) gives

$$\begin{aligned}\dot{z}(t) =& (I_N \otimes \mathbb{A})z(t) - (L \otimes \mathbb{B}(K + \Delta K))z(t_k)\\ &+ (I_N \otimes \mathbb{C})F(t, z(t)),\end{aligned} \tag{6}$$

where $t_k \le t < t_{k+1}$. We rewrite, $z(t_k) = z(t - m(t))$ with $m(t) = t - t_k$, $0 \le m(t) \le \vartheta$, $\dot{m}(t) = 1$ for $t \neq t_k$. Then, (6) can be rewritten as

$$\dot{z}(t) = (I_N \otimes \mathbb{A})z(t) - (L \otimes \mathbb{B}(K + \Delta K))z(t - m(t))$$

$$+ (I_N \otimes \mathbb{C})F(t, z(t)). \tag{7}$$

Definition 1. MAS (2) with protocol (4) is said to achieve non-fragile consensus if for any time-varying matrix $\Delta(t)$ satisfying $\Delta^T(t)\Delta(t) \leq I$, and for any initial condition $z_i(0)$, such that the following condition holds:

$$\lim_{t\to\infty} ||z_i(t) - z_j(t)|| = 0, \ i, j = 1, 2, \dots, N. \tag{8}$$

A widely used approach to guarantee consensus for (7) is to covert the consensus problem into an equivalent stability problem by using a state transformation. In other words, the state transformation is defined in such a way that the stability problem for the transformed system leads to consensus in the original system, i.e., system (7). Let $\hat{L}$ denote the $(N-1)\times N$ dimensional matrix obtained by removing one arbitrary row from Laplacian matrix L. To solve consensus, we propose the following state transformation

$$y(t) = (\hat{L} \otimes I_n)z(t). \tag{9}$$

Lemma 1. If $y(t) = 0$, it holds that $z_1(t) = \cdots = z_N(t)$. In other words, consensus in sense of Definition 1 is satisfied if $y(t) \to 0$.

Proof: If $y(t) = 0$ then $(\hat{L}\otimes I_n)z(t) = 0$, which implies that $z(t)$ belongs to the null space of $\hat{L}\otimes I_n$, i.e., $z(t) \in \text{null}(\hat{L}\otimes I_n)$. The row sum of Laplacian matrix L is zero. Hence, the row some of $\hat{L}$ is also zero. The null space of $\hat{L} \otimes I_n$ is, therefore, given by $\mathbf{1}_N \otimes z_{\text{cns}}(t)$, i.e., $z(t) \in \text{null}(\hat{L}\otimes I_n) = \mathbf{1}_N \otimes z_{\text{cns}}(t)$, where $z_{\text{cns}}(t)$ is the consensus vector to which $z_i(t)$, $(1 \leq i \leq N)$, converges. Therefore, it holds that $z_1(t) = \cdots = z_N(t) = z_{\text{cns}}(t)$. Accordingly, consensus in the sense of Definition 1 is achieved.

In summary, Lemma 1 implies that the consensus problem for (7) is equivalent to the stability problem of the transformed system expressed by $y(t)$. Without loss of generality, to obtain $\hat{L}$ we remove row N from the Laplacian matrix.

Next, we transform system (7) based on transformation (9) which gives way to the following system

$$\begin{aligned}\dot{y}(t) =&(I_{N-1} \otimes \mathbb{A})y(t) + \mathcal{B}y(t - m(t))\\ &+ \mathcal{C}F(t, y(t)),\end{aligned} \tag{10}$$

where

$$\mathcal{B} = (\hat{L} \otimes I_n)(I_N \otimes \mathbb{B}(K + \Delta K))((L\hat{L}^\dagger) \otimes I_n),$$

$$\mathcal{C} = (\hat{L} \otimes I_n)(I_N \otimes \mathbb{C}).$$

Note that $(I_N \otimes \mathbb{B}(K + \Delta K))((L\hat{L}^\dagger) \otimes I_n) = ((L\hat{L}^\dagger) \otimes I_n)(I_{N-1} \otimes \mathbb{B}(K + \Delta K))$ is also used to derive (10).

