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Robust Fault Detection and Isolation of Discrete-Time LPV Systems Combining Set-theoretic UIO and Invariant Sets ⋆

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Abstract: This paper proposes a mixed active/passive robust fault detection and isolation (FDI) method for discrete-time linear paramter varying (LPV) systems based on set-theoretic unknown input observers (SUIO) and invariant sets. The robustness against system uncertainties in FDI of LPV systems can be guaranteed by actively decoupling or passively bounding their effect on residual signal. Furthermore, the quadratic $\mathcal{H}_\infty$ stability condition of the LPV-form state-estimation-error dynamics is established based on a group of linear matrix inequalities (LMIs). Under the precondition of stability, a family of residual sets are constructed to establish set-separation guaranteed fault isolation (FI) conditions using invariant sets off-line. As long as the occurred faults satisfy the guaranteed FI conditions, they can be isolated from each other. At the end, a numerical example is used to illustrate the effectiveness of the proposed method.

Keywords: Fault detection and isolation, LPV systems, Set-theoretic unknown input observer, Invariant sets.

1. INTRODUCTION

As modern engineering systems become more and more complex, system faults will inevitably occur during whole operation stage. In order to improve the safety and reliability of complex systems, fault diagnosis has attracted more and more attention from a great number of researchers (Isermann, 2005).

Fault diagnosis of linear parameter varying (LPV) systems have been widely studied in the literature (Zhang and Yang, 2017; Chouiref et al., 2015). As one of important fault diagnosis methods, many construction methods of invariant sets for linear parameter varying (LPV) systems perturbed by bounded uncertainties have been proposed. In Nguyen et al. (2015), a robust ellipsoidal invariant set method was proposed with respect to maximizing the inclusion of some given reference direction by considering additive disturbances injected into the system dynamics. In Martinez et al. (2018), based on an H_∞ observer gain design and linear matrix inequality (LMI) conditions, an ellipsoidal robust positively invariant (RPI) set is computed, the evolution of which is characterized to bound the estimation error at each time instant. Recently, authors proposed a novel invariant-set computation method based on a shrinking process for LPV systems in Tan et al. (2019), which will be used to construct the invariant sets of LPV-form state-estimation-error dynamics in this paper under the precondition that the dynamics satisfies the quadratic $\mathcal{H}_\infty$ performance γ to guarantee the stability which is first defined in Apkarian et al. (1995).

Different from the invariant sets passively dealing with bounded system uncertainties passively, unknown input observers (UIO) can actively decouple the effect of system unknown inputs on the residual signal to obtain the robustness of FD. The notation of UIO was first proposed for LTI systems and applied for fault detection filters in Chen et al. (1996). However, there is a precondition for the existence of UIO for LTI systems. That is, the number of unknown inputs should not be larger than the number of independent outputs in systems. Otherwise, there does not exist a UIO actively decoupling the effect of all unknown inputs on the residual signal for robust FD, which in fact is a restriction for the application of UIO-based robust FD methods. Xu et al. (2017) proposed to use set-theoretic UIO (SUIO) to implement mixed active/passive robust fault detection and isolation of LTI systems. Xu et al. (2019) extended the application of SUIO to the robust fault detection of LPV systems with inexact scheduling variables. Based on the above works, this paper considers the robust fault detection and isolation of discrete-time LPV systems perturbed by bounded uncertainties and the design method of SUIO for LPV systems is given. The quadratic $\mathcal{H}_\infty$ stability of state-estimation-error dynamics of the designed SUIO can be guaranteed by a group pf

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LMIs. Under the quadratic $\mathcal{H}_\infty$ stability, the invariant sets of state-estimation error of SUIO are constructed similar to the authors' previous work in Tan et al. (2019). Furthermore, we combine the designed SUIO and constructed invariant sets to establish a group of guaranteed FI conditions based on the residual set-separation constraints for discrete-time LPV systems perturbed by bounded uncertainties. Theoretically, as long as the occurred faults satisfy the guaranteed FI conditions, they will be finally isolated during the steady stage of system operation.

The reminder of this paper is organized as follows. Section 2 introduces the plant model and the design of SUIO based on a mixed active/passive decoupling method. Section 3 illustrates the key idea of robust fault detection. The algorithm of robust fault isolation is established in Section 4. A numerical example is used to illustrate the effectiveness of the proposed method in Section 5. This paper is finally concluded in Section 6.

2. SYSTEM DESCRIPTION

2.1 Model of Plant

The discrete-time LPV system under the effect of actuator faults is modeled as

$$x_{k+1} = A(\theta_k)x_k + B(\theta_k)G^i u_k + Ew_k, \tag{1a}$$
$$y_k = Cx_k + F\eta_k, \tag{1b}$$

