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Model-Free Adaptive Fault-Tolerant Control for Multiple Point-Mass Subway Trains With Speed and Traction/Braking Force Constraints ⋆

Haojun Wang * **Zhongsheng Hou** ** **Shangtai Jin** *

* *School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 100044, China (e-mail: 1712078@bjtu.edu.cn; shtjin@bjtu.edu.cn).*
** *School of Automation, Qingdao University, Qingdao 266071, China (e-mail: zhshhou@bjtu.edu.cn; zshou@qdu.edu.cn).*

Abstract: This paper considers the model-free adaptive fault-tolerant control for a subway train based on multiple point-mass model with the actuator fault under the constraints of speed and traction/braking force. The complex subway train model is first transformed into a compact form dynamic linearization (CFDL) data model with pseudo gradient (PG). The actuator fault function is approximated with radial basis function neural network (RBFNN). Finally a fault-tolerant controller only using saturated input/output (I/O) data is designed. The effectiveness of proposed controller is illustrated by a simulation.

Keywords: Fault-tolerant control, model-free adaptive control, subway train, actuator fault, speed and traction/braking constraints, radial basis function neural network

1. INTRODUCTION

Advanced control schemes greatly guarantee the reliability and safety of subway trains. In recent years, some control schemes have been proposed for subway trains [Su et al. (2015); Howlett et al. (2009); Yang et al. (2016); Ning et al. (2015); Sun et al. (2017)]. However these results assume that subway trains operate at normal conditions. In practice, the subway train may have the actuator fault, which poses a great threat to the safety of the subway train. Thus it is necessary to design a fault-tolerant controller for the subway train. Some fault-tolerant control schemes have been proposed for trains [Song et al. (2011a,b); Li et al. (2008); Song et al. (2014); Wang et al. (2011); Li et al. (2017); Gao et al. (2015); Liu et al. (2015); Zhuan et al. (2010); Guo et al. (2017)]. Those results can be divided into two classes. One is the model-based method, and the other is data-driven one. The most model-based fault-tolerant control schemes are adaptive fault-tolerant control schemes. The data-driven fault-tolerant control schemes mainly include the fuzzy adaptive control schemes and neural network adaptive control schemes.

Generally, most existing control schemes for subway trains are based on the single point-mass model [Howlett et al. (2009); Yang et al. (2016); Ning et al. (2015); Sun et al. (2017)]. Compared with single point-mass model, multiple point-mass model can distinguish the difference between locomotives and carriages, and can describe the different resistance of each vehicle [Song et al. (2011a,b)]. Thus, the expression of multiple mass-point model for the subway train is more accurate than single point-mass model [Song et al. (2014); Li et al. (2008); Wang et al. (2011)].

Since the basic resistance and additional resistance of each vehicle are different, and the coefficients of these resistances are also depended on the track and the operation conditions of each vehicle, thus accurate modeling to a subway train is often very difficult [Song et al. (2010, 2011a)]. Further, when a subway train operates many years, the aging and the actuator faults are usually inevitable. Modeling these factors precisely is the another challenge. Therefore, using an inaccurate train model and the existing model-based control methods, to design a train operation control system, whose control performance would be questionable. In order to solve these issues, a data-driven fault-tolerant control scheme will be utilized for the subway train in this paper.

In some fault-tolerant control schemes, usually the unknown nonlinear fault function is approximated by a neural network [Song et al. (2011a,b); Wang et al. (2011); Li et al. (2017); Gao et al. (2015); Liu et al. (2015)]. The BP neural network, radial basis function neural network (RBFNN) and wavelet neural network, are the most popular networks used in this field . Because the RBFNN has simpler structure and stronger learning ability, RBFNN will be utilized to estimate actuator fault in this paper.