2.3 Preliminaries

Definition 2. (Mahmoud and Ismail (2010)) Consider system (10), if there exist some positive constants $\kappa \geq 1$ and $\sigma > 0$ such that

$$||y(t)|| \leq \kappa\psi e^{-\sigma t}, \ \forall\, t \geq 0, \ \psi = \sup_{-\vartheta\leq\theta\leq 0} ||y(\theta)||,$$

then, system (10) is said to achieve exponential consensus with exponential convergence rate σ.

Lemma 2. (Gu et al. (2003)) For any matrix $Z \in \mathbb{R}^{n\times n}, Z = Z^T > 0$, scalar $\vartheta > 0$, and vector function $y : [0, \vartheta] \to \mathbb{R}^n$ such that the integration concerned are well defined, then

$$\left[\int_0^\vartheta y(s)ds\right]^T Z \left[\int_0^\vartheta y(s)ds\right] \leq \vartheta \int_0^\vartheta y^T(s)Zy(s)ds.$$

Lemma 3. (Wang et al. (1992)) Let U, V and $\Lambda(t)$ be real matrices of appropriate dimensions, and $\Lambda(t)$ satisfy $\Lambda^T(t)\Lambda(t) \leq I$, then the following inequality holds for any constant $\epsilon > 0$:

$$U\Lambda(t)V + V^T\Lambda^T(t)U^T \leq \epsilon UU^T + \epsilon^{-1}V^TV.$$

Lemma 4. (Boyd et al. (1994)). Let A, B, C be given matrices such that $C > 0$, then

$$\begin{bmatrix} A & B^T \\ B & -C \end{bmatrix} < 0 \quad \Leftrightarrow A + B^TC^{-1}B < 0.$$

3. NON-FRAGILE SAMPLED-DATA CONTROL

In this section, the sufficient conditions for the consensusability of the considered MASs will be proposed by using the LKF and LMI techniques.

Theorem 1. For desired values of converge rate $\alpha > 0$, sampling upper-bound $\vartheta > 0$, coupling strength $\delta > 0$, $\chi > 0$, $\epsilon > 0$ the matrices M, W, and the controller gain matrix K, MAS (2) achieves non-fragile exponential consensus by protocol (4) with a convergence rate $\sigma = \alpha/2$ if there exist positive-definite matrices $P > 0$, $T > 0$, $Z > 0$, such that the LMI (11) is satisfied:

$$\Xi = \begin{bmatrix} \widehat{\Xi} & \mathcal{A}^T & \mathfrak{M}_1 & \epsilon\mathfrak{N}^T \\ * & -(I_{N-1} \otimes \vartheta Z) & \mathfrak{M}_2 & 0 \\ * & * & -\epsilon I & 0 \\ * & * & * & -\epsilon I \end{bmatrix} < 0, \tag{11}$$

where

$$\widehat{\Xi} = [\Xi_{i,j}], \ i, j = 1, 2, \dots, 4,$$

$$\begin{aligned}\Xi_{1,1} = &I_{N-1} \otimes (P\mathbb{A} + \mathbb{A}^TP + \alpha P + T - \Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)Z\\ &+ \mathbb{G}\mathbb{G}^T),\end{aligned}$$

$$\Xi_{1,2} = -\delta(I_{N-1} \otimes P)\mathcal{B}_1 + (I_{N-1} \otimes \Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)Z), \ \Xi_{1,3} = 0,$$

$$\Xi_{1,4} = (I_{N-1} \otimes P)\mathcal{C}, \ \Xi_{2,2} = -2I_{N-1} \otimes \Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)Z,$$

$$\Xi_{2,3} = (I_{N-1} \otimes \Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)Z), \ \Xi_{2,4} = 0,$$

$$\Xi_{3,3} = -(I_{N-1} \otimes \Big(e^{-\alpha\vartheta}T + \Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)Z\Big), \ \Xi_{3,4} = 0,$$

$$\Xi_{4,4} = -I,$$

$$\mathcal{A} = \left[\, \vartheta(I_{N-1} \otimes Z\mathbb{A}) \ -\delta\vartheta(I_{N-1} \otimes Z)\mathcal{B}_1 \ 0 \ \vartheta(I_{N-1} \otimes Z)\mathcal{C} \,\right],$$

