Inventory model for disassembly systems with price dependent return rate

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Abstract: In this paper, we are proposing a new Economic Order Quantity (EOQ) model for return-driven disassembly systems. End-of-life (EoL) products arrive in the system to be disassembled into parts or material fractions that can be sold in different secondary market or disposed of in an environmental conscious way. The returns are considered controllable with respect to a buyback price. The model can determine if the system is profitable by finding an equilibrium between revenues obtained from the components and buyback, disassembly and inventory costs. The properties of the model are analyzed to derive an efficient solution approach to find the optimal return price and the reorder interval. A sensibility analysis performed on an illustrative example shows the effect of the model parameters.

Keywords: disassembly, EOQ, lot sizing, inventory control, price, end-of-life product.

1. INTRODUCTION

Disassembly is a central activity in product recovery. It connects the collection of end-of-life products at customer locations to the recycling of components with a residual value (by reusing, remanufacturing, material recycling or energy recovering) or to the processing and the conditioning of valueless components and material fractions. The raw material for disassembly systems are the end-of-life products. The volume that enter the system can be unlimited, imposed or controllable depending on the type of product and the environment of the system. The disassembled components are the outputs of the systems Disassembly systems can be considered in different context regarding their output flows. The components can be considered as unit part or as material. There are allocated to different types of recovery channels. Most of the parts or material obtained by disassembly generate incomes. They can be due to the sales in a market (as material, spare parts, or components of a products) or to the transfer of a part of the product price when it includes environmental taxes.

Different context can be encountered in practice but in most of papers on disassembly planning or scheduling, the volume of end-of-life products is considered as unlimited and the system is driven by the demands for the component (assuming there exists). The main specificities of disassembly systems that make them challenging in planning decisions are mainly: (i) the products are the source of the material flows which then diverge to multiple components outputs, (ii) one disassembly operation generated all the components simultaneously, and (iii) the outputs are provided to different market with independent demands.

Under these assumptions, MRP-like algorithm can be used to plan disassembly operations over a discrete and finite planning horizon without cost considerations (Gupta and Taleb, 1994). By considering various type of costs, mathematical programming related approached are mainly used to find solution under different assumptions on the demands (see (Kim et al., 2007) for a literature review of the basic features of the problem, (Ji et al., 2016) or (Liu and Zhang, 2018) for more recent variant of the problem). Planning model under EOQ-like assumptions (constant parameters over an infinite planning horizon) are proposed by Godichaud and Amodeo (2018, 2019). The authors consider disposal option to have a stationary policy given the special specificities of disassembly systems which leads to unnecessary inventories.

In many real cases, the return of products are limited and can be fully, partially or not controllable. In this paper, we consider disassembly system driven by the returns of product under EOQ-like assumptions. Some of the disassembled components can be allocated to a recovery channels which pay a price to acquire them. Inventory systems with return rate arise in reverse logistic in the case of remanufacturing systems (models in (Dobos and Richter, 2004) and (Teunter, 2001) are representative of this context) or closed-loop supply chain (see Saha et al., 2016 or Genc and Giovani, 2017). The case with controllable return rate has been first considered by El Saadany and Jaber (2010). The authors consider that the return rate of used items is variable and dependant of two decision variables, a purchasing price for returned items and an acceptance quality level. In (Teksan and Geunes, 2016), the authors consider an inventory model with a variable supply rate that can be seen as a return rate. They also consider the demand as price sensitive, with the same demand-price function as Ray et al. (2005), so that a functional relationship between supply and selling prices is developed to guaranty an equilibrium between both sides. Our work presented in this paper differ from previous ones with respect to the divergence of the component output flows to different market with independent demands and the returns of product which are not correlated to the demands.

