Iteration-dependent High-order Internal Model based Iterative Learning Control for Discrete-time Nonlinear Systems with Time-iteration-varying Parameter

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Abstract: In this paper, an adaptive iterative learning control (AILC) scheme is designed for discrete-time nonlinear systems with random initial condition and time-iteration-varying parameter. The time-iteration-varying parameter is generated by a general iteration-varying high-order internal model (HOIM) with iteration-varying order and coefficients, and the parameter updating law is designed based on least square method. Compared with the existing works based on iteration-invariant HOIM with fixed order and coefficients, our work significantly extends the application scope of HOIM-based ILC. Using the designed HOIM based iterative learning controller, the learning convergence in the iteration domain is guaranteed through rigorous theoretical analysis under Lyapunov theory. Moreover, an illustrative example is given to demonstrate the effectiveness of the proposed method.

Keywords: Iteration-varying high-order internal model, adaptive iterative learning control, Lyapunov theory, time-iteration-varying parameter, non-repetitiveness.

1. INTRODUCTION

Iterative learning control (ILC) can use the system information in the past iterations to update the control command in the current iteration and improve the control performance. It was first proposed and designed to track the expected target for the repetitive behaviour of industrial robots (Uchiyama, 1978; Arimoto, 1984; Craig, 1984). ILC has unique advantages for controlled objects with repetitive characteristics. It has been applied in various fields, e.g., robot manipulators (Tchoń, 2010; Smith et al., 2015), wastewater treatment (Huang et al., 2011), and multi-agent systems (Li et al., 2015).

Traditional ILC has strict assumptions about the repetitiveness or iteration-invariance of key factors, such as the reference trajectories, system parameters, initial conditions, external disturbances, etc. But in practice, it is difficult to meet the requirements of strict repetitiveness. Non-strict repetitiveness means that a system which runs repeatedly exhibits different dynamic characteristics during each run. For example, control the robot manipulator to continuously draw different circles or carry standard parts with different mass (Xu, 1997; Chen et al., 2002). In addition, systems are generally subject to state disturbance, measurement noise, input disturbance and so on (Saab, 2001; Ahn et al., 2009). Under the condition of non-strict repetitiveness, the ILC design based on strict repetitiveness cannot achieve the prefect control effect, and even make the system divergent.

In fact, non-repetitive or iteration-varying problems can be generally divided into two situations: with known variation pattern or unknown variation pattern (Zhou et al., 2015). In the case of unknown variation pattern, robust ILC is mainly used to overcome the influence of non-repetitiveness (Bu et al., 2011; Yu et al., 2017). In view of the known variation pattern, many researchers have exploited the high-order internal model (HOIM) along the iteration axis to describe the iteration-varying pattern of the system and design the iterative learning control law, to completely eliminate the effects of non-repetitiveness (Yu et al., 2019). The internal model principle provides a theoretical basis for the system to perfectly track the reference signals without steady-state errors (Francis et al., 1975). Chen et al. (2002) analysed the influence of external disturbance on learning performance based on HOIM, and the internal model principle in the iteration domain is used to design the iterative learning control algorithm. HOIM can be also used to describe the iteration-varying desired trajectories of the system, and the internal model coefficients are used to design the learning gain parameters, which makes it satisfy the convergence condition of the contraction mapping method (Liu et al., 2010). The HOIM is introduced to the continuous-time nonlinear dynamic systems in (Yin et al., 2010), in which the iteration-varying unknown parameter is depicted by HOIM and the adaptive ILC (AILC) method is used to guarantee the convergence of the tracking errors. AILC scheme developed hitherto is designed by means of Lyapunov theory (Yu et al., 2012; Chi et al., 2015). HOIM-based ILC is introduced into the discrete-time systems by Zhou et al. (2015), and the zero-error tracking along the iteration direction is ensured with the contraction mapping method.

