Online offset optimization for urban traffic network with distributed model predictive control ⋆

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Abstract:
This paper proposes a distributed control framework to optimize the offset for a path in a traffic network with arbitrary topology. Each intersection along the target path applies the model predictive control to optimize their own phase sequence and green splits with the objective of minimizing the sum of queue lengths. The first intersection on this path is regarded as the main intersection and responsible for optimizing the start green time and duration of the first phase on this path with a weighted objective according to the real-time traffic information, while the other intersections take the constraints of offset imposed by intersections ahead into consideration. The signal cycles of these intersections are fixed but allowed to be different. For computation efficiency, the nonlinear optimization problem is approximately reformulated as a mixed-integer linear programming problem. Numerical experiments on a calibrated network of Caohejing District in Shanghai indicate that our proposed method can effectively decrease delay time and waiting time especially at medium and high traffic loads.

Keywords: Traffic signal control; Online offset; Model predictive control; Distributed control framework; Mixed-integer linear programming

1. INTRODUCTION

Traffic signals were first implemented in the urban traffic network to guarantee no conflict movements at the same time. With the increasing traffic demand and limited road resources, besides the safety consideration, they are expected to improve the traffic flow and alleviate congestion with appropriate signal settings, including the signal cycle length, phase sequences, green time and intersection-to-intersection offsets.

Great efforts have been made to study the settings of signalized intersections (Taale, 2002; Aboudolas et al., 2009; Zhou et al., 2014), which come to the conclusions that: 1) signal settings play a key role in shaping traffic streams and network efficiency in general; 2) the coordination between adjacent intersections with offsets shows advantage over decentralized signal control in terms of increasing traffic throughput. In the literature, Little et al. (1981) proposed a off-line MAXBAND program for setting arterial signals to achieve maximal bandwidth, that is, the length of the time window a vehicle can travel along without stop by red lights. Ying-Ying et al. (2008) established off-line offset optimization models by considering the relationship between the arriving and departing vehicles with the objective to minimize the delay of vehicles. However, due to the random nature of traffic system, the off-line signal settings with one or several fixed numbers of offsets can not express the volatility and random of traffic flow movement.

Some researchers turn to optimize the offsets with online traffic information. In the well-known practical traffic signal control systems SCOOT(Robertson and Bretherton, 1991) and SCATS(Sims, 1979), the online offsets are adjusted with small values such that the timing plans evolve to match the measured traffic data. Gong et al. (2009) adopted the nonlinear cointegration theory and model to optimize real time green light starting time based on a series of strict applicability tests with the practical data. Gomes (2015) proposed a new formulation of bandwidth maximization problem in which a linear program in the case of pulse arrival functions is developed to reduce the computational complexity. These approaches can provide effective online offsets for arterial roads but are incapable of generalizing to networks of arbitrary topology, while in urban traffic network, it is a common situation that several paths not along the arterial roads are with high traffic loads due to the recreational or social activities.

To accommodate networks with arbitrary topology, Coogan et al. (2017) formulated the offset optimization problem as a quadratically constrained quadratic program(QCQP) with the objective of minimizing the queues at all intersec-

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tions in the network, where the cycle lengths of all intersections are assumed to be the same and the intersections are undersaturated. Ouyang et al. (2019) further presented a novel algorithm to solve the above QCQP problem to near-global optimality on a large-scale by using a tree decomposition reduction to relax the nonconvex problem and using randomized rounding to recover a near-global solution. This centralized structure of offset optimization by minimizing the queues of all links puts great burden on the online computing. In addition, the assumption of the same cycle length is unreasonable for the intersections with different link length and number of lanes.

In this paper, we propose a distributed framework to optimize the offset for a path with arbitrary topology based on the model predictive methodology, where the signal cycles for intersections along this path are fixed but allowed to be different. The target path can be with large demands or the major roads taken by vehicles in the network. During the online optimization, the first intersection along the target path is regarded as the main intersection and optimizes its phase sequence and green splits with the objective of minimizing the weighted sum of queues. The following intersections independently optimize the phase sequences and green splits in their own signal cycles by considering the offset constraints from the intersections ahead. For computation efficiency, the optimization problems are approximately reformulated as mixed integer programming problem. The performances of our proposed method are evaluated via simulation in Caohejing District of Shanghai.

