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A Novel State and Parameter Estimation Algorithm for Spark Ignition Engine

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Abstract: The engine control and estimation problem is an important area of research in the automotive industry. Researchers have been working to make the vehicles more efficient and economically friendly while producing lesser pollutants. To reduce emissions, the air-fuel ratio must be controlled to a specific value. The requirement of air-fuel ratio improvement has increased the need for the investigation of engine dynamical models and their parameter estimation. Some of the main parameters affecting the air-fuel ratio are the throttle discharge coefficient, thermal efficiency and volumetric efficiency. The precise values of these parameters are essential for accurate control of the air-fuel ratio of the engine. Under steady state, these parameters are constant but in the long run due to wear and tear of the engine and various uncertainties, their value may change. The main challenges are how to obtain the information of parameters and that of the states under the influence of process noise, measurement noise and parameter uncertainty, which are essential elements to develop an effective control strategy. In this work, the problem of physical parameter estimation of the nonlinear system comprising a throttle, intake manifold, engine speed dynamics and fuel system altogether with unknown states have been considered. A novel method with a unique combination of Unscented Kalman Filter and Recursive Least Squares with forgetting factor for estimation of parameters and states of spark ignition engines has been developed. Simulation results are provided for state and parameter estimation for spark ignition engine model.

Keywords: Spark Ignition engine, Recursive Least Squares method, Unscented Kalman Filter, coefficient of discharge in throttle body, volumetric efficiency, thermal efficiency.

1. INTRODUCTION

Internal combustion engines are very complex. There are many mathematical models available in the literature for combustion engines with different assumptions. Multiple objectives such as optimized fuel economy and lower pollutant emission from the engine requires control strategy and parameter estimation of the engine. Also, knowledge of parameter values is essential for calibration and fault diagnosis. This makes parameter estimation an important part of the automotive engine world.

In the literature, most of the papers focus on controlling the inputs of Spark Ignition (SI) engine to keep the air-fuel ratio equal to the stoichiometric ratio Yildiz et al. (2008). Stoichiometric ratio implies that there is correct amount of air and fuel for a complete combustion in the cylinder. While doing so, some parameters of the model are estimated first and then control strategy is applied. In Chen et al. (2017), recursive least squares and batch least squares is used for estimating the lumped parameters. In Gao et al. (2017), stochastic gradient descent algorithm is used for the parameter estimation. Some papers have employed observer to get estimated states which are used for the control purpose. In Butt et al. (2009), a sliding mode observer has been designed for estimation of coefficient of discharge in automotive gasoline engines. In Cavina and Suglia (2005), parameters of spark advance model are estimated for combustion phase control of gasoline SI engines. There are state estimation based parameter estimation methods for linear systems Ding (2014). The problem of physical parameter estimation of the nonlinear system comprising a throttle, intake manifold, engine dynamics and fuel system altogether with unknown states has not been considered before. In Tang et al. (2009) similar parameters have been adopted from the work done by Khan and Spurgeon Khan and Spurgeon (2003) but with known states. In this work, the problem of model parameter estimation with unknown states has been considered. Model parameters K_a, C_p and C_t which gives information about throttle discharge, volumetric efficiency and thermal efficiency are estimated. The objective is to estimate the parameters for a given set of input-output data. Values of these parameters monitor the condition of the engine. In this case, the parameters are considered as time signals. In many methodologies parameter are estimated as function of other variables or with defined dynamics. In other cases, huge amount of training data is required or the parameters are estimated using the known states. Another case is applying Unscented Kalman Filter (UKF) for joint state and parameter estimation Wan and Van Der Merwe (2000). For dual and joint estimation by any variant of Kalman Filter, parameters are considered as states and hence their dynamics are required. But in this work, parameter dynamics are unknown. EKF and RLS have been combined for synchronous estimation of road grade and vehicle mass for a hybrid electric busYong Sun (2016). The combination of UKF and RLSF for an estimation of parameters of SI engines is the novelty that we explore through this work. RLSF is preferred over Kalman Filter for parameter estimation because Kalman Filter needs dynamics of the parameters and they are not known in this work. Since RLSF requires state information, UKF which is robust to parameters in many cases is used for state estimation.

2. A MEAN VALUE ENGINE MODEL FOR ANY INTERNAL COMBUSTION ENGINE

There are number of engine dynamic models available in the literature. Mean Value Engine Model (MVEM) developed by Hendricks and Sorenson (1990) is considered for this work. It is mathematically compact, serves as a nonlinear dynamic engine model and is suitable for control applications Tang et al. (2009). It comprises of three subsystems: air flow system, engine speed dynamics and fuel system.

