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Repetitive Control of Nonlinear Systems via Feedback Linearization: an Application to Robotics

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Abstract: In this paper, a novel Repetitive Control (RC) scheme for a class of nonlinear systems is presented and discussed. This work generalizes the approach proposed in Biagiotti et al. (2015) where a RC scheme based on the modification of a B-spline reference trajectory has been presented. Also in this case, the generation of the B-splines based on dynamic filters plays a crucial role in the control scheme since it allows to implement a feedforward action that, coupled with an exact feedback linearization and a stabilizing state feedback, makes the RC robustly asymptotically stable. In this manner, the tracking error at the via-points defining the reference trajectory is nullified even if parametric uncertainties on the system model or exogenous (cyclic) disturbances are present. The application to a two-dof robot manipulator shows the effectiveness of the proposed method and its inherent robustness.

Keywords: Iterative learning control, Robotics, Motion Control Systems, Trajectory Tracking and Path Following.

1. INTRODUCTION

In many industrial applications based on robot manipulators, the tasks to be performed are based on the iteration of given motion profiles and are therefore inherently cyclic. For this type of application, the Repetitive Control (RC) approach represents an effective solution to reduce the tracking error over repetitions by learning from previous iterations (Inoue et al. (1981); Hara et al. (1988)). Starting from this consideration, and observing that in practical applications the reference signals are typically defined by means of tools such as Spline, Bezier, Nurbs curves, and other similar functions, in the seminal work of Biagiotti et al. (2015) a RC scheme based on the modification of a B-spline reference trajectory has been proposed with the purpose of improving the tracking accuracy. This idea, initially developed for a single-input single-output minimum-phase plant (a robot joint actuator), has been generalized to multi-input multi-output systems (in Biagiotti et al. (2019b) a robot manipulator with a decentralized control has been considered) and also to non-minimum phase systems, see Biagiotti et al. (2019a). However, in all these works the plant is supposed to be linear, while standard motion control applications are based on nonlinear mechanisms. For this reason, the same design philosophy proposed by Biagiotti et al. (2015) has been adapted to nonlinear systems by exploiting a feedback linearization approach (Isidori (1995)), that allows e.g. to deal with the centralized control of a robot manipulator.

The combination of feedback linearization and RC has been firstly proposed in Ghosh and Paden (2000). However, to guarantee the stability of the control loop the standard time-delay internal model, valid for any periodic signal with period τ, has been approximated by a finite-dimensional system that takes into account only a finite number of harmonics.As a consequence, the perfect tracking of the periodic signal cannot be guaranteed, even in nominal conditions. This choice is driven by the fact that for systems with relative degree other than zero, the asymptotic convergence of the RC loop cannot be achieved (Hara et al. (1985)). Note that the zero relative degree condition is generally not satisfied in robot control applications because of the need for an acceleration measurement. For the same reason, an approach similar to the one used by Ghosh and Paden (2000) has been exploited in the work of Kasac et al. (2008), where a finite set of oscillators are employed to control a robot manipulator. Alternative methods are based on the modification of the basic RC scheme by using an artificial feedthrough term in parallel to the plant as proposed by Hara et al. (1985), or by inserting in the internal model loop a low-pass filter $Q(s)$ in series with the time-delay element. This is what has been done for instance in a recent paper of Zhou et al. (2020), where the RC for nonlinear systems is addressed by using a Lyapunov-based technique.

A completely different approach for solving the issue tied to the relative degree is based on the design of the RC in the discrete-time domain, Tomizuka et al. (1988). The control method proposed in this paper belongs to this class of solutions since the modification of the control points defining the B-spline reference trajectory leads to a discrete-time scheme whose sampling period is the knot span T between the control points. This represents an undoubted advantage of this approach, being the time T generally quite large at least in comparison with the typical sampling periods of discrete-time systems. Note that the use of B-

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spline functions combined with learning mechanisms, such as RC and ILC (Iterative Learning Control) and similar, has been proposed in different works with the purpose of reducing the complexity of the controller and increasing its robustness. For instance, in Sadegh and Guglielmo (1992); Rakprayoon et al. (2011) B-spline functions are employed to parameterize a feedforward control action that improves the position control of a robotic manipulator. The control points defining the spline are modified according to a learning algorithm based on the tracking error, leading to the control scheme called Desired Compensation Learning Law (Sadegh and Guglielmo (1991)). In Wang and Zou (2014), the precise output tracking of a multiaxis nanomanipulation system is achieved by constructing a library of the basic output functions, parameterized as B-spline, and then determining via ILC the corresponding input signals. Finally, the control is deduced by decomposing the desired output trajectory according to the function primitives of the library and by synthesizing the input signal via the superposition principle.