$$\mathfrak{M}_1 = \left[\, -\delta((I_{N-1} \otimes P)\mathcal{B}_2)^T \ 0 \ 0 \ \ 0 \,\right]^T,$$

$$\mathfrak{M}_2 = -\delta\vartheta((I_{N-1} \otimes Z)\mathcal{B}_2),$$

$$\mathfrak{N} = \left[\, 0 \ (\hat{L} \otimes I_n)(I_N \otimes W)((L\hat{L}^\dagger) \otimes I_n) \ 0 \ 0 \,\right],$$

$$\mathcal{B}_1 = (\hat{L} \otimes I_n)(I_N \otimes \mathbb{B}K)((L\hat{L}^\dagger) \otimes I_n),$$

$$\mathcal{B}_2 = (\hat{L} \otimes I_n)(I_N \otimes \mathbb{B}M)((L\hat{L}^\dagger) \otimes I_n).$$

Proof: Consider the following LKF candidate:

$$
\begin{aligned}
V(y_t,t) =& y^T(t)(I_{N-1}\otimes P)y(t) \\
&+ \int_{t-\vartheta}^{t} y^T(s)e^{\alpha(s-t)}(I_{N-1}\otimes T)y(s)ds, \\
&+ \int_{-\vartheta}^{0}\int_{t+\rho}^{t} \dot{y}^T(s)e^{\alpha(s-t)}(I_{N-1}\otimes Z)\dot{y}(s)dsd\rho.
\end{aligned} \tag{12}
$$

It follows from (12) that that $V(y_t,t) \ge \lambda_{min}||y(t)||^2$. Then, the derivative $\dot{V}(y_t,t)$, along the trajectories of the system (7) is

$$
\begin{aligned}
\dot{V}(y_t,t)+\alpha V(y_t,t) \le & 2y^T(t)(I_{N-1}\otimes P)\dot{y}(t) \\
&+ \alpha y^T(t)(I_{N-1}\otimes P)y(t) \\
&+ y^T(t)(I_{N-1}\otimes T)y(t) \\
&- e^{-\alpha\vartheta}y^T(t-\vartheta)(I_{N-1}\otimes T)y(t-\vartheta) \\
&+ \vartheta\dot{y}^T(t)(I_{N-1}\otimes Z)\dot{y}(t) \\
&- e^{-\alpha\vartheta}\int_{t-\vartheta}^{t}\dot{y}^T(s)(I_{N-1}\otimes Z)\dot{y}(s)ds.
\end{aligned} \tag{13}
$$

Using Lemma 2, we have

$$
\begin{aligned}
&-\int_{t-\vartheta}^{t}\dot{y}^T(s)(I_{N-1}\otimes Z)\dot{y}(s)ds \\
&\le -\int_{t-\vartheta}^{t-m(t)}\dot{y}^T(s)(I_{N-1}\otimes Z)\dot{y}(s)ds \\
&\quad -\int_{t-m(t)}^{t}\dot{y}^T(s)(I_{N-1}\otimes Z)\dot{y}(s)ds \\
&\le -\frac{1}{\vartheta}[y(t-m(t))-y(t-\vartheta)]^T(I_{N-1}\otimes Z) \\
&\quad \times[y(t-m(t))-y(t-\vartheta)] - \frac{1}{\vartheta}[y(t)-y(t-m(t))]^T \\
&\quad \times(I_{N-1}\otimes Z)[y(t)-y(t-m(t))].
\end{aligned} \tag{14}
$$

From Assumption 1, we have

$$
\begin{aligned}
&y^T(t)(I_{N-1}\otimes \mathbb{G})(I_{N-1}\otimes \mathbb{G})^T y(t) \\
&\quad - F^T(t,y(t))F(t,y(t)) > 0.
\end{aligned} \tag{15}
$$