2.1 Air flow system

The air mass flowing through the intake manifold is described by the intake manifold pressure dynamics as:

$$\dot{p} = \frac{RT_i}{V_i}(\dot{m}_{at} - \dot{m}_{ap}) \tag{1}$$

$$\dot{m}_{ap} = C_p p n \tag{2}$$

$$\dot{m}_{at} = K_a(1 - cos(\theta - \theta_0))\beta(p) \tag{3}$$

$$\beta(p) = \begin{cases} 1, & \text{for } p_{atm}/2 < p \\ (2/p_{atm})\sqrt{p_{atm}p - p^2}, & \text{for } p \geq p_{atm}/2 \end{cases}$$

where p is the intake manifold pressure, $\dot{m}_{at}$ is the air mass flow rate past throttle plate, $\dot{m}_{ap}$ is the air mass flow rate into cylinder, θ is throttle angle, θ_0 is the minimum possible throttle angle and p_{atm} is the atmospheric pressure. C_p and K_a are the parameters. $C_p = \sqrt{\frac{T_i}{T_a}}\frac{V_d \eta_{vol}}{120RT_i}$, T_a: ambient temperature, V_d: engine displacement, η_{vol}: volumetric efficiency. K_a: throttle discharge coefficient.

2.2 Engine speed dynamics

Engine speed dynamics are described as a function of injected fuel mass flow rate.

$$\dot{n} = \frac{\tau_e + \tau_{pf}}{J} = \frac{C_t}{J}\frac{\dot{m}_f}{n} + \frac{\tau_{pf}}{J} \tag{4}$$

where $\tau_e = C_t \dot{m}_f / n$ is the engine torque, $\tau_{pf} = 1.673 + 0.272n + 0.0135n^2 + p(-0.969 + 0.206n)$ is pumping and frictional torque, n is the engine speed, $\dot{m}_f$ is the injected fuel mass flow rate, J is the moment of inertia and C_t is a parameter. $C_t = H_\mu \eta_i$, H_μ: fuel energy constant, η_i: thermal efficiency.

2.3 Fuel system

The fuel flow is described by wall wetting phenomenon in the intake manifold.

$$\dot{m}_f = \dot{m}_{ff} + \dot{m}_{fv} \tag{5}$$

$$\ddot{m}_{ff} = \frac{1}{\tau_f}(-\dot{m}_{ff} + \chi \dot{m}_{fi}) \tag{6}$$

$$\dot{m}_{fv} = (1 - \chi)\dot{m}_{fi} \tag{7}$$

where $\dot{m}_f$ is the fuel flow rate into the cylinder for combustion, $\dot{m}_{ff}$ is the fuel film flow rate, $\dot{m}_{fv}$ is the fuel vapor flow rate, $\dot{m}_{fi}$ is the injected fuel flow rate, τ_f is fuel evaporation time constant and χ is a parameter. χ: fraction of $\dot{m}_{fi}$ which is deposited on the manifold as a film.
Special Case: The fraction of injected fuel flow rate that is deposited on the manifold film ($\chi \dot{m}_{fi}$) is equal to the fuel film flow rate ($\dot{m}_{ff}$). In othe words, fuel flow rate is not affected by wall wetting phenomenon. Using (7), (5) becomes:

$$\dot{m}_f = \dot{m}_{fi} \tag{8}$$

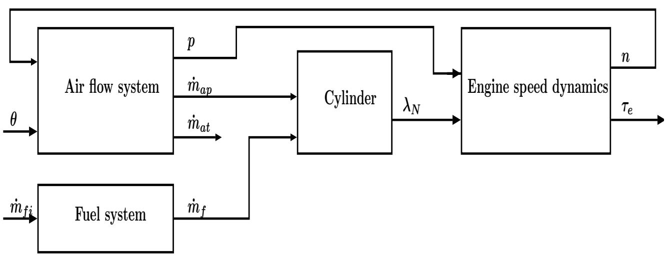

Fig. 1. Spark ignition engine model

In this work, Multi Input-Multi Output (MIMO) system as shown in figure 1, is considered with two inputs (throttle angle and injected mass flow rate), two states (intake manifold pressure and engine speed) and three outputs (normalized air-fuel ratio λ_N, engine torque tor and air mass flow rate past throttle plate $\dot{m}_{at}$).
Inputs: $u_1 = \theta, \quad u_2 = \dot{m}_{fi} = \dot{m}_f$

