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A preventive maintenance strategy for an actuator using Markov models

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Abstract: This paper deals with a proactive maintenance strategy used to increase the reliability of the equipment. A predicting schedule of the renewal interventions is proposed so as to ensure optimal equipment maintenance. Hence, the goal is to find the optimal time which is the most profitable to carry out the equipment renewal operations. For the optimization of the maintenance, the preventive strategy is based on the average maintenance cost. The deterioration process is modeled by a Markov model, which is able to provide information about the tendency of the equipment state. The considered case study is an actuator, used in the sugar industry. A parametric form of the random behavior of the main variables was added to the Markov model, in the particular case of this actuator.

Keywords: preventive maintenance strategy, cost optimization, reliability, Markov model

1. INTRODUCTION

The intensification of the research in the field of reliability, leads to the development of numerous maintenance strategies, each of them with the related maintenance policies. An overview of the maintenance strategies developed to date has been done in Kothmasu, R., et al. (2006) and Endrenyi, J., et al. (2001). Basically, these can be classified into reactive and proactive maintenance strategies. If initially the intervention was carried out on the equipment only if it failed (reactive or not scheduled maintenance), subsequently the maintenance strategies have been improved leading to proactive maintenance strategies. They make use of regular inspections and preventive renewals increasing the availability of the equipment. Proactive maintenance strategies are developed according to different criteria, they involve renewal at certain predetermined time intervals and lead to the total or partial removal of the accumulated wear in the system. This way the system is brought each time into an operation state characterized by the lack of wear or a negligible wear. Proactive maintenance strategies can be also classified as preventive and predictive strategies. Predictive maintenance strategies are based on a continuous monitoring of the system state or its components. The equipment condition is quantified in parameters whose evolution is continuously monitored using appropriate techniques to the application Hasemian, H. M., (2011). Whatever is the motivation behind the development of a maintenance strategy (regardless of priority criteria: economic or operational safety) the final goal remains the same. It aims to develop an optimal maintenance strategy, which means the achieving of the pursued objective with minimum possible cost. The challenging aspect of the maintenance optimization is that the condition of the equipment cannot be accurately predicted throughout its life. The time to reach a fault state is uncertain and varies widely depending on different factors which most often are uncontrollable. Most of the equipment degrades gradually as they get older and fails when exceeding a certain degradation threshold. The value of this threshold and how it should be specified is not clearly established yet Singpurwalla, N. D. (2006). Different models of maintenance optimization focusing on the optimization criteria and the pursued objectives are presented in Van Horenbeek, (2010), as well as in Dekker, (1996). The maintenance optimization using mathematical models has been carried out in numerous applications between the 1970s and 1990s: Pierskalla, W. P., et al., (1976), Sherif, Y.S., et al., (1981), Valdez, F.C., (1989), Wijnmalen, D. J. D., et al., (1992), Amik, G., et al., (2006) provides a comprehensive study of the state of researches in the field of maintenance management, with a review of the researches carried out on all areas related to the maintenance modeling up to that point. A maintenance model involves a system state deterioration model and a decision algorithm. Therefore, a method is needed to model the degradation allowing the incorporation of the uncertainty that characterizes this aspect. In Gorjian, N., et al., (2009) is given a review, a classification and at the same time a series of applications of the degradation models used in reliability, reported by the literature. According to Gorjian, N., et al., (2009) the degradation models used in the reliability analysis can be classified into two broad categories: normal degradation models and accelerated degradation models. In the category of accelerated degradation models there are statistical models and among them, Markov models are widely applied in the degradation modeling used for reliability analysis.

This paper proposes a method to determine the optimal renewal time of an equipment, using its degradation model. It is organized as follows: section 2 presents the concept of the Markov models used in the present work to determine the state probabilities of the system. The mathematical formulation and the reasoning underlying the Markov models are also shown. Section 3 presents the proposed method for the optimization of the maintenance strategy, in section 4 a case study is presented, section 5 deals with the applied methodology and the obtained results. The last section is dedicated to the conclusions.

2. MARKOV MODELS

A Markov model is a stochastic model, in which the current state of the system (at the moment t_n) depends only on its previous state (at the moment t_{n-1}). It is characterized by the following elements: the states of the system, subsystem or component subassemblies (new, faulty or with different wear degrees) at a particular time and the possible transitions between these states. These ones are associated with transition probabilities (for the individual events) or with failure rates and repair rates (for the continuous events).