Adding from (13) to (15), we have

$$
\begin{aligned}
&\dot{V}(y_t,t)+\alpha V(y_t,t) \qquad (16)\\
&\le 2y^T(t)(I_{N-1}\otimes P)\Big[(I_{N-1}\otimes \mathbb{A})y(t) \\
&\quad + \mathcal{B}y(t-m(t)) + \mathcal{C}F(t,y(t))\Big] \\
&\quad + y^T(t)(I_{N-1}\otimes(\alpha P+T))y(t) \\
&\quad - e^{-\alpha\vartheta}y^T(t-\vartheta)(I_{N-1}\otimes T)y(t-\vartheta) \\
&\quad + \vartheta\Big[(I_{N-1}\otimes \mathbb{A})y(t) + \mathcal{B}y(t-m(t)) \\
&\quad + \mathcal{C}F(t,y(t))\Big]^T(I_{N-1}\otimes Z)\Big[(I_{N-1}\otimes \mathbb{A})y(t) \\
&\quad + \mathcal{B}y(t-m(t)) + \mathcal{C}F(t,y(t))\Big] \\
&\quad - \frac{e^{-\alpha\vartheta}}{\vartheta}[y(t-m(t))-y(t-\vartheta)]^T \\
&\quad \times(I_{N-1}\otimes Z)[y(t-m(t))-y(t-\vartheta)] \\
&\quad - \frac{e^{-\alpha\vartheta}}{\vartheta}[y(t)-y(t-m(t))]^T \\
&\quad \times(I_{N-1}\otimes Z)[y(t)-y(t-m(t))] \\
&\quad + y^T(t)(I_{N-1}\otimes \mathbb{G})(I_{N-1}\otimes \mathbb{G})^T y(t) \\
&\quad - F^T(t,y(t))F(t,y(t)).
\end{aligned} \tag{17}
$$

Then, by using the relation (5), we can obtain

$$
\dot{V}(y_t,t)+\alpha V(y_t,t) \le \Psi^T(t)\overline{\Xi}\Psi(t), \tag{18}
$$

where

$$
\overline{\Xi} = \widehat{\Xi} + \mathcal{A}^T(I_{N-1}\otimes\vartheta^{-1}Z)\mathcal{A} + \mathfrak{M}\Delta(t)\mathfrak{N} + (\mathfrak{M}\Delta(t)\mathfrak{N})^T,
$$

$$
\Psi^T(t) = \Big[\, y^T(t)\ y^T(t-m(t))\ y^T(t-\vartheta)\ F^T(t,y(t)) \,\Big].
$$

By applying Lemma 3, for $\epsilon > 0$, $\overline{\Xi}$ in (18) becomes,

$$
\widetilde{\overline{\Xi}} = \widehat{\Xi} + \mathcal{A}^T(I_{N-1}\otimes\vartheta^{-1}Z)\mathcal{A} + \epsilon^{-1}\mathfrak{M}\mathfrak{M}^T + \epsilon\mathfrak{N}^T\mathfrak{N}. \tag{19}
$$

By applying the Lemma 4 to $\widetilde{\overline{\Xi}}$ one can easily obtain Ξ in (11). It follows from (11) that $\dot{V}(y_t,t)+\alpha V(y_t,t) \le 0,\ t \ge 0$, which implies $V(y_t,t) \le V(y_0,0)e^{-\alpha t}$. This leads to $||y(t)|| \le \sqrt{\frac{V(y_0,0)}{\lambda_{min}(P)}}e^{-(\alpha/2)t}$, which completes the proof.

Remark 1. It is noted that, (11) is not an LMI when K is unknown (and to be computed) due to multiplication of decision variables involved in (11). Therefore, we need to obtain an LMI based constraint that promises the system (10) to be exponentially stable. In order to do that, the following theorem is established.