States: $x_1 = p, \quad x_2 = n$

Outputs: $y_1 = \lambda_N, \quad y_2 = \tau_e, \quad y_3 = \dot{m}_{at}$

$$\begin{aligned} \dot{x}_1 &= \frac{RT_i}{V_i}[K_a(1 - cos(u_1 - \theta_0))\beta(x_1) - C_p x_1 x_2] + v_{x1} \\ \dot{x}_2 &= \frac{C_t}{J}\frac{u_2}{x_2} + \frac{\tau_{pf}}{J} + v_{x2} \\ y_1 &= C_p\frac{x_1 x_2}{14.67u_2} + d_{y1} \\ y_2 &= C_t\frac{u_2}{x_2} + d_{y2} \\ y_3 &= K_a(1 - cos(u_1 - \theta_0))\beta(x_1) + d_{y3} \end{aligned} \tag{9}$$

where K_a, C_p and C_t are the parameters.
v_{xi}: process noise corresponding to state x_i, $i = 1, 2$
d_{yi}: observation noise corresponding to output y_i, $i = 1, 2, 3$

3. METHOD FOR PARAMETER ESTIMATION USING STATE ESTIMATES

In the model, the relation between the parameters and the outputs is linear. So Recursive Least Squares (RLS) method can be used for online parameter estimation Isermann and Münchhof (2010). Information of states is required for the estimation. Considering the states are unknown and have known physics based nonlinear dynamics, the states behavior can be modeled with the reasonable values of the parameters. Since Unscented Kalman Filter (UKF) is most widely used for nonlinear state estimation Kandepu et al. (2008), so UKF can be used to get the estimated states which are then fed to RLS with forgetting factor (RLSF) for parameter estimation.

In this methodology, the algorithm starts by initializing the parameters, states and corresponding covariances. Next, input-output data is collected from the plant. State estimation is done using UKF for one time step. The estimated states along with known inputs and outputs are then fed to RLSF for parameter estimation. The estimated parameters are then used for the next time step state estimation. This cycle goes on.

Assume that the states are known. Let the nonlinear system be represented by the following discrete time equations:

$$\begin{aligned} \tilde{x}(k) &= f(\tilde{x}(k-1), \tilde{v}(k-1), \tilde{u}(k-1), \tilde{\vartheta}(k-1)) \\ \tilde{y}(k) &= h(\tilde{x}(k), \tilde{d}(k), \tilde{u}(k), \tilde{\vartheta}(k)) \end{aligned} \quad (10)$$

such that

$$\begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \end{bmatrix} = \begin{bmatrix} \begin{array}{c} \dfrac{RT_i}{V_i}[K_a(k-1)(1-cos(u_1(k-1)-\theta_0)) \\ \beta(\tilde{x}_1(k-1)) - C_p(k-1)\tilde{x}_1(k-1) \\ \tilde{x}_2(k-1)] \\ +v_{x1}(k-1) \\ \\ \dfrac{C_t(k-1)}{J}\dfrac{u_2(k-1)}{\tilde{x}_2(k-1)} + \dfrac{\tau_{pf}}{J} \\ +v_{x2}(k-1) \end{array} \end{bmatrix} \Delta t$$

$$. + \begin{bmatrix} \tilde{x}_1(k-1) \\ \tilde{x}_2(k-1) \end{bmatrix}$$

$$\begin{bmatrix} \tilde{y}_1(k) \\ \tilde{y}_2(k) \\ \tilde{y}_3(k) \end{bmatrix} = \begin{bmatrix} \begin{array}{c} C_p(k)\dfrac{\tilde{x}_1(k)\tilde{x}_2(k)}{14.67u_2(k)} + d_{y1}(k) \\ \\ C_t(k)\dfrac{u_2(k)}{\tilde{x}_{2(k)}} + d_{y2}(k) \\ \\ K_a(k)(1-cos(u_1(k)-\theta_0))\beta(\tilde{x}_1(k)) \\ +d_{y3}(k) \end{array} \end{bmatrix}$$

where Δt is the sample time. Sensors frequency and the method used for discretization takes care of the sample time. $\tilde{y}(k)$ is the measured output.

