Dynamic Event-Triggered Adaptive Control for Robust Output Regulation of Nonlinear Systems with Unknown Exosystems ⋆

Pin Liu *,** **Feng Xiao** *,** **Bo Wei** **

* *State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China*
** *School of Control and Computer Engineering, North China Electric Power University, Beijing, 102206, China*
(e-mails: pinliu@yeah.net; fengxiao@ncepu.edu.cn; bowei@ncepu.edu.cn)

Abstract: In this paper, robust output regulation of nonlinear systems with unknown neutral exosystems is discussed. Based on the internal model and adaptive control theory, we design the output feedback controller with a dynamic event-triggering mechanism. Under the dynamic event-triggering mechanism, the controller can be implemented in the digital platform. The effectiveness of the proposed event-triggering mechanism is illustrated through an example.

Keywords: Robust output regulation; nonlinear systems; adaptive control; internal model; dynamic event-triggering mechanism.

1. INTRODUCTION

Output regulation is to drive the system output to track the trajectory generated by an exosystem, reject it as a disturbance and achieve the stability simultaneously. Since the publication of the pioneering papers (Francis and Wonham, 1975; Isidori and Byrnes, 1990; Serrani et al., 2001; Marconi et al., 2002), the output regulation problems have aroused great interest in recent years. The related work on the output regulation of linear systems or nonlinear systems can be found in Ding (2015); Xi and Ding (2007); Huang (2004). And the results extended to multi-agent systems for seeking a consensus or containment control can be found in Gao et al. (2019); Guo et al. (2016); Su and Huang (2011, 2013).

The above-mentioned works are all based on continuous control laws to achieve the output regulation. In practice, control systems may have limited computation and communication resources, and thus a parsimonious usage of resources is a critical issue. The traditional periodic approach can be implemented to emulate continuous controllers. In spite of the simplistic implementation, the high triggering frequency in the worst cases may lead to many unnecessary triggering instants. An event-triggering mechanism (ETM) is proposed in Tabuada (2007) where only necessary control tasks are executed. Therefore, a reduction of triggering is achieved while stability or performance is guaranteed. In Girard (2014), a dynamic ETM is proposed. When introducing a dynamic variable in event-triggering conditions, the inter-event times of the dynamic ETM can be prolonged compared with the ETM. However, Zeno behaviors may exist in the dynamic event-triggered control. There are two classifications of Zeno freeness: non-Zeno behavior with or without a positive minimum inter-event time (MIET) (Nowzari et al., 2019). The Zeno freeness without a positive MIET could require hardwares to compute arbitrarily fast, which is problematic in the practical implementation. Therefore, guaranteeing a positive MIET in the event-triggered control is a favorable choice.

A digital controller is designed to achieve the robust output regulation of nonlinear systems in Liu and Huang (2017). It is assumed that exosystems are known. In practical industrial applications, exosystems may be unknown. An adaptive controller that automatically tunes the control input to achieve the output regulation with unknown exosystems is proposed in Serrani et al. (2001). Adaptive controllers are widely used in the robust output regulation, but the applications of this kind of adaptive controllers to digital platforms are rare. The adaptive items and internal models in controllers increase the complexity of the design of event-triggering conditions. A digital adaptive controller is applied to achieve the tracking of unknown exosystems in Qian et al. (2019). However, the digital controller is designed for linear systems, and the event-triggering condition cannot be extended to nonlinear systems directly.

Motivated by the above-mentioned papers, we propose a novel ETM to achieve the robust output regulation of a nonlinear system with an unknown exosystem. Adaptive control and the internal model are used for the design of the controller. Compared with the work of Liu and Huang (2017) where known exosystems are considered and the work in Qian et al. (2019) where the linear systems is discussed with an event-triggering mechanism,

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the design process is more complex. Major contributions of this paper are as follows. First, it is the first time that the dynamic event-triggered controller of nonlinear systems is designed to achieve the output regulation. Second, positive minimum inter-event times can be guaranteed and the dynamic variable in the event-triggering condition can prolong the length of inter-event intervals.