With respect to the approach initially proposed in Biagiotti et al. (2015), the method that we are presenting now deals with nonlinear systems leading to a control scheme whose convergence does not depend on the reference input (while in the former approach the stability of the RC loop was influenced by the sampling period T and therefore by the time period between the via-points to be interpolated). The novel scheme maintains a straightforward implementation that does not require the tuning of any parameter. Finally, it is worth highlighting that the use of B-splines is not imposed by the need of reducing the dimensionality of a problem but it is the direct consequence of the adoption of this type of curves for the definition of complex trajectories in many applications involving robots or automatic machines.

2. OVERVIEW OF THE USED TECHNIQUES

2.1 Reference trajectory generation via B-spline filters

In many practical applications, smooth reference signals are defined using spline functions interpolating a set of desired via-points $\boldsymbol{q}_k^\star$, $k = 0, \ldots, N-1$. A typical example is represented by uniform B-splines, i.e. splines in the so-called B-form characterized by an equally-spaced distribution of the knots, which are defined as

$$\boldsymbol{q}(t) = \sum_{k=0}^{N-1} \boldsymbol{p}_k^\star \, B^d(t - kT), \qquad 0 \le t \le (N-1)T \tag{1}$$

where $B^d(t)$ is the uniform B-spline *basis function* of degree d, T the *knot span*, that is the distance between knots, and the constants $\boldsymbol{p}_k^\star$, called *control points*, determine the shape of the curve and are deduced by imposing the interpolation condition on $\boldsymbol{q}_k^\star$, i.e.

$$\boldsymbol{q}(kT) = \boldsymbol{q}_k^\star, \qquad k = 0, \ldots, N-1. \tag{2}$$

Note that the dimension of $\boldsymbol{p}_k^\star \in \mathbb{R}^m$ determines the dimension of the trajectory $\boldsymbol{q}(t)$, that will be defined in a m-dimensional space.

The particular interest towards uniform B-splines is due to the fact that they can be generated online by means of the chain $M^d(s)$, composed by d dynamic filters defined as

$$M(s) = \frac{1 - e^{-sT}}{Ts},$$

fed by the staircase signal $\boldsymbol{p}(t)$ obtained by maintaining the value of each control point $\boldsymbol{p}_k^\star$ for the entire period $kT \le t < (k+1)T$ with a zero-order hold $H_0(s)$ applied to the sequence of impulses $\boldsymbol{p}_k^\star$ of period T (Biagiotti and Melchiorri (2010)). Obviously, for multi-dimensional trajectories a separated chain of filters is necessary for each trajectory component $q_i(t)$, by considering the input sequence $p_{i,k}^\star$, $k = 0, 1, \ldots, N-1$.

The degree d of the spline and therefore the number of filters composing the B-spline generator determines the smoothness of the output trajectory, which will be a function of class C^{d-1}. A fundamental property of the filter for B-spline generation is the possibility to compute online the profiles of all the time derivatives of the trajectory up to the order d, as shown in Fig. 1. In this scheme the B-spline trajectory generator is preceded by a block that takes into account the computation of the control points from the via-points. In the majority of applications this task is performed off-line by solving the linear system deriving from (2). However, as detailed in Biagiotti et al. (2019b), the mapping between a sequence of ordered via-points $\boldsymbol{q}_k^\star$ and the corresponding sequence of control points $\boldsymbol{p}_k^\star$ can be approximated in the discrete-time domain with a FIR (with sampling-period T) defined by

$$\frac{P(z)}{Q(z)} \approx H(z) = \sum_{i=-r}^{r} h(i) \; z^{-i} \tag{3}$$

whose coefficients $h(i)$, in the case of cubic B-splines ($d = 3$) used in Sec. 4, are given by

$$h(i) = \frac{1-\alpha}{1+\alpha} \alpha^{|i|} \quad \text{with} \quad \alpha = -2 + \sqrt{3}. \tag{4}$$

Note that the value of $h(i)$ becomes extremely small in magnitude as $|i|$ grows. This means that for the computation of the control points $\boldsymbol{p}_k^\star$, only the weights of the via-points close to $\boldsymbol{q}_k^\star$ are important, while, from a practical point of view, the others can be neglected with consequent small approximation errors. Moreover, because of its definition, the filter $H(z)$ is not causal and the insertion of a r samples delay is necessary for its practical implementation.

