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Analysis of a bilinear model of an electric power system using spectral decompositions of Lyapunov functions

Alexey B. Iskakov* Igor B. Yadykin**

**V.A. Trapeznikov Institute of Control Sciences RAS, Ul. Profsoyuznaya 65, Moscow 117997, Russia (Tel: +7-495-334-90-30; e-mail: isk_alex@mail.ru)*
***V.A. Trapeznikov Institute of Control Sciences RAS, Ul. Profsoyuznaya 65, Moscow 117997, Russia (Tel: +7-495-334-90-31; e-mail: jadikin1@mail.ru)*

Abstract: In this paper, the spectral decompositions of Lyapunov functions are applied for the first time to the analysis of the behavior of a bilinear model of a two-area electric power system. In contrast to the technique of normal forms and modal series methods, we consider the spectral decomposition not for the dynamics of state variables, but for Lyapunov functions, which characterize the L_2-norms of variables or signals in the time domain. The solution of the generalized Lyapunov equation for a bilinear system is represented as the sum of Hermitian matrices corresponding to individual eigenvalues of the system or their pairwise combinations. An iterative algorithm for calculating spectral terms is developed for stable bilinear systems. In a test experiment for the purpose of transient stability analysis, we evaluate the value of individual eigenmodes and their pairwise combinations depending on the magnitude of bilinear terms. The obtained results are consistent with an intuitive interpretation derived from the model equations and eigenvalue analysis. The spectral decompositions of Lyapunov functions allowed us to indicate the range of applicability of linear model and to reveal dominant eigenmodes in the analysis of transient stability of electric power system.

Keywords: bilinear systems, stochastic linear systems, electric power system, transient stability, generalized Lyapunov equations, Lyapunov functions, spectral decompositions, Gramians.

1. INTRODUCTION

A critical requirement for the introduction of modern smart grid and microgrid technologies is increasing the reliability of *electric power systems* (EPS) and the ability to monitor and control their stability in real time (Häger et al., 2014). For this purpose, *bilinear models* may be useful. In the electric power industry, they are used to evaluate states and parameters, to reduce the order of high-dimensional models, to identify parameters from wide area measurement system (WAMS) measurements, and also to analyze small-signal and transient stability (Arroyo et al., 2007). In (Al-Baiyat et al., 1993), a bilinear model of a two-area power system was used to analyze its stability and reduce its dimensionality. The connection between the generalized Lyapunov equations and the problem of calculating the Gramians of the controllability and observability of a bilinear system was established, and the balanced truncation algorithms were developed to construct simplified models of the energy system. It was shown that the bilinear simplified model gives a more accurate reaction in various modes of EPS operation than a linear high-order model. Later, the same model was used in (Benner and Breiten, 2012) for testing optimal algorithms for decreasing the order of a bilinear model based on the H_2 norm of its transfer function. A new iterative algorithm was developed for optimal interpolation of a bilinear system using Krylov subspaces and Petrov-Galerkin projection methods. A bilinear model of a two-area power system was used in (Al-Baiyat and Bettayeb, 1993) to analyze the static and transient stability of the power system. A mathematical model of a bilinear power system was developed on the basis of linearization method of Carleman and tensor products. For the stability analysis, the nonlinear factors of influence of individual modes on the system states were used. The method showed good results for isolating dominant modes and analyzing inter-area oscillations. Bilinear state estimation was proposed in (Gomez-Exposito et al., 2012; Kumar et al., 2016) as an alternative to the traditional estimation based on the use of the Gauss-Newton method. The use of bilinear models together with the distributed robust method for assessing the state of regional power systems allows one to take into account the nonlinear characteristics of phase measurement units in WAMS.

Lyapunov and Sylvester equations are widely used in modern control theory. The spectral properties of the solutions of the Lyapunov equations are effectively used to reduce the dimensionality of models of large systems in mechanical engineering, power industry, spacecraft control and electronics (Baur et al., 2014). The use of *generalized Lyapunov equations* to analyze the properties of controllability and observability of bilinear systems has also long been known in control theory (Gray, Mesko, 1998). The effective iterative algorithms for solving such equations were proposed in (Zhang, Lam, 2002; Damm, 2008). In (Benner, Damm, 2011), a unified approach to the design of simplified models of linear stochastic and bilinear high-dimensional systems based on the use of energy functionals has been formulated.

