

# Traffic Control on Freeways Using Variable Speed Limits

Lorenz Dörschel\* Dirk Abel\*

\* RWTH Aachen, {L.Doerschel, D.Abel}@irt.rwth-aachen.de

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**Abstract:** A new practical oriented feedback control structure for traffic control on freeways using variable speed limits is presented. Therefore, a simple controller structure which satisfies legal and operator demands for a single road is derived from system analysis of a macroscopic traffic model. Furthermore, this road-controller is accompanied with a sequence control and a road-network-controller. In contrast to existing work, novel control variables are used within controller design and tuning guidelines for non-control-engineers are given. The proposed control structure is investigated in simulations, which have been validated using experimental data from the German autobahn.

*Keywords:* Traffic control, variable speed limits, distributed parameter systems

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## 1. INTRODUCTION

In times of increasing traffic on roads as, e.g., on freeways, solutions are needed to improve the capacity of the existing roads to meet these rising demands. Here novel concepts regarding the traffic control have also gathered increased attention over the last decades as highlighted by the white paper on transport of the European Commission (2011). In science, several approaches and methods have been proposed for traffic control, which can be roughly distinguished by their chosen control variable (ramp metering, variable speed limits (VSL) or route guidance) and their field of application (urban or highway), see e.g. Han (2017). In this context it has been shown by, e.g., Papageorgiou et al. (2008) and Weikl et al. (2012), that variable speed limits have a positive effect onto the traffic flow on motorways. Even though great theoretical results could also be derived for ramp metering as, e.g., in Yu and Krstic (2018), VSL is still preferred by most freeway operators, as the infrastructure for VSL is already installed.

In previous works, several different feedback control algorithms have been proposed for traffic control on freeways using VSL, where first approaches go back to the 1970's with the work of Cremer (1979). At this time several models describing the dynamics and physics of traffic flow had already been presented as, e.g., the fundamental work of Lighthill and Whitham (1955) or Payne (1971). Along the years, several different models have been proposed and many differences between the control strategies in literature are caused by the use of different models. Those models can be roughly distinguished into microscopic and macroscopic models, whereas the latter ones lead to system description based on partial differential equations (PDEs, see, e.g., Nagatani (2002)). While most models just describe the flow, also models for the influence of VSL are

at hand. Macroscopic models concerning the compliance of the driver to stick to the given VSL are not known to the authors. Notable works regarding the controller design are (along with several others) model based control strategies from, e.g., Carlson et al. (2010), Burger et al. (2013), Frejo and Camacho (2015) and Hegyi et al. (2008). Facing the numerous available feedback control strategies, it might surprise, that – with very few exceptions like the Specialists algorithm in the Netherlands from Hegyi et al. (2008) – almost all freeways worldwide are not operated with feedback controllers: e.g. in Germany freeways are still controlled using the legally required feedforward mechanism prescribed by MARZ (1999). Reasons for that are the often too complex structures of the proposed algorithms, which hinder them to be legally imposed and deter operators, as they struggle to understand and tune them properly. Hence, there is need for control algorithms, which are theoretical solidly based but still match requirements of the legislator and the operators.

In this work, a novel feedback control structure is presented, which combines a system theoretical foundation with practical usability. In doing so, new control variables are introduced, which are found by stability analysis and the formulation of the optimality conditions of a macroscopic model including compliance of the drivers. From the control variables, two intuitive actions for traffic control – harmonization and congestion avoidance – can be derived. Furthermore, restrictions for the applicability of the controller can be given, which result in an overlying sequence control of the two actions. In the ongoing paper, the mentioned model is presented and analyzed in Section 2 and 3. In Section 4 and 5 the two actions are derived. Simulation results are then shown and discussed in Section 6, whereas the conclusion in Section 7 summarizes the paper.