2.2 Exact state feedback linearization for output tracking

Given a single-input single-output system in the input-affine state space form, i.e.

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + g(\boldsymbol{x})u \tag{5}$$

$$y = h(\boldsymbol{x}) \tag{6}$$

and assumed that at point $\boldsymbol{x}_0$ its relative degree is $r = n$, being n the order of the system [1], it is always possibile to find a local coordinate transformation $\boldsymbol{z} = \boldsymbol{\Phi}(\boldsymbol{x})$ in a neighborhood of x_0, which is defined as

$$\begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{pmatrix} = \begin{pmatrix} \Phi_1(\boldsymbol{x}) \\ \Phi_2(\boldsymbol{x}) \\ \vdots \\ \Phi_n(\boldsymbol{x}) \end{pmatrix} = \begin{pmatrix} h(\boldsymbol{x}) \\ L_f h(\boldsymbol{x}) \\ \vdots \\ L_f^{n-1} h(\boldsymbol{x}) \end{pmatrix}$$

[1] This condition is not necessary for for input-output linearization but it allows to simplify the analysis. Moreover, it is consistent with the application to robotic systems.

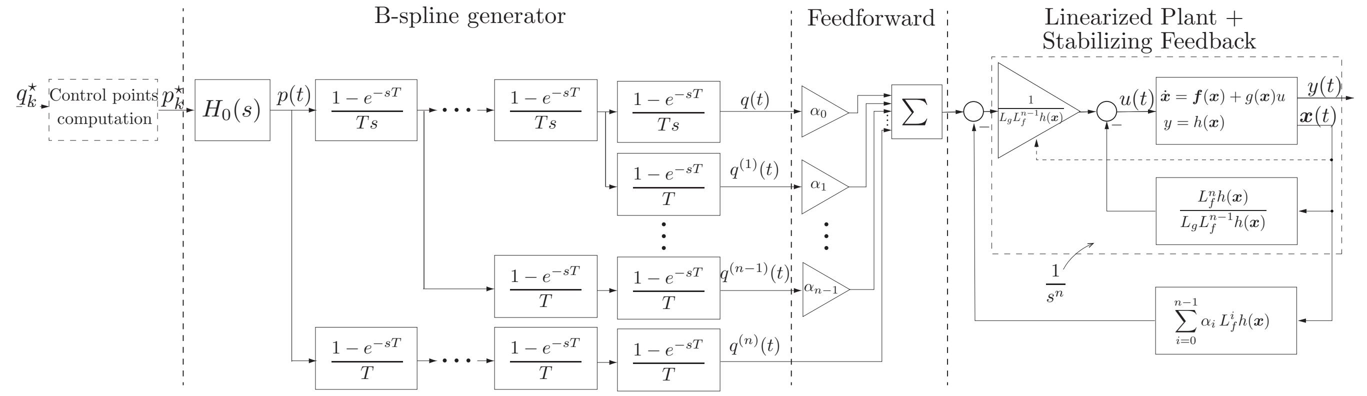


Fig. 1. Block-scheme representation of a control approach based on feedback linearization and B-spline output reference generation.

where $L_f(h(\boldsymbol{x}))$ denotes the Lie derivative of $h(\boldsymbol{x})$ with respect to the vector field $\boldsymbol{f}(\boldsymbol{x})$, that transforms the system (5)-(6) in

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\dots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= b(\boldsymbol{z}) + a(\boldsymbol{z})u \end{aligned}$$

with $a(z) \neq 0$. By assuming now the state feedback control law

$$\begin{aligned} u &= \frac{1}{a(\boldsymbol{z})}(-b(\boldsymbol{z}) + v) \\ &= \frac{1}{a(\boldsymbol{\Phi}(\boldsymbol{x}))}(-b(\boldsymbol{\Phi}(\boldsymbol{x})) + v) \end{aligned} \tag{7}$$

the initial nonlinear system is finally transformed in a linear and controllable system, i.e. a chain of n integrators with input v and output y, that can be stabilized with standard techniques for linear systems like for instance the pole-placement.