Model-free adaptive control (MFAC) is proposed for a class of unknown nonlinear discrete-time systems [Hou et al. (1998, 2011)]. The basic idea of MFAC is that, the controlled plant model is first equivalently transformed into a compact form dynamic linearization (CFDL) data

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model by using a new concept called pseudo gradient (PG). The time-varying PG is then estimated merely using the input/output (I/O) measurement data of the closed-loop system of the controlled plant. Next the adaptive control scheme is designed based on the equivalent CFDL data model. Compared with other model-based control schemes, the MFAC approaches have several advantages, which enable it more suitable for practical control applications. First, MFAC just use the real-time measurement data of the controlled plant to design the control system, which implies that we can directly develop a controller for an industrial practical process bypassing the time-consuming modeling procedure. Second, only the time-varying PG needs to be updated on-line by using the real-time data generated in the control process, which means that MFAC does not require any training process. Third, MFAC is of a simple structure and is easily implemented with a strong robustness. Fourth, under some practical assumptions, the monotonic convergence and bounded-input bounded-output (BIBO) stability of the control system can be guaranteed [Hou et al. (2019)]. Finally, the CFDL based MFAC has been successfully implemented in many practical applications [Hou et al. (2017, 2019)].

Based on the above discussion, a model-free adaptive fault-tolerant control (MFAFTC) scheme is proposed in this paper. The main contributions of this paper are summarized as following aspects:

(1) A MFAC based fault-tolerant controller is designed for the actuator fault with multiple point-mass model of subway train. Both the speed and traction/braking force constraints are considered. (2) Compared with [Song et al. (2011a,b); Wang et al. (2011); Song et al. (2010); Gao et al. (2015)], the number of parameters need to be updated is reduced and the information of the subway train model is not utilized in the design of controller.

The remainder of this paper is organized as follows. In Section II, the complex multiple mass-point subway train model is transformed into an equivalent data model and a fault compensation mechanism is designed. In Section III, a fault-tolerant controller is designed for the subway train. A simulation is provided in Section IV. The main conclusion is expounded in Section V.

2. PROBLEM FORMULATION

This section will introduce a multiple point-mass subway train model, its equivalent CFDL data model as well, and a fault compensation mechanism utilized to compensate the actuator fault.

2.1 Multiple Point-Mass Subway Train Model

Considering a multiple mass-point subway train model consisting of n vehicles (p carriages and q locomotives) connected by $n-1$ elastic couplers, see Fig. 1, where $v_1(km/h)$ is the speed of first vehicle, $x_i(km)$ is the distance between the center of ith vehicle and the reference point, $\triangle x_i = l_i + \triangle x_{di}$ is the length of the connector between the ith vehicle and $(i+1)$th vehicle, l_i is a constant, and $\triangle x_{di}$ is the length of the ith spring connector, which is determined by

$$\triangle x_{di} = x_i - x_{i+1} - d_{i+1} - d_i - l_i, \qquad (1)$$

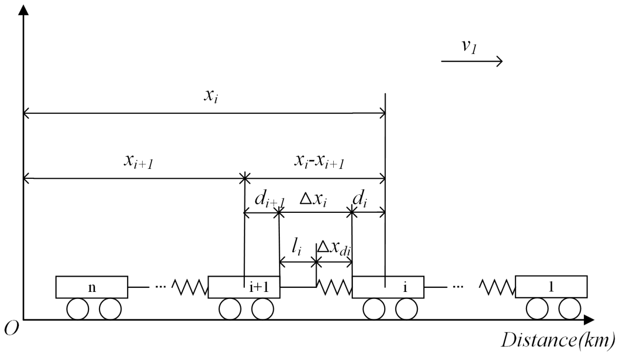

Fig. 1. Multiple mass-point model of subway train

where d_{i+1}, d_i and l_i are constants, so the following can be obtained.

$$\triangle \dot{x}_{di} = \dot{x}_i - \dot{x}_{i+1}, \qquad (2)$$

$$\triangle \ddot{x}_{di} = \ddot{x}_i - \ddot{x}_{i+1}. \qquad (3)$$

Then one can get that

$$\ddot{x}_i = \ddot{x}_1 - \sum_{j=1}^{i-1} \triangle \ddot{x}_{dj}. \qquad (4)$$