However, the existing HOIM methods all assume that the used HOIM is invariant in the direction of iteration, which means, the change rule of non-repetitiveness factors with iteration is invariant, which is also difficult to be fully satisfied in reality. For example, the change rules of reference trajectories or system parameters between successive iterations may be different. Therefore, it is of great significance to consider the HOIM which is iteration-varying. In the case that the HOIM varies with iteration, there have been no research about how to use the known change rule to define and describe the iteration-varying HOIM, which is used to make the system tracking errors asymptotically convergent along the iteration axis. In addition, the applications of HOIM-based ILC in discrete-time systems and nonlinear systems are relatively few, but discrete-time nonlinear systems are widespread in practice.

Therefore, this paper considers a discrete-time nonlinear system with unknown time-iteration-varying parameters, in which the change rule of the unknown parameters with iteration is expressed as an iteration-varying HOIM, and the appropriate control law is designed by using the discrete adaptive learning control method, so that the system tracking errors converge asymptotically to zero along the iteration axis. The contributions of this article are presented as: ⅰ) The HOIM for iteration-dependent parameters is considered as iteration-varying, and the dimension of the HOIM matrix is uniformed by choosing an upper bound for iteration-varying orders of the HOIM. ⅱ) The AILC scheme is applied to the discrete-time nonlinear system with iteration-varying parameters described by an iteration-varying HOIM.

This paper is organized as follows. In Section 2, the problem formulation is given. Next, the iteration-varying HOIM-based adaptive ILC scheme is designed in Section 3. Then, the learning convergence is proved in Section 4. The proposed adaptive ILC scheme is applied in an illustrative example in Section 5. Finally, the conclusion is elaborated in Section 6.

2. PROBLEM FORMULATION

In this paper, we consider a discrete-time nonlinear system as follows:

$$x_k(t+1)=\theta_k(t)\xi(x_k(t),t)+b_k(t)u_k(t) \tag{1}$$

where $k\in Z^+$denotes the k-th iteration and $t\in\{0,1,\dots,T\}$ is the discrete time index. $x_k(t)\in R$ is the measurable state with random initial value $x_k(0)$. $u_k(t)\in R$ is the system input. $\theta_k(t)\in R$ is the unknown time-iteration-varying parameter. $\xi(x_k(t),t)$ is the known nonlinear function. $b_k(t)\in R$ is the iteration-varying gain of the system input. The dimension of the system is one.

The reference trajectory in the k-th iteration is denoted as $y_k^r(t)$.

We have the following assumptions.

Assumption 1 $b_k(t)$ are bounded and non-zero for all $t\in\{0,1,\dots,T\}$.

Assumption 2 $x_k(0)$ and $y_k^r(t)$ are uniformly bounded for all $t\in\{0,1,\dots,T\}$ and $k\in Z^+$, which means $|x_k(0)|\le\alpha_1$ and $|y_k^r(t)|\le\alpha_2$, where α_1,α_2 are constants.

Assumption 3 $\xi(x_k(t),t)$ satisfies the linear growth condition, that is, $\forall t\in\{0,1,\dots,T\}$ and $\forall k\in Z^+$, we have

$$|\xi(x_k(t),t)|\le\beta_1+\beta_2|x_k(t)| \tag{2}$$

where $0<\beta_1<\infty$ and $0<\beta_2<\infty$ are positive constants.

Assumption 4 The time-iteration-varying unknown parameter $\theta_k(t)$ is bounded, and is generated by a general iteration-varying HOIM as

$$\theta_k(t)=h_1(k)\theta_{k-1}(t)+h_2(k)\theta_{k-2}(t)+\cdots+h_{m(k)}(k)\theta_{k-m(k)}(t) \tag{3}$$

where $m(k)$ is the iteration-varying order of the HOIM and $h_l(k), l=1\cdots m(k)$ are coefficients of the iteration-varying HOIM, k is the iteration number.