where $k \in \mathbb{N}$ is the discrete time index. $A(\theta_k) \in \mathbb{R}^{n_x \times n_x}$ and $B(\theta_k) \in \mathbb{R}^{n_x \times n_u}$ are related system matrices dependent on a varying scheduling vector $\theta_k \in \mathbb{R}^{n_\theta}$ able to be measured online at time instant k. $x_k \in \mathbb{R}^{n_x}$ and $y_k \in \mathbb{R}^{n_y}$ are the system states and outputs at time instant k, respectively. The unknown inputs $w_k \in \mathbb{R}^{n_w}$ (including process disturbances, modeling errors, etc.) are contained in a known compact and convex set $\mathbf{W} = \{w \in \mathbb{R}^{n_w} \big| \|w\|_\infty \leq \bar{w}\}$ and $\bar{w}$ is a positive scalar. Similarly, the measurement noises $\eta_k \in \mathbb{R}^{\eta}$ also belong to a given compact and convex set $\mathbf{V} = \{\eta \in \mathbb{R}^{n_\eta} \big| \|\eta\|_\infty \leq \bar{\eta}\}$ and $\bar{\eta}$ is a positive scalar. $C \in \mathbb{R}^{n_y \times n_x}$ is the constant output matrix. The input vector u_k is bounded by the input set $\mathbf{U} = \{u \in \mathbb{R}^{n_u} | \underline{u} \leq u \leq \bar{u}\}$, where $\underline{u}$ and $\bar{u}$ are known vectors. $E \in \mathbb{R}^{n_x \times n_w}$ and $F \in \mathbb{R}^{n_y \times n_\eta}$ are the known constant distribution matrices of w_k and η_k, respectively.

$G^i = \mathbf{diag}(1, ..., f_i, ..., 1) \in \mathbb{R}^{n_u \times n_u}$ is a diagonal matrix modeling the i-th actuator fault mode. If the i-th actuator becomes faulty, f_i takes a value inside the interval $[0, 1)$, where 0 means that the i-th actuator has completely lost its function while a value inside $(0, 1)$ means that the i-th actuator has partially lost its function. For convenience, we let G^0 be the identity matrix I to model the healthy situation. The set of actuator indices is denoted as $\mathbb{I} = \{1, 2, .., n_u\}$.

Assumption 1. There is only one actuator that may become faulty at a time and faults are persistent such that the fault detection and isolation (FDI) module has sufficient time to detect and isolate them.

It is assumed that the n_θ-dimensional scheduling vector θ_k is a convex combination of given vertexes generating a convex set $\mathbf{\Theta} = \mathbf{Conv}\{\theta^1, \theta^2, ..., \theta^{\mathcal{N}}\}$. Therefore, a linear affine function $\Phi(\theta_k)$ of θ_k can be written as the convex combination of vertex matrices: $\Phi(\theta_k) = \sum_{i=1}^{\mathcal{N}} \lambda_i(\theta_k)\Phi^i$, where the weighting coefficients $\lambda_i(\theta_k)$ satisfy $\sum_{i=1}^{\mathcal{N}} \lambda_i(\theta_k) = 1$, $0 \leq \lambda_i(\theta_k) \leq 1$, Φ can represent A and B, and $\Phi^i = \Phi(\theta^i)$ represents the i-th vertex matrix of set $\Phi(\mathbf{\Theta})$.

2.2 Notion of SUIO

When the plant (1) is in the healthy situation, i.e., $G^i = G^0 = I$, the SUIO matching the healthy situation can be designed as

$$z_{k+1} = N(\theta_k)z_k + T(\theta_k)u_k + K(\theta_k)y_k, \tag{2a}$$
$$\hat{x}_k = Mz_k + Hy_k, \tag{2b}$$
$$\hat{y}_k = C\hat{x}_k, \tag{2c}$$

where $z_k \in \mathbb{R}^{n_x}$, $\hat{x}_k \in \mathbb{R}^{n_x}$ and $\hat{y}_k \in \mathbb{R}^{n_y}$ are the state vector of the SUIO, the state and output estimation vectors , respectively. The matrices $N(\theta_k) \in \mathbb{R}^{n_x \times n_x}$, $T(\theta_k) \in \mathbb{R}^{n_x \times n_u}$, $K(\theta_k) \in \mathbb{R}^{n_x \times n_y}$, $M \in \mathbb{R}^{n_x \times n_z}$ and $H \in \mathbb{R}^{n_x \times n_y}$ can be obtained according to the SUIO design procedure. The corresponding state-estimation-error vector is defined as $e_k = x_k - \hat{x}_k$.

Thus, the dynamics of the state-estimation error e_k can be derived as

$$\begin{aligned} e_{k+1} = &(A(\theta_k) - HCA(\theta_k) - MK_1(\theta_k)C)e_k \\ &+ [(A(\theta_k) - HCA(\theta_k) - MK_1(\theta_k)C)M - MN(\theta_k)]z_k \\ &+ [(A(\theta_k) - HCA(\theta_k) - MK_1(\theta_k)C)H - MK_2(\theta_k)]y_k \\ &+ [B(\theta_k) - MT(\theta_k) - HCB(\theta_k)]u_k \\ &+ (E_1 - HCE_1)w_k^1 + (E_2 - HCE_2)w_k^2 \\ &- HF\eta_{k+1} - MK_1(\theta_k)F\eta_k, \end{aligned} \tag{3}$$

with $K(\theta_k) = K_1(\theta_k) + K_2(\theta_k)$ and $Ew_k = [E_1 \;\; E_2] \begin{bmatrix} w_k^1 \\ w_k^2 \end{bmatrix}$.

