A Dedicated Lot Sizing Problem in Tire Industry

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Abstract: In this paper, we consider a dedicated lot sizing and scheduling problem inspired from a real-world application in tire industry. This problem consists in scheduling several products on parallel machines with eligibility constraints within a finite planning horizon. We consider several specific constraints such as the number of simultaneous scheduled products, the number of setup per period and minimum quantities to produce. A mathematical model that determines a production schedule minimizing backlogging, low and high inventory surplus is proposed and tested on real data with up to 210 products, 70 machines and 7 periods. The obtained results show the effectiveness of the proposed model which improve significantly the industrial solution.

1. INTRODUCTION

Production planning and control is a burning issue for most supply chain managers in manufacturing industry and still remain one of the most challenging problems in operations research. Tremendous progresses have been made in information technologies over the past few decades. This is an incredible opportunity for companies to reach new market shares but it has also brought a fierce competition to reduce costs and provide better customer service in a globalized environment. An effective production planning allows companies to meet customer expectations while reducing production and inventory costs.

Lot sizing problems have been studied widely in the literature over the last decades. The expected output of lot sizing is to give a complete picture over a planning horizon of how many parts to produce at each period and how many pieces to carry in inventory. It takes its origin in the well-known Economic Order Quantity (EOQ) model of Harris (1913) under the assumption of single item, constant demand and infinite planning horizon. Since then, numerous researchers have built more realistic models to tackle real world problems. Production capacity limitation is a significant constraint that production managers have to deal with. It first has been addressed in Manne (1958) and is known as the capacitated lot sizing problem (CLSP). The general case of the CLSP is NP-hard Bitran and Yanasse (1982). Since then, various extensions of the lot sizing problem have been studied extensively and can be classified based on several crieria. An exhaustive review on the CLSP can be found in Drexl and Kimms (1997) and Pinedo (2005) and a classification of criteria is presented in Brahimi and Dauzère-Péres (2017).

2. LITTERATURE REVIEW

We consider a dedicated lot sizing problem in the tire industry. Several real-world problem have already been studied. Lasdon and Terjung (1970) used a column generation approach and Dantzig-Wolfe decomposition to solve an industrial case with up to 393 items to schedule over 6 periods in less than 15 minutes. Degraeve and Schrage (1997) also proposed a column generation procedure to produce a schedule for one work shift in tire industry. This approach is also used in a "rolling horizon" fashion. Jans and Degraeve (2004) presented a model with specific extensions such as general startup times, multiple capacitated ressources and backlogging. A column generation based algorithm is proposed and a lagrangian relaxation is used to reduce the degeneracy of the master problem. The algorithm has been tested with up to 30 products and 30 periods. More recently Yalaoui et al. (2013) proposed a MIP model and approximate resolution methods based on genetic and particle swarm optimisation algorithms coupled or not with fuzzy logic control to solve a particular version of the hybrid flow shop scheduling problem from a real application in the automotive industry.

We limit the review to the capacitated versions of lot-sizing problem. Belvaux and Wolsey (2000) propose a prototype modelling and optimization system. They presented the generic version of the lot sizing problem to cover a wide variety of big bucket and small bucket models. The classification of lot sizing problems can be done on several criteria (see Fig. 1) such as number of production stages (single or multi-level), number of products to schedule (single or multi-product), number of machines or resources available (single or multi-machine). In addition to that, nature of demand, planning horizon discretization, capacity constraints, backordering policy and setup structure can be considered.

This paper focus on multi-products models. We invite the reader to refer among others to Brahimi et al. (2017) for a recent and exhaustive survey on single-item models. Multi-products lot sizing problems have been studied extensively in the literature. The multi stage model is the most general