The model can be applied to any disassembly centre processing end-of-used manufactured product like vehicle or electronic devices. The disassembly bill of material can contain spare parts, remanufactured or reused as well as

material fraction that can be recycling. It is for instance the case in end-of-life vehicle recycling where engine, running gear, suspension, doors or gearbox can be used as spare parts or recycled with others metal, plastics or glass fractions. Some other examples can also be found in literature related to other disassembly planning problems. The end-of-use product concerned can be for instance personal computers (Güngör and Gupta, 2002), cell phones (Kalayci and Gupta, 2013), copying machines (Lambert, 2007) or automotive engines (Seidi and Saghari, 2016). In many of the disassembly centres which processed these types of products, the return volume is not unlimited and can be varied according to buying back prices.

The contribution of the paper is to propose a new inventory model under EOQ-like assumption for disassembly systems which are return driven and where components are sold in different secondary market. We analyse the properties of the model to propose an efficient solution method. The model and method aim to be used by decision managers to determine the profitability of disassembling end-life-product given the parameters which have an effect on their return rate and the markets for the components. In section 2, the assumptions of the problem are presented with the return-price function we have selected for this study. The model and its properties are analysed in section 3 to achieve a solution method. An illustrative example with a sensitivity analysis is proposed in section 4 and the conclusion is presented in section 5.

2. PROBLEM STATEMENT

Disassembly planning is based on a bill-of-material of the product to disassemble. For the problem addressed in this paper, two-level bill-of-material are considered. The first level represents the product and the second level represents the components which are obtained by one disassembly operation. A disassembly bill-of-material can include spare parts, parts that can be reused or remanufactured as well as material fractions that can be recycled. One disassembly operation on the product generate simultaneously all the components and the disassembly yield of one leaf item is the number of unit of this one obtained at each disassembly operation. Fig. 1 presents an example of a disassembly system for a product with three components. The disassembly operation is on the product, item 0, and it generates all the components, items 1 to 3, in the quantity noted on the edge (disassembly yields).
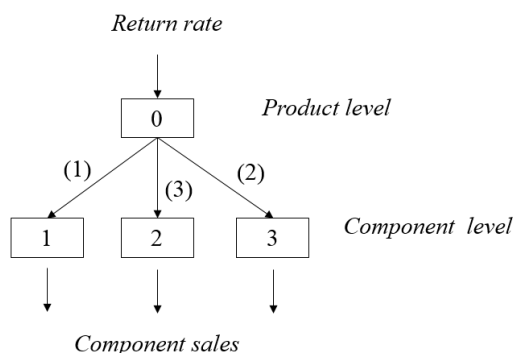
Disassembly systems driven by the return of end-of-life products are considered in this paper. In many real cases, disassembly center performance is based on the volume of product that are processed (achievement of a recovery rate). The end-of-life products arrived in the system according to a return rate (unit of product per unit of time) and are stored before being disassembled. A price is offered by disassembly center to the owners of end-of-life product to acquire it. As the price increases, attraction the disassembly center is improved and it can collect more product. An income is generated for each component disassembled that be allocated to a recovery channel. Based on this description of the system operation, the following assumption are considered (these are basic setting EOQ related settings):

- end-of-life product arrival is constant, continuous and characterized by a constant rate in units per unit time (per year for example),
- the planning horizon is considered as infinite,
- the disassembly yields are known and constant,
- there is a fixed cost for each disassembly operation incurred whenever an order is placed,
- there is an inventory holding cost for each unit in inventory per unit time,
- the disassembly rate is sufficiently high to consider disassembly operations as instantaneous,
- the return rate is an increasing function of a collection price (effort to collect more product).

A disassembly order launches disassembly operations on products, the product inventory is then emptied and the income for each component is collected. The collection price is set for all the planning horizon. The problem is to determine the collection price (or the return rate) simultaneously with the disassembly policy which sets the timing of disassembly orders and the associated quantities. The objective is to maximize a profit function which includes the revenues generated by the sales of the component, the collection effort price, the unit disassembly, inventory holding and order costs. We restrict our attention to stationary policies where the orders are repeated according to a time cycle with a constant length and the product inventory is emptied at each order. The following notations are used:

- $i = 1 ... N$ are the index for the leaf items and $i = 0$ is used for the product,