Notations: $\mathbb{R}_{\geq 0}$ denotes the set of non-negative reals and $\mathbb{Z}_{\geq 0}$ denotes the set of non-negative integers. Let $\mathbb{R}^n$ represent the real space of n-dimensional real vectors. Let $\mathbb{R}^{m\times n}$ denote the set of $m\times n$ real matrices. For a vector or matrix X, X^T represents the transpose of X. A function α: $\mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class K if it is continuous, strictly increasing and $\alpha(0) = 0$. A function α: $\mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be a class K_∞ if it is a class K function and $\alpha(r) \to \infty$ as $r \to \infty$ additionally.

2. PROBLEM SETUP

2.1 The Model Setup

The robust output regulation of nonlinear systems with unknown exosystems is achieved with continuous controllers in Su and Huang (2013) and Liu and Huang (2017). Inspired by these papers, the objective of this work here is to implement the above-mentioned continuous controllers with dynamic event-triggering mechanism. We consider the following nonlinear systems

$$\begin{aligned}\dot{z} &= f(z, y, v, w),\\ \dot{y} &= g(z, y, v, w) + b(w)u,\\ e &= y - q(v, w),\end{aligned} \tag{1}$$

where $z \in \mathbb{R}^n$, $y \in \mathbb{R}$ and $u \in \mathbb{R}$ are the state, the measured output and the actual control input of the nonlinear system respectively. $e \in \mathbb{R}$ denotes the tracking error, variables $q(v, w)$ and v will be explained later. $w \in \mathbb{R}^{n_w}$ is an unknown constant vector. $b(w)$ is a continuous function. f and g are smooth functions and satisfy $f(0, 0, 0, w) = 0$, $g(0, 0, 0, w) = 0$ for all w.

The unknown linear exosystem with an unknown vector $\sigma \in \mathbb{R}^{n_\sigma}$ is given by

$$\begin{aligned}\dot{v} &= S(\sigma)v,\\ y_0 &= q(v, w),\end{aligned} \tag{2}$$

where v is the state, $q(v, w)$ is the output of the exosystem to be tracked by the nonlinear system (1), and S is a constant matrix. Since the vector σ is unknown, the eigenvalues of $S(\sigma)$ and the trajectory of the exosystem (2) are thus unknown. The function q is smooth and continuous with $q(0, w) = 0$ for all w.

Considering the nonlinear system (1) and the unknown exosystem (2), the robust output regulation in this paper is to design the control input to achieve the following two objectives.
(I) Boundedness: trajectories of the closed loop system exists and is bounded;
(II) Practical tracking: the tracking error e satisfies $\limsup_{t\to\infty} |e(t)| < \varepsilon$ and ε is a positive constant.

Our primary task is to propose a novel event-triggering mechanism that can achieve the robust output regulation of the nonlinear system (1) with the unknown exosystem (2). The data transmitted between the plant and controller are not continuous. At the triggering instants $\{t_i\}_{i\in\mathbb{Z}_{\geq 0}} \subset \mathbb{R}_{\geq 0}$ and some other instants, state signals are sampled and the control input is updated.

2.2 Some Standard Assumptions

The followings are some standard assumptions (Liu and Huang, 2017; Serrani et al., 2001) for the output regulation of nonlinear systems.

Assumption 1. The eigenvalues of the exosystem (2) are unknown and all have the zero real parts.

Assumption 2. The function b is continuous and $b(w) > 0$ for all w.