The organization of this paper is as follows. Section 2 provides notations for network description and formulates the model predictive control for a isolated intersection. The distributed control framework and control process to optimize the offset for a path online is developed in Section 3. Numerical results are presented and discussed in Section 4. Finally, conclusions are provided in Section 5.

2. MODEL PREDICTIVE CONTROL FOR A ISOLATED SIGNALIZED INTERSECTION

2.1 Traffic network description

Consider a traffic network composed of a number of intersections and links(roads) whose sets are denoted as N and C, respectively. Each intersection $n \in N$ consists of several input links. Each link $l \in C$ has a number of upstream links Γ_l^{-1} and downstream links Γ_l. There are certain phases associated with each intersection, where each phase is corresponding to a connection between one input link and one output link of this intersection. As shown in Fig. 1, a connection between input link l to output link m is denoted as phase (l, m). One or more phases that can occur simultaneously without conflict compose a stage for a intersection. The sum of the durations of all stages equals to the signal cycle length.

In the traffic network, all the vehicles are assumed to follow their predetermined paths without rerouting until arriving at their destinations. Their path information can be obtained by the vehicle-to-infrastructure(V2I) technology in real-time using several platforms (Dey et al., 2016), such as Dedicated Short Range Communications (DSRC), 4G,

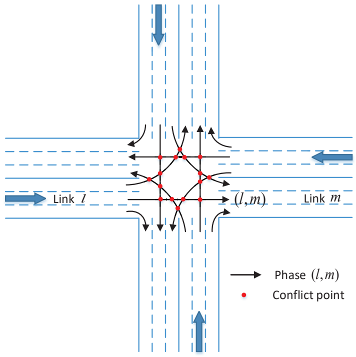

Fig. 1. A typical intersection with 12 available phases

Wi-Fi or Bluetooth. In this paper, we present a path p by an ordered collection of links or phases, where $l \in p$ denotes path p goes through link l; $(l, m) \in p$ expresses path p goes through the phase (l, m).

2.2 Model predictive control for a isolated intersection

Model predictive control(MPC) is a closed-loop control methodology by solving the optimization problem online in a rolling horizon way. At each control step, based on current system information, a optimal control sequence over a finite predictive horizon is obtained, but only the first control step of the optimal solution is implemented. At next control step, the optimization problem is resolved based on new initial condition. This framework has the advantage in handling the deviation between the predicted process and actual behavior due to system-model mismatch or disturbances.

Due to the above characteristics, model predictive control has been extensively studied for intersection control(Lin et al., 2011; Zhou et al., 2014) because the traffic system is essentially random. The optimization variables in the literature are mainly the green splits of ordered phases. In this subsection, for a isolated intersection with a fixed signal cycle, besides the green splits, we simultaneously optimize its phase sequence in a rolling way.

Take intersection n with four input links as shown in Fig. 1 for example. It contains a number of phases (12 phases in Fig. 1) whose set is denoted as P_n. Among these phases, there are several conflict points as indicated in Fig. 1, each of which corresponds to a pair of phases that can not be activated simultaneously. Write the set of conflict points in intersection n as Θ_n and $[(l, m), (l', m')] \in \Theta_n$. Denote $s_{l,m}(t)$ as the signal value for phase (l, m) at time step t and it is a binary variable, where 1 indicates phase (l, m) is activated and 0 is not. Then at time step t, each conflict point in Θ_n is presented by the following constraint:

$$s_{l,m}(t) + s_{l',m'}(t) \leq 1 \tag{1}$$

Considering the frequent switching between phases tends to confuse the drivers and cause potential danger in reality, in every signal cycle with length T_n, the green time for each phase is constrained to be consecutive by

$$\sum_{t=0}^{\tau_n - 2} |s_{l,m}(t_0 + t + 1) - s_{l.m}(t_0 + t)| \leq$$
$$2[1 - s_{l.m}(t_0) \cdot s_{l.m}(t_0 + \tau_n - 1)] \tag{2}$$

where $\tau_n = T_n/\Delta t$ and Δt is the time interval. Note that T_n is the integer multiples of time interval Δt.