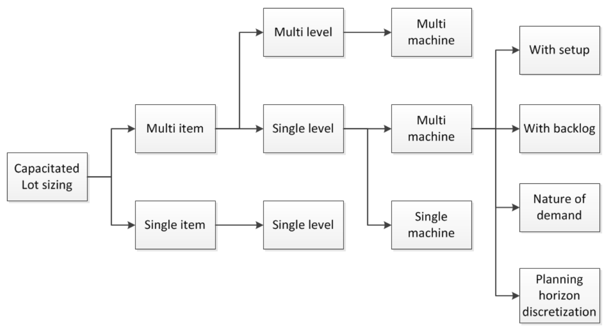

Fig. 1. Capacitated lot sizing models classification

case. The well-known multi-level capacitated lot sizing problem (MLCLSP) was proposed by Billington et al. (1986). The basic principle of this formulation is to link end items demand with internal sub-components needs through a "gozinto" matrix, which is a representation of the bill of material. This problem has been proven NP-hard by Tempelmeier and Derstroff (1996) and the add of setup time constraint make the feasibility problem NP-complete (Maes et al. (1991)). Several approaches have been used to solve the MLCLSP such as mathematical programming, Lagrangian relaxation and decomposition, local search and metaheuristics. For example Helber and Sahling (2010) proposed a new approach called fix and optimize (F&O). They solve to optimality in an iterative fashion sub-problems derived from the MLCLSP. They experimented three ways to define the sub-problems to be treated: product-oriented decomposition, resource-oriented decomposition and process-oriented decomposition. They empirically prove on the MLCLSP that the fix and optimize heuristic outperform the classical rolling horizon based fix-and-relax heuristic (R&F).

However, F&O method can be mixed with the classical R&F heuristic. For instance Toledo et al. (2015) used R&F heuristic to build an initial solution and then improved it by applying F&O heuristic to solve the MLCSLP with backlogging and to apply it to an industrial case called two-stage glass container production scheduling problem. Recently, Behnamian et al. (2017) considered a markovian approach to MLCLSP with sequence-dependent family setup times, setup carry over and uncertainty in levels due to uncertainty in inspection, rework and scrap. They developed a mixed integer linear programming model tested on a numerical example and provided a sensitivity analysis.

According to Drexl and Kimms (1997) the MLCSLP can be considered as the model dedicated to MRP systems, whereas the mono-level multi-product CLSP is viewed as the dedicated MPS model. A review of model and algorithm for the CLSP is presented in Karimi et al. (2003). Multi-item CLSP is difficult to solve to optimality, especially with setup constraints. Miller et al. (2000) provided valid inequalities to improve the resolution of a branch and cut MIP solver.

Single-level multi-item problems can either be single or multi-machines. For instance, Ceschia et al. (2017) proposed a Simulated Annealing(SA) approach and a hybrid method SA/MIP to solve the multi-item single-machine CLSP. Lately, Nobil and Taleizadeh (2016) addressed a multi-item single-machine production system with imperfect products to rework under non-zero setup times and proposed a non-linear programming model to solve it. However, most papers in the literature deal with the multi-machine version of the multi-item CLSP. The relax-and-fix (R&F) solution heuristic is widely used to solve this problem. For example, de Araujo et al. (2007) developed a MIP model with sequence-dependant setup costs and time and then solved it using R&F heuristic in a rolling horizon fashion. To improve the R&F method three variants of local search are presented: descent heuristic, diminishing neighbourhood search and simulated annealing and compared to the commercial solver IBM Ilog Cplex.

Absi and Kedad-Sidhoum (2007) also addressed the multi-item CLSP with setup times, and added shortage costs and safety stock deficit costs. They propose MIP heuristics based on a planning horizon decomposition strategy to find a feasible solution on real-world instances. They provide a R&F heuristic and a double R&F heuristic and use Cplex 9.0. to solve sub-problems to optimality. Recently Ghirardi and Amerio (2019) considered lot sizing problem with back-ordering, setup carry-overs and non identical machines. They suggested three matheuristics to solve this problem from the ideas of variable neighbourhood local search, local branching and feasibility pump. The feasibility pump algorithm outperformed the other algorithms and two different MIP-solvers. In addition Absi and van den Heuvel (2019) provided a worst case analysis of R&F heuristics for lot sizing problems. They showed that even for simple instances with time-invariant parameters, the worst case ratio may be unbounded.

To the best of our knowledge, the specific constraints that we describe in the following section have never been considered together in the literature, even in the real-world problems close to the one we cope with.

The paper is organised as follows. Section 3 describes the global production process of a tire and provide specific details on the curing process. In Section 4, the model is presented and specific constraints are explained. The results of our model are discussed in Section 5. Conclusion is to be found in Section 6.

