A Newsboy formulae to optimize planned lead times for two-level disassembly systems

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Abstract: Disassembly planning and inventory management are important for businesses to provide customers with used components at competitive prices. To achieve this objective, one of the planners’ priorities is to reduce the expected level of inventory in an uncertain environment. This study deals with a single-period disassembly to-order problem with known and fixed demand for components. The disassembly lead time for each component is an independent discrete random variable whose probability distributions are known and bounded. A mathematical model is suggested to determine the disassembly order for the end-of-life product and to calculate the expected total cost. Newsboy formulae for optimal disassembly order determination that minimizes the expected total cost is developed.

Keywords: Reverse supply chain, disassembly-to-order, inventory control, stochastic disassembly lead times, Newsboy model.

1. INTRODUCTION

In the past decades, the remanufacturing field has increased considerably, due to its environmental and economic benefits (Benaisa et al., 2018). Uncertainty management is becoming one of the most important challenges in optimizing the Reverse Supply Chain (RSC). Indeed, uncertainty causes several difficulties in disassembly planning and inventory management. The sources of uncertainty are diverse and can be located to several levels of the RSC: demand variability, machine failures, transport delays, recovery rate, quality problems, etc.

In this paper, we study a two-level disassembly system for the disassembly-to-order (DTO) environment where the products are disassembled for the retrieval of reusable items and resold in order to satisfy a certain demand. The main goal is to determine the time and the quantity of end-of-life (EoL) products to be disassembled in order to meet the requests of each part (Ji et al., 2016; Slama et al., 2019b).

The uncertainties associated with the disassembly DTO problem have not been treated much in the literature. Authors generally considered the problem with uncertainty of yield (number of units of parts obtained from disassembling one unit of parent item) and/or demand. They have applied a stochastic algorithms to determine the optimal schedule of complex returned products (two/multi-level, with/without part commonality and single/multi-type product).

Inderfurth and Langella (2006) considered the disassembly system with multi-type product and parts commonality to solve the single period DTO problem under random disassembly yield. Commonalities of parts means that the EoL products can share a common components. For solving this problem, they introduced a heuristic to reduce the expected disposal, purchasing and disassembly operation costs. Kongar and Gupta (2006) treated the DTO problem by incorporating uncertainty in the number of end-of-life products retrieved and recycled components. They utilized the fuzzy goal programming, which allows the goals of the problem to be characterized using intentional vagueness.

Barba-Gutiérrez and Adenso-Díaz (2009) have incorporated the uncertainty of demand of parts. They used trapezoidal distributions to incorporate the imprecise demand. For solving this problem, they suggested a fuzzy RMRP algorithm. Kim and Xirouchakis (2010) have treated the problem with multi-product type and multi period disassembly system under random demands. They developed a Lagrangian relaxation heuristics to reduce the sum of expected setup, inventory holding, and penalty costs for unmet requests. Inderfurth et al. (2015) proposed mathematical model (with two-root and three-leaf items) to
illustrate the effect of random disassembly yield in stochastically proportional and binomial models. Liu and Zhang (2018) formulated disassembly scheduling problem with the uncertainty of demands and yields as a Mixed Integer Nonlinear Program (MINLP) and proposed an outer approximation approach to resolve it.

In regarding the literature, the lead time of the EoL product in disassembly systems is supposed constant or equal to zero. In the industrial reality, disassembly lead time is far to be constant because of high level of uncertainty of the disassembly process in terms of its timing. In the present work, the disassembly lead time is then defined as the total time required to receive the parts after placing an order of disassembling the EoL product (Kim et al., 2006).

Kim et al. (2007), investigated the effect of uncertainty of lead time on the disassembly planning problem. Their analyses show that disassembly lead time variability strongly affect the system performance. Few works are proposed to help in deciding on disassembly schedule plan under random lead time. Slama et al. (2019a) are the first to treat this problem type. The case of the uncertainty of disassembly lead time of the EoL product is studied. A model is proposed to deal with a multi-period, single product type and two-level disassembly system. The problem is formulated as a minimization problem and then converted to a Monte Carlo-mixed integer programming model. The proposed model is used to determine the optimal quantity for EoL products in order to minimize the average total cost over the planning horizon and all scenarios. Recently, (Slama et al., 2020) proposed a generalization of the discrete Newsboy formulae to find the optimal release date when the time of disassembling the EoL product is random variable. This last study assuming that once the disassembly operation started, the EoL product will simultaneously release all the parts with independent probability discrete distributions. This research was the first one to regard the disassembly systems using the Newsboy model. For a thorough study dealing Newsboy problems, the readers can be referring to the proposed research’s by (Khouja, 1999; Ben-Ammar et al., 2018).

