The method of path planning for AUV-group moving in desired formation in unknown environment with obstacles

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Abstract: The paper proposes a new method for trajectory formation of the AUVs group in "leader-follower" mode in the given formation in the unknown environment containing obstacles. In this mode, the AUV-leader defines the motion trajectory at the safe distance from the detected obstacles in accordance with the given mission. The AUVs-followers follow the leader and during obstacles avoidance they move along set in advance trajectories within the formation to ensure the safe distance between the AUVs-followers. In addition, the motion of the AUVs-followers along the predetermined trajectories allows to avoid additional data exchange between the AUVs with a view to match their positions.

Keywords: formation control, autonomous mobile robots, path planning, obstacle avoidance, unknown environment.

1. INTRODUCTION

The use of autonomous underwater vehicles (AUVs) groups is currently promising approach to enhance the efficiency of the observation missions and search operations in an underwater environment. During these tasks performing, AUVs should move in the unknown in advance environment containing obstacles. Leader-follower formation control strategy is considered to be one of the fundamental method for effective group control. In this mode, the leader robot is sole one that has information about mission and generates its trajectory on the basis of the defined task. The follower robots observe the data about the leader current position and calculate their relative position, maintaining the formation with respect to the leader. This approach ensures the coherent movement of the entire group.

In the process of the movement all robots in the group should bypass obstacles on a safe distance avoiding collisions between each other. Many approaches are currently designed to solve specified issue (Liu, 2018). And in most of them different optimization techniques Spensieri (2015), Langerwisch (2011), Shahriari (2018) are used for path planning for robots. Issue of safe interaction between industrial robots was addressed in particular in Spensieri (2015) that proposes a special movement schedule for carrying out joint technical operations without collisions. However, these methods can be applicable only in the presence of a priori information describing obstacles.

The control problem for the group of robots can be also resolved through the use of algorithms motivated by animal behavior which allow robots to move in an unknown environment Vo (2009), Yong (2017). But these methods are generally designed for swarms of robots that require no formation to be maintained. It is not appropriate for some kind of tasks, since swarming movement involves frequent information interchange among group of robots that is difficult to implement due to the low data throughput of the acoustic communication channels.

The article Liu (2011), Reyes (2015) consider formation movement control in the unknown in advance environment. Collision avoidance with encountered obstacles is supposed to be achieved through changing either specified pattern of the formation or velocity of the robots in the group. Therewith if dynamic obstacle is detected, group should bypass it at a safe distance.

Analysis showed the known approaches of trajectory generating for robots in group suggest each interacting robot to get current position of others. It allows them to adjust trajectories in the process of obstacles avoiding and prevent collisions with other robots belonging to the group. However, this approach is fraught with practical difficulties regarding the AUVs due to the low bandwidth of the acoustic communication channels (just hundreds of bytes per second). Therewith the information exchange between AUVs is effected with delays. Therefore, the method that does not require interacting robots to share significant amount of data should be developed for group control of AUVs.

As a basic method of AUVs path planning in the unknown environment the method described in Filaretov (2016), Filaretov (2017), Yukhimets (2017) will be used. The principal advantage of this method is a possibility of generating smooth spatial trajectories. They ensure quick and accurate motion that is essential for AUVs.

2. TASK DEFINITION

The paper deals with the AUVs formation control problem. The robots form up and move in a specified pattern via virtual leader-follower scheme in the unknown in advance environment containing obstacles. The AUV-leader defines the motion of the formation. It has full information about the mission and computes safe trajectories in accordance with its goal and detected obstacles. The AUVs-followers should follow the leader with respect to the provided place in the defined formation (see fig. 1). In this process, the information about leader current position transmitted through the hydro acoustic communication channels and distances from followers to the detected obstacles obtaining from their set of onboard sensors are used.

The aim is to develop AUVs-followers path planning method that will guarantee collision-free safe motion referred to both obstacles and other AUVs belonging to the group. The method should not require significant increasing data being exchanged between all AUVs of group while the correction of their trajectories is conducting.





3. DESCRIPTION OF THE AUV MOVEMENT FEATURES IN SOLVING THE APPLICATION TASK

Current positions of the AUVs-followers in an absolute coordinate frame (CF) are calculated by the information-control systems according to the leader position by following expression (see fig. 1):

$$X_{Fi} = X_L + R\widetilde{X}_{fi}, i = (\overline{1, N}),$$

where $X_{Fi} \in \mathbb{R}^3$ are current reference coordinates of i-th AUVfollower in absolute CF; $X_L \in \mathbb{R}^3$ is the absolute current AUV-leader position, transmitted to the followers via the communication channels; $\widetilde{X}_{fi} = (\widetilde{x}_{fi}, \widetilde{y}_{fi}, 0)$ is the reference displacement vector relative to the AUV-leader in the horizontal plane that is given in the leader body-fixed CF; $R \in \mathbb{R}^{3x3}$ is the transformation matrix from the body-fixed to absolute CF; N is the number of the followers in the group.