3. PROBLEM DESCRIPTION

Our paper is inspired by a french tire manufacturer in the agricultural field. They face a complex production planning process with a wide portfolio of tires to be produced on unrelated parallel machines with numerous eligibility constraints. The production is based on a make-to-stock inventory policy, so that the inventory level stays between a minimum and a maximum. The whole production process can be divided in 5 major sub-processes (See Fig.2). First, a banbury mixer creates a homogeneous rubber material based on natural rubber, carbon black, resins and other chemicals. This rubber material is the basic raw material to build a tire, with textile layer and steel wire. Second, the rubber is shaped into different sub-components during the extruding and calendaring process. At the same time the textile layers are cut at the right dimensions and the steel wires transformed into bead cores.

Third is the assembly process. All the components are put together so that the tire gets its final form before curing. The tire is often referred to a “green tire”. Fourth is the curing and vulcanizing process. A tire-specific mold is placed into a heater. That heater utilizes steam to heat the green tire. The tacky and pliable rubber is transformed into a non-tacky less pliable long lasting state material during the vulcanization process. Finally inspection and finishing operations remains before the tire is stored in the warehouse.

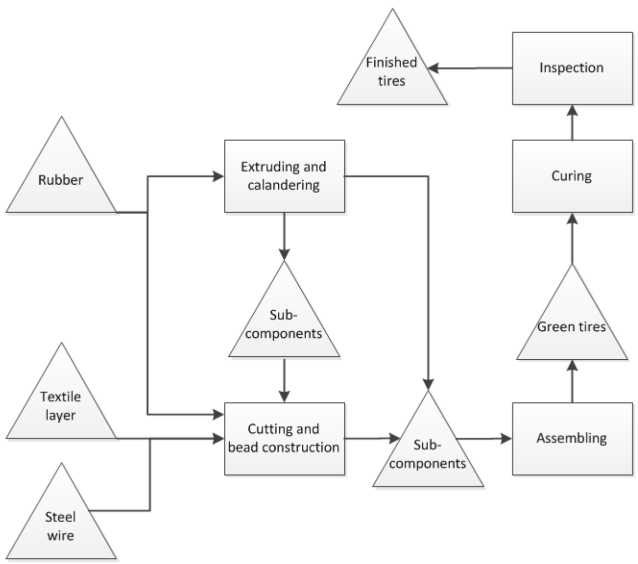

Fig. 2. Tire manufacturing global process

In this paper, we focus on the curing sub-process production planning problem. It is the most important stage as it has been identified as the bottleneck, requires consequent setup times and is highly restricted by tire-heater compatibilities. During the curing process the green tire is put into a mold that provides a specific pattern for the tire. Each mold is tire-specific: it can be used for exactly one type of tire. For some tires, several molds are available. However most tires have only one mold. The mold can be placed in several heaters, each heater containing at most one mold at a time. The curing time depends on the tire produced and the heater used. The heaters capacity links together different tires types that compete for the same resource. Except for the first and the last period of the production campaign, tires are produced in a continuous run and production is always done at full capacity. This type of production is often referred as “all-or-nothing” production. Only one type of tire can be cured in a heater within one period. Thus, our problem is classified as a small-bucket lot sizing problem.

4. MODEL FORMULATION

The production planning problems encountered in the industry may be intractable in numerous situations due to several practical constraints. The demand over the planning horizon is known in advance (deterministic) and changes over time (dynamic). In order to deal with situations where demand cannot be met in time, the company allows backlogging. Specific constraints are also added such as the number of ”campaign endings” (the plan of the end of a campaign) within the planning horizon or the number of different items produced at each period. The resolution approach proposed in this paper is motivated by developing a method that helps to find a good feasible solution for the single-level multi-item multi-machine with dynamic demand, backlogging and a simple setup structure lot sizing problem with specific constraints.

Indexes and sets

A: Number of tires in the portfolio, a = 1 .. A
N: Number of items to plan, i = 1 .. N, $N \geq A$
N_a: Number of processes i for tire a, i = 1 .. N_a
P: Number of curing machines, p = 1 .. P
T: Number of periods in planning horizon (days), t = 1 .. T
H: Number of period sets (weeks), h = 1 .. H
T_h: Set of days t in week h
W: Number of workshops in assembly shop, Θ = 1 .. W
N_Θ: Set of items doable on workshop Θ
N_d: Number of assembly machine resource (drums), d = 1 .. N_d