By considering the original system (5)-(6), the linearizing control law (7) becomes

$$u = \frac{1}{L_g L_f^{n-1} h(\boldsymbol{x})}(-L_f^n h(\boldsymbol{x}) + v), \tag{8}$$

see Isidori (1995). In order to guarantee the convergence of the output function to a prescribed function $y_r(t)$ the auxiliary input v can be assumed as

$$\begin{aligned} v &= y_r^{(n)} + \sum_{i=0}^{n-1} \alpha_i \, (y_r^{(i)} - y^{(i)}) \\ &= y_r^{(n)} + \sum_{i=0}^{n-1} \alpha_i \, (y_r^{(i)} - L_f^i h(\boldsymbol{x})). \end{aligned} \tag{9}$$

By substituting (9) in (8) and then in (5), the error dynamics

$$e^{(n)} + \alpha_{n-1} e^{(n-1)} + \ldots + \alpha_1 e^{(1)} + \alpha_0 e = 0 \tag{10}$$

with $e = y_r - y$, is obtained. Since the roots of the characteristic equation associated with (10) can be freely chosen by acting on parameters α_i, the decay of the error can be made arbitrarily fast. **Remark.** The proposed approach can be extended to multi-input multi-output systems of the form

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}^T(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{h}(\boldsymbol{x}) \end{aligned}$$

provided that the number of inputs $\boldsymbol{u}$ equals the number of outputs $\boldsymbol{y}$. In particular, if $\{r_1, \ldots, r_m\}$ denotes the relative degree at a point $\boldsymbol{x}_0$, the system of order n can be decomposed into m chains of r_i integrators each, i.e.

$$\boldsymbol{G}(s) = \begin{pmatrix} \frac{1}{s^{r_1}} & 0 & \ldots & 0 \\ 0 & \frac{1}{s^{r_2}} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \frac{1}{s^{r_m}} \end{pmatrix},$$

if

$$r_1 + r_2 + \ldots + r_m = n$$

and the *decoupling matrix*

$$\boldsymbol{Q} = \begin{pmatrix} L_{g_1} L_f^{r_1 - 1} h_1(\boldsymbol{x}) & \ldots & L_{g_m} L_f^{r_1 - 1} h_1(\boldsymbol{x}) \\ L_{g_1} L_f^{r_2 - 1} h_2(\boldsymbol{x}) & \ldots & L_{g_m} L_f^{r_2 - 1} h_2(\boldsymbol{x}) \\ \vdots & \vdots & \vdots \\ L_{g_1} L_f^{r_m - 1} h_m(\boldsymbol{x}) & \ldots & L_{g_m} L_f^{r_m - 1} h_m(\boldsymbol{x}) \end{pmatrix}$$

is nonsingular at x_0 (Isidori (1995)).

This is exactly the case of a robot manipulator.

3. FEEDBACK LINEARIZATION-BASED REPETITIVE CONTROL

The application of a B-spline reference signal generated by dynamic filters to a feedback linearized nonlinear system is straightforward, provided that the relative degree r of the plant (which in this case is supposed to be equal to its order n) does not exceed the degree d of the spline, and accordingly the number of filters composing the generator. In Fig. 1 the combination of these two elements is shown in a schematic manner. Note the role of the real-time trajectory generator that, together with the reference spline, provides the value of the derivatives used for the computation of the feedforward control action.