Consider output model

$$\tilde{y}_{1M}(k) = \psi_1(k)\vartheta_1(k) + e_1(k) \quad (11)$$

such that $\psi(k) = \dfrac{\tilde{x}_1(k)\tilde{x}_2(k)}{14.67u_2(k)}$, $\vartheta_1(k)$ is model parameter and $e_1(k) = \tilde{y}_1(k) - \tilde{y}_{1M}(k)$. For sample 1 to k, cost function V(k) with weight matrix W(k) is introduced such that

$$V(k) = E^T(k)W(k)E(k) \quad (12)$$

W(k) : symmetric positive definite matrix,

$$E(k) = Y_{1M}(k) - Y_1(k) = \Psi(k)\vartheta_1(k) - Y_1(k) \quad (13)$$

$Y_{1M}(k) = [y_{1M}(1)\ y_{1M}(2)...y_{1M}(k)]^T$
$Y_1(k) = [\tilde{y}_1(1)\ \tilde{y}_1(2)...\tilde{y}_1(k)]^T$
$\Psi(k)$ is the corresponding data matrix.

W(k) takes care that more weight is given to the recent data. The weight increases exponentially to 1 for the most recent data. To minimize the cost function V(k), take derivative of $V(k)$ with respect to parameter vector in (12) and equate it to zero. This gives,

$$2\Psi^T(k)W(k)E(k) = 0 \quad (14)$$

Let $\hat{\vartheta}_1(k)$ be the estimated parameter vector at kth instant. Using (13) and (14),

$\Psi^T(k)W(k)(Y_1 - \Psi(k)\hat{\vartheta}_1(k)) = 0$
$\hat{\vartheta}_1(k) = (\Psi^T(k)W(k)\Psi(k))^{-1}\Psi^T(k)W(k)Y_1(k)$
Take $P_{\vartheta 1}(k) = (\Psi^T(k)W(k)\Psi(k))^{-1}$
Then,
$\hat{\vartheta}_1(k) = P_{\vartheta 1}(k)\Psi^T(k)W(k)Y_1(k)$
$\hat{\vartheta}_1(k+1) = P_{\vartheta 1}(k+1)\Psi^T(k+1)W(k+1)Y_1(k+1)$

$$\begin{aligned} \hat{\vartheta}_1(k+1) =& P_{\vartheta 1}(k+1)[\Psi^T(k)lW(k)Y(k) + \psi_1(k+1) \\ & .\, w(k+1)y_1(k+1)] \end{aligned}$$

since $w(k+1) = 1$
$\hat{\vartheta}_1(k+1) = P_{\vartheta 1}(k+1)[P_{\vartheta 1}^{-1}(k)\hat{\vartheta}_1(k) + \psi_1(k+1)y_1(k+1)]$

$$\hat{\vartheta}_1(k+1) = \hat{\vartheta}_1(k) + \gamma_{\vartheta 1}(k)(y_1(k+1) - \psi_1(k+1)\hat{\vartheta}_1(k)) \quad (15)$$

where

$$\gamma_{\vartheta 1}(k) = \frac{P_{\vartheta 1}(k)\psi_1(k+1)}{\psi_1(k+1)P_{\vartheta 1}(k)\psi_1(k+1) + l} \quad (16)$$

$$P_{\vartheta 1}(k+1) = (I - \gamma_{\vartheta 1}(k)\psi_1)P_{\vartheta 1}(k)/l \quad (17)$$

This method is known as Recursive Least Squares method with forgetting factor l Isermann and Münchhof (2010). Equation (15)-(17) are the three main equations used in RLS method. In this manner, all the parameters are estimated using one output for one parameter. Since the states are unknown, $\tilde{x}$ is replaced by $\hat{x}$. UKF is used for estimating the states. For a given model (known parameters), the nonlinear space of the states are transferred to sigma points state. The states are updated in the sigma points space and the output is calculated in that space and then transferred back to the original space. This has proven to be better than linearizing the nonlinear function and predicting the states Kandepu et al. (2008).

Since the parameters in this work are unknown, UKF is initialized with a guess of parameters. The guess can be taken a value which is known from history or can be calculated from history Yong Sun (2016). Figure 2 shows the flow chart of the algorithm. The steps for implementing this method are written below:

(1) Initialization of parameters $\hat{\vartheta} = [\hat{\vartheta}_1\, \hat{\vartheta}_2\, \hat{\vartheta}_3]^T$, augmented state $x^a = (\tilde{x}, \tilde{v}, \tilde{d})$ and covariances $P_{\vartheta i}$(for i=1,2,3), P^a at $k = 0$.
$\hat{\vartheta}(0)$, $P_{\vartheta i(0)}$(for i=1,2,3), $\hat{x}_0 = E[\tilde{x}_0]$,
$P_{\tilde{x}(0)} = E[(\tilde{x}(0) - \hat{x}(0))(\tilde{x}(0) - \hat{x}(0))^T]$,
$E[.]$: expectation of (.)
$\hat{x}(0)^a = E[x^a] = E[\hat{x}(0)\ 0\ 0]^T$
$\hat{P}(0)^a = E[(x^a(0) - \hat{x}^a(0))(x^a(0) - \hat{x}^a(0))^T]$
(2) Set $k = k + 1$.
(3) For state estimation, transfer the states to sigma plane. Calculate N (number of states + state noise + output noise + 1) sigma points $X^a(i, k-1)$ and

obtain constraint sigma points $X^{x,C}(i,k-1)$ if there are any state constraints.