Assumption 3. For any constant vector σ, there exists an invariant manifold $(\mathbf{z}(v, w, \sigma), \mathbf{y}(v, w))$ with $\mathbf{z}(v, w, \sigma)$, $\mathbf{y}(v, w)$ satisfying

$$\begin{aligned}\frac{\partial \mathbf{z}(v, w, \sigma)}{\partial v} S(\sigma)v &= f(\mathbf{z}(v, w, \sigma), q(v, w), v, w)\\ \mathbf{y}(v, w) &= q(v, w).\end{aligned} \tag{3}$$

The desired feedforward control input can be described by

$$\begin{aligned}\mathbf{u}(v, w, \sigma) =& b^{-1}(w)\big(\frac{\partial \mathbf{z}(v, w, \sigma)}{\partial v} S(\sigma)v - g(\mathbf{z}(v, w, \sigma),\\ & q(v, w), v, w)\big).\end{aligned}$$

Assumption 4. The desired feedforward control input $\mathbf{u}(v, w, \sigma)$ is a polynomial of v with the coefficients depending on w and σ.

2.3 Parameterization of Internal Model

The parameterization progress of the internal model is proposed in Serrani et al. (2001). Given two nonempty compact sets $\mathcal{P} \subset \mathbb{R}^{n_\sigma}$ and $\mathcal{W} \subset \mathbb{R}^{n_w}$, under Assumption 4, there exist a integer r and a set of real numbers $\varrho_1(\sigma), \varrho_2(\sigma) \cdots \varrho_r(\sigma)$ so that the following equation exists for all $(w, \sigma) \in \mathcal{W} \times \mathcal{P}$,

$$\begin{aligned}\frac{d^r \mathbf{u}(v, w, \sigma)}{dt^r} =& \varrho_1(\sigma)\mathbf{u}(v, w, \sigma) + \varrho_2(\sigma)\frac{d\mathbf{u}(v, w, \sigma)}{dt}\\ & + \cdots + \varrho_r(\sigma)\frac{d^{r-1}\mathbf{u}(v, w, \sigma)}{dt^{r-1}}.\end{aligned} \tag{4}$$

Then for any vector σ, there exists a mapping τ_σ as follows

$$\tau_\sigma(v, w) = \begin{pmatrix} \mathbf{u}(v, w, \sigma) \\ \dfrac{d\mathbf{u}(v, w, \sigma)}{dt} \\ \cdots \\ \dfrac{d^{r-1}\mathbf{u}(v, w, \sigma)}{dt^{r-1}} \end{pmatrix},$$

with

$$\begin{aligned}\dot{\tau}_\sigma(v, w, \sigma) &= \Phi(\sigma)\tau_\sigma(v, w, \sigma),\\ \mathbf{u}(v, w, \sigma) &= \Gamma\tau_\sigma(v, w, \sigma),\end{aligned} \tag{5}$$

where the matrix $\Phi(\sigma)$ and the vector Γ are given by

$$\Phi(\sigma) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ \varrho_1(\sigma) & \varrho_2(\sigma) & \cdots & \varrho_s(\sigma) \end{bmatrix}, \Gamma = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}^T.$$

Lemma 5. (cf. Lemma IV.1 in Serrani et al. (2001)) Given any Hurwitz matrix M and vector N composing the controllable pair (M, N), the Sylvester equation that

$$T\Phi(\sigma) = MT + N\Gamma$$

has an unique solution since $\Phi(\sigma)$ and M have no common eigenvalues. And T is nonsingular due to the observability of the pair $(\Phi(\sigma), \Gamma)$ and the controllability of the pair (M, N).

Based on the parameterized method, the internal model containing the unknown vector can be described by a known transformed matrix pair. By introducing a linear transformation, $\theta(v, w, \sigma) = T\tau_\sigma(v, w, \sigma)$, the internal model can be described by a controllable matrix pair (M, N), where M is Hurwitz. The linear system is given by

$$\begin{aligned} \dot{\theta}(v, w, \sigma) &= (M + N\Gamma T^{-1})\theta(v, w, \sigma) \\ &= M\theta(v, w, \sigma) + N\mathbf{u}(v, w, \sigma), \\ \mathbf{u}(v, w, \sigma) &= \Psi^\sigma \theta(v, w, \sigma), \\ \Psi^\sigma &= \Gamma T^{-1}, \end{aligned} \tag{6}$$

where $\theta(v, w, \sigma) \in R^{n_\theta}$, $M \in R^{n_\theta \times n_\theta}$, $N \in R^{n_\theta}$ and $\mathbf{u}(v, w, \sigma) \in R$.