In nominal conditions, this scheme is sufficient for obtaining asymptotic perfect tracking of the planned B-spline trajectory. However, even if estimation parametric uncertainties and possible exogenous disturbances do not destabilize the system, they rapidly deteriorate the tracking performance. For this reason, the feedback linearization control approach described in Sec. 2.2 is often coupled with a mechanism for improving its robustness, starting from a simple integral control as in Khalil (2002). In this paper, the controlled system and the trajectory generator are both inserted in an outer control loop that includes

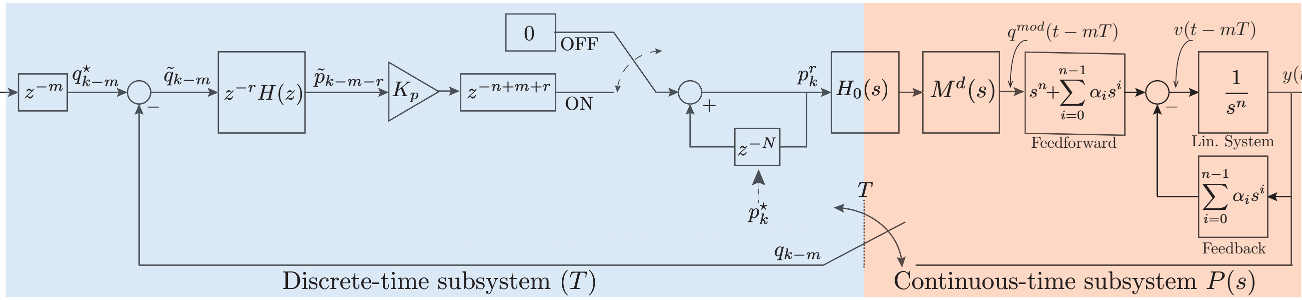


Fig. 2. Discrete-time repetitive control scheme based on B-spline trajectory generator and on feedback linearization.

- the term
$$\frac{1}{1-z^{-N}} \tag{11}$$
where N is the periodicity of the input signal, that in this case coincides with the sequence of the via-points $\boldsymbol{q}_k^\star$;
- a freely modifiable gain K_p;
- the digital filter $H(z)$ defined in (3) for the computation of the control points from the corresponding via-points;
- some additional delays for the synchronization of all the elements in the loop (r is the number of delays that are necessary to make $H(z)$ feasible, $m = \frac{d+1}{2}$ is the delay, caused by the trajectory generator, between a control point application and the corresponding via-point interpolation).

The final scheme is shown in Fig. 2, where the continuous-time plant $P(s)$ and the discrete-time loop are highlighted. According to the internal model principle (Francis and Wonham (1975)), the presence in the loop function of the term (11) assures asymptotic perfect tracking of any periodic signal with period N provided that the system is asymptotically stable. After simple manipulations the scheme can be simplified as in Fig. 3, where $P(z)$ denotes the plant, discretized with sampling time T, i.e.

$$P(z) = \mathcal{Z}\left\{H_0(s)M^d(s)\left[s^n+\sum_{i=0}^{n-1}\alpha_i s^i\right]\frac{\frac{1}{s^n}}{1+\frac{1}{s^n}\sum_{i=0}^{n-1}\alpha_i s^i}\right\}$$
$$= \mathcal{Z}\left\{H_0(s)M^d(s)\right\} \tag{12}$$

A sufficient condition for asymptotic stability of this scheme can be derived by means of classical Nyquist analysis:
$$|L(e^{j\omega T})-1|<1, \quad \forall\omega\in[0,\pi/T]. \tag{13}$$
where $L(z)$ is the loop function except the internal model term (11), i.e.
$$L(z) = H(z)\,K_p\,z^m\,\mathcal{Z}\left\{H_0(s)M^d(s)\right\}.$$
Since the transfer function $\mathcal{Z}\left\{H_0(s)M^d(s)\right\}$ is obtained by discretizing the B-spline trajectory filter with sampling period T, it represents the relationship between the sequence of control points $P(z)$ and the sequence of via-points $Q(z)$, with an additional delay of m samples. As a consequence, by considering the meaning of $H(z)$ it follows that $H(z)\,z^m\,\mathcal{Z}\left\{H_0(s)M^d(s)\right\}\approx 1$ and accordingly $L(z)\approx K_p$. The value $K_p = 1$ allows $L(z)$ to meet (13) with the largest margin. Additional details can be found in Biagiotti et al. (2019b).

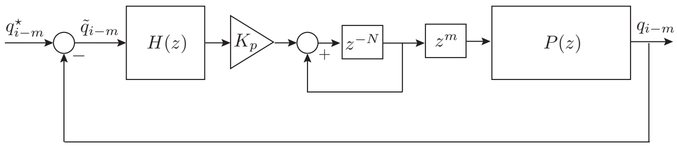


Fig. 3. Equivalent block-scheme representation of Repetitive Control in Fig. 2.