From (3), it can be seen that the parametric matrices of SUIO can be obtained by solving

$$E_1 - HCE_1 = \mathbf{0}, \tag{4a}$$
$$B(\theta_k) - MT(\theta_k) - HCB(\theta_k) = \mathbf{0}, \tag{4b}$$
$$(A(\theta_k) - HCA(\theta_k) - MK_1(\theta_k)C)M - MN(\theta_k) = \mathbf{0}, \tag{4c}$$
$$(A(\theta_k) - HCA(\theta_k) - MK_1(\theta_k)C)H - MK_2(\theta_k) = \mathbf{0}. \tag{4d}$$

Under (4), the dynamics of e_k can be reduced to

$$\begin{aligned} e_{k+1} = &(A(\theta_k) - HCA(\theta_k) - MK_1(\theta_k)C)e_k \\ &+ (E_2 - HCE_2)w_k^2 - HF\eta_{k+1} - MK_1(\theta_k)F\eta_k. \end{aligned} \tag{5}$$

Remark 1. In the dynamics (3), we decompose the term $(E - HCE)w_k$ into $(E_1 - HCE_1)w_k^1 + (E_2 - HCE_2)w_k^2$ such that even though we can not find a proper matrix H to decouple the effect of whole unknown inputs w_k, we can still guarantee that there always exists a proper matrix H to decouple the effect of partial unknown inputs w_k^1 on the dynamics (3), i.e., $E_1 - HCE_1 = 0$. Then, for the remaining unknown inputs w_k^2, we turn to the set theory to passively consider their effect on the dynamics (3). That is reason why we call the observer (2) as SUIO. Readers can refer authors' seminal work in Xu et al. (2016) for more details regarding this point.

By analyzing (4a), in order to ensure the existence of the observer (2), it should be guaranteed that $E_1 - HCE_1 = \mathbf{0}$ is solvable. Then we can compute other parametric matrices $N(\theta_k)$, $T(\theta_k)$, $K(\theta_k)$ and M. The solution of (4a) is $H = E_1[(CE_1)^T CE_1]^{-1}(CE_1)^T + H_0\{I - CE_1[(CE_1)^T CE_1]^{-1}(CE_1)^T\}$,where H_0 is an arbitrary matrix with proper dimensions.

For the purpose of FD, the residual vector corresponding to the healthy situation is defined as

$$r_k = y_k - \hat{y}_k = Ce_k + F\eta_k. \tag{6}$$

Furthermore, we consider using the quadratic $\mathcal{H}_\infty$ performance index to establish the stability condition for the dynamics (5) and (6).

Definition 1. (Apkarian et al., 1995) The LPV system

$$x_{k+1} = \mathcal{A}(\theta_k)x_k + \mathcal{B}(\theta_k)u_k, \tag{7a}$$
$$y_k = \mathcal{C}(\theta_k)x_k + \mathcal{D}(\theta_k)u_k \tag{7b}$$

has the quadratic $\mathcal{H}_\infty$ performance γ if and only if $\|y_k\|_2 \leq \gamma\|u_k\|_2$ along all possible parameter trajectories θ_k.

Theorem 1. Considering the LPV dynamics described by (5) and (6), if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ and a positive scalar $\gamma > 0$ such that

$$\begin{bmatrix} -P + C^TC & C^T\mathcal{F} & \mathcal{A}^{i^T} \\ \mathcal{F}^TC & \mathcal{F}^T\mathcal{F} - \gamma^2 & \mathcal{E}^{i^T} \\ \mathcal{A}^i & \mathcal{E}^i & -P^{-1} \end{bmatrix} \preceq 0 \tag{8}$$

holds for $i = \{1, ..., \mathcal{N}\}$, where $\mathcal{A}(\theta_k) = A(\theta_k) - HCA(\theta_k) - MK_1(\theta_k)C$, $\mathcal{E}(\theta_k) = [E_2 - HCE_2\ -HF\ -MK_1(\theta_k)F]$, $\mathcal{F} = [0\ 0\ F]$ and $\mathcal{A}^i$ and $\mathcal{E}^i$ are the i-th vertex matrices of $\mathcal{A}(\theta_k)$ and $\mathcal{E}(\theta_k)$, respectively, then the dynamics (5) and (6) have a quadratic $\mathcal{H}_\infty$ performance γ.

Proof 1. Let us consider the following Lyapunov function for the dynamics (5):

$$V(e_k) = e_k^T P e_k \geq 0. \tag{9}$$

Suppose there exist $P \succ 0$ and a positive scalar $\gamma > 0$ verifying the following dissipation inequality:

$$V(e_{k+1}) - V(e_k) \leq -r_k^T r_k + \gamma^2 \delta_k^T \delta_k \tag{10}$$

with $\delta_k = \begin{bmatrix} w_k^{2^T} & \eta_{k+1}^T & \eta_k^T \end{bmatrix}^T$. Substituting (9) into (10) and combining the dynamics (5) and (6), we can obtain

$$\begin{aligned} & e_k^T(\mathcal{A}(\theta_k)^T P\mathcal{A}(\theta_k) - P + C^TC)e_k + e_k^T(\mathcal{A}(\theta_k)^T P\mathcal{E}(\theta_k) + C^T\mathcal{F})\delta_k \\ & + \delta_k^T(\mathcal{E}(\theta_k)^T P\mathcal{A}(\theta_k) + \mathcal{F}^TC)e_k \\ & + \delta_k^T(\mathcal{E}(\theta_k)^T P\mathcal{E}(\theta_k) + \mathcal{F}^T\mathcal{F} - \gamma^2)\delta_k \leq 0. \end{aligned} \tag{11}$$