3. DESIGNING DIGITAL CONTROLLERS

3.1 The Design of Event-triggering Condition

In this paper, the event-based controller for the robust output regulation of the nonlinear system with the unknown exosystem is discussed. It it practical that continuous updates of the controller are not necessary. The event-based controller is given by

$$\begin{aligned} \dot{\eta}(t) &= M\eta(t) + Nu(t), \\ u(t) &= -\bar{k}\rho(e(t_i))e(t_i) + \hat{\Psi}(t_i)\eta(t_i), \\ \dot{\hat{\Psi}}(t) &= -e(t)\eta(t), \\ \dot{k}(t) &= \rho(e(t_i))e(t_i)e(t), \end{aligned} \tag{7}$$

where $\rho(e)$ is a smooth function. $\eta(t)$ is introduced as an estimator of $\theta(v, w, \sigma)$. $\hat{\Psi}(t)$ is an estimated value of Ψ^σ for any σ and is served as an adaptive variable. $k(t)$ is a smooth function served as another adaptive variable. $\bar{k}$ is a piecewise function. Introduce a positive constant k_{in} as a threshold. When the signal k arrives at $\bar{k} + k_{in}$, $\bar{k} = k(t)$ and the controller updates the control input again.

For convenience, define $\nu(t) = -\rho(e)e$ and $\nu(t_i) = -\rho(e(t_i))e(t_i)$. Two error variables $\tilde{\nu}(t)$ and $\tilde{k}(t)$ are given by

$$\begin{aligned} \tilde{\nu}(t) &= \nu(t_i) - \nu(t), \\ \tilde{k}(t) &= k(t) - k_0, \end{aligned}$$

where k_0 is a positive constant that $k(t)$ will approach to.

The introduced internal dynamic variable h is given by

$$\begin{aligned} \dot{h}(t) &= -h(t) + f(t) - \delta, \\ f(t) &= |\hat{\Psi}(t_i)\eta(t_i) - \hat{\Psi}(t)\eta(t)| + |k_0 - k_{in}||\tilde{\nu}(t)|, \end{aligned} \tag{8}$$

where δ is a positive constant and is introduced to guarantee a positive lower bound of inter-event times. The next event-triggering instant t_{i+1} is determined by the dynamic event-triggering condition as:

$$\begin{aligned} t_{i+1} &= inf\{t > t_i | \quad f(t) > \delta + h(t)\}, \\ t_0 &= 0. \end{aligned} \tag{9}$$

In this dynamic ETM, even if the value of static part $\delta - f(t)$ is negative, the event may not happen because the variable h is non-negative all the time. This guarantees the inter-event time is large compared with the static ETM. In the static ETM, the value $\delta - f(t)$ needs to be non-negative or be negative shortly before the next execution instant. If the static part of dynamic ETM has no Zeno behavior, the dynamic ETM can guarantee the Zeno freeness.

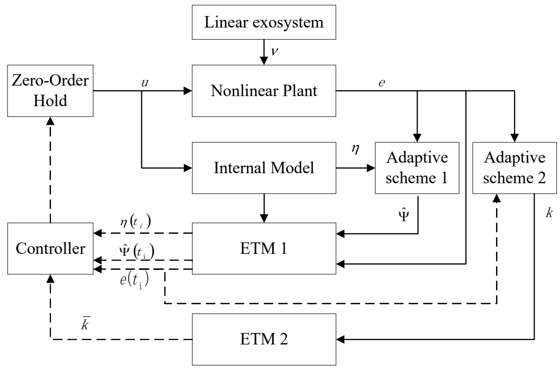

Fig. 1. Event-triggered control for robust output regulation: solid lines stand for continuous signals and the dashed lines stand for digital signals