With the signal value for phase (l, m), according to the store-and-forward model, the traffic flow for phase (l, m) is propagated by

$$x_{l,m}(t+1) = x_{l,m}(t) + \sigma_{l,m}(t)\lambda_l(t)\Delta t - y_{l,m}(t) \quad (3)$$

$$y_{l,m}(t) = \min(x_{l,m}(t), c_{l,m}(t)s_{l,m}(t)) \quad (4)$$

where $x_{l,m}(t)$ is the number of vehicles on link l heading to link m at time step t, $y_{l,m}(t)$ denotes the number of vehicles driving from link l to m within time step t, $c_{l,m}(t)$ represents the saturation traffic flow for movement (l, m) within time step t. $\lambda_l(t)$ is the arrival rate of link l within time step t. There have been many approaches developed to estimate short-term traffic demand for signalised links from either the model based or data-driven perspectives in the literature(Smith and Demetsky, 1994; Vigos et al., 2008). In this paper, we adopt the nonparametric regression method proposed by Smith and Demetsky (1994) to estimate the arrival rates. In Eq.(3), $\sigma_{l,m}(t)$ denotes the turning ratio for the phase (l, m) within time step t and $\sum_{m\in\Gamma_l} \sigma_{l,m}(t) = 1$, which can be estimated from the historical path information of arriving vehicles. Denote the control cycle of intersection n as $T_{c,n}$ and prediction horizon as $T_{p,n}$, both of which are the integer multiple of signal cycle T_n and $T_{p,n} \geq T_{c,n}$. Then the signal control of intersection n can be formulated as follows:

$$\min \quad z = \sum_{t=t_0+1}^{t_0+t_f} \sum_{\forall(l,m)} x_{l,m}(t) \quad \text{(I)}$$

$$\begin{aligned}
s.t. \quad & \text{Flow propagation constraints :} \\
& \forall(l,m) \in P_n, t = t_0, \cdots, t_0 + t_f - 1 : Eq.(3-4); \\
& \text{Signal conflict points constraints :} \\
& \forall[(l,m),(l',m')] \in \Theta_n, t = t_0, \cdots, t_0 + t_f^c - 1 : \\
& Eq.(1); \\
& \text{Consecutive green time constraints :} \\
& \forall(l,m) \in P_n, i = 0, 1, \cdots, T_{c,n}/T_n : \\
& \sum_{t=i\tau_n}^{(i+1)\tau_n - 2} |s_{l,m}(t_0+t+1) - s_{l,m}(t_0+t)| \leq \\
& 2[1 - s_{l,m}(t_0 + i\tau_n) \cdot s_{l,m}(t_0 + (i+1)\tau_n - 1)] \\
& \text{Signal settings constraints from } T_{c,n} \text{ to } T_{p,n} : \\
& \forall(l,m) \in P_n, i = 0, \cdots, \frac{T_{p,n} - T_{c,n}}{T_n}, j = 0, \cdots, \tau_n : \\
& s_{l,m}(t_f^c - \tau_n) = s_{l,m}(t_f^c + i\tau_n + j)
\end{aligned}$$

where $t_f^c = T_{c,n}/\Delta t$ and $t_f = T_{p,n}/\Delta t$. The objective function z aims to minimize the sum of the queue lengths in the input links of intersection n. The above optimization problem I will be solved every control cycle with new initial traffic states and only signal settings in the first cycle are implemented.

3. ONLINE OFFSET OPTIMIZATION WITH DISTRIBUTED MODEL PREDICTIVE CONTROL

In this section, we develop a distributed control framework to optimize the offset for a path online based on the model predictive control of isolated intersection and

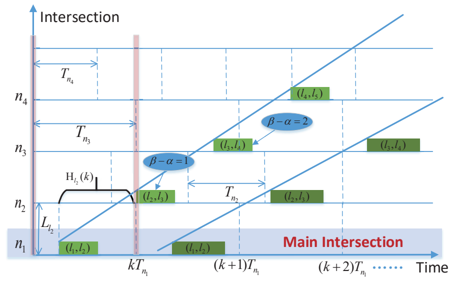

Fig. 2. The illustration of dynamic offsets for one path

approximately reformulate the optimization problem for the controlled intersection to a mixed-integer linear programming problem for computation efficiency.