This inequality can be rewritten into the following matrix form:

$$\begin{bmatrix} e_k \\ \delta_k \end{bmatrix}^T \begin{bmatrix} \mathcal{A}(\theta_k)^T P\mathcal{A}(\theta_k) - P + C^TC & \mathcal{A}(\theta_k)^T P\mathcal{E}(\theta_k) + C^T\mathcal{F} \\ \mathcal{E}(\theta_k)^T P\mathcal{A}(\theta_k) + \mathcal{F}^TC & \mathcal{E}(\theta_k)^T P\mathcal{E}(\theta_k) + \mathcal{F}^T\mathcal{F} - \gamma^2 \end{bmatrix} \begin{bmatrix} e_k \\ \delta_k \end{bmatrix} \leq 0, \tag{12}$$

which holds if and only if

$$\begin{bmatrix} \mathcal{A}(\theta_k)^T P\mathcal{A}(\theta_k) - P + C^TC & \mathcal{A}(\theta_k)^T P\mathcal{E}(\theta_k) + C^T\mathcal{F} \\ \mathcal{E}(\theta_k)^T P\mathcal{A}(\theta_k) + \mathcal{F}^TC & \mathcal{E}(\theta_k)^T P\mathcal{E}(\theta_k) + \mathcal{F}^T\mathcal{F} - \gamma^2 \end{bmatrix} \preceq 0. \tag{13}$$

Furthermore, the matrix form (13) can be rewritten as

$$\begin{bmatrix} -P + C^TC & C^T\mathcal{F} \\ \mathcal{F}^TC & \mathcal{F}^T\mathcal{F} - \gamma^2 \end{bmatrix} + \begin{bmatrix} \mathcal{A}(\theta_k)^T P\mathcal{A}(\theta_k) & \mathcal{A}(\theta_k)^T P\mathcal{E}(\theta_k) \\ \mathcal{E}(\theta_k)^T P\mathcal{A}(\theta_k) & \mathcal{E}(\theta_k)^T P\mathcal{E}(\theta_k) \end{bmatrix} \preceq 0. \tag{14}$$

By using the Schur complement lemma, we can obtain

$$\begin{bmatrix} -P + C^TC & C^T\mathcal{F} & \mathcal{A}(\theta_k)^T \\ \mathcal{F}^TC & \mathcal{F}^T\mathcal{F} - \gamma^2 & \mathcal{E}(\theta_k)^T \\ \mathcal{A}(\theta_k) & \mathcal{E}(\theta_k) & -P^{-1} \end{bmatrix} \preceq 0. \tag{15}$$

Since (15) is a polytopic model, we have

$$\begin{aligned} & \begin{bmatrix} -P + C^TC & C^T\mathcal{F} & \mathcal{A}(\theta_k)^T \\ \mathcal{F}^TC & \mathcal{F}^T\mathcal{F} - \gamma^2 & \mathcal{E}(\theta_k)^T \\ \mathcal{A}(\theta_k) & \mathcal{E}(\theta_k) & -P^{-1} \end{bmatrix} \\ & = \sum_{i=1}^{\mathcal{N}} \begin{bmatrix} -P + C^TC & C^T\mathcal{F} & \mathcal{A}^{i^T} \\ \mathcal{F}^TC & \mathcal{F}^T\mathcal{F} - \gamma^2 & \mathcal{E}^{i^T} \\ \mathcal{A}^i & \mathcal{E}^i & -P^{-1} \end{bmatrix} \preceq 0. \end{aligned} \tag{16}$$

Therefore, we can obtain a group of matrix inequalities (8). The existence of P and γ means that the dissipation inequality (10) holds. In this case, we can conclude that $\mathcal{L}_2$ gain of the input/output mapping is bounded by γ stated in Apkarian et al. (1995) and the dynamics (5) has a quadratic $\mathcal{H}_\infty$ performance γ. □

3. INVARIANT SET-BASED ROBUST FAULT DETECTION

Under the precondition of quadratic $\mathcal{H}_\infty$ stability of the dynamics (5), the existence of invariant sets for the dynamics (5) can be guaranteed. Readers can refer Blanchini (1999) for more details on the relationship between the stability and set invariance. Therefore, we can construct the invariant set $\mathbf{E}$ of the dynamics (5) to generate the corresponding healthy residual set $\mathbf{R}$ to implement robust FD. Let us consider the set version of the dynamics (5): $\mathbf{E}_{k+1} = \bar{\mathcal{A}}(\mathbf{E}_k) \oplus \mathbf{S}$, where $\mathbf{E}_k$ is the set containing the estimation error e_k, $\oplus$ is the Minkowski sum operator, $\bar{\mathcal{A}}(\mathbf{E}_k) = \mathbf{Conv}\left\{\bigcup_{i=1}^{\mathcal{N}} \mathcal{A}^i\mathbf{E}_k\right\}$, $\mathbf{S} = \mathbf{Conv}\left\{\bigcup_{i=1}^{\mathcal{N}} \mathcal{E}^i\mathbf{\Delta}\right\}$, $\mathbf{\Delta} = \mathbf{W}_2 \times \mathbf{V} \times \mathbf{V}$ and $\mathbf{W}_2$ is the set bounding the unknown input vector w_k^2.

Based on the authors' seminal work in Tan et al. (2019), a novel method computing invariant sets for LPV systems is proposed. Considering the space limit, here we only give the final computation result of invariant sets in the following theorem. Readers can refer Tan et al. (2019) for more details on the computation of invariant sets of LPV dynamics perturbed by bounded uncertainties.