The proposed ETMs are shown in Fig. 1. It shows the digital implementation of a continuous controller. At the event instants $\{t_i\}$, the signals η, $\hat{\Psi}$ and e are sampled and the controller is updated. The determination of the event instants $\{t_i\}$ and the data-sampling process are achieved by ETM 1 which is a dynamic event-triggering mechanism. The inter-event times are enlarged due to the introduced dynamic variable. The ETM 2 is to determine when to update $\bar{k}$. It seems that the updated frequencies of the controller are increased when compared with the case that only one ETM exists. However, from the simulation, it can be seen that when the constant threshold k_{in} is enlarged, it has no effect on the output regulation and the update times of the controller can be reduced. The larger k_{in} is, the less $\bar{k}$ updates. In such a way, neither the signals are needed to be transmitted continuously nor the controller needs to be updated continuously.

3.2 Stabilization Analysis

The output regulation problem can be converted to the stability problem via a coordinate transformation (Liu and Huang, 2017). The coordinate transformation is given by

$$\begin{aligned} \bar{z} &= z - \mathbf{z}(v, w, \sigma), \\ \bar{\eta} &= \eta - \theta(v, w, \sigma) - Nb^{-1}(w)e, \\ e &= y - q(v, w). \end{aligned} \tag{10}$$

The objective in this paper is to guarantee that the trajectory $(z(t), \eta(t))$ and the tracking error e are bounded with the control input (7). Define the variable $x_c = col(z, \eta, e)$ and the variable $\bar{x}_c = col(\bar{z}, \bar{\eta}, e)$. If the state variable $\bar{x}_c$ is bounded, the state variable x_c is also bounded.

The dynamics of coordinated variables have the following form:

$$\begin{aligned}\dot{\bar{z}} =&\overline{f}(\bar{z},e,\mu),\\ \dot{\bar{\eta}} =&M\overline{\eta}+MNb^{-1}(w)e-Nb^{-1}(w)\overline{g}(\bar{z},e,\mu),\\ \dot{e} =&\overline{g}(\bar{z},e,\mu)+b(w)\Psi^{\sigma}\overline{\eta}+\Psi^{\sigma}Ne-b(w)\bar{k}\rho(e(t_i))e(t_i)\\ &+b(w)\hat{\Psi}(t_i)\eta(t_i)-b(w)\Psi^{\sigma}\eta, \end{aligned}\tag{11}$$

where $\tilde{\Psi}=\hat{\Psi}-\Psi^{\sigma}$, $\mu=col(v,w,\sigma)$ and

$$\begin{aligned}\overline{f}(\bar{z},e,\mu)&=f(\overline{z}+\mathbf{z},e+q,v,w)-f(\mathbf{z},q,v,w),\\ \overline{g}(\bar{z},e,\mu)&=g(\overline{z}+\mathbf{z},e+q,v,w)-g(\mathbf{z},q,v,w).\end{aligned}\tag{12}$$

The functions $\overline{f}$ and $\overline{g}$ are smooth with $\overline{f}(0,0,\mu)=0$ and $\overline{g}(0,0,\mu)=0$ for all μ. Denote $Z=col(\bar{z},\bar{\eta})$. The tracking error e is given by

$$\begin{aligned}\dot{e}=&\tilde{g}(Z,e,\mu)-b(w)\bar{k}\rho(e(t_i))e(t_i)+b(w)\hat{\Psi}(t_i)\eta(t_i)\\ &-b(w)\Psi^{\sigma}\eta(t),\end{aligned}\tag{13}$$

and

$$\tilde{g}(Z,e,\mu)=\overline{g}(\bar{z},e,\mu)+b(w)\Psi^{\sigma}\overline{\eta}+\Psi^{\sigma}Ne,$$

where $\tilde{g}$ is smooth and $\tilde{g}(0,0,\mu)=0$ for all μ. And Z has the following form:

$$\dot{Z}=F(Z,e,\mu)$$

with $F(Z,e,\mu)=col(\overline{f}(\bar{z},e,\mu),M\overline{\eta}+MNb^{-1}(w)e-Nb^{-1}(w)\overline{g}(\bar{z},e,\mu))$.