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## 2. MODELING

As microscopic models suffer the drawback of restricted system analysis (as important properties like stability depend on spatial or temporal resolution (see Nagatani (2002))), a macroscopic modeling approach is chosen. For sake of simplicity, the traffic flow is modeled along one spatial coordinate with one type of car (hence, all values are understood as passenger cars per lane). Several different macroscopic models have been proposed under these assumptions. With  $z \in [0, L]$  being the spatial coordinate and  $t \in \mathbb{R}^+$  the time almost all of the models from literature have the structure

$$\begin{aligned} \rho_t(t, z) &= -(\rho(t, z) v(t, z))_z = -q_z \\ v_t(t, z) &= -v(t, z) v_z(t, z) + \frac{V(\rho) - v(t, z)}{\tau} + f(v, \rho) \\ q(t, 0) &= q_{in}(t), \quad v(t, 0) = v_{in}(t) \\ v(0, z) &= v_0, \quad \rho(0, z) = \rho_0 \end{aligned} \quad (1)$$

where  $\rho$  denotes the traffic density,  $q$  the traffic volume,  $v$  the traffic velocity,  $V(\rho)$  is the desired velocity of the vehicles (connected to the fundamental diagram  $Q(\rho) = \rho \cdot V(\rho)$ ) and  $\tau$  a time constant. The function  $f$  includes further terms, which differ between the different models from literature and may also include derivatives with respect to space. Depending on  $f$ , a first or second order models arises, which might make additional boundary conditions necessary (e.g.  $v_z(t, L) = 0$ ). Models involving further terms  $f$  (like additional viscosity or pressure terms as in Nagatani (2002)) do often have the problems of identification and calibration of upcoming additional parameters. Nevertheless, additional terms  $f$  are often necessary for a sufficiently good accuracy of the model.

The model from (1) does not include the action of the variable speed limits. Those are most often incorporated with the assumption, that the speed limits change the desired velocity of the vehicles  $V(\rho)$ . Denoting the variable speed limits as  $u(z)$ , the second equation of the model including the VSL becomes

$$v_t = -v v_z + \frac{\hat{V}(\rho, u) - v}{\tau} + f(v, \rho, u) \quad (2)$$

with the new desired velocity  $\hat{V}(\rho, u)$ . Further effects of  $u$  can be included in the function  $f$ . The shape of  $\hat{V}$  depends on the fundamental diagram, but also on the compliance of the drivers. Here, it is meaningful to assume, that the drivers only change their speed, if the VSL is smaller than the desired speed  $V(\rho)$ , as the term  $V(\rho)$  describes the maximal allowable speed given the present traffic situation, at which the driver still feels save. Thus it holds

$$\hat{V}(\rho, u) = V(\rho) \quad \text{if } u \geq V(\rho). \quad (3)$$

Only few validated works exists on compliance of drivers to VSL in a control engineering context. Thus, we state a very simple compliance model as follows:

$$\hat{V}(\rho, u) = (1 - C)V(\rho) + C \min(u, V(\rho)) \quad (4)$$

with the compliance factor  $C \in (0, 1]$ , which fulfills (3).

The presented model still contains the unknown function  $f$ . As mentioned, several different functions  $f$  have been presented in literature and despite comparative studies as, e.g., in Nagatani (2002), no model stands out of the crowd. As all models show good agreement to some extent with measurement data, we assume, that all models cover parts of the true behavior of traffic flow, whereas none covers all effects. Thus we propose, that a controller structure, which

- does clearly not work for some established  $f$ , won't work reliable in practice
- shows clearly positive effect for some established  $f$ , will likely have one in practice.

Of course, it is possible, that a specific controller parametrization may stabilize one model but destabilize the other. But both propositions are made for clear effects and a general controller structure but not a specific configuration.