In this study, we apply for the first time the spectral decompositions of the solutions of generalized Lyapunov equations proposed in (Yadykin, Iskakov, 2019) to the analysis of the behavior of a bilinear model of a two-area electric power system (EPS). Similar spectral decompositions for the solutions of ordinary linear Lyapunov equations have been previously proposed in (Yadykin, Galyaev, 2013; Yadykin, Iskakov, 2017), where these solutions were represented as the sum of Hermitian matrices corresponding to individual eigenvalues of the system or their pairwise combinations. Similar analytical solutions were obtained in (Zubov et al., 2017) using Jordan normal forms. These decompositions have already been applied for the stability analysis of EPS in (Vassilyev et al., 2017). Therefore, we expect that the spectral decompositions of the solutions of the generalized Lyapunov equations for bilinear systems will be useful in the analysis of their transient stability.

Several methods are used to analyze the bilinear effects in power systems, such as the technique of *normal forms* (Jang et al., 1998), *modal series methods* (Pariz et al. 2003), and *bilinear approximation* (Arroyo et al., 2007). In contrast to these methods, in this paper we consider the spectral decomposition not for the instantaneous dynamics of state variables, but for the Lyapunov functions, which characterize the L_2-norms of variables or signals in the time domain. For example, in a linear time-invariant system, eigenmodes do not interact in the instantaneous dynamics, but the time integral of the product of these modes participates in the L_2 norm of the state variable, and this participation can be estimated by the method proposed in this study. In addition, the methods in (Jang et al., 1998; Pariz et al. 2003; Arroyo et al., 2007) evaluate bilinear effects associated with the nonlinear dependence of the model on state variables. This approach allows one to take into account the bilinear coupling between the input control and perturbations of the second and higher orders. The proposed approach uses a more general model of a bilinear system and also allows one to analyse the bilinear coupling between the input control and first-order perturbations. Brief information about the generalized Lyapunov equations is given in Section 2. Section 3 introduces the spectral decompositions for the solutions of these equations. Section 4 introduces an iterative algorithm for computing the spectral components. A bilinear test model of a two-area EPS is described in Section 5. The results obtained in the test experiment are provided in Section 6. Section 7 contains the conclusions drawn from this work.

2. GENERALIZED LYAPUNOV EQUATION FOR BILINEAR DYNAMICAL SYSTEM

Bilinear systems can be characterized by the following equations:

$$\dot{x}(t) = Ax(t) + \sum_{\gamma=1}^{m} N_\gamma\, x(t)\, u_\gamma(t) + B\, u(t),$$
$$y(t) = C\, x(t), \qquad (1)$$

where $x(t) \in \mathbb{R}^n, u(t), y(t) \in \mathbb{R}^m$ are the state, input and output vectors, respectively, and $A, N_1, \dots, N_m, B,$ and C are the real matrices. Gramian of controllability of the bilinear system (1) can be determined using the sequence of kernels of the Volterra matrix series (D'Allesandro, Isidori, and Ruperdi, 1974)

$$P_1(t_1) = e^{At_1}B,$$
$$P_i(t_1, \dots, t_i) = e^{At_i}[N_1P_{i-1}, N_2P_{i-1}, \dots, N_mP_{i-1}],\ \ i = 2, 3, \dots,$$
$$P = \sum_{i=1}^{\infty} \int_0^{\infty} \dots \int_0^{\infty} P_i(t_1, \dots, t_i)\, P_i^T(t_1, \dots, t_i) dt_1 \dots dt_i\,, \qquad (2)$$

provided that this series exists. It can be shown that the Gramian (2) satisfies the following generalized Lyapunov equation

$$AP + PA^T + \sum_{\gamma=1}^{m} N_\gamma P N_\gamma^T = -BB^T \qquad (3)$$

Using the Kronecker product and the vectorization operator, equation (3) can be written as a linear matrix equation

$$\left(A \otimes I + I \otimes A + \sum_{\gamma=1}^{m} N_\gamma \otimes N_\gamma \right) \mathrm{vec}(P) = -\mathrm{vec}(BB^T) \quad (4)$$