- α_i is the yield of component i (quantity of items i that can be obtained from one product),

- h is the inventory cost per unit and per unit time,

- c is the disassembly cost of one unit of the product,

- k is the disassembly order cost for the product,

- x is the return rate (unit per unit of time) for the product,

- p_i is the sale price of component i,

Fig. 1. Disassembly bill of material with three components

- M_i is the maximum output rate to the market for component i (the additional obtained components do not generate an income),

- p_0 is the product collection price,

- X is the maximum return rate,

- a, b are the parameters of the price-return function,

- $F(p_0)$, $G(x)$ are, respectively, the return function of the price and the price function of the return,

- Q is the disassembly quantity per order,

- T is the cycle time (time between two disassembly orders),

- $\Pi(T, x)$ is the total mean profit per unit of time,

- $R(x)$ is the total mean profit per unit of time without inventory costs,

- $C(T, x)$ is the total mean inventory cost.

Based on these assumptions, the evolution of the product return inventory is illustrated in Fig. 2 with two different return rates (x or x') and the same reorder interval T. We note that two decision variables are necessary to set the policy and we use T and x while Q and p_0 can be deduced with $Q = T \cdot x$ and $p_0 = G(x)$.
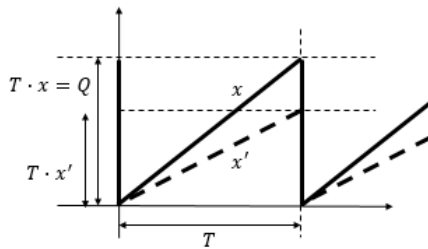

Fig. 2. Evolution of the returned product inventory.

The return-price function mathematically translates the fact that, as the potential price offered to end-of-life product owners increases, the disassembly centre is more attractive and receives more returns. Similarly to price-demand function, the price effect on the returns can be modelled by different upward sloping functions depending on the possibility to give rise to explicit results for the optimal solution, the ease to estimate its parameters in an empirical study and the ability to correspond to reality Huang et al. (2013). In this paper, we adapt a power iso-elastic function defined in (1), with $a > 0$ and $b > 0$, which is one of the most widespread function in pricing literature (Ray et al. 2005). We note that $F(0) = 0$ but if there is a minimal return rate, the function is easily modified and the results are not changed. We also assume that there is an upper bound on the return rate since the field of end-of-life products is limited. More generally, the proposed results in the papers are similar by choosing another reasonable upward sloping function. The shape of the function is illustrated on Fig. 3 with $a = 80$, different value for b and a maximum return rate $X = 5000$.

$$F(p_0) = a \cdot (p_0)^b \qquad (1)$$

The function is easily reversed as display in (2) and the return rate can be equivalently used as decision variable.

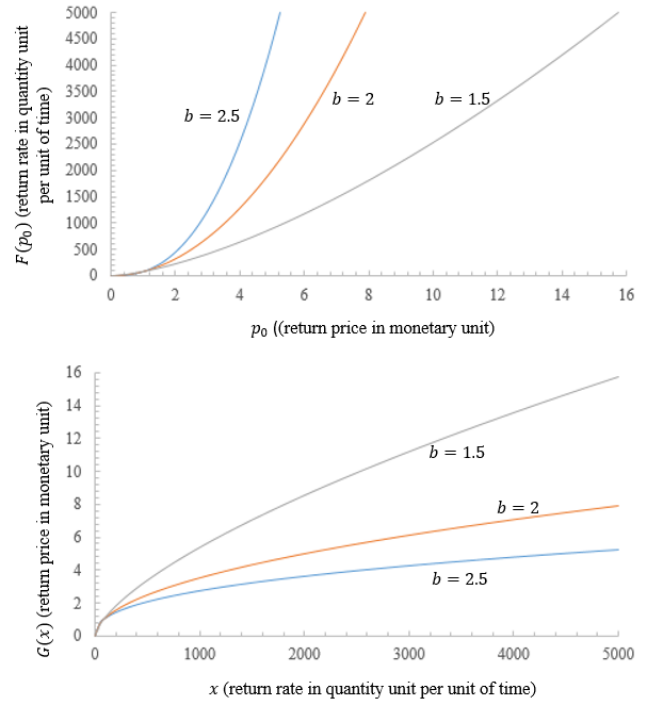$$G(x) = (x/a)^{\frac{1}{b}} \qquad (2)$$

Fig. 3. Shapes of return-price functions (with $a = 80$).