Theorem 2. Given an initial RPI set $\mathbf{E}_0$ for the dynamics (5), the sequence $\mathbf{E}_k$: $\mathbf{E}_{k+1} = \bar{\mathcal{A}}(\mathbf{E}_k) \oplus \mathbf{S}$ ensures that at each iteration $\mathbf{E}_k$ is an RPI set of the dynamics (5) and $\mathbf{E}_\infty \subseteq \mathbf{E}_{k+1} \subseteq \mathbf{E}_k \subseteq \mathbf{E}_0$ holds for $k \geq 1$. Furthermore, we have $\mathbf{E}_\infty = \lim_{k\to+\infty} \mathbf{E}_k = \oplus_{n=0}^{\infty} \bar{\mathcal{A}}^n(\mathbf{S})$, which is the exact minimal robust positively invariant (mRPI) set of the dynamics (5). $\bar{\mathcal{A}}^n(\mathbf{S})$ is computed by the iteration: $\bar{\mathcal{A}}^n(\mathbf{S}) = \mathbf{Conv}\left\{\bigcup_{i=1}^{\mathcal{N}} \mathcal{A}^i\bar{\mathcal{A}}^{n-1}(\mathbf{S})\right\}, n \geq 1$ with $\bar{\mathcal{A}}^0(\mathbf{S}) = \mathbf{S}$. It can be found that the computation of mRPI set $\mathbf{E}_\infty$ involves infinite times of sum, which is not realistic to obtain the exact mRPI set. Based on the idea in Tan et al. (2019), an outer-approximation $\mathbf{E}$ of the mRPI set $\mathbf{E}_\infty$ can be obtained with any given precision in priority.

Based on the computed invariant set $\mathbf{E}$, we can further obtain the residual set in healthy situation: $\mathbf{R} = C\mathbf{E} \oplus F\mathbf{V}$. Then the FD criterion is to real-timely check whether $r_k \in \mathbf{R}$ holds or not. If there is a violation, i.e., $r_k \notin \mathbf{R}$, then we consider that there is a fault occurred in the system (1). Otherwise, we still consider that the system (1) operates in the healthy situation.

4. ROBUST FAULT ISOLATION BASED ON SET-SEPARATION CONSTRAINTS

In this section, we consider establishing guaranteed FI conditions off-line based on set-separation constraints such that the occurred fault can be always isolated as long as it satisfies the established FI conditions. After a fault is detected by using the method proposed in the previous section, i.e., $r_k \notin \mathbf{R}$, the FI mechanism is activated.

Without loss of generality, we assume that the system (1) operates in the i-th actuator fault mode. The SUIO corresponding to the i-th actuator is designed as

$$z^i_{k+1} = N^i(\theta_k)z^i_k + T^i(\theta_k)u_k + K^i(\theta_k)y_k, \tag{17a}$$
$$\hat{x}^i_k = M^i z^i_k + H^i y_k, \tag{17b}$$
$$\hat{y}^i_k = C\hat{x}^i_k. \tag{17c}$$

Similar to (3) and (4), by designing proper parameter matrices, the dynamics of state-estimation error $e^{ii}_k = x_k - \hat{x}^i_k$ can be obtained as

$$\begin{aligned} e^{ii}_{k+1} &= (A(\theta_k) - H^iCA(\theta_k) - M^iK^i_1(\theta_k)C)e^{ii}_k \\ &+ (E_2 - H^iCE_2)w^2_k - H^iF\eta_{k+1} - M^iK^i_1(\theta_k)F\eta_k. \end{aligned} \tag{18}$$

It can be seen that if the SUIO matches the current system mode, the dynamics (18) of state estimation error is similar to the healthy situation (5). We can also compute the invariant set $\mathbf{E}^{ii}$ and construct the corresponding residual set $\mathbf{R}^{ii}$ similar to the previous section. However, if the j-th actuator ($j \neq i$) becomes faulty, the state estimation error of the i-th SUIO will change. The state equation of the plant under the j-th actuator fault is

$$x_{k+1} = A(\theta_k)x_k + B(\theta_k)G^ju_k + Ew_k. \tag{19}$$

Then, the dynamics of the state estimation error of the i-th SUIO changes to

$$\begin{aligned} e^{ji}_{k+1} &= (A(\theta_k) - H^iCA(\theta_k) - M^iK^i_1(\theta_k)C)e^{ji}_k \\ &+ [B(\theta_k)G^j - M^iT^i(\theta_k) - H^iCB(\theta_k)G^j]u_k \\ &+ (E_2 - H^iCE_2)w^2_k - H^iF\eta_{k+1} - M^iK^i_1(\theta_k)F\eta_k. \end{aligned} \tag{20}$$

Thus, for the i-th SUIO, if we consider all the possible actuator faults except for the i-th one, the following dynamics can be obtained to describe the state estimation error under the different faulty situations:

$$\zeta^i_{k+1} = \Psi^i\zeta^i_k + S^i\Delta_k, \tag{21}$$

where

$$\zeta^i_k = \begin{bmatrix} e^{1i^T}_k \cdots e^{i-1,i^T}_k & e^{i+1,i^T}_k \cdots e^{n_u i^T}_k \end{bmatrix}^T, \Delta_k = \begin{bmatrix} u^T_k & \delta^T_k \end{bmatrix}^T,$$
$$\bar{B}^i = B(\theta_k)G^i - M^iT^i(\theta_k) - H^iCB(\theta_k)G^i,$$
$$\Psi^i = \begin{bmatrix} \bar{A}^i & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \bar{A}^i & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \bar{A}^i \end{bmatrix}, \bar{A}^i = A(\theta_k) - H^iCA(\theta_k) - M^iK^i_1(\theta_k)C,$$
$$S^i = \begin{bmatrix} \bar{B}^1 & (E^2_i - H^iCE^2_i) & -H^iF & -M^iK^i_1(\theta_k)F \\ \vdots & \vdots & \vdots & \vdots \\ \bar{B}^{i-1} & (E^2_i - H^iCE^2_i) & -H^iF & -M^iK^i_1(\theta_k)F \\ \bar{B}^{i+1} & (E^2_i - H^iCE^2_i) & -H^iF & -M^iK^i_1(\theta_k)F \\ \vdots & \vdots & \vdots & \vdots \\ \bar{B}^{n_u} & (E^2_i - H^iCE^2_i) & -H^iF & -M^iK^i_1(\theta_k)F \end{bmatrix}.$$

