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Performance Analysis of Long Horizon Predictive Control with Modified Sphere Decoding Algorithm

Mohamed Tamim Touati[1,2]. Shaoyuan Li[1]
Jing Wu[1]

[1] *Department of Automation, SEIEE, Shanghai Jiao Tong University, Shanghai 200240 China (e-mail: syli @sjtu.edu.cn).*
[2] *Algerian Space agency, Omar Aissaoui, El Hammadia, Bouzareah, Algiers, Algeria.*

Abstract: The complexity of the optimization problem arises in multistep model predictive control for power electronics as they are discrete by nature and have predefined control actions given as integer control variables. Generally, Sphere Decoding Algorithm (SDA) is used to solve the optimization problem. In this paper, we present an SDA with an Evolutionary Optimization attitude (EO) to simplify the complex exhaustive search that is brought by the long prediction horizon. The presented technique reconstructs a smaller search area from a large search area which decreases the number of candidate solutions. The performance of the optimization algorithm is evaluated through statistical analysis and computation burden.

Keywords: Long Horizon, Model Predictive Control, Sphere Decoding, Evolutionary Optimization.

1. INTRODUCTION

Model Predictive Control (MPC) has gained a lot of interest in field of power electronics because of the existence of clear mathematical models to predict the behaviour of variables. In power electronics MPC is applied using two main methods, Finite Control Set (FS-MPC) and direct control set MPC, Rodriguez et al. (2013).

FS-MPC takes advantage of the finite number of possible control actions the converter can perform to calculate a limited number of predictions and optimize a limited number of cost functions, Kouro et al. (2015). To this end, with single step prediction horizon FS-MPC's are easily incorporated into multiple power electronics applications to improve their reliability and efficiency while keeping computation complexity low, see Perez et al. (2008), Wilson et al. (2010) and Zhang et al. (2016). Regardless of the single step FS-MPC advantages, its performance is subject to high switching frequency and fast response time. This in certain applications is inconvenient as it increases the switching efforts and accelerates the degradation of the switching components. A high switching effort decrease quickly the life cycle of the switching components.

Low switching frequencies increase the harmonics distortion in the outputs of power electronics systems as there is more time between the switching actions. To address this issue researchers have worked on improving the accuracy MPC optimization in a single switching period, see Hu et al. (2015), Tomlinson et al. (2016), Vargas et al. (2007) and Ma et al. (2018). Long horizon MPC has been proved to perform better than single-step MPC in many applications. Yet, in power electronics it is hard to be applied as the increase in prediction horizon increase the number of possible control input and enlarges the optimization problem, Tiagounov et al. (2003). Transforming the optimization problem into Integer Least Squares (ILS) problem allow the integration of more intelligent optimization techniques such as the branch and bound and Sphere Decoding Algorithms (SDA) as did Geyer et al. (2014), Geyer et al. (2015). Same technique has been presented by Baidya et al. (2018), Karamanakos et al. (2016a), Karamanakos et al. (2016b), Karamanakos et al. (2017), Hassibi et al. (2005) and Vicalo et al. (2005), but despite the achieved performance the optimization accuracy is improved by introducing modifications to the ILS problem. It focuses on matrix decomposition to rearrange the searching space and to simplify optimization.

In this paper, a sphere decoding algorithm is modified to adopt Evolutionary Optimization (EO) behaviour. The EO algorithms have been widely used in multi-objective optimization problems, Deb et al. (2011), Zhou et al. (2011), Z. H. Zhang et al. (2017) and L. Zhang et al. (2017). Unlike other techniques, the proposed algorithm uses a global search region in the first iteration of the searching process then identifies an elite solution vector which is used to reconstruct a local downsized optimization region. The optimal solution is searched inside the smaller search region using a modified sphere decoding algorithm. The main idea is the Combination of EO and SD algorithm to reduce the number of input entries to be processed after the first iteration of the optimization algorithm. Moreover, this reformulation and recombination of the optimization algorithm allow a faster optimization. The proposed approach is evaluated on a seven-level converter in which a long horizon MPC can reduce the current THD associated with low switching frequencies.