The majority of the missions performing by the group assumes that formation operates in horizontal plane. Thus, the z coordinates of the followers current positions are assumed to be equal to the leader one. If the need to implement the spatial motion of the group at a specified height above the surface arises, then each AUV will generate its z coordinate on the basis of method proposed in

Yukhimets (2017). Further, we neglect the coordinate z in coordinate vectors of all AUVs of group.

Each AUV in the group is equipped with onboard sensors which allow to measure distances to the obstacles and form vector $D = (d_1,..,d_n)$ during the motion, where n – the number of corresponding sensors. Since the AUVs operate in unknown in advance environment, every AUV in the group should correct its original trajectory, based on data obtained from onboard range finders, in the case of obstacle detection. The correction must enforce the following requirements.

1. Trajectory of AUV-leader is generated without regard to other AUVs by the method described in Filaretov (2016), Filaretov (2017), Yukhimets (2017). This requirement allows the leader to plane safe trajectories based only on a given goal of the mission and data received from its onboard sensors.

2. All AUVs-followers plan their trajectories independently of each other, using information about the current position of the leader and data obtaining from their onboard sensors.

3. The corrected trajectories of the AUVs-followers also pass at a safe distance from the obstacles detected by them.

4. The corrected trajectories of the AUVs-followers exclude collisions between the AUV group. It is achieved by ensuring the safe distances between these AUVs during their motion.

5. In the process of obstacle avoidance, AUVs-followers are not required to keep given formation. It allows the leader to plan trajectory in the unknown environment regardless of the followers. In addition, the keeping formation can significantly increase the execution time of missions or make it impossible to achieve its goals in the cases when the gap between obstacles is insufficient to be passed for the AUV group in the given formation.

6. During the obstacles avoidance the correction of the trajectories does not lead to increase in the amount of the data transferred through the acoustic communication channels between the AUVs in the group. This requirement follows from the low data throughput of the acoustic communication channels what can reduce the motion safety if the amount of data being exchanged is big.

3. PATH PLANNING METHOD FOR AUV GROUP

The main concern in the path planning of each follower in the process of obstacle avoidance is the coherence of its trajectory with ones of remaining AUVs to avoid collisions between all robots in the group. In this case, every AUV should receive the data about the other AUVs current positions because if one follower changes its position in the formation, it can entail the necessity to change the position of other robots. But since the acoustic communication channels have low bandwidth, data exchange with sufficient frequency is considered to be frequently impossible.

However, because the reference position of all AUVsfollowers is set relative to the AUV-leader, then the only possible trajectory of the motion in the formation can be generated in advance for each follower. This trajectory will provide its safe movement towards other AUVs-followers in accomplishing the specific detected obstacle avoidance. In this case, when highly accurate control systems are used, the current positions of AUVs-followers can be not agreed towards the neighboring robots. It is achieved as all their movements relative to each other within the given formation will be safe due to the preliminary (correct) choice of their trajectories.

The proposed method for the trajectory formation of the AUVs-followers operating in the unknown environment containing obstacles as a part of formation involves two main steps.

3.1 Setting the motion trajectories of each AUV-follower within the formation

Moving away from obstacles AUVs-followers maintain the formation with respect to the leader. If the obstacle avoidance is carried out, followers should change their place in the formation to keep the safe distance between all interacting robots.

The figure 2 shows the motion of the AUVs group in the environment with obstacles. In this figure the solid line demonstrates the trajectory AUVL generating by the AUVleader to bypass obstacles at the safe distance. AUVsfollowers: AUV1 and AUV2 - follow the leader keeping their place (the lower part of the fig. 2) in the formation with respect to the leader (their reference trajectories relative to the leader are shown with the dashed lines). When AUVsfollowers sensors detect the obstacles located close to them, the followers automatically begin the avoidance, shifting from the dashes trajectories toward the dotted ones (see fig. 2). This shifting motion is always conducting in the direction of the leader movement trajectory, passing at the safe distance from the obstacles. That is, during the obstacles avoidance the AUVs-followers perform two concurrent actions: keep moving behind the leader and (if necessary) shift toward its trajectory. This previously unpredictable displacement in the gap between two obstacles carried out instantly by the several followers can lead to their collisions. The problem arises if the followers do not receive data about the position of other robots in the group. This information is hard to transmit at the low data throughput of the acoustic communication channels.