Since u_k and δ_k are bounded by $\mathbf{U}$ and $\mathbf{\Delta}$, respectively, Δ_k is bounded by $\mathbf{U} \times \mathbf{\Delta}$, i.e., $\Delta_k \in \mathbf{U} \times \mathbf{\Delta}$. Given fault magnitudes f_1, f_2,..., f_{n_u}, an RPI set $\bar{\mathbf{E}}^{ii}$ for ζ^i_k can be constructed. Thus, if we can guarantee that

$$\bar{\mathbf{R}}^{ii} \cap \mathbf{R}^{ii} = \emptyset, \tag{22}$$

where $\bar{\mathbf{R}}^{ii}$ is an union of $n_u - 1$ sets of the residual vectors of the i-th SUIO, each set corresponding to a faulty situation in an actuator different from the i-th one, this implies that for any considered fault except for the one in the i-th actuator, as long as it occurs, the residual vector of the i-th SUIO will leave its residual set $\mathbf{R}^{ii}$ and we can compute $\bar{\mathbf{R}}^{ii}$ as

$$\bar{\mathbf{R}}^{ii} = \mathbf{R}^{1i} \cup \cdots \cup \mathbf{R}^{i-1,i} \cup \mathbf{R}^{i+1,i} \cup \cdots \cup \mathbf{R}^{n_u i}, \tag{23}$$

where $\mathbf{R}^{ji}$ represents the set of the residual vector of the i-th SUIO in the j-th actuator-fault mode with $\mathbf{R}^{ji} = C\mathbf{E}^{ji} \oplus F\mathbf{V}$, where $\mathbf{E}^{ji}$ is the RPI set of the state estimation error of the i-th SUIO in the j-th actuator-fault mode.

For each SUIO, we need to guarantee that the corresponding condition (22) is satisfied. In this case, the FI task can be guaranteed. A group of guaranteed FI conditions are summarized in the following theorem.

Theorem 3. For the considered fault magnitudes f_1, f_2,..., f_{n_u}, if the set-separation constraints

$$\bar{\mathbf{R}}^{ii} \cap \mathbf{R}^{ii} = \emptyset \quad \forall\, i = 1, 2, .., n_u \tag{24}$$

are satisfied, as long as one of the faults is detected, it can be guaranteed that the occurred fault can be isolated during steady stage after a time interval.

Proof 2. Let us assume that the i-th actuator is faulty. If (24) is satisfied, according to the property of invariant sets, only the i-th SUIO can generate residual signal r^i_k that always stays in its corresponding residual set $\mathbf{R}^{ii}$ during the steady stage after fault occurrence, while the residual signals $r^j_k (j \neq i)$ generated by the other SUIOs will leave their residual sets $\mathbf{R}^{jj} (j \neq i)$ during steady stage, which guarantees the success of active FI. □

The proposed set-separation FI constraints can guarantee one and only one residual vector among all the SUIOs will always stay the corresponding residual sets during the steady stage, while the other residual vectors will go outside their corresponding residual sets. Furthermore, this residual vector can accurately indicate the fault.

5. AN ILLUSTRATIVE EXAMPLE

Let us consider a discrete-time LPV system (1) under the effect of actuator fault with

$$A(\theta_k) = \begin{bmatrix} 0.5 + 0.2\theta_k(1) & 0.8\theta_k(1) + 0.5\theta_k(2) \\ 0 & 0.4 - 0.3\theta_k(2) \end{bmatrix}, F = \begin{bmatrix} 0.079 & 0.220 \\ 0.041 & 0.1419 \end{bmatrix},$$
$$B(\theta_k) = \begin{bmatrix} 0.81 + 0.2\theta_k(1) & 0.13 + 0.52\theta_k(2) & 0.62 + 0.32\theta_k(1) \\ 0.91 & 0.87 + 0.78\theta_k(2) & 0.15 \end{bmatrix},$$
$$E = \begin{bmatrix} 0.0969 & 0.0954 & 0.562 \\ 0.101 & 0.0878 & 0.146 \end{bmatrix}, C = \begin{bmatrix} 0.9572 & 0.8003 \\ 0.4854 & 0.1419 \end{bmatrix}.$$

The scheduling vector θ_k is contained in the bounded set

$$\mathbf{\Theta} = \mathbf{Conv}\left\{ \begin{bmatrix} -0.15 \\ -0.1 \end{bmatrix}, \begin{bmatrix} -0.15 \\ 0.15 \end{bmatrix}, \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \begin{bmatrix} 0.1 \\ 0.15 \end{bmatrix} \right\}.$$

The unknown inputs w_k and measurement noises η_k are bounded by the sets $\mathbf{W} = \{w \in \mathbb{R}^3 | \|w\|_\infty \leq 0.1\}$ and $\mathbf{V} = \{\eta \in \mathbb{R}^2 | \|\eta\|_\infty \leq 0.1\}$, respectively. The system input vector u_k is given by $[10 - 0.5sin(0.05k)\ 15 - 0.6cos(0.065k)\ -15 - 0.2sin(0.01k)]^T$. Owing to that there are three inputs in the system, we consider three fault magnitudes corresponding to them, respectively, which are given as $f_1 = 0.1$, $f_2 = 0.6$ and $f_3 = 0.7$. Since there are three unknown inputs and only two outputs, we can not decouple the effect of all unknown inputs w_k.