Remark 1 In practice, it is possible that at the *k*-th iteration, the unknown factor $\theta_k(t)$ is related to the process of the previous m iterations, while at the $(k+1)$-th iteration, the unknown factor $\theta_{k+1}(t)$ is related to the process of the previous n iterations, that is, the order $m(k)$ of the HOIM will change with iteration. In addition, it is also possible that the correlation relationship changes, that is, the coefficients of HOIM changes.

Define $\boldsymbol{\theta}_k(t)=[\theta_{k+1-m(k)}(t)\cdots\theta_{k-1}(t)\ \ \theta_k(t)]^{\mathrm{T}}$, and

$$\boldsymbol{A}(k)=\begin{bmatrix}0&1&0&\dots&0&0\\0&0&1&\cdots&0&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&0&1\\h_{m(k)}(k)&h_{m(k)-1}(k)&h_{m(k)-2}(k)&\cdots&h_2(k)&h_1(k)\end{bmatrix} \tag{4}$$

If m(k) is a positive constant, the following relationship can be easily obtained

$$\boldsymbol{\theta}_k(t)=\boldsymbol{A}(k)\boldsymbol{\theta}_{k-1}(t)=\cdots=\textstyle\prod_{i=0}^{k-1}\boldsymbol{A}(k-i)\,\boldsymbol{\theta}_0(t) \tag{5}$$

where $\boldsymbol{\theta}_0(t)=[\theta_{-m(k)+1}(t)\cdots\ \theta_0(t)]^{\mathrm{T}}$ is the time-varying iteration-invariant vector consisting of the unknown basis functions of HOIM and the unknown basis functions are linearly independent.

But when $m(k)$ is iteration-varying, $m(k)$ may be different at each iteration, so that the above HOIM matrixes $\boldsymbol{A}(k)$ $(k\in Z^+)$ cannot be multiplied directly. In practice, there always exists an upper bound M for the order of HOIM such that $m(k)\le M$. We will utilize this relationship to uniform the dimension of $\boldsymbol{A}(k)$.

Then we have

$$\begin{cases}h_{m(k)+1}(k)=\cdots=h_M(k)=0, & m(k)<M\\h_M(k)\neq0, & m(k)=M\end{cases} \tag{6}$$

Redefine $\boldsymbol{\delta}_k(t)=[\theta_{k+1-M}(t)\cdots\theta_{k-1}(t)\ \ \theta_k(t)]^{\mathrm{T}}$, and

$$\boldsymbol{\Lambda}(k)=\begin{bmatrix}0&1&0&\dots&0&0\\0&0&1&\cdots&0&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&0&\cdots&0&1\\h_M(k)&h_{M-1}(k)&h_{M-2}(k)&\cdots&h_2(k)&h_1(k)\end{bmatrix} \tag{7}$$

Then according to (5) and (7), we can easily obtain

$$\boldsymbol{\delta}_k(t) = \boldsymbol{\Lambda}(k)\boldsymbol{\delta}_{k-1}(t) = \cdots = \prod_{i=0}^{k-1} \boldsymbol{\Lambda}(k-i)\, \boldsymbol{\delta}_0(t) \quad (8)$$

where $\boldsymbol{\delta}_0(t) = [\theta_{-M+1}(t) \cdots\ \theta_0(t)]^{\mathrm{T}}$ is the generalized time-varying iteration-invariant vector consisting of the unknown basis functions. Define the last row of matrix $\prod_{i=0}^{k-1} \boldsymbol{\Lambda}(k-i)$ as $\boldsymbol{a}_k = [a_{1,k}\ \ a_{2,k} \cdots a_{M,k}]$, and the following relationship can be obtained according to (8)

$$\theta_k(t) = \boldsymbol{a}_k \boldsymbol{\delta}_0(t) \quad (9)$$

Remark 2 To make HOIM neutrally stable, $h_l(k), l = 1 \cdots M$ are the coefficients of a stable characteristic polynomial $P(z) = z^M - h_1(k)z^{M-1} \cdots - h_M(k)$, where all roots of its characteristic equation lie inside the unit circle or the roots with unit modulus are single.