By Lemma 3.1 in Xu and Huang (2009), we assume that there exists an ISS Lyapunov function V_1 satisfying

$$\begin{aligned}\underline{\alpha}_1(\|Z\|)\le V_1(Z)\le\overline{\alpha}_1(\|Z\|)\\ \frac{\partial V_1(Z)}{\partial Z}F(Z,e,\mu)\le-\Delta(Z)\|Z\|^2+\pi(e)e^2\end{aligned}\tag{14}$$

for all Z, e and μ, where $\underline{\alpha}_1$, $\overline{\alpha}_1$, Δ and π are class K_∞ functions. The existence of the ISS Lyapunov function is a sufficient condition to guarantee the input-to-state stability of the system. And if $\tilde{g}$ is smooth with $\tilde{g}(0,0,\mu)=0$ for all μ, then there exist two smooth positive functions $\varphi(Z)$ and $\chi(e)$ satisfying

$$||\tilde{g}(Z,e,\mu)||^2\le\varphi(Z)||Z||^2+\chi(e)e^2\tag{15}$$

Theorem 6. Under Assumptions 1-4 and the existence of the ISS Lyapunov function V_1, the robust output regulation of the nonlinear systems (1) with the unknown exosystem (2) is achieved globally with the controller (7) under the dynamic ETM (8) and (9) for any positive constants δ and k_{in}.

Proof. Define the Lyapunov function

$$V=V_1(Z)+\frac{1}{2}e^2+\frac{1}{2}b(w)\tilde{\Psi}^T\tilde{\Psi}+\frac{1}{2}b(w)\tilde{k}^T\tilde{k}+h\tag{16}$$

$V_1(Z)$ is positive definite by (14) and $b(w)>0$ for all w under Assumption 2. The function h is a positive function from (8). Therefore, the Lyapunov function V is positive definite. The derivative of V can be calculated as follows

$$\begin{aligned}\dot{V}=&\dot{V}_1(Z)+e\dot{e}+b(w)\tilde{\Psi}^T\dot{\hat{\Psi}}+b(w)\tilde{k}^T\dot{k}+\dot{h}\\ \le&\dot{V}_1(Z)+e(\tilde{g}(Z,e,\mu)-b(w)\bar{k}\rho(e(t_i))e(t_i)+\dot{h}\\ &-b(w)\Psi^{\sigma}\eta(t))+b(w)\tilde{\Psi}^T\dot{\hat{\Psi}}+b(w)\tilde{k}^T\dot{k}\\ &+b(w)\hat{\Psi}(t_i)\eta(t_i)\\ \le&\dot{V}_1(Z)+e\tilde{g}(Z,e,\mu)-b(w)e\nu(t_i)(\tilde{k}-\bar{k})+\dot{h}\\ &+b(w)e(\hat{\Psi}(t_i)\eta(t_i)-\hat{\Psi}(t)\eta(t))\\ \le&\dot{V}_1(Z)+\frac{1}{4}e^2+||\tilde{g}(Z,e,\mu)||^2-b(w)e\nu(t)(k_{in}-k_0)\\ &+\dot{h}+|b(w)e|(|(\hat{\Psi}(t_i)\eta(t_i)-\hat{\Psi}(t)\eta(t))|\\ &+|\tilde{\nu}(t)||k_{in}-k_0|)\\ \le&(\varphi(Z)-\Delta(Z))\|Z\|^2+\big(\pi(e)+\chi(e)+\frac{1}{4}+\frac{1}{4}b_M^2\\ &-b_m(k_0-k_{in})\rho(e)\big)e^2+\delta^2-h\\ \le&-||\bar{x}_c||^2,\quad\forall\bar{x}_c\ge\delta.\end{aligned}\tag{17}$$