Model parameters

$\overline{S}_a$: Maximum stock for tire a
$\underline{S}_a$: Minimum stock for tire a
K_a: Number of curing machine resources (molds) available for tire a
D_{at}: Demand for tire a at period t
M_{ap}: Eligibility matrix tire - heater
ω_i: Unit weight of item i
TU_i: Unit time of production of item i in assembly shop
R_{it}: Daily yield of a curing machine for item i during period t
KM_i: Maximum number of molds that can be planned for item i
M_{pt} Unavailable heater matrix for maintenance or industrial trial
V_t: Targeted weight to produce within one period t
$\overline{v_d}$: Upper bound for targeted weight at day t
$\underline{v_d}$: Lower bound for targeted weight at day t
$\overline{v_w}$: Upper bound for targeted weight at week h
$\underline{v_w}$: Lower bound for targeted weight at week h
$\overline{Sat_\Theta}$: Maximum saturation of workshop Θ in assembly shop (minutes)
K_d: Number of drums d available
S: Maximum number of different items produced per period
$\overline{C}$: Maximum number of molds set up per day t
$\overline{C_w}$: Maximum number of molds set up per week h
Der: Maximum number of campaign ending per week h
τ: Minimum duration of a campaign (days)
τ_1: Duration of campaign suspension to count a mold setup
τ_2: Duration of campaign suspension to count a campaign ending
M: Big number

Decision Variables

I_{at}: Inventory level of tire a at the end of period t
B_{at}: Backorder level of tire a at the end of period t
Y_{ipt}: Binary variable that equals 1 if in period t an item i is produced on machine p; 0 otherwise
X_{ipt}: Quantity of item i to produce on machine p in period t

ϕ_{ipt}: Binary variable that equals 1 if there is a mold set up for item i on heater p in period t; 0 otherwise

Ψ_{it}: Binary variable that equals 1 if item i is being cured at period t; 0 otherwise

Der_{it}: Binary variable that equals 1 if there is a campaign ending for item i in period t; 0 otherwise

Model

The objective of the production planner of the tire company is to optimize several criteria. The main objective is to prevent shortage. Once back-ordering has been minimized, the production planner tries to keep every tire between a minimum and a maximum inventory level set by the supply chain.

Minimize

$\lambda_1 * \sum_{a=1}^{A} \sum_{t=1}^{T} B_{at} + \lambda_2 * \sum_{i=1}^{N} \sum_{t=1}^{T} max(0; \underline{S}_a - I_{at}) + \lambda_3 * \sum_{i=1}^{N} \sum_{t=1}^{T} max(\overline{S}_a; I_{at} - \overline{S}_a)$

with $\lambda_1 + \lambda_2 + \lambda_3 = 1$

$$I_{at-1} + \sum_{i=1}^{N_a} \sum_{p=1}^{P} X_{ipt} - B_{at-1} = I_{at} + D_{at} - B_{at}, a = 1..A, t = 1..T \quad (1)$$

$$X_{ipt} \leq M * Y_{ipt}, i = 1..N, p = 1..P, t = 1..T \quad (2)$$

$$X_{ipt} \geq Y_{ipt}, i = 1..N, p = 1..P, t = 1..T \quad (3)$$

$$\sum_{i=1}^{N} Y_{ipt} \leq 1, p = 1..P, t = 1..T \quad (4)$$

$$\sum_{i=1}^{N_a} \sum_{p=1}^{P} Y_{ipt} \leq K_a, a = 1..A, t = 1..T \quad (5)$$

$$X_{ipt} = R_{it} * Y_{ipt} - Rd_{it} * \Phi_{ipt}, i = 1..N, p = 1..P, t = 1..T \quad (6)$$

Constraint (1) is the inventory balance equation, with considering back-order. Constraints (2) and (3) link the setup binary variable and the production variable together. Constraint (4) allows only one item to be produced on a heater within a single period. Constraints (5) and (6) represent capacity constraints. Given a particular item a, Constraint (5) limits the number of heaters used to the mold capacity of that item (K_a). Constraint (6) describes the "all-or-nothing" policy. The production rate (R_{it}) can be affected by a mold setup (Rd_{it}) or any event planned in advance in the calendar. In addition, no mold setup is allowed during the weekends.