4. FEEDBACK-LINEARIZATION-BASED RC FOR A ROBOT MANIPULATOR

Let's consider the dynamics of a robot manipulator with rigid joints
$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})=\tau. \tag{14}$$
The feedback linearization based control, that in this case is called *inverse dynamics* (Chung et al. (2007)), assumes the form
$$\tau = \hat{\boldsymbol{M}}(\boldsymbol{q})\boldsymbol{v}+\hat{\boldsymbol{C}}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\hat{\boldsymbol{g}}(\boldsymbol{q}) \tag{15}$$
where $\hat{\boldsymbol{M}}(\cdot)$, $\hat{\boldsymbol{C}}(\cdot,\cdot)$ and $\hat{\boldsymbol{g}}(\cdot)$ are approximations of the real functions and the auxiliary variable v is generally assumed to be
$$\boldsymbol{v}=\ddot{\boldsymbol{q}}^\star+\boldsymbol{K}_V(\dot{\boldsymbol{q}}^\star-\dot{\boldsymbol{q}})+\boldsymbol{K}_P(\boldsymbol{q}^\star-\boldsymbol{q}) \tag{16}$$
or with an integral control action
$$\boldsymbol{v}=\ddot{\boldsymbol{q}}^\star+\boldsymbol{K}_V(\dot{\boldsymbol{q}}^\star-\dot{\boldsymbol{q}})+\boldsymbol{K}_P(\boldsymbol{q}^\star-\boldsymbol{q})+\boldsymbol{K}_I\int(\boldsymbol{q}^\star-\boldsymbol{q})dt \tag{17}$$
The two control actions are based on the assumptions that both position and velocity of the robot joints are available. In general, the control based on feedback linearization relies on the knowledge of the full state of the plant. The robotic plant, together with the controller described by (15) and (16), and a cubic B-spline trajectory generator is shown in Fig. 4. This system, inserted in the scheme of Fig. 2 in lieu of continuous-time subsystem $P(s)$, leads to the final RC controller.

4.1 Numerical simulations

Some numerical simulations based on a two-dof robotic manipulator have been performed in order to validate the proposed control scheme. The nominal values of the robot parameters are reported in Tab. 1. Note that, differently from the model (14), in the simulative model the viscous friction at the joints has been considered.
Since the controller is based on feedback linearization (15) the scheme strongly depends on the knowledge of the plant model. For this reason, it is of great interest to investigate how the accuracy of the model influences the performance

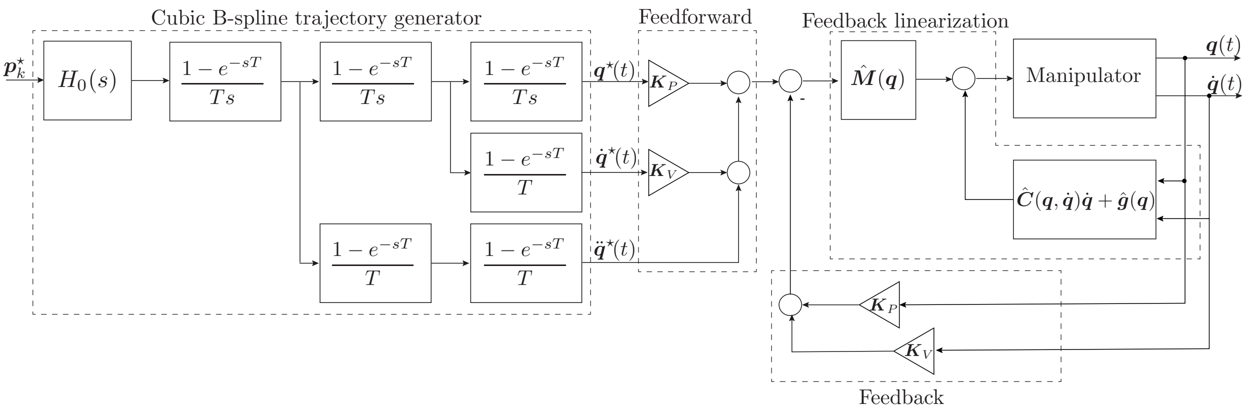


Fig. 4. Block-scheme representation of the robot manipulator with inverse dynamics based controller and cubic ($d = 3$) B-spline trajectory generator.