$$X^a(i,k-1) = \begin{cases} \hat{x}^a(k-1), & \text{for } i=0 \\ \hat{x}^a(k-1)+\gamma S_i, & \text{for } i=1,...,N \\ \hat{x}^a(k-1)-\gamma S_i, & \text{for } i=N+1,...,2N \end{cases}$$

where S_i is the ith column of $S = \sqrt{P^a(k-1)}$, $\gamma = \sqrt{N+\lambda}$ is a scaling parameter,
$\lambda = \alpha^2(N+\mathcal{K})-N$. α and $\mathcal{K}$ are tuning parameters.
$X^{x,C}(i,k-1) = P(X^x(i,k-1))$, $i=0,1,...2N$
where P referes to the projection.

(4) Update state vector using state update function, corresponding mean (apriori state) and covariance:

$$\begin{aligned} X^x(i,k/k-1) =& f_{\hat{\vartheta}}(X^{x,C}(i,k-1), X^v(i,k-1), \\ & .\, u(k-1), \vartheta(k-1)), i=0,1,...2N \\ X^{x,C}(i,k/k-1) =& P(X^x_{i,k/k-1}), i=0,1,...2N \\ \hat{x}^-(k) =& \sum_{i=0}^{2N}(w_m^{(i)} X^{x,C}(i,k/k-1)) \\ \hat{P}_x^-(k) =& \sum_{i=0}^{2N} w_c^{(i)}(X^{x,C}(i,k/k-1)-\hat{x}^-(k)) \\ & .\,(X^{x,C}(i,k/k-1)-\hat{x}^-(k))^T \end{aligned}$$

where $w_m^{(0)} = \dfrac{\lambda}{N+\lambda}$, $i=0$,
$w_c^{(0)} = \dfrac{\lambda}{N+\lambda} + (1-\alpha^2+\beta)$, $i=0$,
$w_m^{(0)} = w_c^{(0)} = \dfrac{\lambda}{2(N+\lambda)}$, $i=1,...,2N$, and β is a parameter.

(5) Calculate output sigma points, corresponding mean and covariance:

$$\begin{aligned} Y(i,k/k-1) =& h_{\hat{\vartheta}}(X^{x,C}(i,k/k-1), X^d(k-1), u(k), \\ & .\, \vartheta(k)), i=0,1,...,2N \\ \hat{y}^-(k) =& \sum_{i=0}^{2N}(w_m^{(i)} Y(i,k/k-1)) \\ P_y^-(k) =& \sum_{i=0}^{2N} w_c^{(i)}(Y(i,k/k-1)-\hat{y}^-(k)) \\ & .\,(Y(i,k/k-1)-\hat{y}^-(k))^T \end{aligned}$$

(6) Calculate cross covariance and kalman gain.

$$\begin{aligned} P_{xy}^-(k) =& \sum_{i=0}^{2N} w_c^{(i)}(X^x(i,k/k-1)-\hat{x}^-(k)) \\ & .\,(Y(i,k/k-1)-\hat{y}^-(k))^T \\ K(k) =& P_{xy}^-(k)(P_{\bar{y}}^-(k))^{-1} \end{aligned}$$

(7) Calculate the state estimate and its covariance.

$$\begin{aligned} \hat{x}(k) =& \hat{x}^-(k) + K(k)(y(k)-\hat{y}^-(k)) \\ P_x(k) =& P_x^-(k) - K(k)P_y^-(k)K^T(k) \end{aligned}$$

(8) For a value of forget factor l, calculate gain $\gamma_{\vartheta 1}$, $\gamma_{\vartheta 2}$, $\gamma_{\vartheta 3}$ for estimation of C_p, C_t, K_a respectively, using estimated states

$$\gamma_{\vartheta i}(k-1) = \frac{P_{\vartheta i}(k-1)\psi_i(k)}{\psi_i(k)P_{\vartheta i}(k-1)\psi_i(k)+l}, \; for\, i=1,2,3.$$

(9) Calculate the estimated parameters and corresponding covariance.

$$\begin{aligned} \hat{\vartheta}_i(k) =& \hat{\vartheta}_i(k-1) + \gamma_{\vartheta i}(k-1)(y_i(k)-\psi_i(k) \\ & .\hat{\vartheta}_i(k-1)), \; for\, i=1,2,3. \\ \hat{\vartheta}(k) =& [\hat{\vartheta}_1(k)\,\hat{\vartheta}_2(k)\,\hat{\vartheta}_3(k)]^T \\ P_{\vartheta i}(k) =& (I-\gamma_{\vartheta i}(k-1)\psi_i(k))P_{\vartheta i}(k-1)/l, \\ & .for\, i=1,2,3. \end{aligned}$$

(10) 10. Collect new data and repeat from step 2.