This equation has a unique solution if and only if the matrix on the left side of (4) is nonsingular. However, solving this equation is usually not effective. If the dynamics matrix A is stable, and equation (3) has a unique solution $P \geq 0$, then the controllability Gramian (2) exists and can be obtained by the following iterative procedure (Zhang and Lam, 2002):

$$AP^{(1)} + P^{(1)}A^T + BB^T = 0\,,$$
$$AP^{(k)} + P^{(k)}A^T + \sum_{\gamma=1}^{m} N_\gamma P^{(k-1)} N_\gamma^T = 0,\ \ k = 2, 3, \dots\,, \quad (5)$$
$$P = P^{(1)} + \sum_{k=2}^{\infty} P^{(k)}\,.$$

3. SPECTRAL DECOMPOSITIONS FOR GENERALIZED LYAPUNOV FUNCTIONS

Consider the generalized Lyapunov equation

$$AP + PA^T + \alpha \sum_{\gamma=1}^{m} N_\gamma P N_\gamma^T = -Q, \qquad (6)$$

where $Q = BB^T$ or $C^T C$, and $\alpha \geq 0$ is a real parameter. According to (Yadykin, Iskakov, 2019), if the linear operator on the left-hand side of (6) is nonsingular and the dynamic matrix A has a simple spectrum $\sigma(A) = \{\lambda_1, \lambda_2, \cdots, \lambda_n\}$, then the spectral decompositions of the solution of equation (6) can be represented as

$$P = \sum_{i=1}^{n} \bar{P}_i = \sum_{i,j=1}^{n} P_{ij}\ ,\ \ \bar{P}_i = \sum_{j=1}^{n} P_{ij}\ , \qquad (7)$$

where the matrices $\bar{P}_i$ and P_{ij} are respectively defined as the solutions of the following equations:

$$A\,\bar{P}_i + \bar{P}_i A^T + \alpha \sum_{\gamma=1}^{m} N_\gamma \bar{P}_i N_\gamma^T = -\,\frac{1}{2}(R_i Q + Q R_i^*)\ , \quad (8)$$

$$A\,P_{ij} + P_{ij} A^T + \alpha \sum_{\gamma=1}^{m} N_\gamma P_{ij} N_\gamma^T = -\,\frac{1}{2}\left(R_i Q R_j^* + R_j Q R_i^*\right) \quad (9)$$

where $(\cdot)^*$ denotes the Hermitian conjugation, and the matrix residues R_i and R_j are the coefficients in the expansion of the resolvent of matrix A:

$$(Is - A)^{-1} = \frac{R_1}{s-\lambda_1} + \frac{R_2}{s-\lambda_2} + \cdots + \frac{R_n}{s-\lambda_n} \qquad (10)$$

In (Yadykin, Iskakov, 2019) the matrices $\bar{P}_i$ and P_{ij} in (8, 9) were called *the sub-Gramians for the corresponding bilinear system*. Since the operator on the left-hand side of (6) is nonsingular, the sub-Gramians $\bar{P}_i$ and P_{ij} are defined uniquely for any matrix Q. The norms of these sub-Gramians allow one to characterize the contribution of individual eigen-components or their pairwise combinations to the asymptotic dynamics of the perturbation energy in deterministic bilinear systems. In particular, the sub-Gramians $\bar{P}_i$ characterize the contribution of linear modes $\lambda_1, \lambda_2, \cdots, \lambda_n$ of bilinear system, whereas the pairwise sub-Gramians P_{ij} characterize the contribution from the interaction modes of the form $\lambda_i + \lambda_j$. Therefore, let us call equations (8) and (9) *the generalized modal Lyapunov equations.* When $\alpha = 0$ we obtain in (8, 9) the spectral decompositions for the ordinary Lyapunov algebraic equation, which coincide with those obtained earlier in (Yadykin, Iskakov, 2017).