2. INTEGER QUADRATIC PROGRAMMING PROBLEM

A three-phase converter ($i \in \{a,b,c\}$) optimization problem with a long prediction horizon is formulated as an Integer Least Squares (ILS) problem as follows:

$$U_{opt}(k) = \arg\left\{ \min_{U(k)} \left\| HU(k) - \bar{U}_{unc}(k) \right\|_2^2 \right\} \quad (1)$$

subject to

$$U(k) = \left[\boldsymbol{u}^T(k), \ldots, \boldsymbol{u}^T(k+N-1) \right]^T \in \boldsymbol{\mu}. \quad (2a)$$

$$u_i \in \mathbf{U} = \{-3,-2,-1,0,1,2,3\}, i \in \{a,b,c\} \quad (2b)$$

$$\|\Delta \boldsymbol{u}(l)\|_\infty \leq 1, \forall l = k, \ldots, k+N-1. \quad (2c)$$

where, U_{opt} is the optimal control sequence, H is lower triangular matrix, $u_i \in \mathbf{U}$ is the control input which denotes the converter seven voltage levels $\{-3,-2,-1,0,1,2,3\}$, $U_{unc}(k)$ is the global unconstrained solution or control vector for the converter each phase, N is the prediction horizon and μ is the N-time Cartesian product of the set $\mathbf{U}$.

3. SDA OPTIMIZATION

Contrary to exhaustive search where the whole optimization problem is solved as one entity by trying multiple solutions in an exhaustive manner, the idea of SDA focus on decomposing the search problem into small optimization problems. Then the problem is solved through multiple steps and each step or iteration is used to find a single optimal element of the solution vector. SD algorithm defines a sphere of a radius $\rho(k)$ that is centred at $\bar{U}_{unc}(k)$ the unconstrained solution. This sphere contains all the optimal and sub-optimal solutions. Each element of the system finite control set will be tested one by one in order to distinguish the ones that belong inside the sphere. The radius of the sphere is reduced after each iteration to eliminate the sub-optimal solutions and to conserve only the optimal solutions from the searching process. Hence, the radius value is critical in this method, as it should allow the existence of one optimal solution at least inside the sphere at each sample k. the value of $\rho(k)$ is initialized as follows

$$\rho(k) = \left\| HU_{ini}(k) - \bar{U}_{unc}(k) \right\|_2^2 \quad (4)$$

The value of U_{ini} is the previous step optimal solution vector.

$$U_{ini}(k) = U_{opt}(k-1) \in \mathbf{U} \quad (5)$$

The sphere decoding algorithm searches for optimal elements of the control vector $U(k)$ in sequential computation, where the constraint (2b),(2c) and the following condition should be fulfilled

$$\left\| HU(k) - \bar{U}_{unc}(k) \right\|_2^2 \leq \rho(k) \quad (6)$$

Since H is a lower triangular matrix, then (6) is rewritten in a recursive manner

$$\rho^2(k) \geq (\bar{U}_1 - H_{(1.1)}U_1)^2 + (\bar{U}_2 - H_{(2.1)}U_1 - H_{(2.2)}U_2)^2 + \ldots \quad (7)$$

where U_i is the ith of the control sequence $U(k)$, $H_{(i.j)}$ is the $(i.j)^{\text{th}}$ element of matrix H, and $\bar{U}_i$ denotes the ith element of $\bar{U}_{unc}(k)$.

In conventional SDAs, an over-all initial squared distance $\rho^2(k)$ is calculated for all $3N$ elements of U_{ini} at once which gives a large real number. Therefore, the condition in (7) can be satisfied by suboptimal solutions at the first elements and iteration. This results in a need of more iteration to shrink the sphere. This increases computations and brings uncertainty issues.

3.1 Proposed Sphere Decoding Algorithm

The proposed modifications aim to improve the SDA searching process, meanwhile, enhance or keep the same performance of the conventional SD algorithms. Researchers have been suggesting algebraic reformulations of the ILS problem to reconstruct a more computationally efficient optimization. However, these techniques focus only of the formulation but ignore the number of input entities to be