Fig. 1 presents the simplest Markov model, of second order. The example (Fig. 1), the notations used through section 2, and the integration of the model were presented in Mărăşescu, N., (1999). The model has 3 states: S_0 – new/as new/renewed, S_1 - state of wear and S_2 – faulty state. Each transition between the states of the system is characterized by a transition rate (noted in Fig. 1 $\varphi(t)$, $z_0(t)$ and $z_1(t)$).

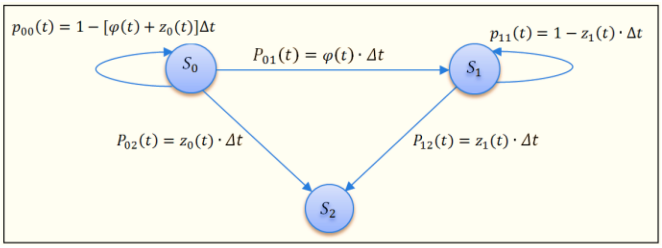

Fig. 1. Second order Markov model

The structure was adapted to the purpose of this paper, so there is no transition from state 1 to state 0. The system may be damaged either due to the accumulated wear or due to the sudden appearance of a "heavy" fault. The system can pass into the state S_2, both from the state S_0 and from the wear state S_1. The transition rates towards state S_2 are failure rates. Usually, the failure rate of the system in case of wear, marked $z_1(t)$ on the graph, is higher than the failure rate of the system in case of accidental occurrence of a fault, marked on the graph with $z_0(t)$. The wear of the system is gradually installed, producing a probable transition from state 0 to state 1, with the transition rate $\varphi(t)$.

If the parameters of the Markov model are independent and evolve slowly, it can be described by a linear model. Its continuous-time space-state model is presented in (1) and (2). In these equations, P_0 and P_1 are the probabilities of the states S_0 and S_1, respectively. The probability of the state S_2 is derived from an algebraic equation, knowing that the sum of the probabilities equals 1, as in (3).

$$\frac{dP_0(t)}{dt} = -[\varphi(t) + z_0(t)]P_0(t) \tag{1}$$

$$\frac{dP_1(t)}{dt} = \varphi(t)P_0(t) - z_1(t)P_1(t) \tag{2}$$

$$\sum_{j=1}^{n} P_j = 1 \tag{3}$$

When integrating the state-space model, the solution expressed by (4) and (5) is obtained:

$$P_0(t) = e^{\int -[\varphi(\tau)+z_0(\tau)]d\tau} \tag{4}$$

$$P_1(t) = e^{\int -z_1(\tau)d\tau} + \int \varphi^k(\tau) \cdot e^{-\int_0^t [z_0(v)+\varphi^k(v)-z_1(v)]dv} d\tau \tag{5}$$

This solution applies to linear models only. Even in this case, it raises serious practical problems, because the prediction based on (4) and (5) assumes to know all the primitives of the integral forms. In the cases of nonlinear or higher order models, this solution cannot be used anymore. Instead, a discrete time model is the convenient approach: it simplifies the integration, as in all simulation programs, and it allows nonlinear behavior to be modeled. The usual approximation considers that the states and the values of the transition rates do not change, during the sampling period. According to this approximation, the transitions between states are characterized by transition probabilities, as in (6), (7), (8), where T_s denotes the sampling period and k denotes the discrete time.

$$P_{01}(k) = \varphi(k) \cdot T_s \tag{6}$$

$$P_{12}(k) = z_1(k) \cdot T_s \tag{7}$$

$$P_{02}(k) = z_0(k) \cdot T_s \tag{8}$$

For each state i, the transition probabilities observe the restriction:

$$\sum_{j=1}^{n} P_{ij} = 1 \tag{9}$$

The model presented in Fig. 1 is described in a recursive form by the eqs. (10), (11). The integration of the model is quite simple. Again, the probability of the state S_2 is a function derived from the functions of P_0, P_1.