4. ITERATIVE ALGORITHM FOR COMPUTING SUBGRAMIANS

In this section, by analogy with the iterative procedure (5), we develop an iterative procedure for calculating the sub-Gramians $\bar{P}_i$ and P_{ij} from (8) and (9), respectively. The main idea of the proposed algorithm is to solve at each iteration the ordinary linear algebraic Lyapunov equation with the right-hand side determined from the previous iteration. Moreover, the calculation of the matrix sub-Gramians by components in the eigenvector basis takes a particularly simple form. This approach allows us to simplify the calculation of bilinear Gramians and sub-Gramians and to evaluate the limits of applicability of the linear analysis.

Assume that the matrix A in (6) has n distinct eigenvalues $(\lambda_1, \lambda_2, \cdots, \lambda_n)$ and the following eigenvalue decomposition:

$$A = U\Lambda V,\quad U\,V = V\,U = I\,, \qquad (11)$$

where $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$, the columns of the matrix U are the normalized right eigenvectors of the matrix A, and the rows of the matrix V are its normalized left eigenvectors. Then iterative procedure (5) for solving equation (3) can be represented in a simpler form:

$$\Lambda \tilde{P}^{(1)} + \tilde{P}^{(1)} \Lambda^* = -\,\tilde{Q}\,,$$

$$\Lambda \tilde{P}^{(k)} + \tilde{P}^{(k)} \Lambda^* = -\sum_{\gamma=1}^{m} \tilde{N}_\gamma \tilde{P}^{(k-1)} \tilde{N}_\gamma^*,\ \ k = 2, 3, \ldots, \quad (12)$$

$$\tilde{P} = \tilde{P}^{(1)} + \sum_{k=2}^{\infty} \tilde{P}^{(k)}\ ,\ \ P = U\,\tilde{P}\,U^*\,,$$

where $\tilde{P}^{(k)} = VP^{(k)}V^*$, $\tilde{Q} = VBB^TV^*$, $\tilde{N}_\gamma = VN_\gamma V^*$. Let $\tilde{\nu}_{\gamma pr} = \left(\tilde{N}_\gamma\right)_{pr}$ be the components of the matrices $\tilde{N}_\gamma$. Then (12) can be written in terms of matrix components in the form

$$\left(\tilde{P}^{(1)}\right)_{pr} = \frac{-1}{\lambda_p + \lambda_r^*}\left(\tilde{Q}\right)_{pr},$$

$$\forall\, k > 1:\ \left(\tilde{P}^{(k)}\right)_{p_k r_k} =$$

$$\sum_{\gamma=1}^{m}\ \sum_{p_{k-1}, r_{k-1}=1}^{n} \frac{-\,\tilde{\nu}_{\gamma p_k p_{k-1}} \tilde{\nu}^*_{\gamma r_k r_{k-1}}}{\lambda_{p_k} + \lambda^*_{r_k}} \left(\tilde{P}^{(k-1)}\right)_{p_{k-1} r_{k-1}},$$

$$P = U\left(\sum_{k=1}^{\infty} \tilde{P}^{(k)}\right) U^*\,. \qquad (13)$$

By analogy with the expansions in (Yadykin and Galyaev, 2013), this can also be generalized to the case of multiple eigenvalues. Equations (8) and (9) for the sub-Gramians $\bar{P}_i$ and P_{ij} differ from equation (6) for the Gramian P only in their right-hand side. In the eigenvector basis, the matrix $\tilde{Q} = VQV^*$ is replaced by the matrices

$$\tilde{Q}_i = \frac{1}{2}\left(\mathbf{1}_{ii}\tilde{Q} + \tilde{Q}\mathbf{1}_{ii}\right)\ and$$

$$\tilde{Q}_{ij} = \frac{1}{2}\left(\mathbf{1}_{ij}\left(\tilde{Q}\right)_{ij} + \mathbf{1}_{ji}\left(\tilde{Q}\right)_{ji}\right),$$

for $\bar{P}_i$ and P_{ij} respectively, where $\mathbf{1}_{ij} = \boldsymbol{e}_i \boldsymbol{e}_j^T$ is the product of the unit basis vectors. Substituting matrices $\tilde{Q}_i$ instead of $\tilde{Q}$ into the iterative procedure (13), we obtain an iterative procedure for calculating sub-Gramians $\bar{P}_i$ in (8)