To eliminate the specified drawback in the process of obstacles avoidance, the AUVs-followers are proposed to be shifted toward the leader trajectory on the basis of predetermined strategy (see dot-and-dash lines in the upper part of the fig. 2) so that the safe distance is always kept both between AUVs within the formation and the AUVs and the obstacles. As a result, for each follower it enables to independently generate obstacle avoidance trajectory without additional data exchange among them.

It should be noted that AUV-followers can detect with their sensors both obstacles and other AUV of groups. At the same time, it is not possible to differ these objects.



Fig. 2. Movement of AUV group during obstacle avoidance

However, the AUV trajectories of the group are formed so that these AUVs are always at a safe distance between themselves. Therefore, in the case when the on-board sensors of one AUV detect a different AUV of the group, the correction of the trajectory is not required.

The possible displacement law of AUVs-followers in the leader body-fixed CF $\tilde{x}\tilde{y}$ is shown in the figure 3 by the dashed line, where the points $\tilde{X}_{fi} = (\tilde{x}_{fi}, \tilde{y}_{fi})$ correspond to the reference virtual position of *i*-th AUV-follower in the absence of the obstacles, the \tilde{x} axis is always oriented along the leader current motion. These displacements occur along parallel lines passing through the points \tilde{X}_{fi} and $\tilde{X}_{fi}^0 = (\tilde{x}_{fi} \pm D_a/2, 0)$, the distance between which should not be less than D_{amin} to completely eliminate the collision between the PR groups. The " + "sign in the coordinate of the point \tilde{X}_{fi}^0 indicates that the AUV-follower is located to the left of the axis \tilde{x} , and the" - " that the corresponding follower is to the right of this axis \tilde{x} .



Fig. 3. Defining AUV-followers trajectories during obstacles avoidance

The points \widetilde{X}_{fi}^0 determine the maximum possible displacement of the *i*-th AUV-follower from its original location in the formation.

The shifting occurs within the formation during avoidance of various obstacles from the initial state. The used trajectory of displacement of the *i*-th AUV-follower within the formation is given by the equation of a line that passes the two points \tilde{X}_{fi}^{0} and \tilde{X}_{fi} :

$$\pm D_a(\widetilde{y} - \widetilde{y}_{fi})/2 + \widetilde{y}_{fi}(\widetilde{x} - \widetilde{x}_{fi}) = 0, \qquad (1)$$

which will be used to calculate the coordinates of program points for each follower when it avoids the detected obstacles.

3.2 Calculation of program positions of the AUVs-followers within the formation when they bypass obstacles detected by rangefinders

AUVs-followers determine the presence of the obstacles with the help of the onboard range finders. When a signal from any rangefinder appears, it is necessary to determine how close to the trajectory of the follower is the point on the obstacle detected by this rangefinder. If this distance is less than the allowable one D_{min} , then the program point of the AUV-follower must be shifted along the path described by equation (1), so that the distance between the new position of the program point and the point detected by the rangefinder is equal to D_{min} . In the case when several rangefinders is triggered, it is necessary to measure the distance from all points detected by them to the follower trajectory. Then the closest to the trajectory point is selected to calculate the new position of the program point.

If the distance between all detected by rangefinders points and the follower trajectory exceeds the minimal allowed value D_{min} , the program point is not shifted. The AUVfollower keeps moving behind the leader, keeping defined place in the formation, unless an obstacle located unacceptably close will be detected by any rangefinder. When obstacle avoidance is accomplished, the program point of AUV-follower moving along the trajectory given by equation (1) returns to the specified position within the formation.

The AUV-follower moves to the program point, whose position is set based on the leader current position. Therewith the follower does not have information about the leader trajectory, and therefore does not know its further trajectory in advance. Thereby, to determine the proximity of the detected points on the obstacles to the AUVs-followers trajectories, the ones will be represented as a straight lines parallel to the \tilde{x} axis and passing through the current positions of the AUVs-followers program points (the dashed line in the figure 4).

The figure 4 shows a graphical representation of the displacement value calculation strategy for program point of the one AUV-follower during obstacle avoidance. Since the trajectories of all AUVs-followers will be generated by a

similar way, then the indices corresponding to the number of the follower will be omitted.