Remark 3 From (9), we divide the unknown time-iteration-varying unknown parameter into the product of a known iteration-varying vector and unknown time-varying iteration-invariant basis function; thus, the learning objective can be shifted from $\theta_k(t)$ to $\boldsymbol{\delta}_0(t)$ that is time-varying only (Yu et al., 2017).

Denote $e_k(t) = y_k^r(t) - x_k(t)$ and $e_k(t+1) = y_k^r(t+1) - x_k(t+1)$. According to (1) and (9), we obtain that

$$\begin{aligned} e_k(t+1)/b_k(t) &= [y_k^r(t+1) - \boldsymbol{a}_k \boldsymbol{\delta}_0(t)\xi(x_k(t),t)]/b_k(t) - u_k(t) \\ &= \frac{y_k^r(t+1)}{b_k(t)} - \Phi_k(t)\varphi(t) - u_k(t) \end{aligned} \quad (10)$$

where $\Phi_k(t) = \boldsymbol{a}_k \xi(x_k(t),t)\ /\ b_k(t)$ and $\varphi(t) = \boldsymbol{\delta}_0(t)$, and the argument $x_k(t)$ is omitted where no ambiguity arises.

The control objective is to design a control input $u_k(t)$ ($t \in \{0,1,\dots,T-1\}$), so that the actual system output $x_k(t)$ tracks the reference trajectory $y_k^r(t)$, asymptotically as the iteration number $k \to \infty$.

3. ILC SCHEME DESIGN

The designed controller is as follows

$$u_k(t) = y_k^r(t+1)\ /\ b_k(t) - \Phi_k(t)\hat{\varphi}_k(t) \quad (11)$$

where $\hat{\varphi}_k(t)$ is the estimate of $\varphi_k(t)$ at *k*-th iteration. Based on the least square method, the updating law for $\hat{\varphi}_k(t)$ is designed as

$$\hat{\varphi}_k(t) = \hat{\varphi}_{k-1}(t) - P_{k-1}(t)\Phi_{k-1}^{\mathrm{T}}(t)e_{k-1}(t+1)\ /\ b_k(t) \quad (12)$$

where $P_{k-1}(t)$ is a positive-definite learning gain matrix and is updated iteratively as

$$P_{k-1}(t) = P_{k-2}(t) - \frac{P_{k-2}(t)\Phi_{k-1}^{\mathrm{T}}(t)\Phi_{k-1}(t)P_{k-2}(t)}{1+\Phi_{k-1}(t)P_{k-2}(t)\Phi_{k-1}^{\mathrm{T}}(t)} \quad (13)$$

Remark 4 The initial value $P_{-1}(t)$ and $\hat{\varphi}_0(t)$ can be given arbitrarily.

Remark 5 The traditional discrete-time adaptive control method which updates point by point in the time domain is not applicable to this problem because the uncertain parameters are discrete-time-varying and a prefect tracking is required on a finite time interval. In addition, uncertain parameters are iteration-varying, the discrete adaptive ILC method proposed in (Chi et al., 2008) is also not applicable to this problem. Since $\theta_k(t)$ satisfies a certain HOIM in the iteration domain, the proposed parameter learning law is a format of parallel updating of a group of vectors.

4. LEARNING CONVERGENCE ANALYSIS

Theorem 1 For discrete-time nonlinear systems (1) with assumptions 1-4, the proposed AILC law (11) with learning updating law (12) and (13) can guarantee that the tracking error $e_k(t)$ converges to zero asymptotically as iteration number *k* approaches infinity for all $t \in \{1,2,\cdots,T\}$, that is,

$$\lim_{k\to\infty} e_k(t) = 0, \forall t \in \{1,2,\cdots,T\} \quad (14)$$

Proof This proof consists of two parts.