$$\tilde{\bar{P}}_i^{(1)} = -\frac{1}{2}\sum_{l=1}^{n}\left(\mathbf{1}_{il}\frac{\left(\tilde{Q}\right)_{il}}{\lambda_i + \lambda_l^*} + \mathbf{1}_{li}\frac{\left(\tilde{Q}\right)_{li}}{\lambda_l + \lambda_i^*}\right),$$

$$\forall\, k > 1:\ \left(\tilde{\bar{P}}_i^{(k)}\right)_{p_k r_k} =$$

$$\sum_{\gamma=1}^{m}\ \sum_{p_{k-1}, r_{k-1}=1}^{n} \frac{-\,\alpha\,\tilde{\nu}_{\gamma p_k p_{k-1}} \tilde{\nu}^*_{\gamma r_k r_{k-1}}}{\lambda_{p_k} + \lambda^*_{r_k}} \left(\tilde{\bar{P}}_i^{(k-1)}\right)_{p_{k-1} r_{k-1}},$$

$$\bar{P}_i = U\left(\sum_{k=1}^{\infty} \tilde{\bar{P}}_i^{(k)}\right) U^*. \qquad (14)$$

Similarly, to calculate pairwise sub-Gramians P_{ij} in (9), we obtain the following iterative procedure:

$$\tilde{P}_{ij}^{(1)} = -\frac{1}{2}\left(\mathbf{1}_{ij}\frac{(\tilde{Q})_{ij}}{\lambda_i + \lambda_j^*} + \mathbf{1}_{ji}\frac{(\tilde{Q})_{ji}}{\lambda_j + \lambda_i^*}\right),$$

$$\forall\, k > 1:\ \left(\tilde{P}_{ij}^{(k)}\right)_{p_k r_k} =$$

$$\sum_{\gamma=1}^{m}\sum_{p_{k-1}, r_{k-1}=1}^{n}\frac{-\alpha\, \tilde{v}_{\gamma p_k p_{k-1}}\tilde{v}^*_{\gamma r_k r_{k-1}}}{\lambda_{p_k} + \lambda^*_{r_k}}\left(\tilde{P}_{ij}^{(k-1)}\right)_{p_{k-1} r_{k-1}},$$

$$P_{ij} = U\left(\sum_{k=1}^{\infty}\tilde{P}_{ij}^{(k)}\right)U^*. \qquad (15)$$

The necessary conditions for the applicability of iterative procedures (14) and (15) are the same as for the iterative procedure (5) established in (Zhang and Lam, 2002), namely, the dynamics matrix A is stable, and equation (6) has a unique solution $P \geq 0$.

5. TEST MODEL OF A TWO-AREA POWER SYSTEM

As a test bilinear model, we use the 17th order model from (Al-Baiyat et al., 1993) for two interconnected power systems, each area having one steam and one hydro unit. This model is developed to describe the electromechanical transients and suitable for the study of primary and secondary load and frequency control. It includes turbine governors for steam and hydro units, hydraulic installations and turbines, steam turbines with the installations for steam generation, power plants, consumers, the transmission network with system inertia, and the tie-line.

This system model has been bilinearized around a chosen operating point and represented in the form of equation (6). There are 17 state variables, which represent variations of frequencies; hydro turbine gate openings; steam turbine valve openings; tie-line power exchange; high, intermediate and low pressure outputs of steam turbines; flows and dashpot piston positions of hydro turbines. There are 4 control input variations, one for each hydro or steam unit. There are 8 bilinear terms associated with the variations in frequencies and hydro turbine gate openings. The detailed description of the model equations, state variables, and parameters is given in (Al-Baiyat et al., 1993). In this paper we use the same state variables and parameters taken for $k_1 = k_2 = k_3 = k_4 = 1$. The corresponding values of nonzero coefficients for the matrices $A \in \mathbb{R}^{17\times17}, B \in \mathbb{R}^{17\times4}, N_1, N_2, N_3, N_4 \in \mathbb{R}^{17\times17}$ are given in Appendix A.