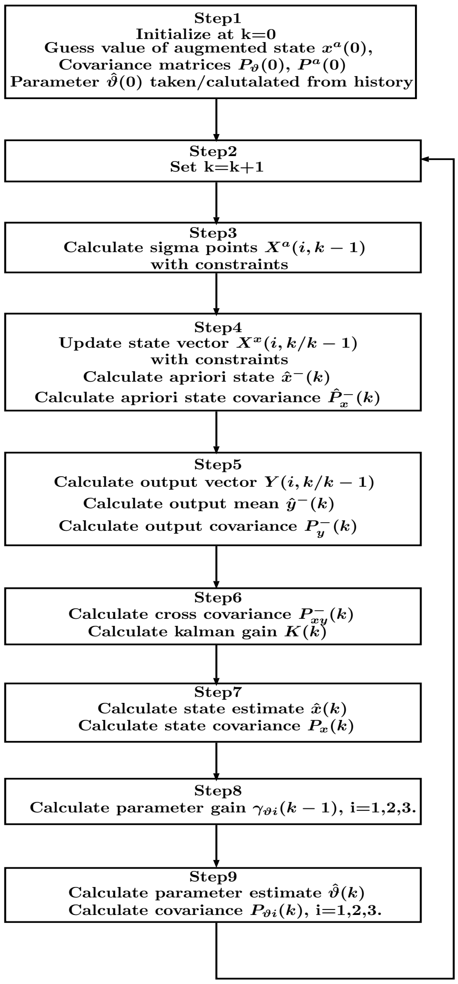

Fig. 2. Algorithm Flow Chart

4. SIMULATION RESULTS

4.1 Implementation of the algorithm

The data is collected by running the Simulink model for the following values of the parameters: $C_p = 0.0113$, $C_t \in [12000, 11998]$, $K_a = 0.6(1+10^{-5}x_2) + 9\times 10^{-5}(1+0.0002x_2)u_1 + 9\times 10^{-5}(1+0.0001x_2)u_1^2$ (polynomial function of throttle angle and engine speed Franchek et al. (2007)). The simulation is run for 50 sec with time step $= 0.005$ sec. $x_1(0) = 0.7$, $x_2(0) = 5$. Process noise: $x_1 \sim \mathcal{N}(0,10^{-2})$, $x_2 \sim \mathcal{N}(0,10^{-2})$. Measurement noise: $y_1 \sim \mathcal{N}(0,10^{-6})$, $y_2 \sim \mathcal{N}(0,10^{-6})$, $y_3 \sim \mathcal{N}(0,10^{-4})$. First 50 data points are considered as history (states are known). The algorithm starts after the first 50 data points. The parameters are initialized by applying Least Squares method on the history points. Initial state values taken:

$\tilde{x}_1(0) = 0.2$, $\tilde{x}_2(0) = 10$. $l = 0.9$. $P_{\tilde{x}}(0) = diag(0.6, 5)$, $P_{\vartheta i}(0) = 10^5$ for i=1,2,3. Constrains applied: $0 \le x_1 \le 1$, $x_2 \ge 10^{-6}$.