Table 1. Nominal parameter values of the two-link manipulator.

Description	Symbol	Value	
Joint inertia	J_i	1.15e-2	$\mathrm{kg\,m^2}$
Joint viscous friction coeff.	d_i	0.005	$\mathrm{N\,s\,m^{-1}}$
Joint mass	m_i	0.54	kg
Link center	$l_{c,i}$	0.085	m
Link length	l_i	0.3	m

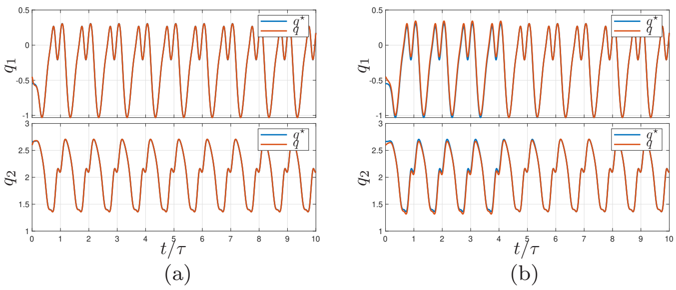


Fig. 5. Reference Trajectory $q^\star(t)$ and joint positions $q(t)$ of the robot manipulator with integral (a) and repetitive (b) control based on feedback linearization (parameters uncertainty $\pm 10\%$).

of the overall algorithm. The tracking performance obtained with the sole controller (17) is compared with the one produced by (16) combined with the RC, both in nominal conditions and in case of estimation errors on the model parameters. In particular, random variations of all parameters, with the exception of geometric quantities, i.e. links lengths, that are usually known with good precision, have been taken into account.

The matrices $\boldsymbol{K}_V$ and $\boldsymbol{K}_P$ (and $\boldsymbol{K}_I$) that appear in (16) and (15) have been chosen so that all the roots of the characteristic equation of the error dynamics are located in $\lambda = -8$. The reference trajectory has been defined by means of 9 control points that form a cyclic motion. The knot span T is assumed equal to 0.5s.

In nominal conditions, both the controllers, with integral control action and with RC, lead to a perfect tracking of the reference input.

In Fig. 5 the reference trajectory and the actual joints position obtained with the two controllers, deduced with an uncertainty of $\pm 10\%$ in the parameters' values, are illustrated as a function of the cycle number t/τ, being τ the total duration of the iterated trajectory, while the related tracking errors are shown in Fig. 6. Note that

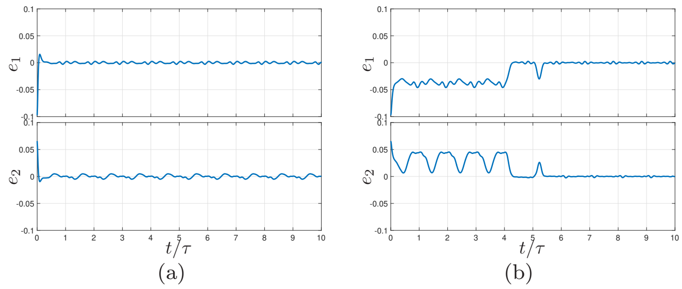


Fig. 6. Tracking error of the robot manipulator with integral (a) and repetitive (b) control based on feedback linearization (parameters uncertainty $\pm 10\%$).

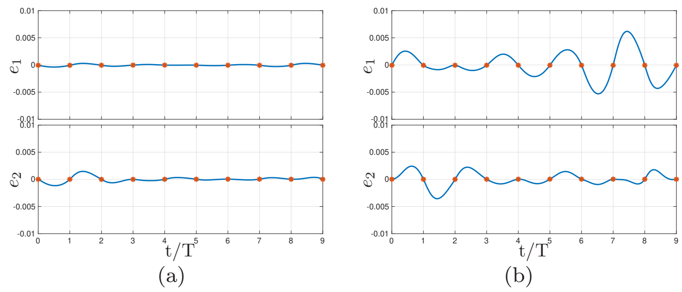
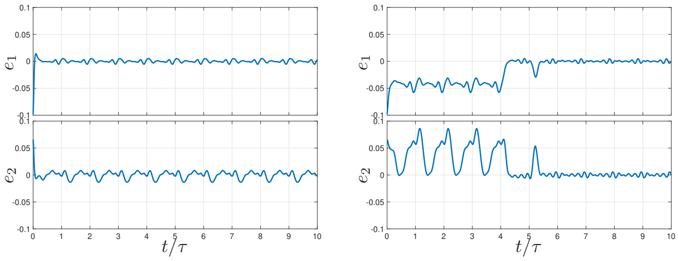