Algorithm 1 FDI Algorithm

Initialization (the plant, SUIOs, etc.)
At time instant k: $r_k \in \mathbf{R}$, FD = FALSE
while FD $\neq$ TRUE **do**
  $k \leftarrow k+1$
  Obtain r_k
  if $r_k \notin \mathbf{R}$ **then**
    FD $\leftarrow$ TRUE
    break
  end if
end while
while FD = TRUE **and** During Steady Stage **do**
  for each $i \in \mathbb{I}$ **do**
    if $r_k^i \notin \mathbb{R}^{ii}$ **then**
      Remove i from $\mathbb{I}$
    end if
  end for
  if length($\mathbb{I}$)=1 **then**
    The unique element in $\mathbb{I}$ indicates the fault
    break
  else
    $k \leftarrow k+1$
  end if
end while

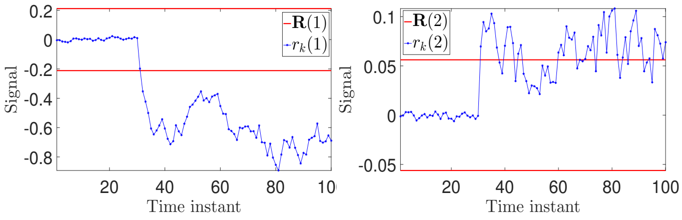
Therefore, we only consider actively decoupling the effect of one unknown input w_k^1 on residual signal, i.e., $E_1 = \begin{bmatrix} 0.0969 \\ 0.101 \end{bmatrix}, E_2 = \begin{bmatrix} 0.0954 & 0.562 \\ 0.0878 & 0.146 \end{bmatrix}$. Furthermore, the parametric matrices of SUIO (2) for robust FD are given by

$$M = \begin{bmatrix} 0.6580 & 0.2918 \\ 0.5629 & 0.6223 \end{bmatrix}, H = \begin{bmatrix} 0.3022 & 0.7241 \\ 0.2277 & 1.0019 \end{bmatrix},$$

$$K_1(\theta_k) = \begin{bmatrix} 2.493\theta_k(1) + 2.648\theta_k(2) + 0.6743 & -4.195\theta_k(1) - 5.223\theta_k(2) - 2.664 \\ -3.867\theta_k(1) - 4.109\theta_k(2) - 1.619 & 6.508\theta_k(1) + 8.102\theta_k(2) + 3.234 \end{bmatrix}.$$

In order to verify the quadratic $\mathcal{H}_\infty$ performance proposed in Theorem 1, we let the positively definite matrix $P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$. Then, by solving the LMIs (8), we can obtain $\gamma = 1.3285$. For the sake of verifying the effectiveness of robust FD, we set the following fault scenario. Form the time instant $k = 0$ to $k = 30$, the system (1) operates in the healthy situation, while after $k = 30$ we inject the fault f_1 into the system during the whole operation stage. The results of robust FD are shown in Fig. 1. It can be seen that from the time instant $k = 0$ to $k = 30$, the residual signal r_k is always contained in the residual set $\mathbf{R}$, i.e., $r_k \in \mathbf{R}$, and we consider that the system operates in the healthy situation during this time interval. After the fault occurs at time instant $k = 30$, it is detected that $r_{32}(1) \notin \mathbf{R}(1)$, which indicates that the fault has been detected at time instant $k = 32$.

After the robust FD, we consider designing a group of SUIOs corresponding to three actuator faults f_1, f_2 and f_3 to implement FI. The parameter of the three SUIOs are designed as



(a) (b)

Fig. 1. Results of robust FD

$$M^1 = \begin{bmatrix} 0.2436 & 0.8725 \\ 0.7562 & 0.1829 \end{bmatrix}, H^1 = \begin{bmatrix} 0.3494 & 0.5908 \\ 0.4137 & 0.4756 \end{bmatrix},$$

$$K_1^1(\theta_k) = \begin{bmatrix} -1.427\theta_k(1) - 1.516\theta_k(2) - 0.9877 & 2.401\theta_k(1) + 2.99\theta_k(2) + 1.283 \\ 1.017\theta_k(1) + 1.08\theta_k(2) + 0.2199 & -1.711\theta_k(1) - 2.131\theta_k(2) - 1.218 \end{bmatrix},$$

$$M^2 = \begin{bmatrix} 0.8871 & 0.1113 \\ 0.3216 & 0.2760 \end{bmatrix}, H^2 = \begin{bmatrix} 0.3322 & 0.6395 \\ 0.2814 & 0.8499 \end{bmatrix},$$

$$K_1^2(\theta_k) = \begin{bmatrix} 1.216\theta_k(1) + 1.292\theta_k(2) + 0.2917 & -2.047\theta_k(1) - 2.548\theta_k(2) - 1.328 \\ -4.937\theta_k(1) - 5.245\theta_k(2) - 2.697 & 8.308\theta_k(1) + 10.34\theta_k(2) + 3.651 \end{bmatrix},$$

$$M^3 = \begin{bmatrix} 0.2959 & 0.1030 \\ 0.7206 & 0.3293 \end{bmatrix}, H^3 = \begin{bmatrix} 0.4842 & 0.2094 \\ 0.4924 & 0.2532 \end{bmatrix},$$

$$K_1^3(\theta_k) = \begin{bmatrix} 12.54\theta_k(1) + 13.33\theta_k(2) + 1.783 & -21.11\theta_k(1) - 26.28\theta_k(2) - 11.98 \\ -30.02\theta_k(1) - 31.89\theta_k(2) - 6.148 & 50.52\theta_k(1) + 62.89\theta_k(2) + 28.78 \end{bmatrix}.$$