## 3. OPTIMALITY CONDITIONS

In order to design an efficient controller, we first analyze the models to deduce a simple but efficient controller structure. As the goal of traffic control is to optimize the traffic flow, we define the Lagrange function

$$\begin{aligned} \mathcal{L} &= J(\rho, v, u) + \int_0^T \int_0^L \lambda(t, z) (\rho_t - (\rho v)_z) dz dt + \\ &\int_0^T \int_0^L \sigma(t, z) \left( v_t - \left( -v v_z + \frac{\hat{V} - v}{\tau} + f \right) \right) dz dt \end{aligned} \quad (5)$$

where  $J$  is a cost function of choice,  $\lambda$  and  $\sigma$  are the adjoint states and  $T$  the time horizon for the optimization. A direct derivation of the optimality conditions (see, e.g., Hinze et al. (2009)) leads to strongly coupled terms between  $\rho$ ,  $v$  and  $u$ , which makes analysis difficult. This problem can be overcome leading to well-arranged equations by introducing the variable

$$\tilde{u} = C(V - u), \quad \tilde{u} \geq 0 \quad (6)$$

leading to  $\hat{V} = V - \tilde{u}$ . The inequality  $\tilde{u} \geq 0$  corresponds to  $u \leq V(\rho)$ , which can always be fulfilled by choosing  $u = V(\rho)$  instead of  $u > V(\rho)$  with identical dynamics due to (4). With  $\pi$  as adjoint state for the inequality condition  $\tilde{u} \geq 0$  the adjoint equations are given as

$$\begin{aligned} \int_0^T \int_0^L \lambda_t dz dt &= \int_0^T \int_0^L \frac{\partial J}{\partial \rho} - v \lambda_z - \frac{\sigma}{\tau} \frac{\partial V}{\partial \rho} - \sigma \frac{\partial f}{\partial \rho} dz dt \\ \int_0^T \int_0^L \sigma_t dz dt &= \int_0^T \int_0^L \frac{\partial J}{\partial v} - \rho \lambda_z - v \sigma_z + \sigma \left( \frac{1}{\tau} - \frac{\partial f}{\partial v} \right) dz dt \\ 0 &= \int_0^T \int_0^L \frac{\partial J}{\partial \tilde{u}} + \frac{\sigma}{\tau} + \pi - \sigma \frac{\partial f}{\partial \tilde{u}} dz dt. \end{aligned} \quad (7)$$

For the inequality condition it has to hold:

$$\pi \geq 0, \quad \tilde{u} \geq 0, \quad \pi \cdot \tilde{u} = 0 \quad (8)$$

The boundary conditions are at  $z = 0$  and the initial conditions at the final time  $t = T$ . Hence, for a well-posed

problem, the characteristic curves have to travel from left ( $z = 0$ ) to right ( $z = L$ ). For a general  $f$ , this is not the case, if  $v < 0$  holds (see adjoint equation for  $\lambda$ ). Thus, a controller working with the presented model will only work for positive vehicle speeds.

The adjoint equations in (7) are formulated for general functions  $f$ . For a robust controller, which can optimize the traffic, the adjoint equations should be well-posed for meaningful  $f$ , which are available in literature, as the true dynamics of the real world traffic are not known. For the ARZ-model from Aw and Rascle (2000) and Zhang (2002), the function  $f$  can be written as

$$f = -\rho \frac{\partial V}{\partial \rho} \cdot v_z. \quad (9)$$

This leads to

$$\sigma_t = \frac{\partial J}{\partial v} - \rho \lambda_z - \underbrace{\left( v + \rho \frac{\partial V}{\partial \rho} \right)}_{>0} \sigma_z + \sigma \frac{1}{\tau}. \quad (10)$$

Hence, for the ARZ-model, the controller will only work sufficiently well, as long as the angular point of the fundamental diagram is not exceeded.

Furthermore, for  $\frac{\partial f}{\partial \tilde{u}} \neq 0$  (which is meaningful if one assumes, that drivers behave different to VSL than to the general traffic situation) one can formulate slightly different  $f$  (see Schwietering et al. (2019)), where the adjoint equation can be rearranged to

$$\lambda_t = \frac{\partial J}{\partial \rho} - v \lambda_z - \frac{\partial F}{\partial \rho} \pi. \quad (11)$$

Hence, if the VSL is activated ( $\tilde{u} \neq 0 \Rightarrow \pi = 0$ ), the solution for  $\lambda$  equals zeros, if  $\frac{\partial J}{\partial \rho} = 0$  would hold. Hence, if the cost function does not include terms depending on the density of the traffic but only on the speed and the actuation, there exists a model, for which the adjoint density  $\lambda$  vanishes from the optimality conditions leading to a ill-posed optimization problem. Accordingly, it is meaningful to ensure  $\frac{\partial J}{\partial \rho} \neq 0$ , i.e. the control variable should depend on the density  $\rho$ .