Owing to complexity of the disassembly process, the time of disassembling each component might be a random variable. Once disassembly has started, each part can be available after a random lead time. To the best of our knowledge, no one has treated this problem type under uncertainty disassembly lead time for each part. This paper is a continuation of our earlier work (Slama et al., 2020). Two new contributions and innovations are investigated: (1) The uncertainty of disassembly time of each component is defined as independent random variables with known and limited probability distributions. (2) An approach based on a Newsboy-like analytical formulae is suggested to determine an efficient disassembly plan. To close to real industrial planning approach, an integer decision variable with a discrete lead times distribution is proposed.

The rest of the paper is organized in six sections. The studied problem is described in section 2. The mathematical formulation of the problem is proposed in section 3. A Newsboy model is given in section 4. Section 5 shows some results. Finally, the conclusion and some perspectives for future research are given in section 6.

2. PROBLEM DESCRIPTION

In this present work, the case of two-level disassembly system is studied. As shown in Fig. 1, the EoL product (first level) is disassembled on individual components/parts: \(i = 1, \ldots, n\) (second level). The demand on components is known and fixed. Without loss of generality, we are assuming only one component for each demand. These demands \(D_i\) must be delivered on predefined delivery dates \(T_i\).

![Fig. 1. Two-level disassembly system.](image)

This study is supposing:

1. There aren’t precedence relations among disassembly tasks yielding the \(n\) parts;
2. Once the EoL product is started for disassembly, each component is available after a random disassembly time;
3. The disassembly lead time, \(L_i\) of each part \(i\) is a random discrete variable with a known probability distribution and a bounded over known interval: \([L_i^-, L_i^+]\);
4. The probability distribution for each part is not identically distributed, that’s to say, the disassembly lead time don’t follow the same probability distribution for each part.

The risk of an uncertainty disassembly lead times entails costs of backlogging and storage. In fact, if the request for the parts is not delivered at the desired time, a backlogging cost is incurred. In the same manner, an inventory cost appears if some parts arrive before the expected delivery date. For this reason, the objective of this present work is to minimize the expected value of the total cost that is equal to the sum of the expected costs of holding and backlogging of all components.
3. PROBLEM FORMULATION

The aim of this research is to develop a mathematical model for two-level disassembly systems under a fixed part’s demand and uncertainty of parts disassembly lead times. Before providing it, the notation can be summarized below:

**Parameters:**
- \( i \) Index of items, \( i = 1, \ldots, n \),
- \( T_i \) Delivery date for each part \( i \),
- \( D_i \) External demand for item \( i \),
- \( h_i \) Inventory holding cost of one unit of \( i \),
- \( b_i \) Backlogging cost of one unit of \( i \),
- \( L_i \) Real disassembly lead time for leaf item \( i \),
- \( M_i \) Real receipt date of \( i \), \( M_i = X + L_i \).

**Decision variable:**
- \( X \) Starting date of disassembling the EoL product.

**Functions:**
- \( E(\cdot) \) Expected value,
- \( F_i(\cdot) \) Distribution function of the random variable,
- \( T^+_i \) \( \max \{M_i, T_i\} \),
- \( T^-_i \) \( \min \{M_i, T_i\} \),
- \( (Z)^+ \) Maximum between 0 and \( z \),
- \( (Z)^- \) Minimum between 0 and \( z \).

The total cost is equal to the sum of backlogging and inventory holding costs for all components.

**Proposition 3.1.** For the system described in the previous section, the total cost, noted by \( TC(X, \mathcal{L}) \), is as follows:

\[
TC(X, \mathcal{L}) = \sum_{i=1}^{n} (b_i + h_i) (T^+_i - M_i - h_i - b_i T_i) \tag{1}
\]

with \( X \in \left[ \min_{i \in [1,n]} \{T_i - L_i^+\} ; \max_{i \in [1,n]} \{T_i - L_i^-\} \right] \) and \( \mathcal{L} = \{L_1, \ldots, L_n\} \).