$$P_0(k+1) = P_0(k)[1 - P_{01}(k) - P_{02}(k)] \tag{10}$$

$$P_1(k+1) = P_0(k) \cdot P_{01}(k) + P_1(k)[1 - P_{12}(k)] \tag{11}$$

3. PREVENTIVE MAINTENANCE STRATEGY CONCEPT

Preventive maintenance strategies provide for the scheduled shutdown of the equipment before it reaches a failure state. The purpose is to perform a renewal of the equipment. Preventive maintenance strategies can be broadly classified into periodic and non-periodic ones. Non-periodic strategies consist of interventions on the equipment at certain instants determined according to relevant random variables (such as the age of the equipment, its wear etc.). As these processes are stochastic processes, the functions that describe the system reliability over a period of time are the following (as presented in Rausand, M., Hoyland A. (2004):

- the reliability function $R(t)$, defined as the probability that the system does not fail within the range $(0, t)$;
- the probability density function of the failure, in the neighborhood of time t. It is a random variable denoted by $f(t)$, and it is the derivative of $R(t)$, multiplied by -1.

The maintenance strategies efficiency is evaluated regardless of the type of strategy, according to the system maintenance cost / unit of time. The cost of the renewal in case of breakdown and the cost of the preventive renewal are considered. It is also considered that any external intervention on the equipment in order to restore its performance takes place at the moment of equipment failure and it is performed in a negligible time. If a reference interval $(0, t)$ is considered, in which n equipment renewals take place, the time evolution of the states of this equipment is represented by the succession of its renewal moments and the intervals between these renewals. The totality of renewals within this interval is a discrete random process, consisting of all the random variables that represent the intervals between renewals. Thus, the renewal process can be characterized by the average and the variance of the number of renewals in the interval $(0, t)$. The average number of renewal actions within the range $(0, t)$ is called the renewal function and is denoted by $H(t)$. Its derivative is called renewal density and is denoted by $h(t)$. The average cost of the equipment maintenance in a time interval t is given by (12):

$$C_{av} = \frac{C_f * H(t) + C_{pv}}{t}, \tag{12}$$

where C_{av} represents the maintenance average cost of the equipment in a time interval t, C_f – the cost of a renewal in case of failure, C_{pv} – the cost of a preventive renewal and $H(t)$ – the renewal function for the interval $(0,t)$. The instant when the value of the average cost is minimum will indicate the optimal time at which it is most profitable to carry out the renewal operation.

4. CASE STUDY

The case study for the degradation model is the benchmark DAMADICS (acronym for *Development and Application of Methods for Actuator Diagnosis in Industrial Control Systems*) - Bartys, M., et al., (2001, 2002, 2003, 2006). This benchmark is a software product developed in Matlab-Simulink graphical programming environment, designed to allow the study and the comparison of a wide range of fault detection and isolation methods for an actuator. In this study the focus is on the actuator responsible with controlling the thin juice flow rate into the first stage of the evaporation station of a sugar plant. The fault *f2* (valve plug or valve seat sedimentation) was simulated, as a fault with gradual evolution. Fig. 2 presents the general scheme of the actuator, which includes the pneumatic servomotor, the positioner and the valve itself, where one can notice the valve seat and plug. In Fig. 3 the systemic scheme can be found, including the measurable variables: the control input CV, the liquid pressures p_1 and p_2 (on the inlet and outlet of the valve respectively), the liquid temperature T_1, the liquid flow rate F and the displacement of the servomotor stem X. The vector f is a parameter of the Matlab program that indicates the simulated fault.

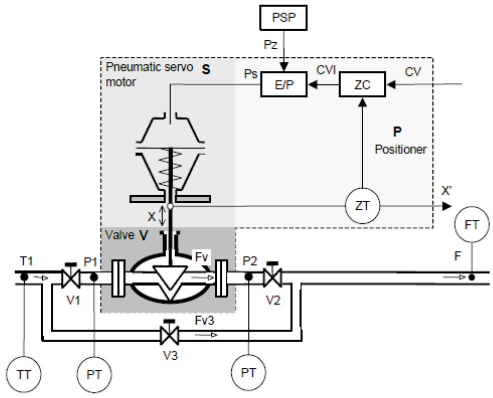

Fig. 2. General scheme of the actuator - Bartys, M., et al. (2006)

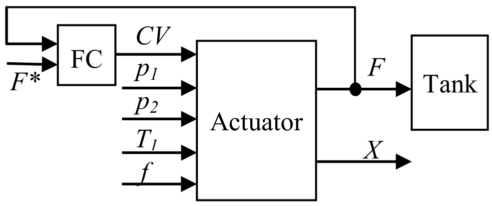

Fig. 3. Systemic scheme of the actuator, in a flow control loop – adapted from Bartys, M., et al. (2006)

The physical variables that allow the continuous monitoring of the equipment state and that were used to determine the Markov model are: F, X, p_1, p_2 and T_1. In this study, the temperature T_1 has been considered a constant.