The forces imposed by ith vehicle have traction/braking force $u_{fi}(kN)$, in-train force $f_{i-1}(kN)$ and $f_i(kN)$, additional resistance $f_{ai}(kN)$, basic resistance $f_{bi}(kN)$, support force $N_i(kN)$ and gravity $G_i(kN)$. According to the Newton's second law, the multiple point-mass model could be written as follows

$$\begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_n, \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \begin{bmatrix} u_{f1} \\ u_{f2} \\ \vdots \\ u_{fn} \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ f_1 \\ \vdots \\ f_{n-1} \end{bmatrix} - \begin{bmatrix} f_1 \\ \vdots \\ f_{n-1} \\ 0 \end{bmatrix} - \begin{bmatrix} f_{a1} \\ f_{a2} \\ \vdots \\ f_{an} \end{bmatrix} - \begin{bmatrix} f_{b1} \\ f_{b2} \\ \vdots \\ f_{bn} \end{bmatrix}, \qquad (5)$$

where $m_i(ton)$ is the mass of ith vehicle, $\lambda_i > 0$ is a distribution constant of the ith vehicle. Assume that $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 1$ in this paper. The expression of f_i is a nonlinear and uncertain function. It is noteworthy that the in-train force obeys the "action and reaction" rule, which motivates us to apply the summation on the both sides of (5). Then the following multiple mass-point with single-coordinate subway train model can be obtained.

$$M\ddot{x}_1 = \boldsymbol{\lambda}^T \boldsymbol{u_f} - F_a - F_b + F_r, \qquad (6)$$

where $M = \sum_{i=1}^{n} m_i$, $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_n]^T$, $\boldsymbol{u_f} = [u_{f1}, u_{f2}, \cdots, u_{fn}]^T$, $F_a = \sum_{j=1}^{n} f_{ai}$, $F_b = \sum_{j=1}^{n} f_{bi}$ and $F_r = \sum_{i=1}^{n-1} \triangle \ddot{x}_{di} \sum_{j=i+1}^{n} m_j$. Therefore, the nonlinear and uncertain in-train force is offset and multi-input multi-output (MIMO) system (5) of the subway train is transformed into a multi-input single-output (MISO) system (6).

The formulations of basic resistance f_{bi} and additional resistance f_{ai} in (6) are given as (7) and (8), respectively.

$$f_{bi} = a_0 + a_1 v_i + a_2 v_i^2, \tag{7}$$

$$f_{ai} = w_{ri} + w_{ci} + w_{ti}, \tag{8}$$

where a_{0i}, a_{1i} and a_{2i} are the resistive coefficients, w_{ri}, w_{ci} and w_{ti} are ramp resistance, curve resistance and tunnel resistance of ith vehicle, respectively..

In practice, the traction/braking actuator may lose its efficiency or even completely collapse during operation due to various kinds of reasons, such as overvoltage in traction transformer, overcurrent in traction converter, and overheat in asynchronous motor and so on [Song et al. (2014)]. The expression of the actuator fault could be described as

$$u_{ai} = p_{1i} u_{fi} + p_{2i}, \tag{9}$$

where u_{ai} is actual force applied to ith vehicle, $p_{1i} \in (0, 1]$ and $p_{2i} \in R$ are health factor and additive fault coefficient of ith vehicle, respectively [Li et al. (2017); Gao et al. (2015); Guo et al. (2017)].

Therefore, the subway train model with actuator fault can be given as

$$M\ddot{x}_1 = \boldsymbol{\lambda}^T \boldsymbol{u_a} - F_a - F_b + F_r, \tag{10}$$

where $\boldsymbol{u_a} = [u_{a1}, u_{a2}, \cdots, u_{an}]^T$ is actual force applied to the subway train.

It should be noted that a_{0i}, a_{1i} and a_{2i} in (7) and w_{ri}, w_{ci} and w_{ti} in (8) are difficult to obtain accurately [Song et al. (2011a,b)], which leads to the precise mathematical expressions of f_{ai} and f_{bi} are not easy to establish. Further, the mass of ith vehicle m_i may change due to passengers getting on and off the subway train. And the health factor p_{1i} and additive fault coefficient p_{2i} are also unknown in subway train operation. Therefore, the exact expression of (10) is difficult to obtain and it is unknown to the designer. However, the subway train generates a large amount of discrete time I/O data in operation, where input data is traction/braking force and output data is speed information. The I/O data of system will be used to realize the fault-tolerant control of the subway train in this paper.