$$\Phi_{ipt} \geq Y_{ipt} - Y_{ipt-1} + \frac{1}{\tau_1 * N} * \sum_{o=t-\tau_1}^{t-1} \sum_{j \neq i} Y_{jpo} - 1, \quad i = 1..N, p = 1..P, t = 1..T \quad (7)$$

$$\Phi_{ipt} \geq Y_{ipt} - \sum_{o=t-\tau_1}^{t-1} Y_{ipo}, i = 1..N, p = 1..P, t = 1..T \quad (8)$$

$$\sum_{i=1}^{N} \sum_{p=1}^{P} \Phi_{ipt} \leq \overline{C}, t = 1..T \quad (9)$$

$$\sum_{i=1}^{N} \sum_{p=1}^{P} \sum_{t \in T_h} \Phi_{ipt} \leq \overline{Cw}, h = 1..H \quad (10)$$

Each item changeover from one period to another incurs a mold setup, which requires human resource and affect the heater productivity during the changeover. To respect the changeover capacity two parameters have been set: the maximum number of mold setup per day ($\overline{C}$) and per week ($\overline{Cs}$). Constraints (7) to (9) limit the number of mold setup per period (days) whereas constraint (7), (8) and (10) limit the number of mold setup per set of periods (weeks). A heater is allowed not to produce an item during a few periods(τ_1) without considering a mold setup if the mold stays in the heater. It means that the heater cannot be used to produce another item during these periods.

$$\sum_{i=1}^{N} \sum_{p=1}^{P} \sum_{t \in T_h} X_{ipt} * \omega_i \leq \sum_{t \in T_h} V_t + \overline{v_w}, h = 1..H \quad (11)$$

$$\sum_{i=1}^{N_i} \sum_{p=1}^{P} \sum_{t \in T_h} X_{ipt} * \omega_i \geq \sum_{t \in T_h} V_t - \underline{v_w}, h = 1..H \quad (12)$$

$$\sum_{i=1}^{N_i} \sum_{p=1}^{P} X_{ipt} * \omega_i \leq V_t + \overline{v_d}, t = 1..T \quad (13)$$

$$\sum_{i=1}^{N_i} \sum_{p=1}^{P} X_{ipt} * \omega_i \geq V_t - \underline{v_d}, t = 1..T \quad (14)$$

Every week, the factory commits to the supply chain a targeted total weight of items to produce. The workforce needed is determined to satisfy this target. Respecting these constraints is very important to keep cost of production the lowest possible. Constraints (11) to (14) make sure the targeted total weight of items produced is stays between upper and lower bounds, per days and per week where ω_i represents the unit weight of item i, $\underline{v_w}$ and $\overline{v_w}$ weekly bounds and $\underline{v_d}$ and $\overline{v_d}$ daily bounds.

$$X_{ipt} \leq M_{ap} * M, a = 1..A, i = 1..N_a, p = 1..P, t = 1..T \quad (15)$$

$$X_{ipt} \leq M_{pt} * M, i = 1..N, p = 1..P, t = 1..T \quad (16)$$

Constraint (15) states that the allocation of items must respect the tire-heater eligibility matrix. Similarly, Constraint (16) makes sure that no item is allocated on a heater shut down for maintenance, industrial trial or any other planned event.

$$\Psi_{it} \geq \frac{1}{P} * \sum_{p=1}^{P} Y_{ipt}, i = 1..N, t = 1..T \quad (17)$$

$$\Psi_{it} \leq \sum_{p=1}^{P} Y_{ipt}, i = 1..N, t = 1..T \quad (18)$$

$$\sum_{i=1}^{N} \Psi_{it} \leq S, t = 1..T \quad (19)$$

Constraints (17) to (19) ensure that the number of different items produced in the same period do not exceed the limit set by the industrial (S). This limit is empirical and makes sure that the workshops before the curing process (assembling, semi-finished items production) are not saturated.

$$Der_{it} \geq \frac{1}{P} * \sum_{p=1}^{P} Y_{ipt-\tau_2} - P * \sum_{o=t-\tau_2+1}^{t} \sum_{p=1}^{P} Y_{ipo}, \quad i = 1..N, t = 1..T \tag{20}$$

$$\sum_{i=1}^{N} \sum_{t \in T_h} Der_{it} \leq Der, h = 1..H \tag{21}$$

When a production campaign comes to an end, all the semi-finished products needs to be produced at the exact quantity to minimize material loss. It requires a particular follow up by the agents in the factory and is quite difficult to manage properly. Thus, a limit of campaign ending per week Der has been added. Similarly, to the mold setup constraints, when a production campaign is suspended for a few days(τ_2), it does not count as a campaign ending. Constraints (20) and (21) states that the number of campaign ending do not exceed a certain limit Der per week.