3. MODEL AND SOLUTION APPROACH

The policy is completely defined by setting the value of T and x according to the previous assumptions and Fig. 2. The profit function is a mean profit per unit of time is defined in (3) subject to a constraint on the maximum potential return rate.

$$\Pi(T, x) = (p - G(x) - c) \cdot x - \frac{k}{T} - \left(\frac{hx}{2}\right) \cdot T \qquad (3)$$

s.t. $x \leq X$

The profit function in (3) includes:

- The total revenues per unit of time generated by the components, px with $p = \sum_{i=1}^{N} \max\{\alpha_i p_i, M_i\}$,

- The collection and disassembly costs per unit of time, $(G(x) + c) \cdot x$,

- The order and inventory holding costs per unit of time, $(k/T) + (hxT/2)$.

The two last terms of $\Pi(T, x)$ are the same as the basic EOQ model. For any given value of x, the optimal reorder interval $T^*(x)$ is defined in (4).

$$T^*(x) = \sqrt{2k/(hx)} \qquad (4)$$

By replacing T by $T^*(x)$, the profit function with respect to x only is obtained in (5).

$$\Pi(x) = (p - G(x) - c) \cdot x - \sqrt{2khx} \qquad (5)$$

The following analysis of $\Pi(x)$ shows that a line search can be used to find the optimal return rate x^* that maximise $\Pi(x)$. We note that $\Pi(x)$ can be written:

$\Pi(x) = R(x) - C(x)$, with $R(x) = (p - G(x) - c) \cdot x$ and $C(x) = \sqrt{2khx}$.

$C(x)$ contains the inventory costs (order and holding) and is the basic square root EOQ formulae with x as variable. $C(x)$ is concave, increasing with respect to x without stationary point.

$R(x)$ is the profit function without inventory costs. $R(x)$ is first used in a sequential decision process to determine the return rate x which is used then as an input data to the disassembly reorder interval. We show that this approach is not optimal, the return rate is reduced by considering simultaneously collection effort profit and inventory costs. Proposition 1 states the optimum of $R(x)$ in a closed form equation.

Proposition 1. The profit function without inventory costs, $R(x)$, is concave and attains it maximum at:

$\bar{x} = b \cdot (p - c)/(b + 1)$.

Proof. Denoting by $R'(x)$ and $R''(x)$ the first and second, respectively, derivative of $R(x)$ with respect to x:

$R'(x) = p - c - G(x) \cdot \left(\frac{b+1}{b}\right)$ and $R''(x) = -\frac{G(x)}{bx}\left(\frac{b+1}{b}\right)$.

It is easy to see that $R''(x) < 0$ and $R'(x) = 0$ gives $\bar{x}$. □

Proposition 1 and 2 are for $b > 1$. In the case of $b < 1$, d must be set as small as possible, which is an uninteresting scenario.

Proposition 2. The profit function including inventory costs, $\Pi(x)$, can be positive only for a contiguous range of x and attains its maximum, on this range when it exists, at the largest value of x such that $\Pi'(x) \geq 0$ (with $\Pi'(x)$ the first derivative of $\Pi(x)$ wit respect to x).