This short analysis shows the following:

- It cannot be expected of the controller to work for all vehicle speeds.
- Thus, the controller should be accompanied with a sequence control, which turns the controller off if certain bounds are exceeded.
- From system analysis, those bounds are given by  $0 < v < \rho \frac{\partial V}{\partial \rho}$ .
- Furthermore, the control structure  $u = V - \frac{1}{C} \tilde{u}$  (see (6)) seems beneficial.
- This is equivalent to a feedforward control using the fundamental diagram and feedback control depending on the compliance of the drivers.
- The control variable should depend on the density  $\rho$ .

Those statements are very general, as they apply to a wide bunch of models. They will be used in the following as guideline for the controller design.

#### 4. HARMONIZATION STRATEGY

In the previous section, it has been identified, that the angular point of the fundamental diagram is a crucial upper bound for the optimizing traffic. This bound is well known in literature, as traffic tends to become unstable, if the density  $\rho$  exceeds the density connected to the angular point  $\rho_{crit}$  (e.g., Nagatani (2002)). The analysis from section 3 also shows, that the violation of the condition at a single point may already be crucial. On the other hand, the capacity and performance of the road is highest, if the traffic volume is highest (which is at the angular point of the fundamental diagram). Obviously, the density can be maximal in mean without validating the constraint at single points, if the density is constant in space. Thus, to avoid exceeding  $\rho_{crit}$  while keeping good performance of the road it is meaningful, to harmonize the density of the vehicle along the road. Thus, a first promising control variable  $y$  is  $y = \rho_z$ . We modify this slightly and state the feedback law

$$u = V(\rho) - K \frac{\rho_z}{\rho}, \quad (12)$$

with the controller gain  $K$ . Note that the control variable meets the formulated requirements.

Analyzing the stability of the closed loop system with the proposed control law gives

$$v_t = -v v_z + \frac{V(\rho) - v}{\tau} - \frac{K}{C \tau} \frac{\rho_z}{\rho} + f. \quad (13)$$

Kerner and Konhäuser (1993) performed a linear stability analysis of a similar type of systems. Transferring their results to the given system and assuming, that  $f$  is sufficiently small one can derive the stability condition

$$C \tau \left( \rho_0 \cdot \frac{\partial V}{\partial \rho} \Big|_{\rho_0} \right)^2 < K \quad (14)$$

with  $\rho_0$  being the steady-state density. The condition obviously can be fulfilled for  $K$  large enough if  $C \neq 0$ . With (14) and  $\rho_0 = \rho_{crit}$  also a lower bound for  $K$  can be given.

##### 4.1 Justification for operators

The stability analysis may satisfy the scientist, but for real world applicability, also the operators have to be convinced of the plausibility of the control law. Looking at popular models  $f$ , second order models are often given with

$$f = \mu \frac{v_{zz}}{\rho} - c_0^2 \frac{\rho_z}{\rho}, \quad (15)$$

where  $\mu$  denotes a viscosity for safety-oriented driving and  $c_0$  the speed of evolving pressure waves, which represents predictive driving. Hence, choosing the control variable  $y = \frac{\rho_z}{\rho}$  allows the controller to increase the predictive driving capability. This also makes the controller amplification  $K$  is more easy to tune as the parameter

$$\tilde{K} = 1 + \frac{K}{c_0^2 \cdot C \cdot \tau} \quad (16)$$

can be understood as amplification of the natural predictive driving capability of the drivers (e.g.  $\tilde{K} = 2$  means, that the related  $K$  will double the natural predictive driving capability).