$$\sum_{i \in N_\theta} \sum_{p=1}^{P} \sum_{t \in T_h} X_{ipt} * TU_i \leq Sat_\Theta, h = 1..H, \Theta = 1..W \tag{22}$$

Thanks to Constraint (22) the number of minutes planned on each workshops of the assembly shop do not exceed the saturation limit Sat_Θ. This constraint is the reason why the differentiation through two sets of items a in $1..A$ and i in $1..N$ is necessary. The same tire from client view can be made on two different assembly shop workshops. Thus we need to consider the same tire as two different ones depending on the shop it is made in. It allows the company to balance the saturation of the different assembly shops and to provide better the curing process bottleneck.

$$\sum_{p=1}^{P} Y_{ipt} * M_{di} \leq K_d * KM_i, d = 1..N_d, i = 1..N, t = 1..T \tag{23}$$

There is a limited number K_d of each type of drum d in the assembly shop. Constraint (23) ensures that for each drum d, the overall production planned in curing process can be absorbed by the assembly shop, where KM_i represents the maximum number of molds that can be planned for item i on drum d.

$$\sum_{o=t-\tau+1}^{t} \Phi_{ipo} \leq Y_{ipt}, i = 1..N, p = 1..P, t = 1..T - \tau + 1 \tag{24}$$

Due to human resources, a minimum campaign duration τ is considered by the company. The minimum duration of a production campaign is guaranteed by Constraint (24).

$$Y_{ipt}, \Phi_{it}, \Psi_{it}, Der_{it} \in \{0;1\} \tag{25}$$

$$X_{ipt}, I_{at}, B_{at} \in \mathbf{N} \tag{26}$$

Finally, Constraint (25) defines the Boolean decision variables and Constraint (26) the production, inventory and backorder decision variables.

5. COMPUTATIONAL RESULTS

The proposed model is tested on real data instances (up to 210 items) on a 8-week planning horizon, knowing that the first 2 weeks of the planning horizon are fixed. Due to the complexity of our problem, the model is applied on each week. The outputs of all weeks are grouped to build the global production plan. Four sets of data were selected to test the model. Those sets represent the tires of three assembly shops and the global portfolio (respectively 14, 57, 102 and 210 items). The commercial solver CPLEX 12.9 was used to solve the problem, limited to 60 minutes of computational time. The main parameters of the model are also reported (See Table 1).

Table 1. Results before preprocessing

	Items	S	Der	$\overline{C}$	$\overline{Cs}$	Gap (%)	CPU (s)
Inst. 1	14	5	1	1	3	0	44
Inst. 2	57	12	3	3	6	0	1216
Inst. 3	102	30	9	3	15	0,87	3600
Inst. 4	210	43	13	5	20	∅	3600

To obtain a feasible production planning on Instance 4, we applied a preprocessing algorithm. Indeed, many tires of the portfolio do not have demand all year long. So each tire with demand to 0 on the week to be planned with an inventory level greater than the minimum stock can be removed (See Table 2).

Table 2. Results after preprocessing

	Items	S	Der	$\overline{C}$	$\overline{Cs}$	Gap (%)	CPU (s)
Inst. 1	14	5	1	1	3	0	1086
Inst. 2	57	12	3	3	6	0	1372
Inst. 3	102	30	9	3	15	0,75	3600
Inst. 4	210	43	13	5	20	1,07	3600

Thanks to the preprocessing algorithm, we obtained a feasible solution in 1 hour on Instance 4 which represents a real-world lot sizing problem.

6. CONCLUSION

In this paper, we addressed a dedicated lot sizing problem inspired from the tire industry. This problem is a single-level multi-item multi-machine new lot sizing problem with dynamic demand, backlogging, a simple setup structure and specific constraints. We propose a mathematical model to solve a real world problem with up to 210 items on 70 machines and 7 periods. The contribution of our work is twofold. First, we solved a relevant real-world problem. Second, we introduced and modeled specific constraints. To the best of our knowledge, respecting a maximum number of simultaneous items per period, a maximum number of production campaign ending and number of setups within a sets of periods have never been dealt with in the literature. In the next steps of our research we will be looking for valid inequalities to speed up the model resolution and to test our tool in the long term to implement the solution in the factory.

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