5. RESULTS AND DISCUTIONS

5.1 Identification of the Markov model

The purpose is to determine the reliability model of the actuator mentioned in section 4 and the optimal renewal time, using the Markov model. The model is presented in Fig. 4; it contains the transition probabilities during the sampling period T_s, as introduced in (6) – (8) and marked on the transition arcs.

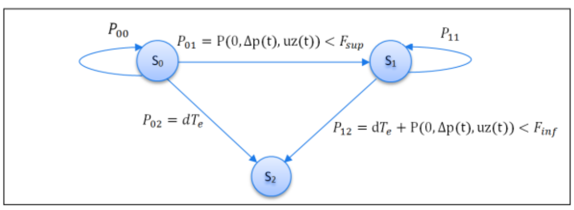

Fig. 4. Second order Makov model for discrete time

To define the equipment states, the following information known from the operating technique of the plant were considered:

1. Failure occurs as a result of several phenomena. One of them is the equipment's positive wear which is

accumulated in a progresive mode. The other ones are accidental/impredictible phenomena;

2. In the present case study, it was considered that the wear is produced by the progressive sedimentation of solid salt particles on the valve seat and/or valve plug which results in a decrease of the sugar juice flow rate, for the same set of varible values (X, $\Delta p = p_1 - p_2$ and T_1);
3. The wear increases monotonously;
4. From the equipment maintenance perspective, equipment states are:

- S_0 (almost new condition): the valve is in the „new" state and it is able to provide the maximum flow rate required by the plant (denoted by F_{sup}), even at the minimum value of the pressure difference Δp_{min}, at the usual temperature;
- S_1 (wear condition): the valve is not able to provide the maximum flow rate F_{sup}, for any value of the pressure difference, but it provides the minimum flow rate necessary to operate the equipment in safe conditions (denoted by F_{inf}), for any value of the pressure difference, higher than Δp_{min};
- S_2 (fault condition): the equipment is close to failure; the valve is not able to provide the safety flow rate, F_{inf}, for some pressure differences values, higher than Δp_{min}.

For modeling purpose, the values of the measurable variables were extracted from the data recorded by Bartys et al., (2002). Their statistical properties justify the following simplifying assumptions:

a. T_s represents the sampling period (the equipment state does not change during T_s). The operating time of the valve, t, is expressed as a discrete time, as follows:

$$t = k \cdot T_s \tag{13}$$

b. The variable that describes the actuator wear is denoted by $uz(t)$ and it is considered a random variable within the interval [0,1]. The value 0 coresponds to the total absence of wear (new valve/lack of wear) and the value 1 to the total obturation of the valve. It is modeled by the equation (14), as long as the product $v \cdot t < 1$:

$$uz(t) = v \cdot t, \tag{14}$$

where v is the salt sedimentation rate considered as a constant. The value of the parameter v has the meaning of the inverse of the valve life time, assuming that there is no accidental failure.

c. Starting from the initial state S_0, the accidental fault occurs with the constant probability, independent of the wear degree accumulated by the equipment, as in (15):

$$P_{02}(\mathrm{k}) = d \cdot T_s \tag{15}$$

where: P_{02} is the failure probability within the interval k-$(k+1)$; d – accidental failure rate;

d. The probability of the equipment in the state S1 is the probability that maximum flow rate provided by the valve to be lower than F_{sup} limit. We denote the function describing the flow rate in the stationary regime as:

$$F = F(X, \Delta p, uz) \tag{16}$$

e. Taking into account that the maximum flow rate through the valve is obtained when the servomotor stem is completely raised ($X = 0$), the transition probability of the equipment from S_0 to S_1 is :

$$P_{01}(k) = P(F(0, \Delta p(t), uz(t) < F_{sup}) \tag{17}$$

The probability P_{12} is the sum of the accidental failure probability and the probability that the maximum flow rate provided by the valve is lower than F_{inf}, when the wear has not reached his upper limit. This means that the following condition holds true:

$$P_{12}(k) = d * T_s + P(F(0, \Delta p(t), uz(t) < F_{inf}) \tag{18}$$

If the above mentioned sum is higher than 1, $P_{12}(k) = 1$.