6. RESULTS OF TEST EXPERIMENT

In a test experiment, we characterize the contribution of linear eigenmodes and their pair interactions to the small-signal and transient perturbation energy of the system (6) depending on the coefficient α, which characterizes the magnitude of bilinear terms. At $\alpha = 0$, the system is purely linear, and with increasing α, bilinear effects increase. For each α, we compute the sub-Gramians $\bar{P}_i$ and P_{ij} defined by (8) and (9), respectively.

A list of eigenmodes that are most sensitive to the parameter α is presented in Table 1.

Table 1. Test system modes and eigenvalues.

Mode	Eigenvalue	Type and principal variables
S_1	-0.0503	Inter-area: hydro turbine dashpot piston positions
S_4/S_5	-0.139 ± 0.187 *j*	Inter-area: frequencies and hydro turbine dashpot piston positions
S_6	-0.1396	Inter-area: steam turbine intermediate pressure outputs and hydro turbine dashpot piston positions
S_{10}	-1.923	Local for system 1: the valve opening and low pressure output of the steam turbine 1 and the flow in hydro turbine 1
S_{13}	-2.038	Local for system 1: the valve opening and low pressure output of the steam turbine 1 and the flow in hydro turbine 1
S_{14}	-4.35	Inter-area: hydro turbine gate openings
S_{15}	-4.45	Inter-area: hydro turbine gate openings

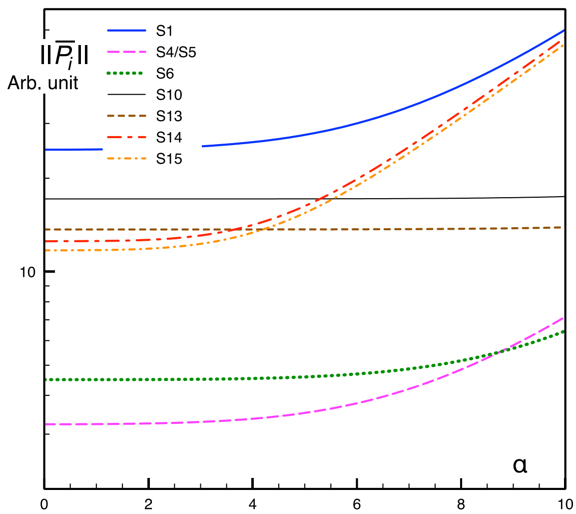

Fig. 1. Frobenius norm of the sub-Gramians $\bar{P}_i$ for linear eigenmodes as a function of weighting coefficient α.

The Frobenius norms of the sub-Gramians $\bar{P}_i$ for linear eigenmodes obtained by (8) are shown in Figure 1 as a function of the weighting coefficient α. Eigenmodes not shown in the figure are weakly dependent on α. The obtained graphs show the general effect of bilinear terms for the analysis of dynamic small-signal and transient stability of the power system. In particular, these graphs allow one to specify the range of applicability of linear model for the analysis of transient stability. It can be seen that the linear model accurately reproduces the behaviour of all modes in a fairly wide range of variation of the weight coefficient α from 0 to 4. Figure 1 also reveals the linear eigenmodes most sensitive to bilinear effects. They are the inter-area modes S_1, S_4/S_5, S_6, S_{14}, and S_{15}. The observed dynamics has an intuitive explanation. The bilinear terms in this model are associated with variations of frequencies and hydro turbine gate openings. As can be seen from Table 1, the oscillation S_4/S_5 has a high participation of frequencies. The aperiodic modes S_{14} and S_{15} are mainly associated with variations of the hydro turbine gate openings. And the eigenmodes S_1, S_4/S_5, and S_6 are determined by the hydro turbine dashpot piston positions, the dynamics of which in the bilinear model (Al-Baiyat et al. 1993) are directly related to the variations of hydro turbine gate openings.