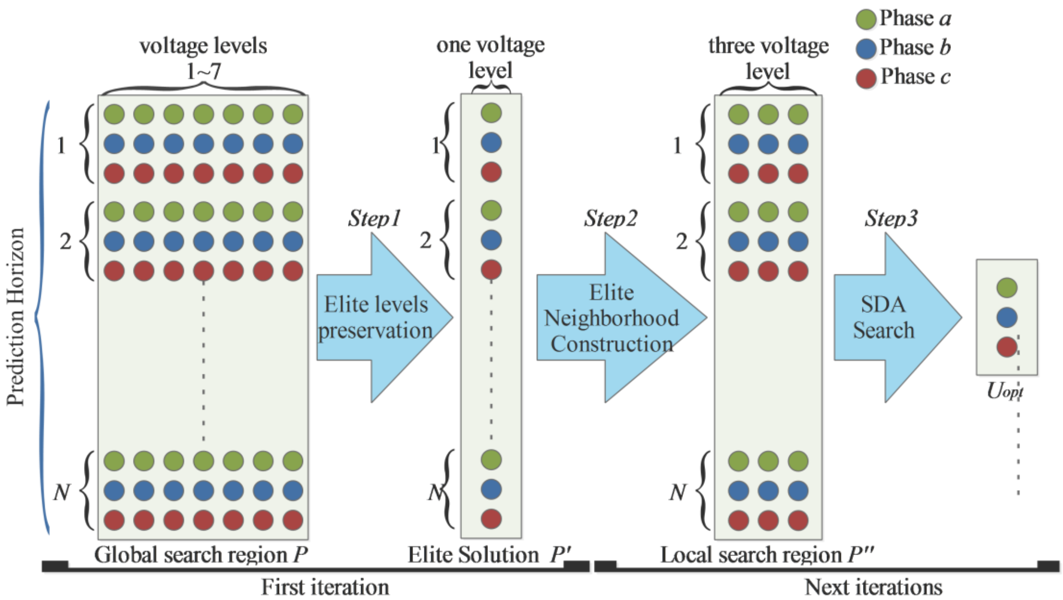
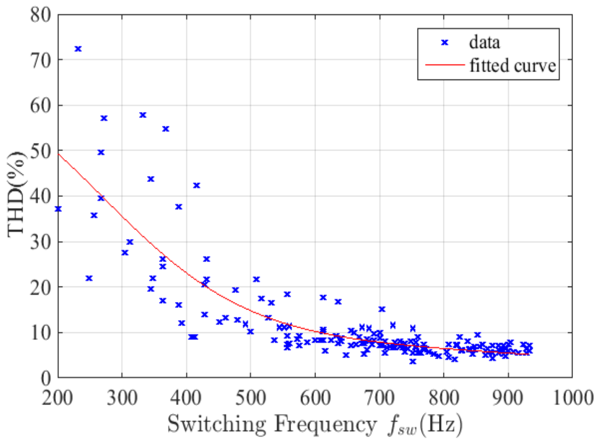
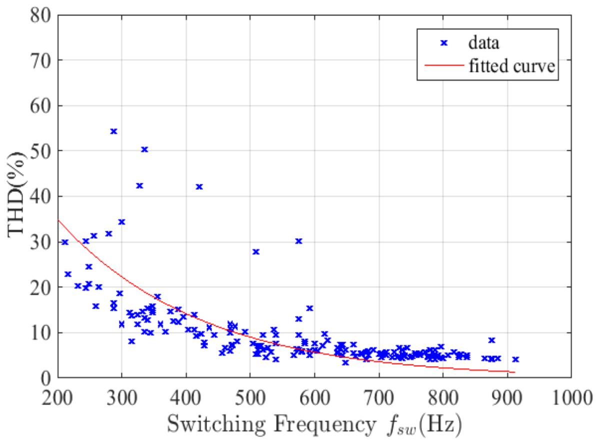


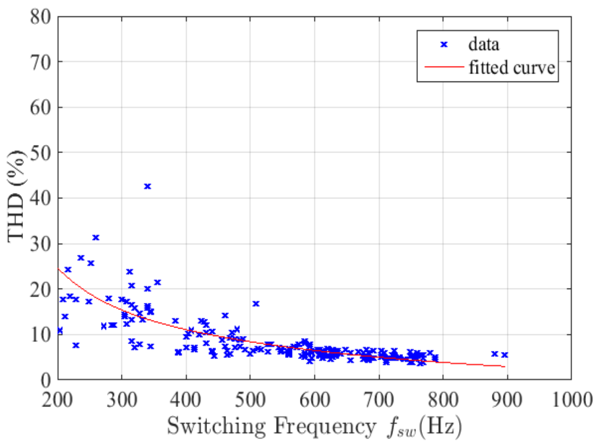
Fig. 1. Proposed Algorithm flow diagram.