We further look at the control law and a group of cars with the density depicted in Fig. 1. It is clear, that the controller will accelerate the cars leaving the group (as  $\rho_z < 0$  hold there) and decelerate the cars, which tend to enter the group. This stretches the group and prevents it from rising, which gives a intuitive explanation of the controller to operators and also judges.

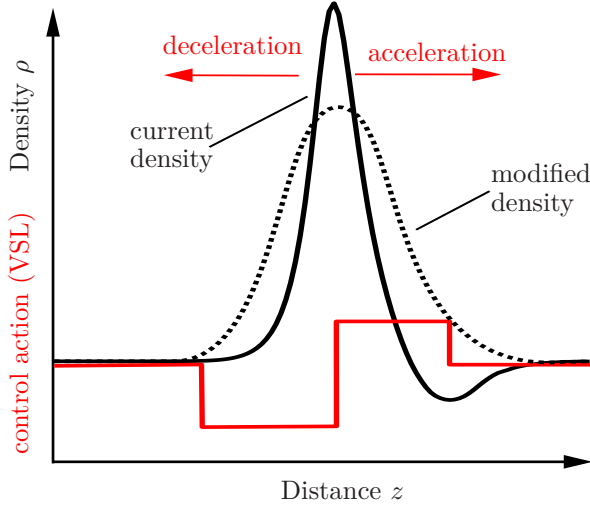


Fig. 1. Visualization of the control actions for operators

## 5. CONGESTION AVOIDANCE STRATEGY

The presented harmonization strategy tries to keep the traffic density constant to prevent the density to exceed a critical density  $\rho_{crit}$ . Hence the strategy ensures a good traffic behavior inside the road, where the controller is active. But this is only possible, if the incoming traffic into this road does not exceed  $\rho_{crit}$  for too long. In those cases, the controller will be deactivated (see also Section 3) by the sequence control. Situation with a mean density higher than  $\rho_{crit}$  cannot always be avoided with VSL and occur in times of high traffic demand. Nevertheless a strategy is needed to reduce the probability of non suitable incoming traffic. This strategy has to rely on an operator-friendly indicator, which allows to predict, whether the incoming traffic will likely be too high.

For finding such an indicator, measurement data from German and Swiss roads have been analyzed (see Schwietering et al. (2019)). The analysis indicates, that - among others - a 'distance factor'  $d_f$  defined by

$$d_f = \frac{q \cdot (d(v) + l)}{v}, \quad q \text{ in } \frac{\text{Fz}}{\text{km}}, l \text{ in m}, v \text{ in } \frac{\text{km}}{\text{h}}, \quad (17)$$

where  $l$  is the (average) length of the cars and  $d(v)$  is the desired maximum distance between cars depending on the velocity. According to §4 StVO in Germany it holds:  $d(v) = v/2 \cdot \frac{\text{h}\cdot\text{m}}{\text{km}}$ , i.e. driving  $100 \frac{\text{km}}{\text{h}}$  should lead to a distance of 50m. Hence, the distance factor  $d_f$  describes the ratio between the necessary space for safe driving and the available space. For the operator  $d_f \leq 1$  means, that the cars stick to §4 StVO, whereas  $d_f > 1$  means, that the allowable distance  $d(v)$  is violated. A comparison with measurements shows, that the traffic tends to congestion for  $d_f > 1.5$  and almost surely congests for  $d_f > 2$  (see

Schwietering et al. (2019)). To the knowledge of the authors, the distance factor is new to the control community and has not been used for control purpose yet.