The figure 4 illustrates the case when the obstacle was concurrently detected by several onboard rangefinders. To estimate the required displacement of the program point, it is primarily necessary to measure the distance from the points detected by the rangefinders to the predicted trajectory of the AUV-follower. Since the displacement trajectory (1) of the program point is set in the AUV-leader body-fixed CF, then the coordinates of detected by rangefinders points must be presented in the same CF. This transformation is performed in two stages. At first, the coordinates X_{di} of these points are presented in the absolute CF:



Fig. 4. The path planning AUV-followers during obstacles avoidance

where X_{di} – the coordinates of point detected by *j*-th rangefinder in the absolute CF; X_L , X_F – the coordinates of the leader and the follower, respectively; ψ_{j} – the current yaw angle of the follower (the angle between the AUV-follower roll axis (the gray solid line in figure 4) and *x* axis); d_j – the distance to the obstacle, estimated by *j*-th rangefinder; $a_j = \text{const}$ – the orientation angle of the *j*-th rangefinder relative to the longitudinal axis of the AUV-follower (the gray solid line in figure 4).

Then, the coordinates of the X_{di} points obtained with (2) are recalculated to the leader body-fixed CF:

$$\widetilde{X}_{di} = \begin{bmatrix} \cos\psi_L & \sin\psi_L \\ -\sin\psi_L & \cos\psi_L \end{bmatrix} (X_{dj} - X_L), j = (\overline{1, n}), \qquad (3)$$

where $\widetilde{X}_{dj} = (\widetilde{x}_{dj}, \widetilde{y}_{dj})^T$ - the coordinates of the X_{di} point, rewritten in the leader body-fixed CF; ψ_L - the current yaw angle of the leader (the angle between the \widetilde{x} and x axes).

The point coordinates obtained with (3) are used to determine obstacles that are unacceptably close to the desired trajectory of the AUV-follower. Since the predicted trajectory of the followers is a straight line parallel to the axis \tilde{x} , then the proximity of the detected points to the specified trajectory can be estimated using the expression:

$$\delta_j = S(\widetilde{y}_{dj})(\widetilde{y}_{dj} - \widetilde{y}_f), \ j = (\overline{1, n}), S(\widetilde{y}_{dj}) = \begin{cases} 1, \ if \ \widetilde{y}_{dj} \ge 0\\ -1, \ if \ \widetilde{y}_{dj} < 0 \end{cases}$$
(4)

The term $S(\tilde{y}_{dj})$ allows to consider how the obstacle detected by *j*-th rangefinder is located towards the AUVfollower (from the left or right side). Therewith the negative value of the δ_j indicates that the point detected on the obstacle lies between the AUV-leader and the AUV-follower. It can also be noted that if the AUV-follower and the obstacle will be located on different sides of the AUV-leader trajectory, then the presence of such obstacles will not lead to the displacement of the followers within the formation, since the distance to this obstacle will obviously be greater than D_{min} .

The calculated values δ_j , $j = (\overline{1,n})$ are used for selecting the closest to the AUV-follower trajectory point $\widetilde{X}_{dc} = (\widetilde{x}_{dc}, \widetilde{y}_{dc})$:

$$\widetilde{X}_{dc} = \widetilde{X}_{dj}, \text{ if } \delta_j = \min(\delta_j), j = (\overline{1, n}).$$
(5)

The coordinates \widetilde{X}_{dc} and the expression (1) are applied to compute the new position of the program point at a safe distance from the detected obstacle:

$$\widetilde{y}_{f}^{*} = \begin{cases} \widetilde{y}_{dc} - sign(\widetilde{y}_{dc})D_{\min}, if \min(\delta_{j}) < D_{\min} \\ \widetilde{y}_{f}, if \min(\delta_{j}) < D_{\min} \end{cases}, j = (\overline{1, n}) \\ \widetilde{x}_{f}^{*} = \widetilde{x}_{f} \pm D_{a}(\widetilde{y}_{f} - \widetilde{y}_{f}^{*})/(2\widetilde{y}_{f}), \end{cases}$$
(6)

where $\widetilde{X}_{f}^{*} = (\widetilde{x}_{f}^{*}, \widetilde{y}_{f}^{*})$ is the adjusted position of the AUV-follower reference point located at a safe distance from the obstacle (see figure 4).