Proof. The first and second derivatives of $\Pi(x)$ with respect to x are:

$$\Pi'(x) = R'(x) - C'(x) = p - c - \left(\frac{b+1}{b}\right)\left(\frac{x}{a}\right)^{1/b} - \sqrt{\frac{kh}{2x}}$$

$$\Pi''(x) = R''(x) - C''(x) = \frac{1}{x} \cdot \left[-\frac{G(x)}{b}\left(\frac{b+1}{b}\right) + \frac{C(x)}{4x}\right]$$

We note that $\Pi''(x) = 0$ has one solution and $\Pi''(x)$ is first positive and then negative as x increases. The analysis according to the shape of $R(x)$ and $C(x)$ complete the analysis and gives indications to design a solution method:
(1) $R(x)$ is concave with a maximum $x = \bar{x}$ and $R(0) = 0$,
(2) $C(x)$ is strictly increasing concave with no stationary point,

(3) Based on (1) and (2), there is 0 or 2 intersection points of $R(x)$ and $C(x)$. With 0 intersection point: Π is always negative ($C(x)$ is always above $R(x)$ and $C(x) > R(x)$) and the solution is to disassemble nothing with $x = 0$. With 2 intersections points, X_{min} and X_{max}: Π is positive between X_{min} and X_{max} only ($R(x)$ is first below $C(x)$ then above and below again). □

Fig. 4 (a) (b) and (c) illustrate proposition 2 with $a = 80$, $b = 2.5$, $c = 1$, $p = 7$ and different values for k and h such that we observe the variation of the positive of the profit function. $R(x)$ is not change in the three cases while the shape of $C(x)$ is varied. In the two cases (a) and (b), there are two intersection points between $R(x)$ and $C(x)$ and $\Pi(x)$ is first negative and concave then positive and convex then negative and convex. We note that by decreasing k and, consequently decreasing $C(x)$, the first negative part of $\Pi(x)$ is reduced. Alternatively, by increasing k and h, the first negative part of $\Pi(x)$ is increased until, as in case (c), there is no intersection point between $C(x)$ and $R(x)$ ($C(x)$ is always above $R(x)$) and it is not profitable to perform any disassembly order.

Proposition 1 and 2 can be used to develop an efficient line search method to find the optimal return rate denoted by x^*. We first note that $\Pi'(\bar{x}) = -C'(\bar{x}) < 0$ and, then, $\Pi(x)$ is decreasing at $\bar{x}$. We also note that $\Pi'(x) < 0$ for $\bar{x} < x$. $\bar{x}$ is then on the concave part of $\Pi(x)$ and is an upper bound for x^* (i.e. $x^* < \bar{x}$). The line search method start at $\bar{x}$, which is directly found according to proposition 1, and x is decreasing until finding the first value of x such that $\Pi'(x) \geq 0$ (the first derivative $\Pi'(x)$ is easily found as in the proof of proposition 2).
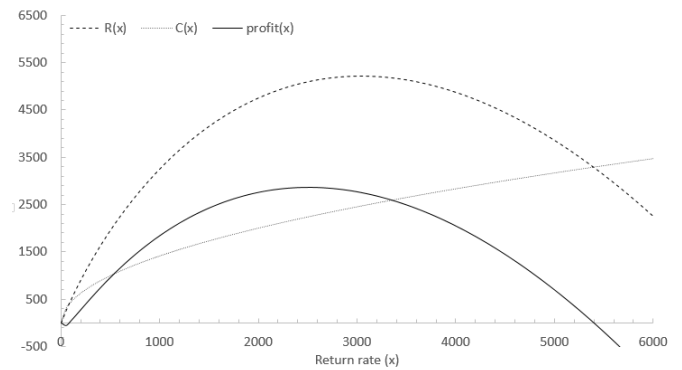

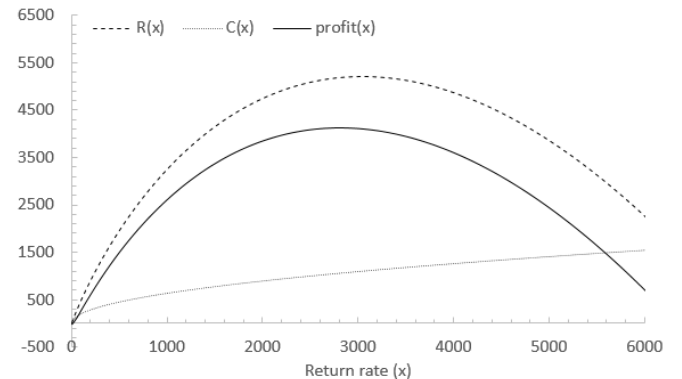Fig. 4(a). Profit and cost functions, 2 intersection points (with $k = 500$ and $h = 2$).