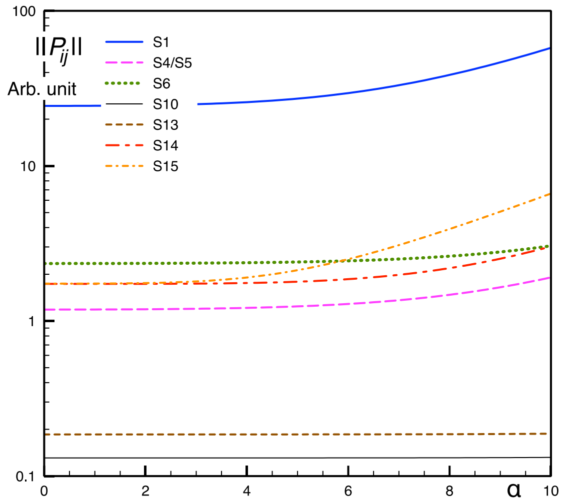

Fig. 2. Frobenius norm of the pair sub-Gramians P_{ij} for S_1 mode and other eigenmodes as a function α.

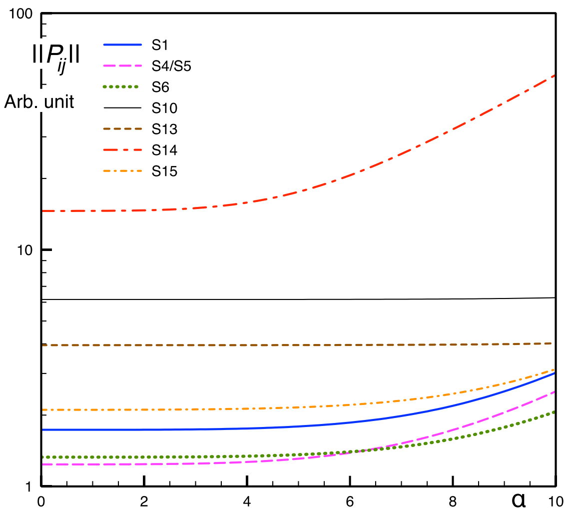

Fig. 3. Frobenius norm of the pair sub-Gramians P_{ij} for S_{14} mode and other eigenmodes as a function α.

Figures 2, 3, and 4 show the Frobenius norms of the pair sub-Gramians P_{ij} (9) for the modal interactions between the modes i = 1, 14, and 15 respectively, and other eigenmodes j as a function of α. The pair sub-Gramians characterize the degree of pairwise interaction between linear eigenmodes. In this case, it can be seen that with an increase in bilinearity, the modes S_1, S_{14}, and S_{15} increase the interaction between themselves, as well as with the modes S_4/S_5 and S_6. Thus, the interaction increases between those modes that are most sensitive to the bilinearity parameter α. We also note that the norms of diagonal pairwise sub-Gramians $P_{11}, P_{14\,14}$, and $P_{15\,15}$ increase most rapidly.

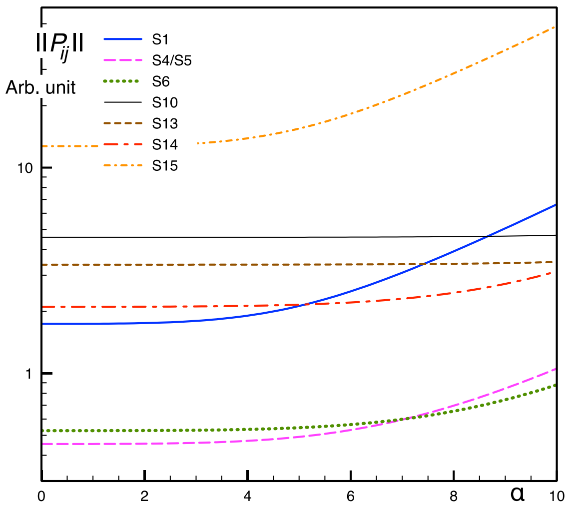

Fig. 4. Frobenius norm of the pair sub-Gramians P_{ij} for S_{15} mode and other eigenmodes as a function α.