The last inequality is based on the constraints: $\rho(e)\ge\frac{\pi(e)+\chi(e)+\frac{1+b_M^2}{4}+2}{b_m(k_0-k_{in})}$, $\Delta(Z)\ge\varphi(Z)+2$, $h\ge0$ and $0<k_{in}<k_0\le\lambda k_{in}$. Since $\bar{k}$ is updated when $k(t)=\bar{k}+k_{in}$, $k(t)\le\bar{k}+k_{in}$ always holds. From the last inequality of (17), $||\bar{x}_c||$ is bounded. There exists a constant δ_c that is larger than δ. $\bar{x}_c(t)$ will approach the set $\Omega_{\delta_c}=\{\bar{x}_c:||\bar{x}_c||\le\delta_c\}$ in a finite time T_δ and stay in the set for all $t\in[T_\delta,\infty)$. The analysis is based on the fact that the derivative of V is less than zero when $||\bar{x}_c||$ is outside the boundary $||\bar{x}_c||=\delta$, which implies $||x_c||$ is also bounded.

Remark 7. Seen from the proof of Theorem 6, the nonlinear system under the event-triggering conditions (8) and (9) cannot achieve the exact tracking of the exosystem. It is because constant δ appears in the right side of (9).

3.3 Zeno Freeness

Theorem 8. Under the same assumptions of Theorem 6, the Zeno freeness is guaranteed under the dynamic ETM (8) and (9).

Proof. The minimum inter-event time is defined by $\tau_{miet}\le inf_{i\in\mathbb{N}}(t_{i+1}-t_i)$. We need to show that a positive minimum inter-event time is guaranteed under (8) and (9). Once a positive MIET under the static part of the dynamic ETM is guaranteed, a positive MIET can be guaranteed under the dynamic ETM. The rest analysis relies on the static part. The result is relatively more conservative. In this regard, we can modify the event-triggering condition as

$$t_{i+1}=inf_{i\in\mathbb{N}^+}\{t>t_i|\quad f(t)>\delta\}\tag{18}$$

This kind of triggering condition can be named the absolute triggering. For absolute triggering, if the derivative of $f(t)$ is bounded in the time period $[t_i,t_{i+1})$, Zeno freeness is achieved.

$$\begin{aligned}d\frac{f(t)}{dt}\le&|\dot{f}(t)|\le|\dot{\hat{\Psi}}(t)\eta(t)|+|\hat{\Psi}(t)\dot{\eta}(t)|+|k_0-k_i||\dot{\nu}(t)|\\ \le&|e(t)\eta^2(t)|+|\hat{\Psi}(t)(M\eta(t)+Nu(t))|\\ &+|\rho(e(t))\dot{e}(t)|+|\frac{d\rho(e(t))}{dt}e(t)|.\end{aligned}\tag{19}$$

Since $\bar{x}_c$ is bounded, $\bar{z}(t)$, $\bar{\eta}(t)$ and $e(t)$ are bounded. $x_c(t)$ is bounded, so $z(t)$ and $\eta(t)$ are bounded. $\rho(e)$ and $d\frac{\rho(e(t))}{dt}$

are bounded in $[t_i, t_{i+1})$. $V(t)$ is bounded from (17), and $\tilde{\Psi}(t)$ and $\hat{\Psi}(t)$ are thus bounded. $\dot{e}(t)$ is bounded from (14). There exists a constant ϵ such that the derivative of $|f(t)|$ is bounded by ϵ. This implies the MIET is lower bounded by $\frac{\delta}{\epsilon}$. The Zeno freeness with a positive MIET is guaranteed for the static part. The dynamic ETM (8) and (9) thus can avoid the Zeno behavior with a positive MIET.