Discretizing multiple mass-point with single-coordinate model (10), taking the traction/braking force $\boldsymbol{u_f}(k)$ and speed $v_1(k)$ as input and output, respectively, and considering the unavailable coefficients and the nonlinearity of the subway train dynamics, the general dynamics with actuator fault of a subway train can be formulated as following general form

$$v_1(k+1) = g(v_1(k), \cdots, v_1(k-n_v), \boldsymbol{u_f}(k), \cdots, \boldsymbol{u_f}(k-n_u), \boldsymbol{f_s}(k)), \tag{11}$$

where $g(\cdot) \in R$ is an unknown nonlinear function, n_v and n_u are two unknown positive integers, $\boldsymbol{f_s}(k) = [f_{s1}(k), f_{s2}(k), \cdots, f_{sd}(k)]^T \in R^d$ is the unknown fault function vector caused by the actuator fault, d is the number of fault functions. The following assumptions are introduced for the subway train system (11).

Assumption 1 [Hou et al. (1998, 2011)]. The partial derivation of $g(\cdot)$ with respect to the $(n_v + 2)$th variable and the $(n_v + n_u + 3)$th variable are continuous.

Assumption 2 [Hou et al. (1998, 2011)]. The system (11) satisfies generalized *Lipschitz* condition, that is, $|\triangle v_1(k+1)| \leq b \|\triangle \boldsymbol{U}(k)\|$, for any k and $||\triangle \boldsymbol{u_f}(k)|| \neq 0$, where $\triangle v_1(k+1) = v_1(k+1) - v_1(k)$, $\triangle \boldsymbol{U}(k) = [\triangle \boldsymbol{u_f}(k), \triangle \boldsymbol{f_s}(k)]^T$, $\triangle \boldsymbol{u_f}(k) = \boldsymbol{u_f}(k) - \boldsymbol{u_f}(k-1)$, $\triangle \boldsymbol{f_s}(k) = \boldsymbol{f_s}(k) - \boldsymbol{f_s}(k-1)$ and b is a positive constant.

Theorem 1. Consider the nonlinear system (11) satisfying *Assumption 1-2*. If $||\triangle \boldsymbol{u_f}(k)|| \neq 0$, then there exists a time-varying vector $\boldsymbol{\varphi_1}(k) \in R^n$, called PG, and a time-varying vector $\boldsymbol{\varphi_2}(k) \in R^d$, such that system (6) can be described as following CFDL data model,

$$\triangle v_1(k+1) = \boldsymbol{\varphi_1}(k) \triangle \boldsymbol{u_f}(k) + \boldsymbol{\varphi_2}(k) \triangle \boldsymbol{f_s}(k), \tag{12}$$

where $\boldsymbol{\varphi_1}(k) = [\varphi_{11}(k), \cdots, \varphi_{1n}(k)]$, $\boldsymbol{\varphi_2}(k) = [\varphi_{21}(k), \cdots, \varphi_{2d}(k)]$, $\|[\boldsymbol{\varphi_1}(k), \boldsymbol{\varphi_2}(k)]\| \leq b$.

Since the speed of subway train should not exceed the allowable maximum speed, and the traction/braking force should not exceed the corresponding maximum force. The constraints of speed and traction/braking force are given as follows.

$$\bar{v}(k) = sat_v(v_1(k)) = \begin{cases} v_{max}, & v_1(k) > v_{max} > 0 \\ v_1(k), & otherwise \end{cases} \tag{13}$$

$$\bar{u}_{fi}(k) = sat_{u_f}(u_{fi}(k)) = \begin{cases} u_{ti}, & u_{fi}(k) \geq u_{ti} > 0 \\ u_{bi}, & u_{fi}(k) \leq u_{bi} < 0 \\ u_{fi}(k), & otherwise \end{cases} \tag{14}$$

where v_{max} is the maximum speed allowed, $\bar{v}(k)$ is the measured speed, u_{ti} and u_{bi} are the maximum traction force and braking force provided of ith vehicle, respectively. $\bar{u}_{fi}(k)$ is saturated force applied to ith vehicle.

Lemma 1. According to (13), one can obtain the expression of relationship between $\bar{v}(k)$ and $v_1(k)$ as

$$\triangle \bar{v}(k) = h(k) \triangle v_1(k), \tag{15}$$

where $0 \leq h(k) \leq 1$.