3.1 Distributed control framework

A path is generally connected by a number of intersections with continuously ordered phases. Denote the intersections on path p as $\{n_i, i = 1, 2, \cdots\}$ in order and the corresponding phases as $\{(l_i, l_{i+1}), i = 1, 2, \cdots\}$. We regard the first intersection n_1 as the main intersection. To be specific, intersection n_1 optimizes its signal settings according to the optimization problem I in a rolling way with the following objective:

$$z = \sum_{t=t_0+1}^{t_0+t_f} \delta_{l_1,l_2} x_{l_1,l_2}(t) + \sum_{t=t_0+1}^{t_0+t_f} \sum_{\forall(l,m)/(l_1,l_2)} x_{l,m}(t) \quad (5)$$

where (l_1, l_2) is the first phase on path p and $\delta_{l_1,l_2} \geq 1$. Then its green time of phase (l_1, l_2) at each signal cycle and the corresponding start time are obtained. Write the green time of phase (l_1, l_2) at kth cycle as $g_{l_1,l_2}(k)$ and its start time as $b_{l_1,l_2}(k)$ relative to the reference time T_0. Note that the reference time is the common signal cycle start time of all intersections on path p.

Intersection n_1 will send the optimized setting of phase (l_1, l_2) to intersection n_2, i.e., $b_{l_1,l_2}(k)$ and $g_{l_1,l_2}(k)$. Accordingly, intersection n_i, $i = 2, 3, \cdots$ will send these information to its adjacent intersection n_{i+1} but along with the travel time of link l_i which is set as the offset between intersection n_i and n_{i+1} in this paper. The real-time travel time of link l_i can be estimated by the Bureau of Public Roads (BPR) function(Riemann et al., 2015) based on the current traffic states by

$$H_{l_i}(k) = L_{l_i}/[u_{l_i}(1 - a(Q_{l_i}(k)/\bar{Q}_{l_i})^b)] \quad (6)$$

herein, a, b are the model parameters; u_{l_i} is the free-flow speed of link l_i; $Q_{l_i}(k)$ is the total number of vehicles in link l_i at start of kth signal cycle and $\bar{Q}_{l_i}$ is the capacity of link l_i.

The above one-way transmission is conducted after the signal optimization of the first intersection n_1 is finished at its every control cycle. The shades of red in Fig. 2 illustrate the time for information sharing, where the control cycle is equal to the signal cycle, i.e., $T_c = T_{n_1}$. Besides the first intersection n_1, the other intersections need to consider the offset constraints from the main intersection while optimizing signal settings with their own signal cycle. To design the green wave for path p, the phase (l_i, l_{i+1}), $i = 2, \cdots$, needs to be activated by intersection n_i at time

$$t^{n_i}_{l_i,l_{i+1}}(k) = b_{l_1,l_2}(k) + \sum\nolimits_{j=2}^{i} H_{l_j}(k) \tag{7}$$

and the green time length is $g_{l_1,l_2}(k)$. Then there exists the following relationships along the time axis of intersection n_i:

$$\alpha T_{n_i} \le t^{n_i}_{l_i,l_{i+1}}(k) \le t^{n_i}_{l_i,l_{i+1}}(k) + g_{l_1,l_2}(k) \le \beta T_{n_i} \tag{8}$$

where $\alpha, \beta(\alpha \le \beta)$ are two positive integers and

$$t^{n_i}_{l_i,l_{i+1}}(k) - \alpha T_{n_i} < T_{n_i}$$
$$\beta T_{n_2} - t^{n_i}_{l_i,l_{i+1}}(k) + g_{l_1,l_2}(k) < T_{n_i}$$

Due to different values of $t^{n_i}_{l_i,l_{i+1}}(k)$, $g_{l_1,l_2}(k)$ and the signal cycles, there are several cases for the signal optimization of intersection n_i:

- $\beta - \alpha = 1$: The green time of phase (l_i, l_{i+1}) is inside one signal cycle of intersection n_i as illustrated in Fig. 2. Then in the model predictive control for intersection n_i, the signal settings of phase (l_i, l_{i+1}) are constrained by
$$s_{l_i,l_{i+1}}(t) = 1, \quad t = t_1, \cdots, t_2 \tag{9}$$
where
$$t_1 = \text{floor}(\frac{t^{n_i}_{l_i,l_{i+1}}(k)}{\Delta t}), t_2 = \text{floor}(\frac{t^{n_i}_{l_i,l_{i+1}}(k) + g_{l_1,l_2}(k)}{\Delta t})$$
- $\beta - \alpha = 2$: The green time of phase (l_i, l_{i+1}) is across two signal cycles of intersection n_i as illustrated in Fig. 2. If the signal settings of these two signal cycles are controlled in one optimization problem, the constrains of phase (l_i, l_{i+1}) are the same with Eq.(9). Otherwise, For the optimization of αth signal cycle, $s_{l_i,l_{i+1}}(t) = 1$ with $t \in [t_1, t_3]$ where $t_3 = (\alpha + 1)T_{n_i}/\Delta t - 1$; For the optimization of $(\alpha+1)$th signal cycle, $s_{l_i,l_{i+1}}(t) = 1$ in time interval $t \in [t_3 + 1, t_2]$.
- $\beta - \alpha > 2$: The green time of phase (l_i, l_{i+1}) across more than two signal cycles of intersection n_i, which tends to deteriorate the performance of other phases in intersection n_i. In this paper, to avoid this case, we constrain the green time of phase (l_1, l_2) in first intersection n_1 by
$$\sum\nolimits_{t=j\tau_{n_1}}^{(j+1)\tau_{n_1}-1} s_{l_1,l_2}(t) \le \gamma \min(\tau_{n_1}, \tau_{n_2}, \cdots) \tag{10}$$
where $\tau_{n_1} = T_{n_1}/\Delta t$, $j = 0, 1, \cdots, T_{c,n_1}/T_{n_1} - 1$ and $\gamma \in (0, 1)$. These constraints can be add to the optimization problem I for the signal optimization of main intersection n_1.

3.2 Distributed control process for online offset optimization

To sum up, at the beginning of each control cycle of main intersection n_1, the optimization problem I with the added linear constraints Eq.(10) is solved by intersection n_1 to get its signal settings. After getting the solution, intersection n_1 sends the settings of phase (l_1, l_2) that is going to be implemented to intersection n_2. Then intersection n_i, $i = 2, 3, \cdots$, continuously sends these information along with the estimated travel time of links $l_2, l_3, \cdots, l_i$ to next intersection n_{i+1}. Besides main intersection n_1, each intersection makes a record of the green time for exact phase on the path with offset optimization. At next control cycle of main intersection n_1, the whole process will repeat again.

From the perspective of intersection n_i, $i = 2, 3, \cdots$, they optimize the signal settings with their own signal cycles by solving the optimization problem I plus with the linear constraints on phase (l_i, l_{i+1}). The exact formulations of these constraints are determined by the values of $t^{n_i}_{l_i,l_{i+1}}(k)$, $g_{l_1,l_2}(k)$ and the signal cycles as discussed above, such as Eq.(9).

Remark 1. Since each intersection is optimized at the beginning of their own signal cycle, to make sure the optimized setting of the first phase (l_1, l_2) in the main intersection n_1 can be timely catched by following intersections, the implemented signal length(the integer multiples of signal cycle) of the main intersection at its every control cycle needs to be lager than other intersections'.

3.3 Linearization of optimization problem

The above path-based dynamic offset optimization process proposes a high online computing speed requirement. However, the optimization problem I that needs to be solved by each intersection is essentially a mixed-integer programming problem with nonlinear constraints: the minimum function Eq.(4) and Eq.(2) with the absolute term, which is difficult to be efficiently solved by existing optimizers, such as Cplex and Gurobi(Atamtürk and Savelsbergh, 2005). Therefore, it is necessary to reformulate the original problem I to improve the computational efficiency. We first equivalently represent Eq.(2) with several linear constraints:

$$s_{l,m}(t_0 + i) + s_{l,m}(t_0 + i + j) - s_{l,m}(t_0 + i + 1) \le 1 \tag{11}$$

with $j = 2, \cdots, \tau_n-1$, $i = 0, \cdots, \tau_n-j-1$. For clarity, the proof about the equivalence between Eq.(2) and Eq.(11) is omitted here. The minimum function Eq.(4) is relaxed by a set of linear inequalities together with the item to maximize $y_{l,m}(t)$ in the objective. Then the original optimization problem I can be reformulated as a mixed-integer linear programming problem for computation efficiency as follows:

$$\min \quad z = \sum_{t=t_0+1}^{t_0+t_f} \sum_{\forall(l,m)} x_{l,m}(t) - \sum_{t=t_0}^{t_0+t_f-1} \sum_{\forall(l,m)} y_{l,m}(t) \tag{II}$$