6. CONCLUSIONS

To analyse the dynamic behaviour of bilinear systems, novel spectral decompositions were proposed in (Yadykin, Iskakov, 2019) for the solutions of the corresponding generalized Lyapunov equations. These solutions (or Gramians) were represented as a sum of spectral components, which were called *sub-Gramians*. Sub-Gramians characterize the contribution of linear eigenmodes $\lambda_1, \lambda_2, \cdots, \lambda_n$ or their pair modal interactions of the form $\lambda_i + \lambda_j$ to the asymptotic dynamics of the perturbation energy in a bilinear system.

In this paper, for the first time we apply the proposed decompositions to the analysis of the behaviour of a bilinear model of a two-area electric power system (EPS). An analysis of the spectral decompositions of the controllability Gramian revealed an intuitively expected result. With an increase in bilinear effects, we observe an increase in the role of eigenmodes that are associated with the variables involved in bilinear terms. Similarly, the interaction between the eigenmodes, which are most sensitive to the bilinearity parameter α, increases. In the simulation experiment, an analysis of the sub-Gramian norms allowed us to determine the limits of applicability of linear model and to reveal dominant modes in the analysis of the transient stability of EPS. However, for a more detailed study and verification of the proposed method, further test experiments and comparison with the results of the known methods of normal forms and modal series are needed, which is the subject of our further study.

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Appendix A. NONZERO COEFFICIENTS FOR THE BILINEAR MODEL MATRICES

$A_{1\,1} = -2.0$; $A_{1\,8} = -4.0$; $A_{2\,1} = 4.8$; $A_{2\,2} = -5.0$; $A_{3\,2} = 0.2$;

$A_{3\,3} = -0.2$; $A_{4\,3} = 2.0$; $A_{4\,4} = -2.0$; $A_{5\,2} = -0.08$;

$A_{5\,3} = -0.08$; $A_{5\,4} = 0.11$; $A_{5\,5} = -4.0$; $A_{5\,6} = 10.0$;

$A_{5\,7} = -0.93$; $A_{5\,8} = -9.1$; $A_{5\,9} = 0.67$; $A_{6\,5} = 0.2$; $A_{6\,6} = -0.5$;

$A_{7\,5} = 1.32$; $A_{7\,7} = -1.4$; $A_{7\,8} = -0.28$; $A_{8\,2} = 0.01$; $A_{8\,3} = 0.01$;

$A_{8\,4} = 0.014$; $A_{8\,5} = -0.06$; $A_{8\,7} = 0.12$; $A_{8\,8} = -0.11$;

$A_{8\,9} = -0.08$; $A_{9\,8} = 22.2$; $A_{9\,10} = -22.0$; $A_{10\,9} = 0.08$;

$A_{10\,10} = -0.1$; $A_{10\,11} = 0.09$; $A_{10\,13} = -0.05$; $A_{10\,14} = 0.02$;

$A_{10\,15} = 0.01$; $A_{10\,16} = 0.01$; $A_{11\,10} = -0.28$; $A_{11\,11} = -1.4$;

$A_{11\,13} = 1.32$; $A_{12\,12} = -0.5$; $A_{12\,13} = 0.2$; $A_{13\,9} = -0.67$;

$A_{13\,10} = -0.91$; $A_{13\,11} = -0.74$; $A_{13\,12} = 10.0$; $A_{13\,13} = -4.1$;

$A_{13\,14} = -0.14$; $A_{13\,15} = -0.09$; $A_{13\,16} = -0.10$; $A_{14\,14} = -2.0$;

$A_{14\,15} = 2.0$; $A_{15\,15} = -0.17$; $A_{15\,16} = 0.17$; $A_{16\,16} = -5.0$;

$A_{16\,17} = -4.8$; $A_{17\,10} = -4.0$; $A_{17\,17} = -2.0$.

$B_{1\,1} = 4.0$; $B_{5\,2} = 10.0$; $B_{13\,3} = 10.0$; $B_{17\,4} = 4.0$.

$N1_{5\,5} = 0.067$; $N2_{5\,8} = 0.067$; $N1_{8\,5} = -0.008$; $N2_{8\,8} = -0.008$

$N3_{10\,10} = -0.008$; $N4_{10\,13} = -0.008$; $N3_{13\,10} = 0.067$;

$N4_{13\,13} = -0.067$.