Substituting (11) into (10), it can be obtained that

$$e_k(t+1)\ /\ b_k(t) = \Phi_k(t)\left(\hat{\varphi}_k(t) - \varphi(t)\right) = \Phi_k(t)\tilde{\varphi}_k(t) \quad (15)$$

Then

$$\begin{aligned} \tilde{\varphi}_k(t) &= \hat{\varphi}_k(t) - \varphi(t) \\ &= \tilde{\varphi}_{k-1}(t) - P_{k-1}(t)\Phi_{k-1}^{\mathrm{T}}(t)e_{k-1}(t+1)\ /\ b_k(t) \end{aligned} \quad (16)$$

According to matrix inverse lemma (Goodwin et al., 1984), we have

$$P_{k-1}^{-1}(t) - P_{k-2}^{-1}(t) = \Phi_{k-1}^{\mathrm{T}}(t)\Phi_{k-1}(t) \quad (17)$$

Part 1 The boundedness of $\hat{\varphi}_k(t)$

Define the Lyapunov function at the *k*-th iteration as

$$E_k(t) = \tilde{\varphi}_k^T(t)P_{k-1}^{-1}(t)\tilde{\varphi}_k(t) \quad (18)$$

where $E_k(t)$ is non-negative because $P_{k-1}(t)$ is a positive-definite matrix and its inverse matrix is positive-definite.

The difference of $E_k(t)$ along the iteration axis is

$$\begin{aligned} \Delta E_k(t) = E_k(t) - E_{k-1}(t) &= \tilde{\varphi}_k^T(t)P_{k-1}^{-1}(t)\tilde{\varphi}_k(t) \\ &-\tilde{\varphi}_{k-1}^T(t)P_{k-2}^{-1}(t)\tilde{\varphi}_{k-1}(t), k \in Z^+ \end{aligned} \quad (19)$$

Substituting (16) and (17) into (19), it can be derived that

$$\begin{aligned} \Delta E_k(t) &= \left(\tilde{\varphi}_{k-1}(t) - P_{k-1}(t)\Phi_{k-1}^{\mathrm{T}}(t)e_{k-1}(t+1)/b_k(t)\right)^T \\ &\times P_{k-1}^{-1}(t)\left(\tilde{\varphi}_{k-1}(t) - P_{k-1}(t)\Phi_{k-1}^{\mathrm{T}}(t)e_{k-1}(t+1)/b_k(t)\right) \\ &-\tilde{\varphi}_{k-1}^T(t)P_{k-2}^{-1}(t)\tilde{\varphi}_{k-1}(t) \\ &= \tilde{\varphi}_{k-1}^T(t)\Phi_{k-1}^{\mathrm{T}}(t)\Phi_{k-1}(t)\tilde{\varphi}_{k-1}(t) \\ &-2e_{k-1}(t+1)\Phi_{k-1}(t)\tilde{\varphi}_{k-1}(t)/b_k(t) \end{aligned}$$

$$+\Phi_{k-1}(t)P_{k-1}(t)\Phi_{k-1}^{\mathrm{T}}(t)(e_{k-1}(t+1)/b_k(t))^2 \quad (20)$$

Form (12), the following equation can be obtained

$$1-\Phi_{k-1}(t)P_{k-1}(t)\Phi_{k-1}^{\mathrm{T}}(t)=\frac{1}{1+\Phi_{k-1}(t)P_{k-2}(t)\Phi_{k-1}^{\mathrm{T}}(t)} \quad (21)$$

Form (15), (20) and (21), we have

$$\Delta E_k(t)=(\Phi_{k-1}(t)P_{k-1}(t)\Phi_{k-1}^{\mathrm{T}}(t)-1)(e_{k-1}(t+1)\,/\,b_k(t))^2$$

$$=-\frac{(e_{k-1}(t+1)/b_k(t))^2}{1+\Phi_{k-1}(t)P_{k-2}(t)\Phi_{k-1}^{\mathrm{T}}(t)} \quad (22)$$

Therefore, we can get $\Delta E_k(t)\le 0$, which means $E_k(t)\le E_{k-1}(t)\le\cdots\le E_0(t)$.