To avoid unfavorable incoming traffic into a road, the distance factor shall be used in the following for avoiding those situations and thus congestion. Therefore we don't look at a single road, but at a road-network with  $N$  road segment given by

$$\begin{aligned} \rho_t^i &= -q_z^i, \quad v_t^i = -v^i v_z^i + \frac{\hat{V}(\rho^i, u^i) - v^i}{\tau} + f(v^i, \rho^i, u^i) \\ q^i(t, 0) &= \sum_{j=1}^N \mathcal{G}_{ij} q^j(t, L^j), \quad \rho^i(t, 0) = \sum_{j=1}^N \mathcal{G}_{ij} \rho^j(t, L^j) \end{aligned} \quad (18)$$

with  $z^i \in [0, L^i]$  and  $\mathcal{G}$  being the matrix decoding all connections of the roads segments, i.e.

$$\mathcal{G}_{ij} = p \in [0, 1] \Leftrightarrow p\% \text{ of segment } i \text{ going to segment } j.$$

We first observe, that for the distance factor

$$d_f^j(t, 0) = \sum_{i=1}^N \mathcal{G}_{ij} d_f^i(t, L^i) \quad (19)$$

holds, e.g. the distance factor entering a segment  $i$  is a linear function in the  $d_f$  leaving the other segments. To ensure good incoming traffic into each road segment the condition

$$d_f^j(t, 0) \leq d_{f,crit} \quad \forall j \text{ with } \sum_{i=1}^N \text{sign}(\mathcal{G}_{ij}) > 1 \quad (20)$$

has to hold, i.e.  $d_{f,crit}$  shall not be exceeded at the inflow of every road segment, where multiples segments flow together. Here not all segments are analyzed, as in a situation of a inflow from a single segment, the condition cannot be satisfied using VSL on long terms, as  $d_f$  depends on  $q$  and cars do not vanish.

Formulating a model for  $d_f$  assuming  $v \approx \hat{V}(\rho, u)$  gives

$$(d_f)_t \approx - \left( u + 2l + \rho \frac{\partial V}{\partial \rho} \right) \cdot (d_f)_z \quad (21)$$

Comparing (21) with (10) gives insight, that also  $d_f$  travels from left to right in the operating range of the controller, whereas the traveling speed can be influenced by the controller. Note, that (21) does not depend on  $\hat{K}$ . The congestion avoidance strategy then solves the optimization problem

$$u_{opt}^i = \arg \min_{u^i} \int_0^T \int_0^{L^i} (V(\rho^i) - u^i)^2 dz dt, \quad d_f^j(t, 0) \leq d_{f,crit}$$

$$\text{subject to } (d_f^i)_t = - \left( u^i + 2l + \rho^i \frac{\partial V(\rho^i)}{\partial \rho} \right) (d_f^i)_z, \quad (22)$$

thus - if possible - tries to shift incoming  $d_f$ -peaks to not exceed the given threshold. The congestion avoidance strategy can be combined with the harmonization strategy using

$$u = u_{opt} - K \frac{\rho_z}{\rho}. \quad (23)$$

Note, that for  $d_f \leq d_{f,crit}$  without congestion avoidance, the same controller as in (12) results.

## 6. RESULTS

### 6.1 Operator Demands and Model

As first and crucial step, the presented strategies and algorithms were given to operators of VSL-facilities in Germany. In discussion it was confirmed, that both strategies meet the requirements of the operators regarding comprehensibility and applicability although the congestion avoidance was criticized due to the underlying optimization, which might become problematically concerning legal issues. The intuitiveness of the tuning parameters (fundamental diagram,  $\hat{K}$  and  $d_{f,crit}$ ) was confirmed. Consequently, the algorithms were tested in simulation. Therefore, measurement data of Autobahn-segment of the Autobahn 5 between Friedberg and Bad Homburger Kreuz were used to set up a simulation. Along the Autobahn-segment 12 cross sections for detection are installed on a total length of about  $L = 9$  km. The fundamental diagram was identified using the measurement data. The validation of the model was conducted using measurement data from a day not used for identification. The model receives the incoming traffic flow and speed and tries to predict the traffic situation at the outflow (i.e. after 9 km). Further parameters are  $C = 0.5$ , where other simulation parameters were chosen according to Damrath and Rose (2002). Results are shown in Fig. 2. Despite visible deviations between measurement and simulation the overall dynamic behavior of the system can be caught by the model. Thus, a simulative testing of the controller may indicate also performance in a real world scenario.