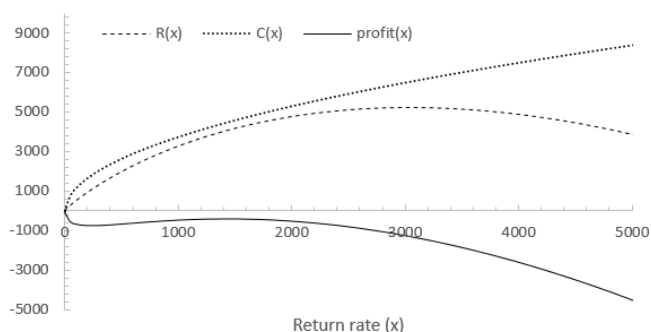Fig. 4(b). Profit and cost functions, 2 intersection points (with $k = 100$ and $h = 2$).

Fig. 4(c). Profit and cost functions, 0 intersection points (with $k = 1000$ and $h = 7$).

4. ILLUSTRATIVE EXAMPLE

An illustrative example is proposed in this section to highlight the shape of the functions and to present the application of the solution method. The data for the example are the following: $a = 80, b = 2.5, c = 1, p = 7, h = 2, k = 500$. The solution algorithm is implemented in JAVA code and the CPU to obtain the optimal solution is negligible. The optimal found value for x is $x^* = 2507$ and the profit is $\Pi(x^*) = 2858.37$.

A sensitivity analysis is performed on the numerical examples to have an insight of the effect of the changes in parameter values. The same type of experimental design of Taleizadeh et al. (2015) is performed. It consists of changing a single parameter's value at a time by a given percentage. The effect on the cost values and the decision variables are computed in percentage according to the base cases presented in the previous sections. Each parameter is changed from -20% to +20% by step of 5%. The main objectives for decision makers are to see the effect of a parameter estimation error and the

importance of a parameter according to the optimal decision and cost. The results are related to the examples but the proposed methods are however simple to compute and fast enough (even on a spreadsheet) to be reproduced by a user on any application. The results are presented on Fig. 5 for the variation of the profit and on Fig. 6 for the variation of the return rate (decision variable) in percentage according to the variation of one parameter in percentage.

We can note on Fig. 5 and Fig. 6 that the profit and the return rate are highly sensitive with respect to the price of the component (p) and the price elasticity of the return (b). The estimation of these parameter must be made carefully and, for the parameter b, it justifies the use of a price-return function with easily estimated parameters. This is the case for the iso-elastic function where the parameters can be estimated with a simple linear function with logarithmic scale (Huang et al., 2013). We also note that the variation of the profit and the return rate are limited with respect to the inventory parameters. For greater variation (out of poor estimation of the parameters), the profit is reduced until the inventory costs is the case while the inventory cost do not exceed the revenues (see Fig. 4).