So

$$\tilde{\varphi}_k^T(t)P_{k-1}^{-1}(t)\tilde{\varphi}_k(t)\le\tilde{\varphi}_{k-1}^T(t)P_{k-2}^{-1}(t)\tilde{\varphi}_{k-1}(t)\le\cdots$$
$$\le\tilde{\varphi}_0^T(t)P_{-1}^{-1}(t)\tilde{\varphi}_0(t) \quad (23)$$

And according to (17), we can get

$$\lambda_{min}(P_{k-1}^{-1}(t))\ge\lambda_{min}(P_{k-2}^{-1}(t))\ge\cdots\ge\lambda_{min}(P_{-1}^{-1}(t))>0 \quad (24)$$

Then, we have

$$\tilde{\varphi}_k^T(t)P_{k-1}^{-1}(t)\tilde{\varphi}_k(t)\ge\lambda_{min}\left(P_{k-1}^{-1}(t)\right)\parallel\tilde{\varphi}_k(t)\parallel^2$$
$$\ge\lambda_{min}\left(P_{-1}^{-1}(t)\right)\parallel\tilde{\varphi}_k(t)\parallel^2>0 \quad (25)$$

$$\tilde{\varphi}_k^T(t)P_{k-1}^{-1}(t)\tilde{\varphi}_k(t)\le\tilde{\varphi}_0^T(t)P_{-1}^{-1}(t)\tilde{\varphi}_0(t)$$
$$\le\lambda_{max}(P_{-1}^{-1}(t))\parallel\tilde{\varphi}_0(t)\parallel^2 \quad (26)$$

From (25) and (26), the following equation is true

$$\parallel\tilde{\varphi}_k(t)\parallel^2\le\kappa\parallel\tilde{\varphi}_0(t)\parallel^2 \quad (27)$$

where $\kappa=\frac{\lambda_{max}(P_{-1}^{-1}(t))}{\lambda_{min}(P_{-1}^{-1}(t))}$ is a constant.

Due to $\varphi_0(t)$ and $\tilde{\varphi}_0(t)$ is bounded, therefore, $\parallel\tilde{\varphi}_k(t)\parallel$ is bounded, which means $\tilde{\varphi}_k(t)$ is bounded for all $t\in\{1,2,\cdots,T\}$ and $k\in Z^+$.

Part 2 The asymptotical convergence of tracking error $e_k(t)$

To sum both sides of equation (22) finitely, we can get

$$E_k(t)=E_0(t)-\sum_{j=1}^{k}\frac{\left(e_{j-1}(t+1)/b_k(t)\right)^2}{1+\Phi_{j-1}(t)P_{j-2}(t)\Phi_{j-1}^{\mathrm{T}}(t)} \quad (28)$$

Because $E_k(t)$ and $E_0(t)$ is non-negative and bounded for all $t\in\{1,2,\cdots,T\}$ and $k\in Z^+$, it is easily derived that

$$\lim_{k\to\infty}\sum_{j=1}^{k}\frac{\left(e_{j-1}(t+1)/b_k(t)\right)^2}{1+\Phi_{j-1}(t)P_{j-2}(t)\Phi_{j-1}^{\mathrm{T}}(t)}<\infty \quad (29)$$

That is

$$\lim_{k\to\infty}\frac{e_{k-1}(t+1)/b_k(t)}{(1+\Phi_{k-1}(t)P_{k-2}(t)\Phi_{k-1}^{\mathrm{T}}(t))^{1/2}}=0 \quad (30)$$

According to **Assumption 1**, we can obtain that $\frac{1}{b_k(t)}$ is bounded, which can be expressed as $|\frac{1}{b_k(t)}|\le\hat{\beta}$, where $\hat{\beta}$ is a constant.