The objective of this paper is to design a fault-tolerant controller for the multiple mass-point subway train under the constraints of speed (13) and traction/braking force (14) only using the measured I/O data of the subway train, which can control actual speed of the subway train to track the desired speed and overcome the impact of the actuator fault on the subway train.

2.2 Fault Approximation Mechanism

RBFNN has the capability of on-line approximation for nonlinear function, which is utilized to compensate fault function $\boldsymbol{\varphi_2}(k) \triangle \boldsymbol{f_s}(k)$. For convenience, $\boldsymbol{\varphi}_2(k) \triangle \boldsymbol{f_s}(k)$ is represented as $\xi(k)$.

The simple structure of RBFNN is shown in Fig. 2, where $R(k) = [\bar{v}_1(k), \varepsilon(k)]^T$ is the input, $\varepsilon(k) = v_1^*(k) - \bar{v}_1(k)$, $v_1^*(k)$ is the desired speed of the subway train, $\psi_i(k)$ is the radial basis function, $w_i(k)$ is the weight, $\xi(k)$ is the output layer that is expressed as

$$\xi(k) = \boldsymbol{w}^{*T}(k) \boldsymbol{\psi}(k) + \varsigma(k), \tag{16}$$

where $\boldsymbol{w}^*(k) = [w_1^*(k), \cdots, w_l^*(k)]^T$ is ideal weight vector, $\boldsymbol{\psi}(k) = [\psi_1 k, \cdots, \psi_l(k)]$ is Gaussian function vector and

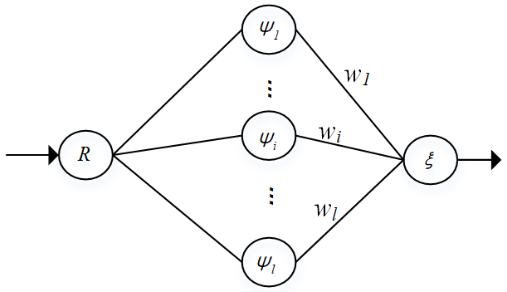

Fig. 2. The structure of RNFNN

$\varsigma(k)$ is the error of compensation[Li et al. (2017); Liu et al. (2015)]. The expression of $\psi_i(k)$ is selected as

$$\psi_i(k) = exp\left(-\frac{\|R(k)-c_i\|^2}{2\sigma_i^2}\right), \quad (17)$$

However, The ideal weight vector $\boldsymbol{w}^*(k)$ is not easy to obtain. The estimation of ideal weight vector $\hat{\boldsymbol{w}}(k) = [\hat{w}_1(k), \cdots, \hat{w}_l(k)]^T$ is calculated as

$$\triangle\hat{\boldsymbol{w}}(k) = \varsigma(v_1^*(k) - v_1(k))\boldsymbol{\psi}(k), \quad (18)$$

where $\triangle\hat{\boldsymbol{w}}(k) = \hat{\boldsymbol{w}}(k) - \hat{\boldsymbol{w}}(k-1)$.

So the estimation of $\hat{\xi}(k)$ can be expressed as

$$\hat{\xi}(k) = \hat{\boldsymbol{w}}^T(k)\boldsymbol{\psi}(k) \quad (19)$$

3. DESIGN OF FAULT-TOLERANT CONTROLLER

In this section, a MFAFTC scheme is designed for the actuator fault with the RBFNN. Both the speed and traction/braking force constraints are considered in the design of the controller, which can guarantee the safety of the subway train.

The following criterion function is introduced for design of controller.

$$Q(\boldsymbol{u_f}(k)) = (v_1^*(k+1) - v_1(k+1))^2 + \rho_1\|\triangle\boldsymbol{u_f}(k)\|^2, (20)$$

where $\rho_1 > 0$ is a weight coefficient.

The first item of (20) refers to the square of difference between the actual speed and the desired speed. Its numerical value can reflect the ability of the subway train to track the desired trajectory. And the second item can reflect the stationarity of the subway train operation.