**Proof.** The total cost is composed of inventory holding cost \( (C_H(X, \mathcal{L})) \) backlogging cost \( (C_B(X, \mathcal{L})) \). As shown in Fig. 2, if a component \( i \) is available before \( T_i \), it is stored during the periods \( T_i^+ - T_i^- \):

\[
C_H(X, \mathcal{L}) = \sum_{i=1}^{n} h_i (T_i^+ - T_i^-) \tag{2}
\]

If a part \( i \) is available after \( T_i \), it is backordered during the periods \( T_i^+ - T_i^- \):

\[
C_B(X, \mathcal{L}) = \sum_{i=1}^{n} b_i (T_i^+ - T_i^-) \tag{3}
\]

The total cost is equal to the sum of \( C_H(X, \mathcal{L}) \) and \( C_B(X, \mathcal{L}) \). Knowing that \( T_i + M_i = T_i^+ + T_i^- \), then by using equations (2-3), the value of the total cost can be easily deduced.

The disassembly lead times \( L_i, \forall i = 1, \ldots, n \) are random discrete variables. Therefore, this total cost is a random variable with a finite number of possible values. Thus, we can calculate its mathematical expectation.

**Proposition 3.2.** The mathematical expectation of the total cost, noted by \( E(C(X, \mathcal{L})) \), is given by the following expression:

\[
E(C(X, \mathcal{L})) = \sum_{i=1}^{n} (b_i + h_i) \left( \sum_{s \geq T_i} \{1 - F_i(s - X)\} \right) - \sum_{i=1}^{n} h_i (X + E(L_i) - T_i) \tag{4}
\]

**Fig. 2.** Composition of the total cost.

On one hand:

\[
P(max(M_i, T_i) \leq s) = P(M_i \leq s; T_i \leq s)
\]

A component \( i \) is available at period \( M_i \) and this period does not depend on \( T_i \), so:

\[
P(max(M_i, T_i) \leq s) = P(M_i \leq s) \times P(T_i \leq s)
\]

and:

\[
E(T_i^+) = \sum_{s \geq 0} 1 - P(M_i \leq s) \times P(T_i \leq s)
\]

On the other hand, the delivery date desired by the customer \( T_i \) is known and superior to 0:

\[
\begin{cases} 
P(T_i \leq s) = 0 & \forall s > T_i \\
P(T_i > s) = 1 & \forall s \leq T_i 
\end{cases}
\]

So \( \forall i = 1, \ldots, n \):
\[
\mathbb{E}(T_i^+) = \sum_{s \geq 0} (1 - P(M_i \leq s) \times P(T_i \leq s)) \\
= \sum_{0 \leq s \leq T_i} (1 - P(M_i \leq s) \times P(T_i \leq s)) + \sum_{s \leq T_i} (1 - P(M_i \leq s) \times P(T_i \leq s))
\] (6)

Knowing that \( \forall i = 1, \ldots, n, M_i = X + L_i \). By using equation (6), the expression of \( \mathbb{E}(C(X, L)) \) can be easily determined.

In this section, we present the mathematical model which calculates the expected total cost. The question then becomes: When should we start the disassembly operation for minimizing the expected cost?

4. NEWSBOY MODEL

In order to determine the optimal starting date of disassembling the EoL product that minimizes \( \mathbb{E}(C(X, L)) \) expressed in equation (4), we propose a Newsboy formulation.

**Proposition 4.1.** The optimal solution \( X^* \) that minimizes equation (4) is unique and verifies the following expression:

\[
\sum_{i=1}^{n} (b_i + h_i)F_i(T_i - X^* - 1) \leq \sum_{i=1}^{n} b_i \leq \sum_{i=1}^{n} (b_i + h_i)F_i(T_i - X^*)
\] (7)

where \( F_i(\cdot) \) is the cumulative distribution function of the lead time \( L_i \).