When the condition of product are variable, the model can be applied to a given product and a given quality level. For application, the model is address to decision makers in disassembly centres. It gives economical indications on the profitability of disassembling a product by finding an equilibrium between revenues obtained from the components and buyback, disassembly and inventory costs. The method is considered as efficient because it provides optimal solutions in small computational time. Decision makers can then investigated different disassembly configuration for various products.
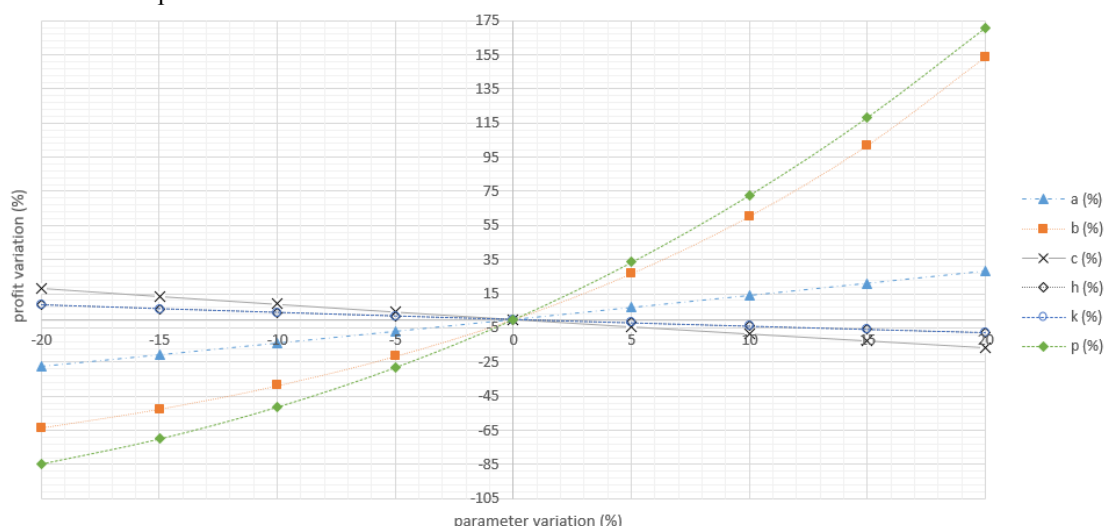

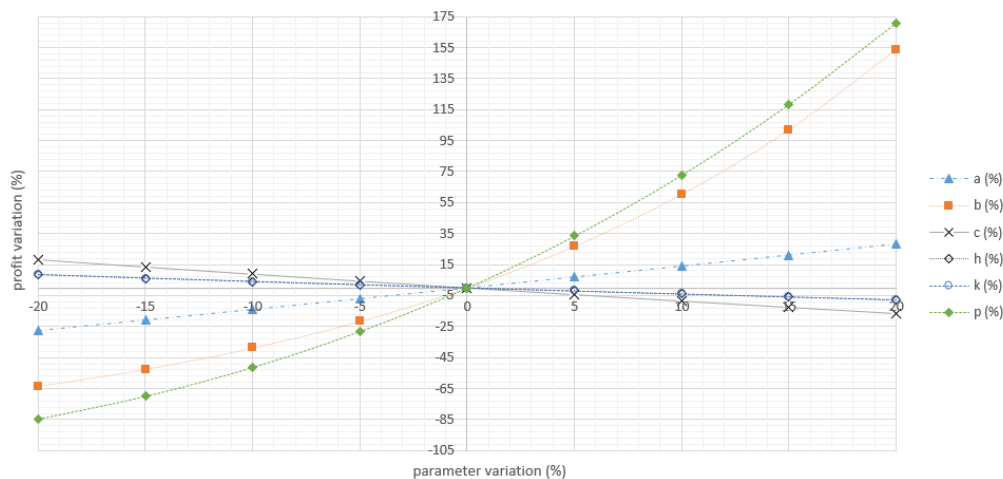Fig. 5. Illustration of profit variation with respect to the parameters.

Fig. 6. Illustration of (optimal) return rate variation with respect to the parameters.

5. CONCLUSIONS

Disassembly systems necessitates new models for their inventory management due to their specificities. One these specificities is to coordinate return flow of end-of-life products and the demands of the secondary markets for the disassembled components. The research question is to determine if an EOQ-pricing policy is relevant for these systems. In this context, we propose a model to evaluate the profit with respect to a reorder time interval of product to disassemble and a buyback price pay to encourage the returns. An efficient method is proposed to find optimal solutions which can be used in practice to evaluate the profitability of disassembly operations. This work can be extended in several ways by considering different variants for the assumptions.

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