(a) SD-MPC method with N=1.



(b) SD-MPC method with N=5.



(c) SD-MPC method with N=8.

Fig.2. Sampling Frequency impact on switching frequency and THD.

searched. The increasing number of inputs complicates the searching and the optimization which makes it exhaustive and time consuming.

In this paper, the number of control entries is optimized and reduced using an Evolutionary Optimization Algorithm (EOA), Deb et al. (2011). The efficient combination of SD algorithm and EOA involves both Optimization and level entries reduction at the same time. It uses a population-based approach in which all the possible solutions (Control inputs) are evaluated and tested at the first iteration, and then it uses a new smaller population set (new set of inputs) in the next iterations. In other words, the algorithm preserves only the elite control actions that satisfy the condition in (7) and defines a local search region of probable solutions from it. The new local search region or the control input set is considered as a global search region in the next iteration. This operation is repeated until only one optimal solution vector satisfies the squared distance condition.

The initialization equation of the sphere radius is reformulated to allow the use of the squared distance of each element as a quality factor to select the elite solutions. This enables the classification of the voltage levels from the best solution to the worst solution. The new formulation calculates a single element squared distance at each step of the regression operation by taking advantage of the recursive computation of equation (7) and set it as $d_{p,i}(k)$ in a vector $\mathbf{d}_p(k)$. The new initial vector of squared distances replaces the old total squared distance $\rho^2(k)$. The whole Vector $\mathbf{d}_p(k)$ is initialized as follows:

$$\mathbf{d}_p = \begin{bmatrix} d_{p,1} & d_{p,2} & \cdots & d_{p,3N} \end{bmatrix}^T \tag{8a}$$

subject to:

$$\begin{aligned} &(\bar{U}_1 - H_{(1,1)} U_{ini1})^2 = d_{p1}^2 \\ &d_{p1}^2 + (\bar{U}_2 - H_{(2,1)} U_{ini1} - H_{(2,2)} U_{ini2})^2 = d_{p2}^2 \\ &\vdots \\ &d_{p3N-1}^2 + (\bar{U}_{3N} - H_{(3N,1)} U_{ini1} - \cdots - H_{(3N,3N)} U_{ini3N})^2 = d_{p3N}^2 \end{aligned} \tag{8b}$$

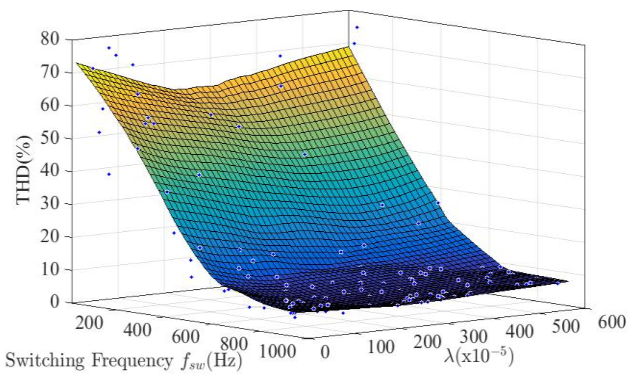
The overall algorithm proceeds through a path of three steps as illustrated in Fig.01. The algorithm is presented as pseudocode in the following steps:

- *Step One*: The algorithm starts the search from the overall possible solutions $\mathbf{U}$ (global population) in the first iteration by applying a decomposed version of equation (7) as:

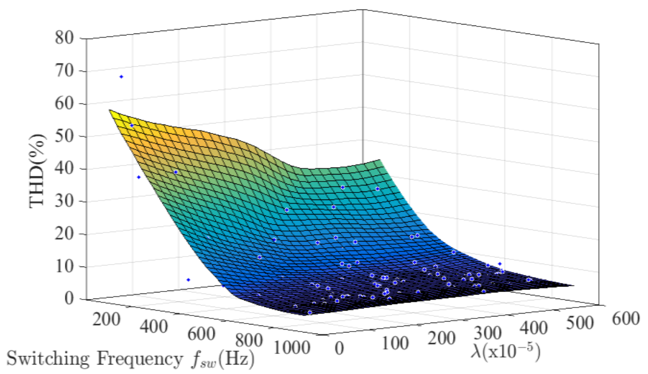
$$\begin{aligned} &(\bar{U}_1 - H_{(1,1)} U_1)^2 \le d_{p1}^2 \\ &(\bar{U}_1 - H_{(1,1)} U_1)^2 + (\bar{U}_2 - H_{(2,1)} U_1 - H_{(2,2)} U_2)^2 \le d_{p2}^2 \\ &\vdots \\ &(\bar{U}_1 - H_{(1,1)} U_1)^2 + \cdots + (\bar{U}_{3N} - H_{(3N,1)} U_1 - \cdots - H_{(3N,3N)} U_{3N})^2 \le d_{p3N}^2 \end{aligned} \tag{9}$$

This compares the distance calculated at each element to the initial values in (8a). Then each element is linked to a squared distance value that is considered its quality factor. It allows the determination and the grouping of the feasible elements according to their quality. The best quality elements are selected and preserved in an elite solution vector. The vector is defined as follows:

$$U_{p'} \in \mathbb{Z}^{1 \times 3N} \in \mathbf{U} \text{ s.t (2b), (2c) and (9).}$$



(a) SD-MPC method with N=1.



(b) SD-MPC method with N=8.

Fig.3. Weighting Factor impact on Current THD and Switching frequency

Hence, the algorithm used the same deterministic transition technique as SD algorithm rather than a probabilistic transition of conventional evolutionary algorithms.

➢ *Step Two*: Elite solution vector makes it easier to reconstruct an elite neighbourhood, in which the elements of $U_{P'}$ are extended element by element to form a local searching region. Thus, each elite element is then transformed into a vector of three elements as follows:

$$U_{p'',i}(k+1)=\begin{bmatrix} U_{P',i}(k)+1 \\ U_{P',i}(k)+0 \\ U_{P',i}(k)-1 \end{bmatrix} \in \mathbf{U}, i\in(1,...,3N) \tag{10}$$

subject to contain (2c) which states that only one level change is considered in all directions. This preserves a small and manageable local search region for any number of input entries. The optimal solution vector is located inside the local solutions region.

➢ *Step Three*: The search for the optimal voltage vector $U(k)$ is conducted by repeating step1 on the vector $U_{P''}(k+1)$ (the local solution region) to determine a new elite solution vector $U_{P'}(k+1)$, from which a new local search region $U_{P''}(k+2)$ is also defined by repeating step 2. Therefore, step 3 will be repeated until only one possible voltage vector satisfies the membership criteria in (9) and the constraints (2b) and (2c). That remaining solution that satisfies all conditions after all iterations is considered as the only optimal solution $U_{opt}(k)$.

4. ANALYSIS

In this section, the proposed technique is investigated on a case study seven-level inverter. Simulation analyses are conducted to understand the controller performance.

4.1 Simulations

A Monte Carlo simulation is processed by using a randomly generated sampling frequencies that vary from [2kHz to 10kHz] and random weighting factor values of λ from $[10\cdot10^{-5}$ to $600\cdot10^{-5}]$, statistical simulation are obtained using the SD-MPC with N=1 and N=8. Results are illustrated first in term of switching frequency versus total harmonic distortion (THD) on the output currents as shown in Fig.2, where fit curves (in red) were added in the scattered plots. For N=1 in Fig.2(a), the THD values tend to be all below the 10% only after reaching the switching frequency f_{sw} of 750Hz, while for N=5 the same THD level is achieved by a