$$\begin{aligned}
s.t.\quad & \text{Relaxations of flow propagation constraints :}\\
& \forall(l,m) \in P_n, t = t_0, \cdots, t_0 + t_f - 1 :\\
& y_{l,m}(t) \le x_{l,m}(t)\\
& y_{l,m}(t) \le c_{l,m}(t)s_{l,m}(t)\\
& \text{Signal conflict points constraints :}\\
& \forall[(l,m),(l',m')] \in \Theta_n, t = t_0, \cdots, t_0 + t^c_f - 1 :\\
& Eq.(1);\\
& \text{Consecutive green time constraints :}\\
& \forall(l,m) \in P_n, t = t_0 + i\tau_n, i = 0, 1, \cdots, T_{c,n}/T_n :\\
& s_{l,m}(t + i_1) + s_{l,m}(t + i_1 + i_2) - s_{l,m}(t + i_1 + 1)\\
& \le 1, i_2 = 2, \cdots, \tau_n - 1, \quad i_1 = 0, \cdots, \tau_n - i_2 - 1\\
& \text{Signal settings constraints from } T_{c,n} \text{ to } T_{p,n} :\\
& \forall(l,m) \in P_n,\\
& i = 0, \cdots, \frac{T_{p,n} - T_{c,n}}{T_n}, j = 0, \cdots, \tau_n :\\
& s_{l,m}(t^c_f - \tau_n) = s_{l,m}(t^c_f + i\tau_n + j)
\end{aligned}$$

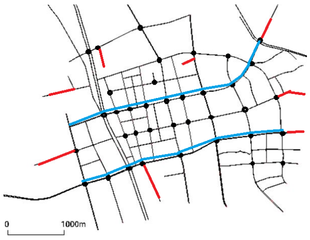

Fig. 3. Road network with 38 signalized intersections and 519 links

4. NUMERICAL STUDY

4.1 Simulation settings

The simulation using the SUMO traffic simulator is performed by a road network from Caohejing District in Shanghai, which is downloaded from the OpenStreetMap. Its simplified network is shown in Fig. 3, containing 38 signalized intersections and 519 links. The red bold links in Fig. 3 denote the origins and destinations which are the entrances or exits of the parking lots, exhibition center or the residential areas in reality. With these origins and destinations, we consider 30 paths in the simulation. Each path is allocated with the low(400 veh/h), medium(700 veh/h) and high(1000 veh/h) traffic loads for performance study, respectively. According to the path distribution, Yishan Road and Caobao Road illustrated by the bold blue lines in Fig. 3 are with the relatively high traffic loads.

In the simulation, we choose fixed-time control (denoted as FTC) as the benchmark, where the possible phases of each signalized intersection are activated in a predetermined periodic way, similar to what is used in common practice. Their cycle length and green splits are determined based on Webster's formula(Webster, 1958) using the average traffic flow. In the simulation their cycle lengths are the integer multiples of time interval Δt (3s) and from 60 seconds to 120 seconds. To make fair comparison, we test our method by comparing the following three methods. Note that only the intersections on the Caobao Road and Yishan Road are controlled and other intersections are under FTC with the same signal settings in each method. All the simulations are performed for one hour.

- Actuated signal control(Taale, 2002)(denoted as ASC): ASC takes into account the actual traffic demand and the presence of vehicles to determine the changes of the traffic signals, where their stage settings and phase sequences are the same with FTC. The presence of vehicles on each lane is detected every 10 seconds by the detectors at the stoplines. The maximum green time for each phase is set to be 40 seconds.
- Distributed signal control with online offset optimization (denoted as DSCO): DSCO is our proposed method as described in Subsection 3.1 and 3.2. The signal cycle length T_u of each controlled intersection is preset and the same with FTC. The control cycle $T_{c,n}$ is equal to the prediction horizon $T_{p,n}$. The weight δ_{l_1,l_2} in the optimization objective for the first intersections on above two controlled roads are set as 2. The parameter γ in the green time length constraints is set as 0.6.