Fig. 7. Tracking error of the robot manipulator with RC based on feedback linearization during 10-th iteration. Parameters uncertainty $\pm 10\%$ (a) and $\pm 50\%$ (b).

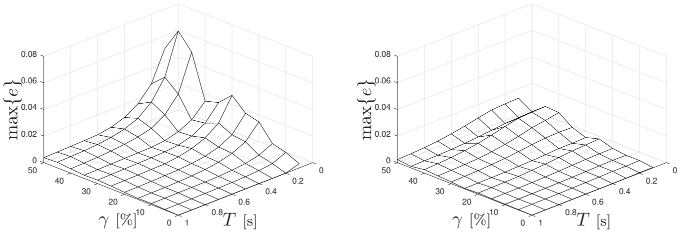
the RC is activated after 4 iterations in which only the inverse dynamics control, without integral, acts on the robotic plant. A moderate error on the model parameters causes small tracking errors in both cases (slightly smaller for RC), even if it is worth noticing that at least at the via-points the RC guarantees a perfect interpolation, see Fig. 7(a) where the tracking errors during the 10-th iteration is shown. When the uncertainty on the parameters knowledge grows, like in case of Fig. 8 where a random error of $\pm 50\%$ is considered, the tracking error increases as well but the level obtained with the RC is considerably smaller and, in any case, is vanishing at the via-points, see Fig. 7(b). In Fig. 9 the maximum tracking error [2] obtained with the two control schemes considered in this section is reported as a function of the uncertainty on the parameters $\pm\gamma$ and of the knot span T, that determines the

[2] For the scheme based on RC, the error during the 10-th iteration is considered.



(a) (b)

Fig. 8. Tracking error of the robot manipulator with integral (a) and repetitive (b) control based on feedback linearization (parameters uncertainty $\pm 50\%$).



(a) (b)

Fig. 9. Maximum error as a function of parameters uncertainty $\pm\gamma$ and of knot span T for schemes with integral control (a) and with RC (b).

duration of the trajectory. In both cases the error increases when the uncertainty grows[3] and the duration of the trajectory becomes smaller. In particular, the combination of fast trajectories and large uncertainty is quite critical for the scheme with integral control (see Fig. 9(a)), while the approach based on RC is significantly more insensitive (Fig. 9(b)).

5. CONCLUSIONS

In this paper, a novel control scheme has been presented that guarantees asymptotic perfect tracking of a set of desired via-points, used for defining the interpolating B-spline trajectory, by a nonlinear (input affine) system. The proposed approach is based on the application of repetitive control to the sequence of control points of the B-spline curve, supposed cyclic, so that the plant, provided with a standard feedback linearization control, exactly crosses the via-points despite parametric uncertainties and periodic exogenous disturbances. With respect to previous control schemes based on a similar concept, the new method offers the advantage of being able to deal with plants with nonlinear dynamics, leading to a control scheme whose stability is guaranteed for any value of the "sampling time" T and without the need for special tuning operations. Its structural simplicity and the lack of free parameters that require a complicated selection procedure make this scheme ideal for practical applications, e.g. with robotic systems, in which B-splines are employed as reference trajectories. The fact that the tracking error is nullified only at the via-points does not seem a limitation, since usually in this type of applications the via-points are the only input provided, while the specific shape of the continuous trajectory joining these points is not unique.

[3] Note that the relationship between γ and the maximum errors does not seems to be monotonic, but this is due to the fact the γ represents the maximum variation of the parameters and not their actual variation.

In any case, the RC scheme causes a significant reduction of the tracking error also during the intersamples. Finally, the fact the sampling period T of the outer control loop is very large can be profitably exploited in those applications in which are used "slow" sensors, like cameras or motion tracking systems.

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