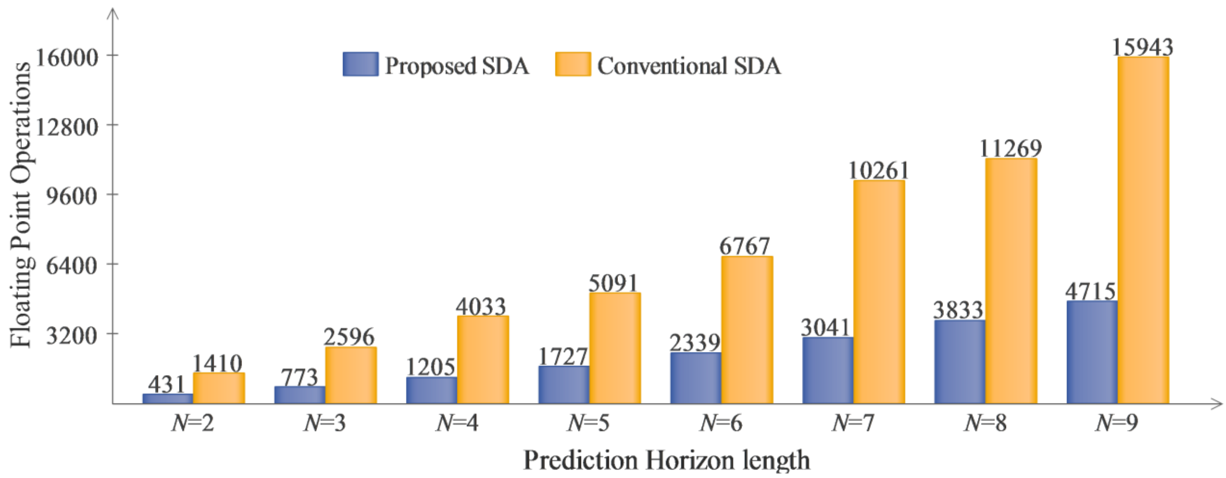


Fig. 4. Computation efforts presented in term of flops.

switching frequency of 600 Hz as seen in Fig.2(b). For N=8 these THD values are reached at a switching frequency of 510Hz in Fig.2(c), this is a decrease of around 32% in the switching efforts. It is also noticed that for N=8 most of the THD results tend to be less than 40% for switching frequencies while they reach levels of up to 60% for N=1.

Table 1. The number of voltage level entries in the worst-case scenario.

Prediction Horizon N	Conventional MPC	Sphere Decoding	Proposed, SDA Algorithm
1	7	21	9
2	49	42	18
5	16,807	105	45
8	5,764,801	168	72

Furthermore, a 3-D scatter plots with a lowess fit (linear regression smooth) are presented in Fig.3 to show the impact of the weighting factor on the output quality. The weighting factor λ values are very important and they can allow very good output current even with short prediction horizon and large sampling interval. It has been noticed that the weighting factor does not have a remarkable impact on one step SD-MPC, because it has an impact only on single step control inputs. However, in the case of $N>1$, the weighting factor has an impact on the constrained solution which affects the definition of the optimal solutions. Increasing λ values have improved the response time resulting in high switching frequency and low THD especially for large prediction horizons, this trade-off relation is clearly seen in Fig.3(b) for N=8.

4.2 Computation efforts

Floating point operations (Flops) measurements are performed on the proposed SDA as well as the conventional one, in order to analyse the impact of the modifications on computations complexity. In computation programming, most of the execution time is spent on 10% of the code, see Aho et al. (1994). The computation analysis has shown that the most effort is put in calculating the squared distance, which is dependable on the number of possible control entries to be tested at each element. In the worst-case scenario, the proposed algorithm has 45 possible entries in case of N=5, while conventional SDA has 105 entries. According to theoretical data in Table.1, the number of voltage level entries required is decreased by 57% in the worst-case scenario. Flops measurements have validated the theoretical hypothesis as illustrated in Fig.4. In which, the number of flops increases exponentially for both SD-algorithms, even though, the proposed technique has reduced the number of flops with an average of 67% for all tested prediction horizons.

5. CONCLUSION

This paper presented a long prediction horizon Finite Set Model Predictive Control (FS-MPC) with an SDA algorithm modified to behave as an Evolutionary Optimization (EO) algorithm. The proposed technique reduces the computation burden associated with increased possible control states in long horizon FS-MPC. This approach permitted the MPC to define the precise control action in a reasonable amount of time, which allowed a better performance with low switching frequencies and low sampling frequencies. This improvement results in the decrease of harmonic distortion on the output currents of multilevel converters even with low switching frequencies. The weighting factor has illustrated a significant effect on the controller precision ability. Therefore, its selection needs to be adaptive and automated in order to limit any negative impact of the optimization feasibility.

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