Markov model initialization is the following:

$$P_0(0) = 1, P_1(0) = 0 \tag{19}$$

The integration of the model in (10) and (11) provides the state probabilities. The integration requires numerical values of the parameters and the probabilities of the two conditions in (17) and (18), both depending on F. The numerical values have been extracted from the benchmark DAMADICS simulation results. These numerical values can be also approximated, based on an expert knowledge related to the large-sized valve reliability.

Based on the state probabilities of the equipment, the statistical functions that characterize the reliability of the equipment are computed in the following order: $R(t) \rightarrow f(t) \rightarrow h(t) \rightarrow H(t)$. Knowing that $R(t) = P_0(t) + P_1(t)$, $f(t)$ is determined:

$$f(t) = \frac{dP_2(t)}{dt} \tag{20}$$

$h(t)$ is computed by solving the equation (21) Rausand, M., Hoyland A. (2004):

$$h(t) = f(t) + f(t) \otimes h(t) \tag{21}$$

where the symbol $\otimes$ represents the convolution. $H(t)$ is the integral of $h(t)$. It is further used to calculate the renewal average cost function as in (12), based on which the optimum renewal time will be determined.

5.2 Numerical results

It is assumed that:

- sampling period: $T_s = 1$ hour
- wear rate: $v = \frac{1}{5000}$ hours^{-1}
- accidental failure rate: $d = \frac{1}{2000}$ hours^{-1}
- failure cost (C_f): 3500 $
- preventive renewal cost (C_{pv}): 1000 $
- the limits of the flow rate are $F_{sup} = 140$ t/hour and $F_{inf} = 100$ t/hour

The simulations considered 3 values of the pressure difference and 2 values of the wear degree. Figs. 5 and 6 show the simulation results, where the colors mean: black for $\Delta p = 600kPa$ (up), green for $\Delta p = 400kPa$, and red for $\Delta p = 200kPa$ (down).

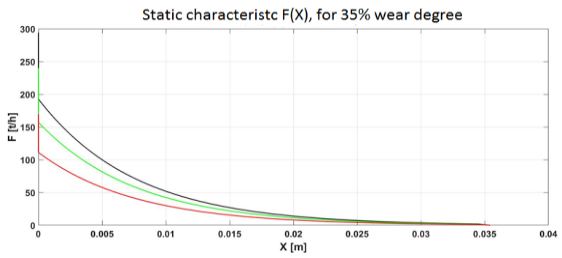

Fig. 5. Static characteristics F(X) for 35% wear degree

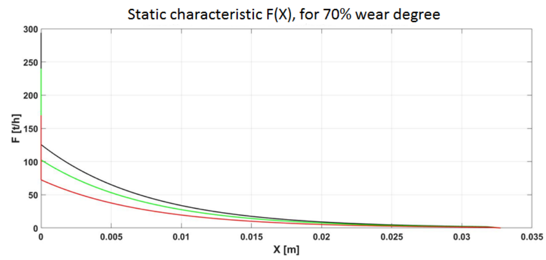

Fig. 6. Static characteristics F(X) for 70% wear degree

In order to illustrate the model integration, another two hypotheses referring to the transition probabilities were made:

1. it is assumed that the pressure difference, Δp, is a random variable, uniformly distributed within the range $[p_{min}, p_{max}]$, as in Fig. 7.

From DAMADICS benchmark simulation results, the following thresholds are established: $\Delta p_{min} = 200$ kPa and $\Delta p_{max} = 600$ kPa.

2. starting from the static characteristics (Figs. 5 and 6) and assuming that the servomotor stem is completely raised (X = 0), the dependence of the sugar juice flow rate F will be parametrized as a function of variables Δp and uz.