Theorem 2. For desired values of converge rate $\alpha > 0$, sampling upper-bound $\vartheta > 0$, coupling strength $\delta > 0$, $\chi > 0$, $\epsilon > 0$ the matrices M, W, MAS (2) achieves non-fragile exponential consensus by protocol (4) with a convergence rate $\sigma = \alpha/2$ if there exist matrices $\mathbb{P} > 0$, $\mathbb{T} > 0$, Y, such that the LMI (20) is satisfied:

$$
\Omega = \begin{bmatrix} \Omega^1 & \Omega^2 & \Omega^3 \\ * & \Omega^4 & \Omega^5 \\ * & * & \Omega^6 \end{bmatrix} < 0, \tag{20}
$$

where

$\Omega^1 = [\Omega_{i,j}],\ i,j = 1,2,\dots,4,$

$\Omega^2 = \big[\, \vartheta\chi(I_{N-1}\otimes\mathbb{AP})\ -\delta\vartheta\chi\overline{\mathcal{B}}_1\ 0\ \vartheta\chi\mathcal{C} \,\big]^T,$

$\Omega^3 = \Big[\, \overline{\mathfrak{M}}_1\ \epsilon\overline{\mathfrak{N}}^T\ \overline{G} \,\Big],\ \Omega^4 = (I_{N-1}\otimes -\vartheta\chi\mathbb{P}),$

$\Omega^5 = \big[\, \overline{\mathfrak{M}}_2\ 0\ 0 \,\big],\ \Omega^6 = diag\{-\epsilon I, -\epsilon I, -I\},$

$\Omega_{1,1} = I_{N-1}\otimes(\mathbb{AP}+\mathbb{PA}^T+\alpha\mathbb{P}+\mathbb{T}-\chi\Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)\mathbb{P}),$

$\Omega_{1,2} = -\delta\overline{\mathcal{B}}_1 + (I_{N-1}\otimes\chi\Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)\mathbb{P}),\ \Xi_{1,3} = 0,$

$\Omega_{1,4} = \mathcal{C},\ \Omega_{2,2} = -2I_{N-1}\otimes\chi\Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)\mathbb{P},$

$\Omega_{2,3} = I_{N-1}\otimes\chi\Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)\mathbb{P},\ \Omega_{2,4} = 0,$

$\Omega_{3,3} = -I_{N-1}\otimes\Big(e^{-\alpha\vartheta}\mathbb{T}+\chi\Big(\frac{e^{-\alpha\vartheta}}{\vartheta}\Big)\mathbb{P}\Big),$

$\Omega_{3,4} = 0,\ \Omega_{4,4} = -I,$

$\overline{G} = \big[\, (I_{N-1}\otimes\mathbb{PG})^T\ 0\ 0\ 0 \,\big]^T,$

$\overline{\mathfrak{M}}_1 = \big[\, -\delta(\mathcal{B}_2)^T\ 0\ 0\ 0 \,\big]^T,\ \overline{\mathfrak{M}}_2 = -\delta\vartheta\chi(\mathcal{B}_2),$

$\overline{\mathfrak{N}} = \big[\, 0\ (\hat{L}\otimes I_n)(I_N\otimes W)((L\hat{L}^\dagger)\otimes I_n)(I_{N-1}\otimes\mathbb{P})\ 0\ 0 \,\big],$

$\overline{\mathcal{B}}_1 = (\hat{L}\otimes I_n)(I_N\otimes\mathbb{B}Y)((L\hat{L}^\dagger)\otimes I_n),$

$\mathcal{B}_2 = (\hat{L}\otimes I_n)(I_N\otimes\mathbb{B}M)((L\hat{L}^\dagger)\otimes I_n).$

and the sampled-data gain matrix can be estimated by $K = Y\mathbb{P}^{-1}$

Proof: Let $\mathbb{P} = P^{-1}$, $\mathbb{T} = \mathbb{P}T\mathbb{P}$, $Y = K\mathbb{P}$, $Z = \chi P, \chi > 0$, $\mathbb{S} = \mathbb{P}S\mathbb{P}$, $\Gamma = I_{N-1} \otimes diag\{\mathbb{P}, \mathbb{P}, \mathbb{P}, I_n, \mathbb{P}, I_n, I_n\}$. From Ξ in (11), calculating $\Omega = \Gamma\Xi\Gamma$ we can obtain Ω in (20), which completes the proof.

4. NUMERICAL EXAMPLE

In order to validate the effectiveness of the proposed control scheme, we consider the following numerical example.