From **Assumptions 2** and **3**, the following relationship can be derived that

$$|\xi(x_k(t),t)|\le\beta_1+\beta_2(|e_k(t)|+|y_k^r(t)|)$$
$$\le\bar{\beta}+\beta_2 max_{\tau=1,\cdots t}|e_k(\tau)| \quad (31)$$

where $\bar{\beta}=\beta_1+\beta_2 e_k(0)+\beta_2|y_k^r(t)|$ is bounded, and we can also know $|\xi(x_k(t),t)|$ is bounded.

From **Assumption 4**, $\boldsymbol{a}_k$ is bounded, which can be written as $\parallel\boldsymbol{a}_k\parallel\le\hat{\alpha}$, where $\hat{\alpha}$ is a constant.

Therefore, we can have

$$\left(1+\Phi_{k-1}(t)P_{k-2}(t)\Phi_{k-1}^{\mathrm{T}}(t)\right)^{\frac{1}{2}}$$
$$\le 1+\lambda_{max}(P_{k-2}(t))|\frac{1}{b(t)}|\cdot\parallel\boldsymbol{a}_k\parallel\cdot|\xi(x_k(t),t)|$$
$$\le 1+\hat{\alpha}\hat{\beta}\lambda_{max}(P_{k-2}(t))|\xi(x_k(t),t)| \quad (32)$$

Using key technology lemma (Goodwin et al., 1984), and from (30), (31) and (32), we can get that $\lim_{k\to\infty}e_{k-1}(t+1)=0$, that is,

$$\lim_{k\to\infty}e_k(t)=0 \quad (33)$$

which implies the asymptotical convergence of tracking error $e_k(t)$ as *k* goes to infinity for all $t\in\{1,2,\cdots,T\}$.

5. ILLUSTRATIVE EXAMPLE

Consider a discrete-time nonlinear system as follows

$$x_k(t+1)=\frac{\theta_k(t)\sin\left(x_k^2(t)\right)}{10}+b_k(t)u_k(t) \quad (34)$$

$$y_k^r(t)=1+2\sin(0.05\pi t) \quad (35)$$

where $t\in\{1,2,\cdots,100\}$ is the sampling instant , $\frac{\sin\left(x_k^2(t)\right)}{10}$ is the known nonlinear function, and $b_k(t)$ is randomly varying in the interval $[-1,0)\cup(0,1]$.

The unknown time-iteration-varying parameter $\theta_k(t)$ is generated by an iteration-varying HOIM as follows

$$\begin{cases}\theta_k(t)=-2\cos(0.05)\,\theta_{k-1}(t)-\theta_{k-2}(t), \\ \quad k\ is\ an\ odd\ number \\ \theta_k(t)=-\theta_{k-1}(t)-2\cos(0.05)\,\theta_{k-2}(t)-\theta_{k-3}(t), \\ \quad k\ is\ an\ even\ number\end{cases} \quad (36)$$

where the upper limit of the iteration-varying order M is 3. The initial time-varying parameters for $\theta_k(t)$ are $\theta_{-2}(t)=0.01sin(t)$, $\theta_{-1}(t)=1.4sin(0.02\pi t)$, and $\theta_0(t)=$

$0.3cos(0.01\pi t)$. The magnitude variation profile of the parameters generated by the iteration-varying HOIM is shown in Fig.1.

The system initial condition $x_k(0)(k \in Z^+)$ is randomly varying in the interval $[-0.5,0) \cup (0,0.5]$.

Define the maximum absolute tracking error as

$$sup|e_k| = \sup_{t\in\{1,2,\ldots,100\}} |y_k^r(t) - x_k(t)| \quad (37)$$

Using the designed AILC control law (11) and parameter learning law (12) and (13), the tracking error $sup|e_k|$ is shown in Fig. 2. The maximum absolute tracking errors along the iteration axis converge to zero asymptotically.