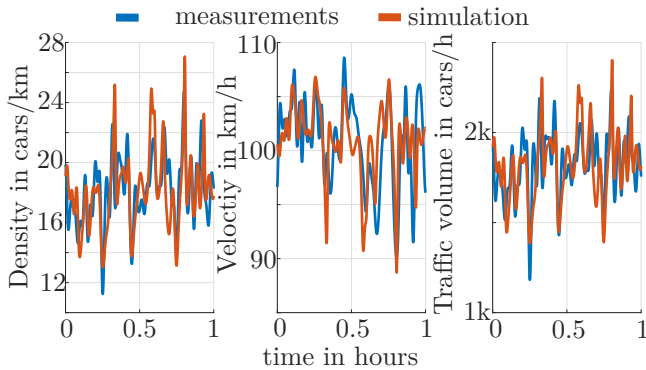


Fig. 2. Comparison of measurement and simulation for validation data

### 6.2 Harmonization Strategy

The harmonization strategy was tested via simulation and inflow conditions from measurements of two different days. As parametrization  $\hat{K} = 5$  was chosen by suggestion of the operators. Displays for the VSL were implemented every 2 km (which is the mean distance on German autobahn with VSL). The controller was called every minute as specified by the German rulework MARZ (1999). Velocities were displayed as  $60 \frac{\text{km}}{\text{h}}$ ,  $80 \frac{\text{km}}{\text{h}}$ ,  $100 \frac{\text{km}}{\text{h}}$  and  $120 \frac{\text{km}}{\text{h}}$  by rounding the output of the controller. A comparison of the open and closed loop dynamics are shown in Fig. 3 and Fig. 4 for the end of the road  $z = L$ . On the first

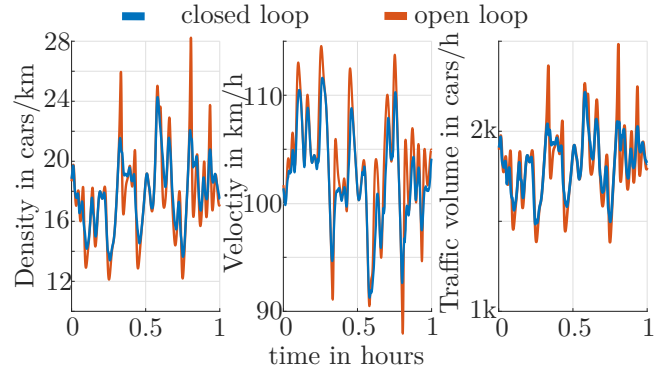


Fig. 3. Performance of the harmonization controller without congestion.

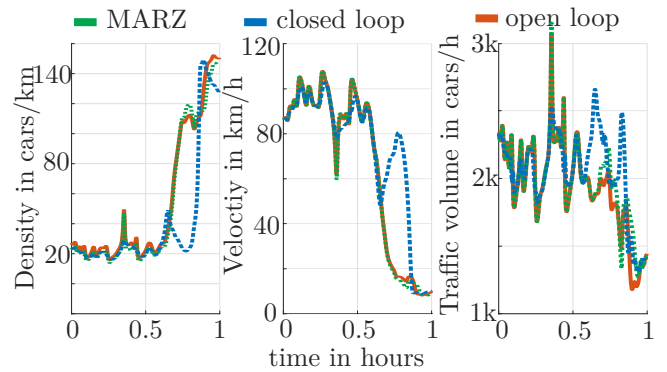


Fig. 4. Performance of the harmonization controller with congestion.

day, no congestion occurred. Here, it can be clearly seen, that the controller is able to harmonize to traffic as high peaks in density, velocity and traffic volume are occurring less often. This can also be quantified by investigating a measure for homogeneity defined as

$$J = \int_0^T \int_0^L \left( \frac{\rho_z}{\rho} \right)^2 dz dt. \quad (24)$$

The use of the proposed controller can (without much tuning) improve the measure from (24) in a mean over many simulative studies by 41.21% compared to the open loop case. For the case with congestion, the open loop, the standard-control for VSL on German autobahn according to MARZ (1999) and the newly proposed controller were compared. It is clearly visible, that the new controller is able to delay the collaps of the traffic by almost 20 min in this scenario.