Example 1. Consider a nonlinear MAS (2) consists of 6 agents. The parameters of (2) are as follows:

$$\mathbb{A} = \begin{bmatrix} -1 & -0.2 & 0.1 \\ 0 & -1 & 0.2 \\ 0.1 & 0 & -1 \end{bmatrix}, \ \mathbb{B} = \begin{bmatrix} 0.5 & 0.2 & -0.2 \\ -0.2 & -0.5 & 0.1 \\ 0.1 & 0.1 & -0.5 \end{bmatrix},$$

$$\mathbb{C} = \begin{bmatrix} 0.2 & -0.2 & -0.2 \\ -0.2 & 0.3 & -0.4 \\ -0.2 & -0.4 & 0.4 \end{bmatrix}, \ \mathbb{G} = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}.$$

And the Laplacian matrix is shown as follows:

$$L = \begin{bmatrix} 4 & 0 & -1 & -1 & -1 & -1 \\ 0 & 2 & 0 & -1 & 0 & -1 \\ -1 & 0 & 2 & 0 & -1 & 0 \\ -1 & -1 & 0 & 2 & 0 & 0 \\ -1 & 0 & -1 & 0 & 2 & 0 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

The initial states of MAS (2) are chosen as

$$z_1(0) = [\,1 \ \ 2 \ \ 3\,]^T, \ z_2(0) = [\,4 \ \ 4 \ \ 0\,]^T,$$
$$z_3(0) = [\,1 \ \ 5 \ \ 2\,]^T, \ z_4(0) = [\,8 \ \ -2 \ \ -3\,]^T,$$
$$z_5(0) = [\,-1 \ \ 6 \ \ -5\,]^T, \ z_6(0) = [\,-4 \ \ -4 \ \ 1\,]^T.$$

On the other hand, the nonlinear dynamics function and the time-varying matrix in non-fragile consensus protocol (4) are chosen as

$$f(t, z(t)) = [0.5\sin(t) \ \ \cos(t) \ \ 0.5\sin(2t)]^T,$$
$$\Delta(t) = diag\{0.5\sin(t), \ 0.5\cos(t), \ 0.5\sin(2t)\}.$$

In this example, the following two cases are considered.

4.1 Case I: Without uncertain perturbation

Let $\Delta K = 0$. By solving Theorem 2 with $\delta = 0.5$, $\chi = 0.5$, $\sigma = 0.1$, $\epsilon = 1$, and the upper bound of sampling interval $\vartheta = 0.9$, we obtain the consensus protocol K as,

$$K = \begin{bmatrix} 0.6174 & -0.1301 & 0.0249 \\ -0.1498 & -0.6114 & -0.3035 \\ 0.0343 & -0.3137 & -0.6562 \end{bmatrix}.$$

State trajectories of the consensus are shown in Figure 1, the control inputs are shown in Figure 2 and Figure 3 shows the sampling instants for each agent during the process for Case I.

4.2 Case II: With uncertain perturbation

When $\Delta K \neq 0$, the MAS (2) satisfies the relation (5) with M and W as follows:

$$M = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \ W = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}. \quad (21)$$