The target point can change its position dramatically upon detection of the obstacle. It can lead to sharp fluctuations in the AUV-follower movement, which are inappropriate for many tasks. This situation may also occur due to the presence of noise in the measurements of the onboard rangefinders. To solve this problem, it is proposed to smooth out sharp changes in the position of the target point using an expression describing a low-frequency filter:

$$\hat{X}_{f}(k) = \hat{X}_{f}(k-1) + \beta(\tilde{X}_{f}^{*}(k) - \hat{X}_{f}(k-1)) \quad , \tag{7}$$

where $\hat{X}_f(k) = (\hat{x}_f(k), \hat{y}_f(k))$ – the position of the AUVfollower target point at the current step of the trajectory formation system operation; $0 \le \beta \le 1$ – the smoothing factor.

The use of the expression (7) allows to avoid the sharp change in the position of the AUV-follower target point upon detection of the obstacle. Therewith the target point \hat{X}_{f}

itself will begin to move toward the point \widetilde{X}_{f}^{*} along the AUV-follower specified trajectory within the formation at a velocity that depends on the value of the coefficient β . The smaller the value β , the lower the motion velocity of the specified point and the better the follower trajectory will be smoothed.

Thus, the use of the expressions (1) - (7) allows to form AUVs group motion in the defined formation under the conditions of the environment containing unknown obstacles. Therewith, in the process of obstacle avoidance, the AUVs-followers do not need to communicate, since they will move within the formation along predetermined safe trajectories.

4. SIMULATION RESULTS

The mathematical simulation was carried out via the V-REP software for investigation of the proposed method. During the simulation the movement of the group of three AUVs operating in the formation of a triangle type was studied. The parameters of the triangle were the follows: $\widetilde{X}_{f1} = (-1.5,1), \ \widetilde{X}_{f2} = (-1.5,-1).$ The AUVs-followers trajectories within the formation was given by expression (1), the permissible distance between the AUVs was assumed to be 0.75 m, and the safe distance to the obstacles was 1 m. Therewith all the AUVs-followers was proposed to have high-precision follow-up control system Yukhimets (2011), Lebedev (2015). All AUVs-followers were equipped with 7 rangefinders arranged evenly in the front hemisphere of each AUV-follower. The reference trajectory of the group passed through two base points with coordinates: WP1 = (7.7, 6.6), WP2 = (3.5, 8.7). Such small distances between AUVs are due to the small size of the landfill, however, this does not affect the results of checking the workability of the proposed algorithm.

The motion trajectories of the group of the AUVs are shown in the figure 5. Curve 1 corresponds to the AUV-leader trajectory, generated via the algorithm proposed in Filaretov (2017), and curves 2 and 3 correspond to the trajectories of the left (AUV1) and right (AUV2) AUVs-followers, respectively. Their trajectories are formed in the process of the movement according to the proposed algorithm. It can be seen from the figure 5 that in the process of moving the AUV-leader finds the safe path to the first base point (WP1) between the two obstacles, bypassing the right obstacle on the left at the safe distance. AUVs-followers cannot pass between these obstacles, keeping the defined formation. Therefore, the right AUV-follower also bypasses the obstacle on the left keeping the safe distance. During the obstacle avoidance, its trajectory almost coincides with the leader one on some site. The left AUV-follower avoids the left obstacle on the right and in this process its trajectory approaches to the AUV-leader trajectory. Only the right AUV-follower passes the second obstacle, shifting towards the AUV-leader trajectory, and the left AUV-follower keeps moving behind the leader keeping defined place in the formation.

The figure 6 shows the change the AUV formation when avoidance of the detected obstacles is carrying out. It can be

seen from the figure 6a that passing through the gap between two obstacles AUVs-followers line up behind the AUVleader and then, when obstacles avoidance is accomplished, come back to an original configuration (see figure 6b).



Fig. 5. Trajectory of AUV-group during movement in unknown environment with obstacles



Fig. 6. Formation changing during obstacle avoidance

Thus, the simulation results confirm the efficiency and effectiveness of the proposed algorithm for trajectory formation of the AUVs group under the conditions of the unknown environment containing obstacles.

CONCLUSIONS

The paper proposes a new method for trajectory formation of the AUVs group in "leader-follower" mode in the given formation under the conditions of the unknown environment containing obstacles. In this mode, the AUV-leader defines the motion trajectory at the safe distance from the detected obstacles in accordance with the given mission. The AUVsfollowers follow the leader in an orderly manner bypassing the obstacles detected by their rangefinders. Therewith in the process of obstacles avoidance they move along set in advance trajectories within the formation to ensure the safe distance between the AUVs-followers. In addition, the motion of the AUVs-followers along the predetermined trajectories allows to avoid additional data exchange between the AUVs with a view to match their positions. The simulation results confirm the efficiency of the proposed method for trajectory formation of the AUVs group.

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