4. EXAMPLE

We illustrate the effectiveness of the proposed control law by a simulation example. The model is borrowed from Xu and Huang (2009) as follows.

$$\begin{aligned}\dot{z}_1 &= a_1 z_1 - a_1 y\\ \dot{z}_2 &= a_2 z_2 + z_1 y\\ \dot{y} &= a_3 z_1 - y - z_1 z_2 + bu\\ e &= y - v_1\end{aligned}$$

where z_1, z_2 are the states, y and e are the measured output and the output of the nonlinear system respectively. Let $a = (a_1, a_2, a_3, b)$ be the parameter variable of the nonlinear system, $a = \bar{a} + w$ with $\bar{a} = (\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{b})$ and $w = (w_1, w_2, w_3, w_4)$ is the uncertainty. The system matrix of the linear exosystem with the unknown constant σ is assumed to be $S = \begin{bmatrix} 0 & \sigma \\ -\sigma & 0 \end{bmatrix}$.

Choose the parameterized matrix pair (M, N) as follows

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -12 & -13 & -6 \end{bmatrix}, N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}^T$$

Then matrix $T = [4 - 9\sigma^4, 12, 13 - 10\sigma^2, 6]$. The controller is the same as (7) with $\rho(e(t)) = 5(e^6(t) + 1)$, and the constants can be chosen to be $k_{in} = 1$, $k_0 = 17.5$, $\sigma = 0.8$ and $\delta = 0.02$. $a = (-10, -8/3, 28, 1)$, $[z_1(0), z_2(0), y(0)] = [3, -1, -2]$, $v(0) = [9, 0]$ and $\eta(0) = \hat{\Psi}(0) = [0, 0, 0, 0]$. Fig. 2 and Fig. 3 show the tracking error and the tracking process, respectively.

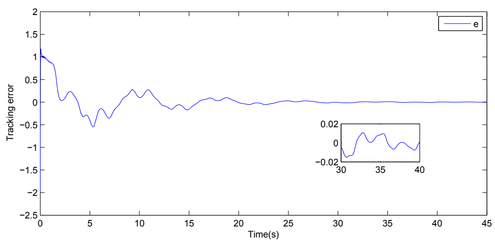

Fig. 2. The tracking error

As shown in Fig. 4, the estimator $\hat{\Psi}$ converges to the vector $\Psi = [0.3136, 12, 6.6, 6]$ with $\sigma = 0.8$. The event numbers of ETM 1 and ETM 2 are 44933 and 15 respectively in the first $45s$.

5. CONCLUSION

In this paper, we have studied the digital implementation of the continuous controller proposed in Xu and Huang

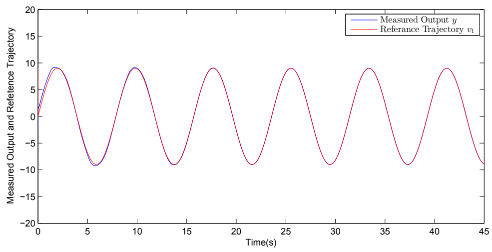

Fig. 3. The tracking process

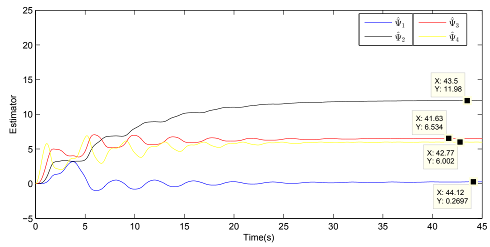

Fig. 4. The estimator $\hat{\Psi}$

(2009). The proposed event-triggers can guarantee the output regulation of the nonlinear system with the unknown linear exosystem, which has a positive minimum inter-event time. In particular, the dynamic event-triggering condition can enlarge the inter-event times.

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