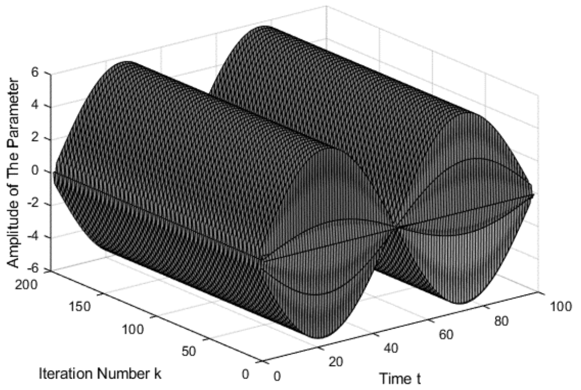

Fig. 1 Time-iteration-varying parameter $\theta_k(t)$

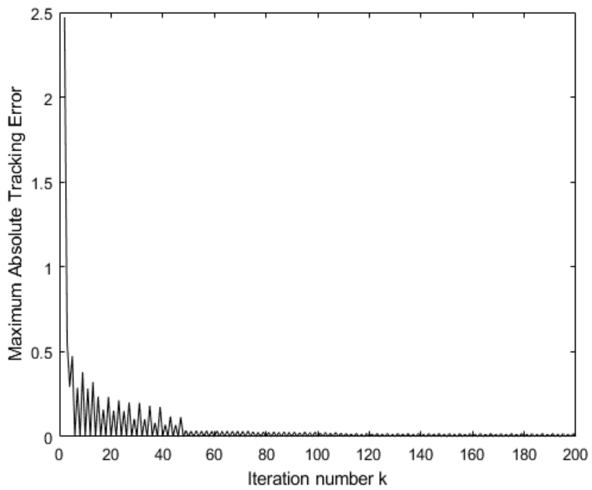

Fig. 2. The maximum absolute tracking error $sup|e_k|$

The effectiveness of the proposed algorithm can be clearly seen for the discrete-time nonlinear system with random initial condition and time-iteration-varying unknown parameters.

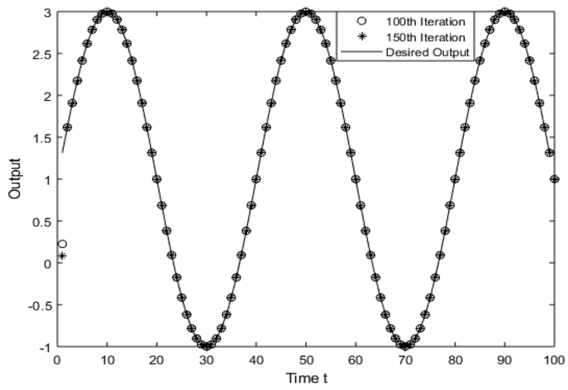

Fig. 3. The system output at 100th and 150th iteration

According to Fig. 3, it can be seen that, although there is some deviation in the initial state, for the discrete-time system, prefect tracking can be always achieved at all discrete-time points after iterative learning.

6. CONCLUSIONS

We have developed an iteration-varying HOIM based adaptive ILC for the discrete-time nonlinear systems with iteration-varying initial condition and time-iteration-varying unknown parameters. Both the theoretical analysis and the simulation results demonstrate the effectiveness of the proposed method. We have extended the iteration-invariant HOIM based ILC to the iteration-dependent HOIM based ILC for discrete-time systems, and it can be also further extended to continuous-time systems. In addition, we just consider the HOIM of time-iteration-varying parameters is iteration-varying, but the HOIM for non-repetitive reference trajectories could also be iteration-varying, which means the change rule of the reference trajectories is expressed as the iteration-varying HOIM. It can be considered to design the learning gain law by using the λ-normal-based contraction mapping method.

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