Based on the guaranteed FI conditions in Theorem 3, the residual sets of the SUIOs are constructed as in Fig. 2. Notice that, in Fig. 2, the first, second and third subplots correspond to the first, second and third SUIOs, respectively. Taking the first SUIO as an example, we can see that $\mathbf{R}^{11} \cap \mathbf{R}^{12} = \emptyset$ and $\mathbf{R}^{11} \cap \mathbf{R}^{13} = \emptyset$, which implies that $\mathbf{R}^{11} \cap \bar{\mathbf{R}}^{11} = \emptyset$. Furthermore, in the remaining subplots, it can be seen that $\mathbf{R}^{22} \cap \bar{\mathbf{R}}^{22} = \emptyset$ and $\mathbf{R}^{33} \cap \bar{\mathbf{R}}^{33} = \emptyset$. Thus, the set-separation constraints are satisfied and the robust FI can be guaranteed.

The simulation results of robust FI corresponding to three actuator faults f_1, f_2 and f_3 are shown in Figs. 3-5. We take the FI for the first actuator fault f_1 as an example. It can be found from Fig. 3 during the steady stage, only the residual signal r_k^1 generated from the first SUIO is still inside the corresponding residual set $\mathbf{R}^{11}$, i.e., $r_k^1 \in \mathbf{R}^{11}$, while the remaining residual signals r_k^2 and r_k^3 are not contained by their corresponding residual sets $\mathbf{R}^{22}$ and $\mathbf{R}^{33}$, i.e., $r_k^2 \notin \mathbf{R}^{22}$ and $r_k^3 \notin \mathbf{R}^{33}$, which indicate that the first fault f_1 has been isolated. Similar analysis can be conducted for isolating the faults f_2 and f_3 from Figs. 4 and 5, respectively.

6. CONCLUSION

In this paper, a new robust FDI method combining SUIOs and invariant sets is proposed to detect and isolate faults of discrete-time LPV systems, where we divide the unknown inputs into two groups. The first group includes the unknown inputs that can be actively decoupled by the designed observers, while for the second group, their bounding sets are used to passively propagate their effect

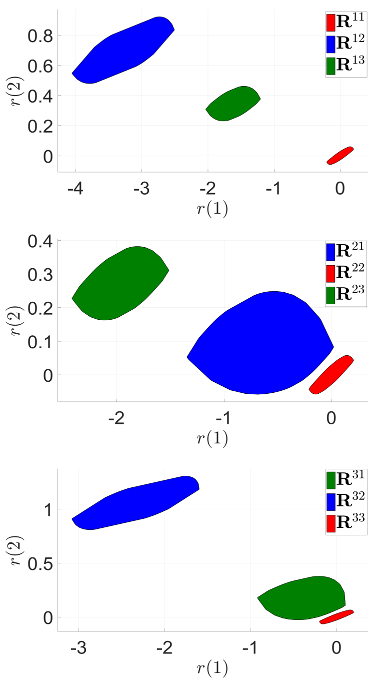
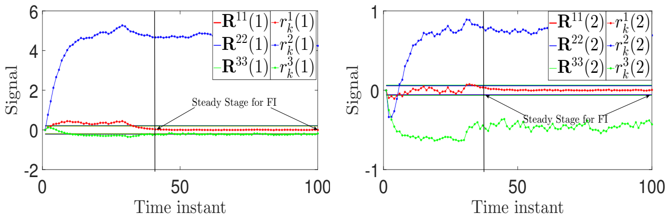
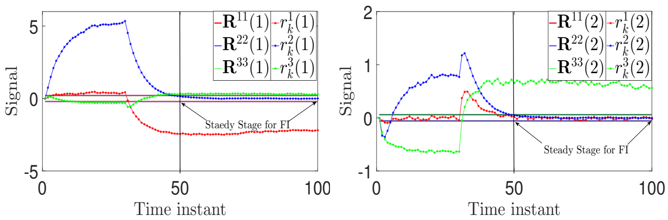


Fig. 2. Verification of guaranteed FI conditions



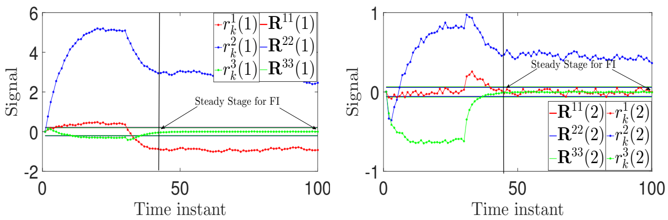
(a) (b)

Fig. 3. Robust FI for the first actuator fault f_1



(a) (b)

Fig. 4. Robust FI for the second actuator fault f_2



(a) (b)

Fig. 5. Robust FI for the third actuator fault f_3

on the residual sets. The stability of the state estimation error of SUIOs is guaranteed by establishing a quadratic $\mathcal{H}_\infty$ performance. Under the precondition of the quadratic $\mathcal{H}_\infty$ stability, a family of guaranteed FI conditions are established using residual set-separation constraints based on invariant-set theory off-line. Any actuator fault satisfying the guaranteed FI conditions can be isolated during the steady stage. In the future, we will consider combining SUIO and invariant sets to implement fault-tolerant control of LPV systems.

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