**Proof.** Let \( G(X) \) be a function such as:

\[
G(X) = \mathbb{E}(C(X + 1, L)) - \mathbb{E}(C(X, L))
\]

We can easily prove that:

\[
G^+(X) = \sum_{i=1}^{n} b_i - \sum_{i=1}^{n} (b_i + h_i)F_i(T_i - X - 1)
\]

Let \( X^* \) be the optimal solution. Then, It must be shown that \( G(X^*) \) is positive and \( G(X^* - 1) \) is negative. If these are verified, then the necessary and sufficient conditions for the existing of a value \( X^* \) minimizing \( \mathbb{E}(C(X, L)) \) are as follows:

\[
\begin{cases}
\mathbb{E}(C(X^*, L)) \leq \mathbb{E}(C(X^* + 1, L)) \\
\mathbb{E}(C(X^*, L)) \leq \mathbb{E}(C(X^* - 1, L))
\end{cases}
\]

It amounts to showing that \( G(X^*) \geq 0 \) and \( G(X^* - 1) \leq 0 \). In other words, demonstrating that:

\[
\begin{cases}
\sum_{i=1}^{n} b_i - \sum_{i=1}^{n} (b_i + h_i)F_i(T_i - X^* - 1) \geq 0 \\
\sum_{i=1}^{n} b_i - \sum_{i=1}^{n} (b_i + h_i)F_i(T_i - X^*) \leq 0
\end{cases}
\]

Then, equation (7) can be deduced and it represents the optimality condition for the discrete Newsboy model.

Theoretically, the integer \( X^* \) which gives the optimum is not unique. To prove the uniqueness of this optimal solution, we must prove the convexity of the objective function. To do this, we introduce two functions \( R(X^*) \) and \( R(X^* - 1) \) verifying the following equalities:

\[
R(X^*) = G(X^* + 1) - G(X^*)
\]

\[
= \sum_{i=1}^{n} (b_i + h_i)(F_i(T_i - X^* - 1) - F_i(T_i - X^* - 2))
\]

\[
R(X^* - 1) = G(X^*) - G(X^* - 1)
\]

\[
= \sum_{i=1}^{n} (b_i + h_i)(F_i(T_i - X^*) - F_i(T_i - X^* - 1))
\]

We know that \( F_i(\cdot) \) is growing. So, we can easily prove that \( R(X^*) \) is positive and \( R(X^* - 1) \) is negative; deduce that \( \mathbb{E}(C(X, L)) \) is convex and verify the uniqueness of the optimal solution.

5. COMPUTATIONAL EXPERIMENTS

5.1 Design of experiments

The mathematical model and the Newsboy formulae were implemented in C++ and computational experiments were performed on an Intel (R) Core i7-5500 processor at 2.4 GHz clock-speed and with 8 Go of memory.

5.2 Numerical example

A numerical example is given to test the solution approach. A small instance, with a disassembly system composed of 10 components, is considered. For each component \( i = 1, \ldots, n \) the following parameters are given in table 1: the unit inventory holding cost \( h_i \), the unit backlogging cost \( b_i \) and the delivery date desired by the customer \( T_i \). The related probability distributions are listed in table 2.

For example, the real disassembly lead time for component 1 varies between 1 and 5, for component 2 it varies between 2 and 6 and so on.

<table>
<thead>
<tr>
<th>Component</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_i )</td>
<td>5</td>
<td>9</td>
<td>5</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>( b_i )</td>
<td>100</td>
<td>120</td>
<td>106</td>
<td>125</td>
<td>162</td>
</tr>
<tr>
<td>( T_i )</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_i )</td>
<td>8</td>
<td>5</td>
<td>11</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>( b_i )</td>
<td>269</td>
<td>285</td>
<td>100</td>
<td>107</td>
<td>201</td>
</tr>
<tr>
<td>( T_i )</td>
<td>9</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

In order to obtain the Pareto front, we execute all possible solutions (see Fig. 3). The optimal solution corresponds to the couple (1,558.45) which corresponds to the optimal starting date of disassembling the EoL product \( X^* \) minimizing the expected total cost. We note that less than 1 second is needed to obtain the optimal solution.
Table 2. Disassembly lead time probability distributions.

<table>
<thead>
<tr>
<th>ω</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(L1 = ω)</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>P(L2 = ω)</td>
<td>-</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>P(L3 = ω)</td>
<td>0.8</td>
<td>0.1</td>
<td>0.05</td>
<td>0.03</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>P(L4 = ω)</td>
<td>-</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.96</td>
</tr>
<tr>
<td>P(L5 = ω)</td>
<td>0.85</td>
<td>0.05</td>
<td>0.05</td>
<td>0.07</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>P(L6 = ω)</td>
<td>-</td>
<td>0.01</td>
<td>0.1</td>
<td>0.2</td>
<td>0.99</td>
<td>-</td>
</tr>
<tr>
<td>P(L7 = ω)</td>
<td>0.1</td>
<td>0.9</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>P(L8 = ω)</td>
<td>0.11</td>
<td>0.6</td>
<td>0.1</td>
<td>0.09</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>P(L9 = ω)</td>
<td>-</td>
<td>0.08</td>
<td>0.02</td>
<td>0.2</td>
<td>0.25</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table 3. Data generation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>h₁</td>
<td>D ∼ U(5, 100)</td>
</tr>
<tr>
<td>b₁</td>
<td>h₁ × D ∼ U(1, 30)</td>
</tr>
<tr>
<td>T₁</td>
<td>D ∼ U(5, 20)</td>
</tr>
</tbody>
</table>