### 6.3 Congestion Avoidance

For testing the congestion avoidance the road network depicted in Fig. 5 was investigated. All parameters for the simulation and the controller were the same as in the previous test cases. By suggestion of the operators  $d_{f,crit} = 1.5$  was chosen. The constraints were implemented as soft constraints to ensure feasible solutions. In the chosen scenario, the traffic volume in roads 1 and 2 change according to Fig. 6. The cases with and without congestion are also shown in Fig. 6. It is clear, that the controller is able to avoid the congestion. This is done by shifting

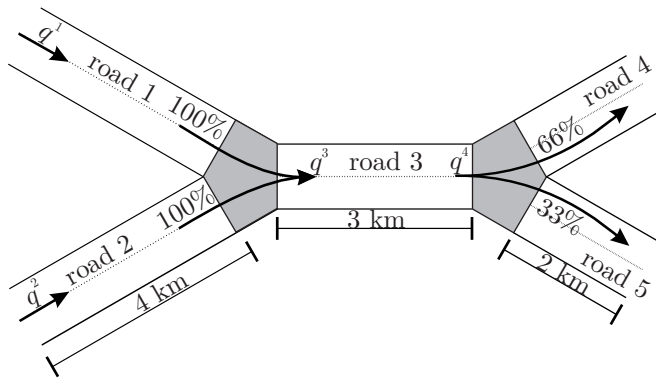


Fig. 5. Investigated road network.

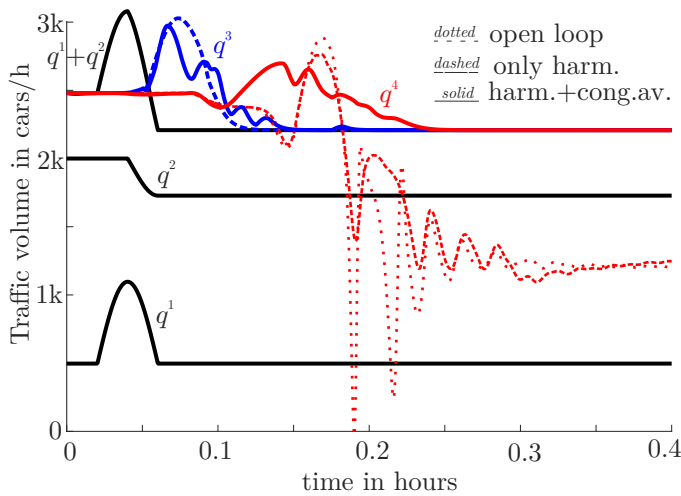


Fig. 6. Traffic volume inside the network with and without congestion avoidance.

the incoming distance factors to avoid too dense traffic situations. In this setup, the congestion avoidance delays the traffic in road 1 by displaying  $80 \frac{\text{km}}{\text{h}}$  to the cars. This shift can be seen in  $q^3$ , where the solid and the other lines significantly differ from each other. Note, that for the harmonization-case (dashed line) the controller is deactivated after 0.2 h by the sequence control.

## 7. CONCLUSION

In this work, two strategies for controller design using VSL on highways have been presented. Both (harmonization strategy and congestion avoidance strategy) are intuitive to operators and at the same time scientifically justified. In simulation with validated simulation models both strategies could in combination increase the capacity of the highway with less or later occurring congestion. Due to the promising results in simulation and the acceptance of the strategies by the operators, it is aimed to implement the controller in a prototype. Before doing so, several tasks are necessary to bring the controller more close to application. This includes the validation on a microscopic model, the analysis of randomly distributed driving behavior, trucks and multiples lanes, the implementation of an observer and the consideration of the time delays, which will occur in application and can be quite high. Those topics will be part of future research.

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