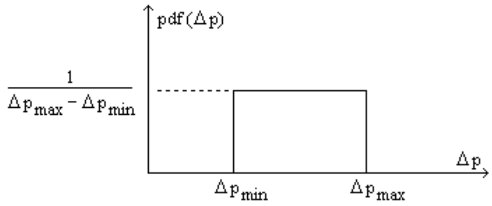

Fig. 7. Probability density of the random variable Δp

A mathematical expression that approximates well enough the relation $F = F(0, \Delta p, uz)$, is:

$$F(0, \Delta p, uz) = \Delta p/\Delta p_{min} \cdot \sqrt[3]{(1 - uz)^2} \cdot F(0, \Delta p_{min}, 70\%) \quad (22)$$

For a fixed value of wear, the random variable F has the same statistical proprieties as the random variable Δp. From the first hypothesis and the equation (22) it can be established:

$$P(F(0, \Delta p, uz) < F_{sup}) = P(\Delta p < \frac{F_{sup} \cdot \Delta p_{min}}{F(0, \Delta p_{min}, 70\%)}) \quad (23)$$

If noted:

$$\Delta p_{lim} = \frac{F_{sup} \cdot \Delta p_{min}}{F(0, \Delta p_{min}, 70\%)} \quad (24)$$

the equation (23) can be written as:

$$P(F(0, \Delta p, uz) < F_{sup}) = P(\Delta p < \Delta p_{lim}) \quad (25)$$

The probability from the right side of the equation (25) is the integral of the probability density from Fig. 7, between 0 and Δp_{lim}.

With the numerical values chosen above, the evolution of state probabilities (P_0, P_1, P_2 - the probabilities of the system to be in the state S_0, S_1, S_2, respectively) has the aspect shown in Fig. 8. It can be noticed that the valve condition is continuously degrading, due to the wear. If no accidental failure occurs, after 1600-1700 hours, the valve reaches the sate S_1 (satisfactory state). After 3000 hours of operation the failure probability of valve (P_2) tends to 1.

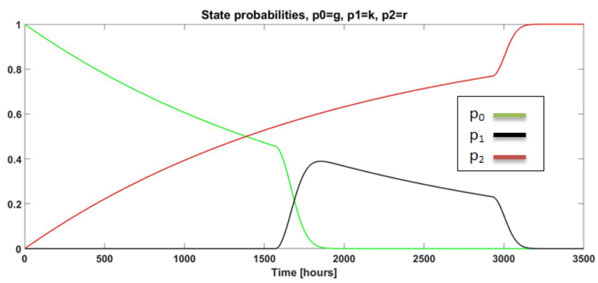

Fig. 8. The evolution of the state probabilities

An average cost with the shape presented in Fig. 9 was obtained, where the horizontal axis means the time interval between preventive renewals. The minimum value of the average cost corresponds to the optimal time at which the preventive renewal should be carried out. The minimum value of the average cost is 2.09 \$/operating hour and the optimal time at which it is profitable to carry out the preventive renewal of the equipment (T^*) is 2932 hours.

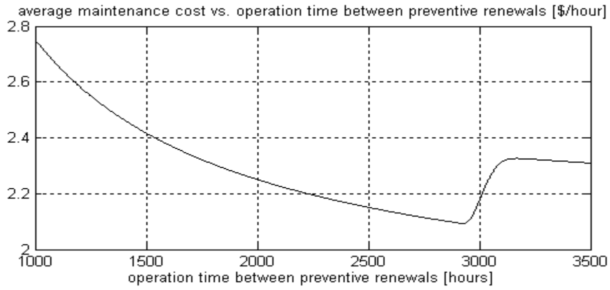

Fig. 9. The evolution of the average cost within the interval [1000, 3000] hours

6. CONCLUSIONS

In this paper, a preventive maintenance strategy was applied in the case of an electro-pneumatic actuator, frequently used in industrial environment. The objective is to obtain the optimal time when it is cost-efficient to stop the equipment and to renew its condition. The method to determine it was proposed in the paper. As a case study the benchmark DAMADICS was used. It allows the simulation of different degradation types of the system state, including the monotonous increasing wear of the considered type (progressive sedimentation of solid salt particles upon the valve seat and/or valve plug). It can be concluded that Markov models are an appropriate approach for modeling the system state degradation and that they provide a well enough prediction of the system state. A parameterization of the statistical properties of the main variables was proposed (they are usually neglected), using the data recorded by DAMADICS. The simulation results fit with the method of computing the optimal renewal time and with the proposed parameterization, for this particular case (extracted from the practice of sugar juice flow control).

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