Fig. 3. Pareto front of all possible solutions.

5.3 Performance tests

In order to analyse the effect of lead times variability on the robustness and the stability of the proposed Newsboy model, the effect of variance (VAR) is treated according to the number of parts (10, 50, 100 and 500). To do so, we generate 400 different instances. The unit inventory and backlogging costs and the delivery due dates are randomly generated as detailed in table 3. In this table, D ∼ U(a, b) means that the parameter follows the discrete uniform distribution characterized by the interval [a, b]. For lead times, we consider the same distribution probability for all components. This distribution, noted by VAR (a), is the reference case and characterized as follows: \( P(L_i = 1) = 0.245, P(L_i = 2) = 0.48, P(L_i = 3) = 0.255, \)
\( P(L_i = 4) = 0.01 \) and \( P(L_i = 5) = 0.01 \).

Let \( X^* \) be the optimal order release date and \( E(C(X^*, L)) \) be the related optimal expected total cost corresponding to VAR (a). In order to evaluate the robustness of the optimal solution found by the proposed approach, we vary the variance of the lead times between \(-75\%\) and \(+75\%\) as detailed in table 4.

As Fig. 4 witnesses, the variation of the expected total cost (ETC) decreases when the number of components increases. However, this average variation remains less than 5% whatever the number of components in the second level of the disassembly system. In the worst case, the maximum absolute variance of ETC does not exceed 14% (see Fig. 5). This proves that our approach remains robust even if the variance of lead times reaches ±75%.

![Graph of lead time variance effect on expected total cost](image)

Fig. 4. Effect of lead time variance on the expected total cost.

![Graph of expected total cost variance](image)

Fig. 5. Effect of lead time variance on the expected total cost.
Let us now look at the effect of the backlogging cost variation. To do this, we only study disassembly systems with 10 components. The unit inventory and backlogging costs are randomly generated as mentioned in table 3. For the delivery due dates, 4 cases are considered: $T_i \sim U(5, 5)$, $T_i \sim U(5, 10)$, $T_i \sim U(5, 20)$ and $T_i \sim U(5, 50)$. For each case, 100 instances have been tested. We note that for each component, the real lead time varies between 1 and $D \sim U(2, 5)$, and considered to have a random discrete distribution.

We consider $b_i = 2h_i$ as reference case. Then, we vary the ratio $b_i/h_i$ between 1 and 9. As can be seen in Fig. 7, the variation of ETC increases not only when the delivery date interval decreases, but also when the ratio $b_i/h_i$ increases. It proves that the proposed approach seems to be less robust when the unit backlogging costs are underestimated and the delivery dates are very close to each other (all due dates are equals to 5).

In our future research, we will analyze in more details the robustness of our approach in order to know why for certain variations of lead times, the variation of ETC reach 14%. Then, we will extend the proposed model to integrate capacity constraint. Finally, we will try to integrate the uncertainty of demand and/or yield in the study of disassembly systems in DTO environments.

6. CONCLUSION AND PERSPECTIVES

This preliminary research deals with the modeling and optimization of two-level disassembly systems under random components disassembly lead times. We have developed a mathematical model to study a one-period planning for the disassembly-to-order environment. An approach based on a Newsboy formulae is developed to minimize the mathematical expectation of the total cost and determine the optimal order release date of the EoL product.

In order to test the robustness of the proposed approach, a first sensitivity study on the probability distributions of disassembly lead times is conducted. These findings highlight that it is important to obtain good statistical data to get a reliable estimate of the probability distributions of lead times. The second sensitivity analysis focuses on the effect of backlogging cost variance on the quality of the solution. It suggests the importance of taking into account the ratio $b_i/h_i$ in the decision making in order to optimise the disassembly planning.

REFERENCES