According to $\partial Q(\boldsymbol{u_f}(k))/\partial\boldsymbol{u_f}(k) = 0$, the following controller can be obtained.

$$\boldsymbol{u_f}(k) = \boldsymbol{u_f}(k-1) + \frac{\beta_1\boldsymbol{\varphi_1}^T(k)}{\rho_1 + \|\boldsymbol{\varphi_1}(k)\|^2} \times (v_1^*(k+1) - v_1(k) - \boldsymbol{\varphi_2}(k)\triangle\boldsymbol{f_s}(k)), \quad (21)$$

where $\beta_1 \in (0, 1]$ is a step-size constant.

The cost function of the estimation of $\boldsymbol{\varphi_1}(k)$ is designed as follows.

$$Q(\boldsymbol{\varphi}_1(k)) = (v_1(k) - v_1(k-1) - \boldsymbol{\varphi_1}^T(k)\triangle\boldsymbol{u_f}(k) - h(k) \times\hat{w}^T(k-1)\psi(k-1))^2 + \rho_2\|\boldsymbol{\varphi_1}(k) - \hat{\boldsymbol{\varphi}}_1(k-1)\|^2, (22)$$

where $\rho_2 > 0$ is a weight coefficient.

The first item of (22) includes the actuator fault compensation, which can overcome the impact of actuator fault to the subway train. And the (22) can solve the problem that the estimation of $\boldsymbol{\varphi_1}(k)$ is sensitive to some inexact sampling data.

According to minimization of (22), the following simplified PG estimation algorithm is used for the estimation

$$\hat{\boldsymbol{\varphi}}_1(k) = \hat{\boldsymbol{\varphi}}_1(k-1) + \frac{\beta_2\triangle\boldsymbol{u_f}^T(k-1)}{\rho_2 + \triangle\|\boldsymbol{u_f}(k-1)\|^2}(\triangle v_1(k) - h(k) \times\hat{w}^T(k-1)\psi(k-1) - \hat{\boldsymbol{\varphi}}_1(k-1)\triangle\boldsymbol{u_f}(k-1)), (23)$$

where $\beta_2 \in (0, 2]$ is a step step-size constant and $\hat{\boldsymbol{\varphi}}_1(k)$ is the estimation of $\boldsymbol{\varphi}_1(k)$.

Integrating (13), (14), (19) and (20)-(23), the MFAFTC scheme is constructed as follows.

$$\bar{u}_{fi}(k) = sat_{u_f}(u_{fi}(k)) = \begin{cases} u_{ti}, u_{fi}(k) \geq u_{ti} > 0 \\ u_{bi}, u_{fi}(k) \leq u_{bi} < 0 \\ u_{fi}(k), otherwise \end{cases} \quad (24)$$

$$\boldsymbol{u_f}(k) = \boldsymbol{u_f}(k-1) + \frac{\beta_1\boldsymbol{\varphi_1}^T(k)}{\rho_1 + \|\boldsymbol{\varphi_1}(k)\|^2} \times (v_1^*(k+1) - \bar{v}_1(k) - \hat{w}^T(k)\psi(k)), \quad (25)$$

$$\hat{\boldsymbol{\varphi}}_1(k) = \hat{\boldsymbol{\varphi}}_1(k-1) + \frac{\beta_2\triangle\bar{\boldsymbol{u}}_{\boldsymbol{f}}^T(k-1)}{\rho_2 + \triangle\|\bar{\boldsymbol{u}}_{\boldsymbol{f}}(k-1)\|^2}(\triangle\bar{v}_1(k) - h(k) \times\hat{w}^T(k-1)\psi(k-1) - \hat{\boldsymbol{\varphi}}_1(k-1)\triangle\bar{\boldsymbol{u}}_{\boldsymbol{f}}(k-1)), (26)$$

$$\hat{\varphi}_{1i}(k) = \hat{\varphi}_{1i}(1), if|\hat{\varphi}_{1i}(k)| \leq \delta, or|\triangle\bar{u}_{fi}(k)| \leq \delta, \quad (27)$$

4. SIMULATION

In this section, the performances of three controllers including MFAFTC (24)-(27), PI in [Li et al. (2006)] and the prototype MFAC in [Hou et al. (2011)] are compared by simulation results, which is illustrated the effectiveness of MFAFTC.