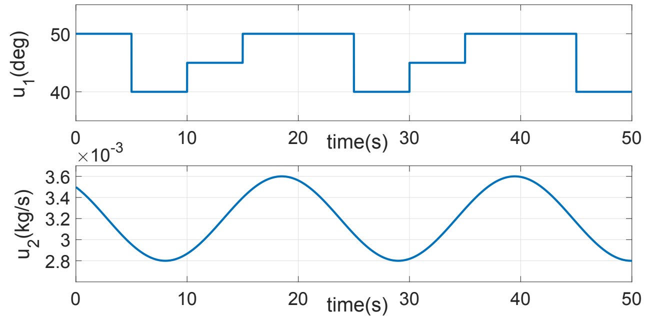

Fig. 3. Input data

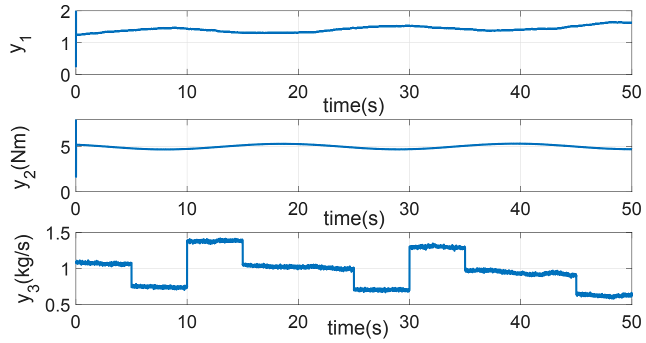

Fig. 4. Output data

Figure 3 shows the input data collected from the simulink model. u_1 is throttle angle (deg) and u_2 is fuel mass flow rate injected (kg/s). Figure 4 shows the collected output from the simulink for the input data. y_1 is normalized air-fuel ratio, y_2 is engine torque (Nm) and y_3 is air mass flow rate past throttle (kg/s). The simulink acts as plant for the experiment which produces synthetic data.

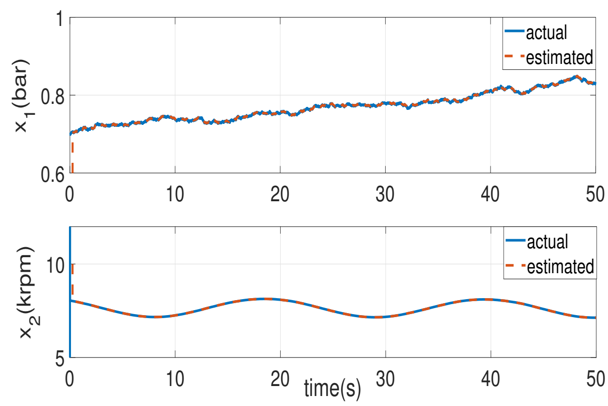

Fig. 5. State estimation

In figure 5, the first plot shows the estimated manifold pressure with orange curve and simulated manifold intake pressure in blue. Both the curves are almost coinciding. Initial value of the manifold pressure taken is 0.7 bar whereas the simulated value is 0.2 bar. The second plot in the figure shows estimated engine speed in orange and simulated speed in blue. The initial value for the speed is taken as 5000 rpm whereas the simulated value is 10000 rpm. The estimated speed converges to the simulated speed with time. In figure 6, behavior of the parameters with time is shown. C_p is a constant whereas C_t and K_a are time-varying. The estimated parameters converges towards the actual parameters in small time. The estimates track the actual values within a very small error band.

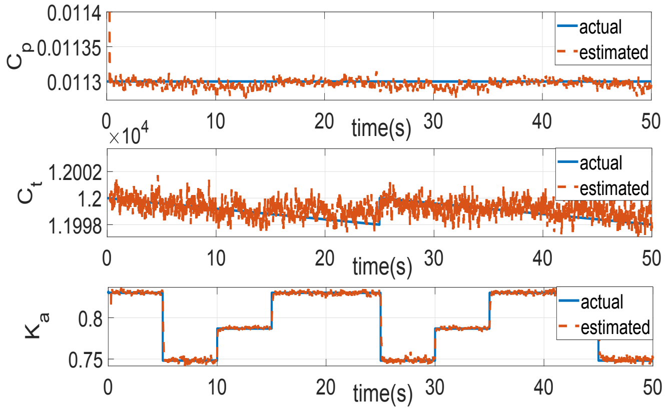

Fig. 6. Parameter estimation

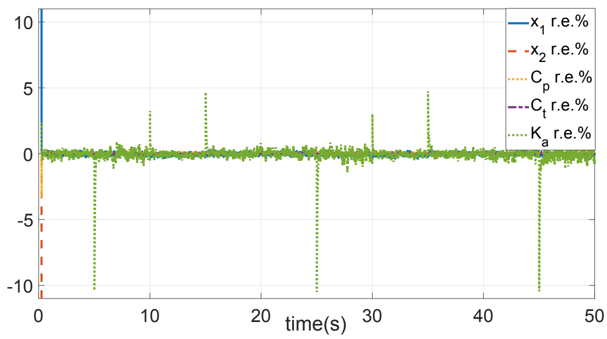

Fig. 7. Relative error percentage of states and parameters

Figure 7 shows the relative error percentage for the state and parameter estimation. The error is very small and acceptable. It can be observed from the figure that the algorithm converges for this case. Relative error percentage for both the states and parameters C_p and C_t is less than 0.4%. Relative error percentage for K_a is less than 11%.