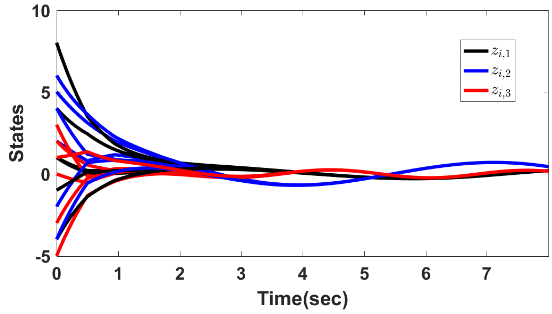

Fig. 1. Evolution of state consensus using Theorem 2 for Case I

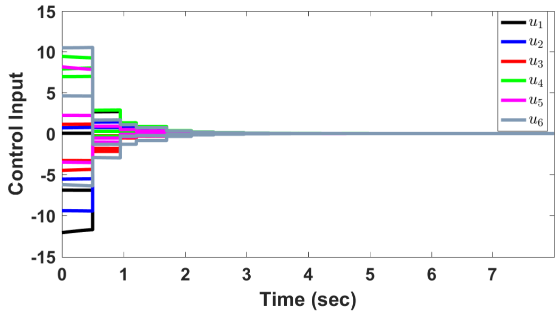

Fig. 2. Control inputs $u_i(t)$ for Case I

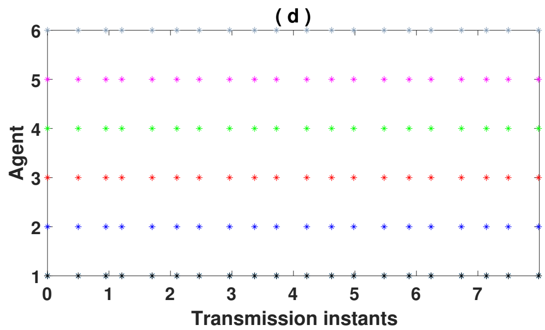

Fig. 3. Sampling instants for the six agents for Case I

Then, by solving Theorem 2 with $\delta = 0.5$, $\chi = 0.5$, $\sigma = 0.1$, $\epsilon = 0.1$, and the upper bound of sampling interval $\vartheta = 0.5$, we obtain the consensus protocol K as,

$$K = \begin{bmatrix} 0.3809 & -0.0939 & 0.0099 \\ -0.1051 & -0.5408 & -0.2511 \\ 0.0079 & -0.2682 & -0.5511 \end{bmatrix}.$$

State trajectories of the consensus are shown in Figure 4, the control inputs are shown in Figure 5 and Figure 6 shows the sampling instants for each agent during the process for Case II.

5. CONCLUSION

In this study, the non-fragile exponential consensus problem of nonlinear MASs has been investigated via sampled-

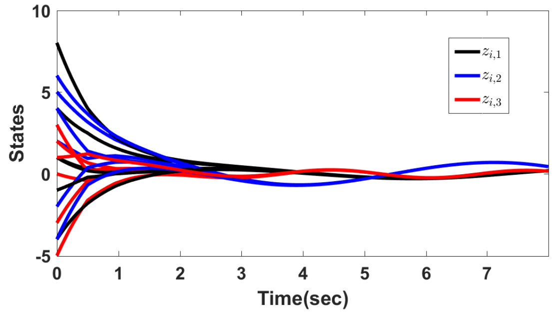

Fig. 4. Evolution of state consensus using Theorem 2 for Case II

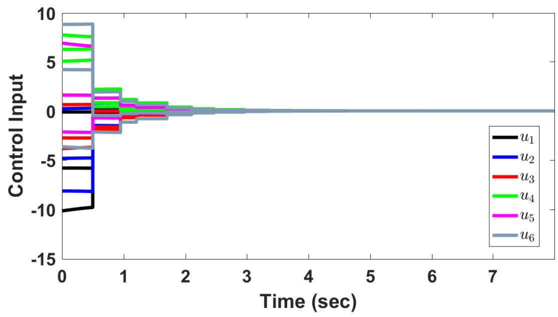

Fig. 5. Control inputs $u_i(t)$ for Case II

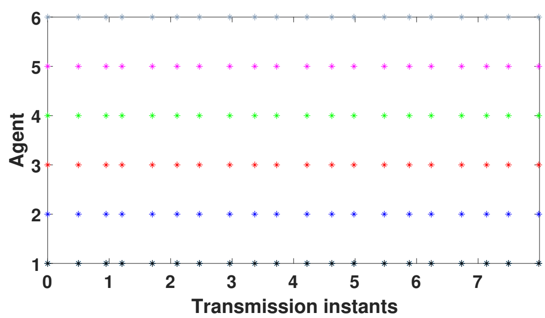

Fig. 6. Sampling instants for the six agents for Case II

data mechanism. Each agent's control input is based on the corresponding information of neighboring agents in the discrete sample events. The exponential consensusability of the sampled-data control system has been analyzed with a constant input delay by using LKF and LMI technique. Further non-fragile sampled-data controller design measures have also been proposed. Simulation results are provided to validate the effectiveness of the proposed method.

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