The number of vehicles is 6 (2 carriages and 4 locomotives), the parameters of the subway train are shown in Table 1. The each elastic coupler and basic resistance are given as (28), (29) and (30), respectively. The sampling time is 1s.

Table 1. Parameters Setting for The Subway Train

Comment	Symbol	Value
Maximum traction force (kN)	u_t	60
Maximum braking force (kN)	u_b	-60
Mass of each vehicle (ton)	M	50
Running line (m)	L	2391
Running time (s)	T	140

$$\triangle x_{d1} = \triangle x_{d3} = \triangle x_{d5} = 0.8sin(2k), \quad (28)$$

$$\triangle x_{d2} = \triangle x_{d4} = 0.25cos(k). \quad (29)$$

$$F_b = 1.02 + 0.0035v_1(k) + 0.000426v_1^2(k). \quad (30)$$

Assume that the actuator fault occurs at 70s and the actuator fault is given as

$$\boldsymbol{u_a}(k) = \begin{bmatrix} 0.90 & & & & & \\ & 0.70 & & & & \\ & & 1.00 & & & \\ & & & 0.76 & & \\ & & & & 0.78 & \\ & & & & & 0.86 \end{bmatrix} \boldsymbol{u_f}(k) + \begin{bmatrix} -5.30 \\ 3.52 \\ 3.39 \\ -6.57 \\ 0.71 \\ -2.88 \end{bmatrix} \quad (31)$$

The parameter settings of three controllers are chosen by trial and error approach. The initial values of input of MFAC and MFAFTC schemes are set as $\boldsymbol{u_f}(1) = [0.01, 0.01, 0.01, 0.01, 0.01, 0.01]^T$ and $\boldsymbol{u_f}(2) = [9.41, 0.01, 0.64, 0.64, 0.01, 0.35]^T$. Initialling the values of output to $v_1(1) = 0.01$, $v_1(2) = 0.01$ and $v_1(3) = 2.43$. Setting the initial value of PG as $\boldsymbol{\varphi_1}(1) = [0.58, 0.95, 0.73, 1.08, 0.09, 0.08]^T$. In addition, setting the values of the remaining parameters as $\beta_1 = 0.07$, $\rho_1 = 3.52$, $\beta_2 = 0.99$, $\rho_2 = 0.01$. The parameters of PI are set as $K_P = [33.25, 28.73, 25.23, 30.25, 30.66, 28.44]^T$ and $K_I = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1]^T$.

The RBFNN is utilized to approximate the actuator fault in MFAFTC. The value of weights are initialized as zero. The number of the hidden layer neurons is a free design parameter. It is set as 7 in this paper. The input of RBFNN are set as $R(k) = [\bar{v}_1(k), \varepsilon(k)]^T$. The width σ and center c of Gaussian function are given as follows.

$$\sigma = [0.41 \quad 0.31 \quad 0.73 \quad 0.93 \quad 0.49 \quad 0.76 \quad 0.72]^T.$$

$$c = \begin{bmatrix} 69.00 & 67.94 & 65.67 & 63.44 & 69.74 & 60.18 & 67.37 \\ 0.50 & 0.01 & 1.46 & 0.38 & 0.01 & 3.96 & 2.61 \end{bmatrix}.$$

Fig. 3-7 show the simulation results of three control schemes. The tracking performances of three control schemes are shown in Fig. 3. Fig.4 gives the tracking errors of three control schemes. The traction/braking forces of MFAC, PI and MFAFTC are exhibited in Fig.5, Fig.6 and Fig.7, respectively. From Fig.3 and Fig.4, we can get that the speed tracking error and the range of speed fluctuation in MFAFTC scheme are less than them in PI and MFAC scheme, thus the tracking ability of MFAFTC scheme is better than the others. The traction/braking force with MFAFTC scheme has been constrained within the given range, shown in Fig.7, but the tracking force with the prototype MFAC exceeds the threshold due to the actuator fault, which can been seen in Fig.5.