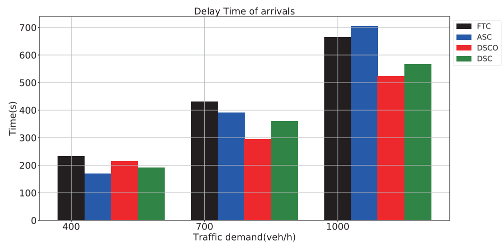

Fig. 4. Average delay time per vehicle over simulation time with low, medium and high traffic loads

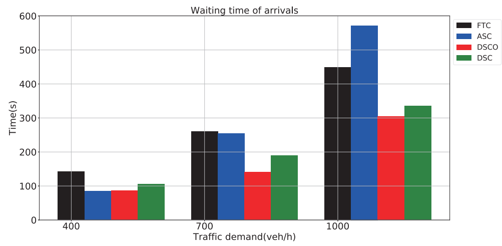

Fig. 5. Average waiting time per vehicle over simulation time with low, medium and high traffic loads

- Decentralized signal control with model predictive control(denoted as DSC): DSC is applied to control the intersections on Yishan Road and Caobao Road separately by solving the optimization problem II at each control cycle $T_{c,n}$ to get their phase sequence and green splits. The signal cycle T_u, control cycle $T_{c,n}$ and prediction horizon $T_{p,u}$ for each intersection are the same with DSCO for comparison.

4.2 Performance study

In Fig. 4 and Fig. 5, we plot the average delay time and waiting time per vehicle under different control methods with low, medium and high traffic loads, respectively. Here the delay time essentially equals to the real travel time minus the ideal travel time which is the time a vehicle travels in free flow speed without signal control. The waiting time is the time of a vehicle whose speed is below 0.1m/s. The average values in Fig. 4 and Fig. 5 are taken over both the simulation time and number of arriving vehicles.

At low loads, it can be easily seen from Fig. 4 and Fig. 5 that ASC provides the shortest delay time and waiting time, and FTC gets the longest. This is due to the fact that under ASC, the traffic states are detected every 10 seconds and the phases with no vehicles are allowed be skipped, which is of common occurrence at low loads, while other methods make decisions of signal settings every signal cycle($\geq 60s$). FTC actives the phases with preset sequence and green time, incapable of considering the random arrivals, which tends to allocate green time to the phase with empty queues. DSCO and DSC also allow the phases to be skipped in the optimization problem, but the deviation between the predicted states and actual behavior makes the signal settings not timely as ASC. Comparing DSCO and DSC, we can find that at low loads, DSC outperforms DSCO in terms of delay time but the performance of waiting time is inverse. This can be explained by that the online offsets optimized in DSCO increase the throughput along Yishan Road and Caobao Road, and decrease the waiting time. However, the optimized offset is very likely not be fully occupied at low traffic loads, which further affects the performance of other phases in the intersections behind on these two paths due to the imposed restrictions.

At medium traffic loads, DSCO yields the best performance in terms of both delay time and waiting time, followed by DSC, ASC and FTC. Compared with FTC and ASC which make the signal timing plans only based on current measured traffic states, DSC optimizes the signal settings using the model predictive control by taking the traffic loads into consideration. DSCO further coordinates the intersections along two main roads to dynamically generate the green wave and improve the network throughput.

At high traffic loads, the performance of different control methods have the same tendency with the medium traffic loads, except the comparison between FTC and ASC, where FTC provides better performance than ASC. This is because the advantage of ASC explained above is gradually weakening as traffic loads increase since the possibility of being idling will be greatly decreased at high traffic loads. Under ASC, the stages with large traffic loads tend to be continuously allocated the green time until the maximum green time, making the vehicles of other stages keep waiting. Another interesting observation from Fig. 4 and Fig. 5 is that the advantage of DSCO at medium traffic loads is more obvious than at high traffic loads in terms of delay and waiting time. This is possibly because at high traffic loads, the vehicles are likely to be stuck due to traffic jam, making the designed offset between two adjacent intersections hard to be realized in reality, while these situations are of rare occurrences at medium loads.

5. CONCLUSION

This paper proposes a distributed framework to optimize the offset for a path with arbitrary topology based on the model predictive methodology, where the signal cycles for intersections along this path are fixed but allowed to be different. During the online control, each intersection separately optimizes the phase sequence and green splits in their own signal cycles with specific objective and constraints, where the first intersection along the target path is regarded as the main intersection with the weighted objective and other intersections with the offset constraints from the intersections ahead. The simulation results on two main roads in Caohejing District of Shanghai show the advantage of our proposed method over other compared methods in terms of delay time and waiting time, especially at medium and high traffic loads.

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