$$r.e.\% = \frac{actual\,value - estimated\,value}{actual\,value} 100$$

4.2 Joint estimation using UKF

For the same data and initial conditions, joint state and parameter estimation is done using UKF for comparison. In joint estimation, states and parameters are estimated jointly by using one UKF. The parameters are considered as state variables for the estimation Wan and Van Der Merwe (2000). The system model of the system defined by (10) becomes:

$$\tilde{x}^p(k) = f(\tilde{x}^p(k-1), \tilde{v}(k-1), \tilde{u}(k-1))$$
$$\tilde{y}(k) = h(\tilde{x}^p(k), \tilde{d}(k), \tilde{u}(k))$$

where state vector $\tilde{x}^p(k) = (\tilde{x}(k), \tilde{\vartheta}(k))$. Process noise corresponding to additional states: $C_p \sim \mathcal{N}(0, 0.001)$, $C_t \sim$

$\mathcal{N}(0,100)$, $K_a \sim \mathcal{N}(0,10)$. $P_{\tilde{x}}(0) = diag(0.6, 5, 0.1, 100, 0.1)$. Constrains applied: $0 \leq x_1 \leq 1$, $x_2 \geq 10^{-6}$, $x_3 \geq 10^{-6}$, $x_4 \geq 10^4$, $x_5 \geq 10^{-6}$. Dynamics of the parameters fed to UKF:

$$\dot{C}_p = v_{Cp};\ \dot{C}_t = v_{Ct};\ \dot{K}_a = v_{Ka}$$

where v_{Cp}, v_{Ct}, v_{Ka} represents the process noise corresponding to C_p, C_t, K_a respectively.

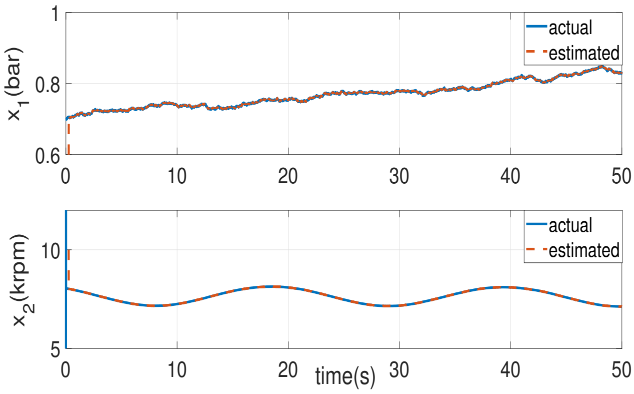

Fig. 8. State estimation using UKF for joint estimation

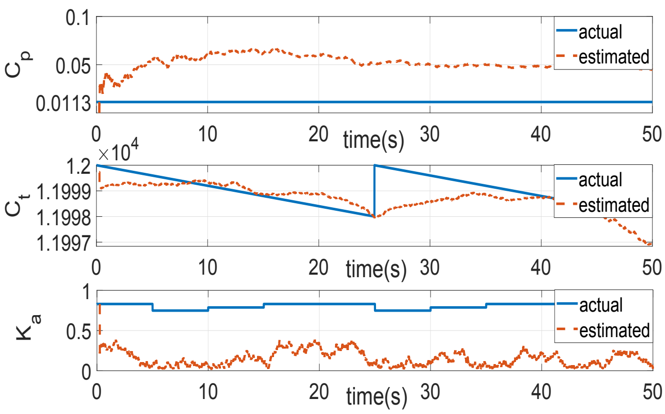

Fig. 9. Parameter estimation using UKF for joint estimation

Figure 8 shows the state estimation and figure 9 shows the parameter estimation using UKF for joint estimation. The parameter estimates tries to track the behavior but does not converges to their actual values. However, the state estimates converges to the actual states. This is because of two reasons: 1). we did not provide the true dynamics of C_t and K_a to UKF, 2). since UKF is robust to parameters within some range, so even with the estimates as in figure 9, the states with known dynamics i.e., intake manifold pressure and engine speed are estimated correctly.

5. CONCLUSION

Parameter estimation with unknown states has been done for SI engine model under the influence of process noise and measurement noise. The method estimates the parameters by using state estimates. States are estimated by UKF and parameters are estimated by RLSF. The results shows that the algorithm converges for the combination of constant and time varying parameters. In this case where one parameter is constant and two parameters are time varying, joint state and parameter estimation by using UKF does not converge. Further work will focus on parameter estimation with unknown states and wall wetting phenomenon.

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