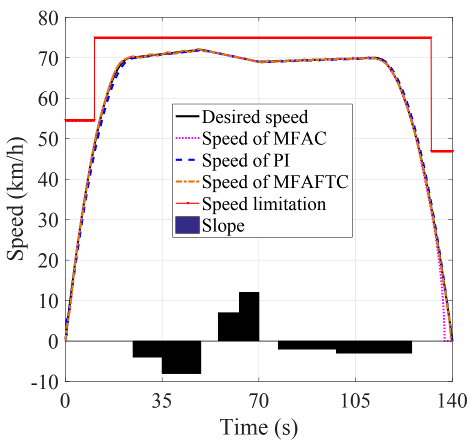

Fig. 3. Tracking performances

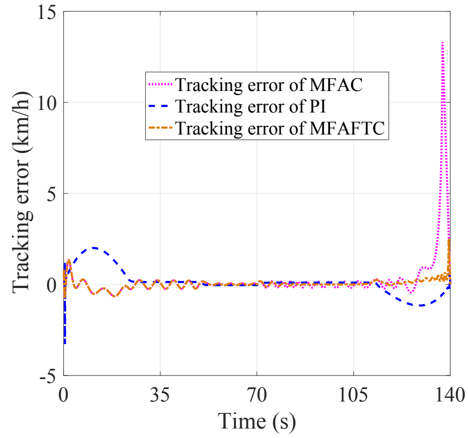

Fig. 4. Tracking errors

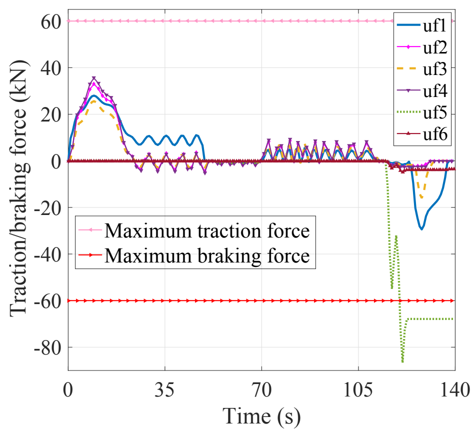

Fig. 5. Traction/braking force of MFAC

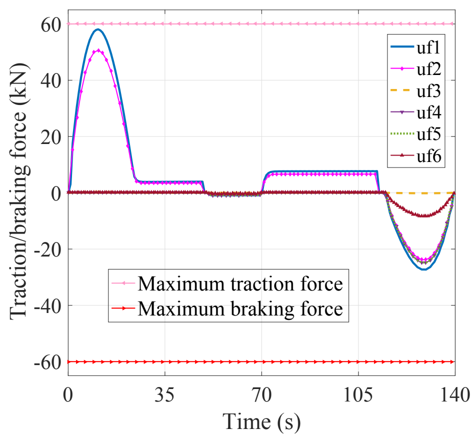

Fig. 6. Traction/braking force of PI

Integral of absolute error (IAE) is used to measure the tracking ability of two control schemes, the IAE is defined as follows.

$$IAE = \sum_{k=1}^{T} |v_1^*(k) - \bar{v}_1(k)|. \quad (32)$$

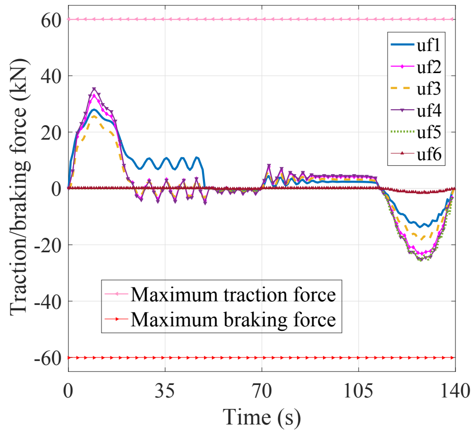

Fig. 7. Traction/braking force of MFAFTC

According to (32), the results of IAE for MFAC, PI and MFAFTC are 60.45, 57.10 and 21.65, respectively. It can be concluded that the tracking ability of MFAFTC is much better than that of MFAC and PI.

5. CONCLUSION

The MFAFTC scheme is designed for the multiple mass-point subway train model with speed and traction/braking force constraints in this paper. An equivalent CFDL data model with PG is proposed for the multiple point-mass with single-coordinate subway train model with the actuator fault. The actuator fault function is compensated by RBFNN. The simulation results are offered to verify the effectiveness of proposed control scheme. In future, we will consider the possible field application.

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