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Adaptive stabilization of minimum-phase systems under quantized measurements

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Abstract: We construct an adaptive controller for a linear minimum-phase system of an arbitrary relative degree with an unknown bounded disturbance and dynamically quantized measurements. The key novelty is the extension of the shunting method (parallel feedforward compensator) to plants with bounded disturbances. This method leads to an augmented system of relative degree one that is stabilized by a passification-based adaptive controller. Moreover, we design a switching procedure for the controller parameters and the quantizer's zoom that ensures the state convergence from an arbitrary set to an ellipsoid whose size depends on the disturbance bound. The results are demonstrated by an example of an aircraft flight control.

1. INTRODUCTION

Compared to linear feedback, adaptive controllers achieve better performance in the presence of uncertainties. However, the price for this is higher complexity. Namely, the dynamical order of adaptive systems is typically several times higher than that of linear ones, which makes them more sensitive to disturbances, noise, unmodeled dynamics, etc. Consequently, quite a number of studies were devoted to designing simple adaptive controllers with low dynamical orders (Fradkov (1974, 1976); Barkana and Kaufman (1985); Kaufman et al. (1998); Iwai and Mizumoto (1992); Deng et al. (2001); Dolinar et al. (2000); Cho and Burton (2011); Amini and Javanbakht (2014)).

The class of adaptive systems proposed in Fradkov (1974, 1976) is based on passification: design of feedback rendering the closed-loop system passive. Such feedback exists if and only if the system is hyper-minimum-phase, i.e., it is minimum-phase and has relative degree one (Fradkov (1974, 1976)). This property is also called *strict passivity* (Barkana and Kaufman (1985); Kaufman et al. (1998)).

Since the 1970s, quite a number of adaptive control and synchronization problems have been solved for passifiable systems and networks (see the references in Fradkov (2003); Andrievskii and Fradkov (2006); Bobtsov et al. (2014); Selivanov et al. (2015); Andrievskii and Selivanov (2018); Pyrkin et al. (2018)). In Rahnama et al. (2018) and Zhu et al. (2017) a concept of passivity indices was proposed and used for passivation-based event-triggered control. However, those passivation results are based on introducing the feedthrough terms making relative degree zero, which is sometimes nonrealistic. In Selivanov et al. (2016), the passification method was extended to systems with quantized measurements and bounded disturbances. The main restriction of that paper is the "relative degree one" assumption that limits potential applications.

In this paper, we remove the "relative degree one" assumption by using the so-called shunting method (parallel feedforward compensator) in the form proposed in Fradkov (1994). This allows us to obtain a hyper-minimum-phase augmented system, which is further stabilized by a passification-based adaptive controller. By constructing a switching procedure for the adaptive controller parameters and quantizer's zooming, we ensure convergence of the system state from an arbitrary set to an ellipsoid, whose size depends on the disturbance bound. The results are demonstrated by an example of an aircraft flight control.

2. PLANT DESCRIPTION AND PRELIMINARIES

Consider an uncertain linear system

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u(t) + w_p(t), \\ y_p(t) &= C_p x_p(t) \end{aligned} \tag{1}$$

with the state $x_p \in \mathbb{R}^n$, control input $u \in \mathbb{R}$, measured output $y_p \in \mathbb{R}^l$, unknown disturbance $w_p \in \mathbb{R}^n$, and uncertain matrices A_p, B_p, C_p.

Assumption 1. There exists $\Delta_w > 0$ such that

$$\|w_p(t)\| \le \Delta_w, \quad \forall t \ge 0.$$

Assumption 2. There exists $g_p \in \mathbb{R}^l$ such that $g_p^T W_p(s) = g_p^T C_p(sI - A_p)^{-1} B_p$ is minimum-phase, i.e., its numerator is Hurwitz.

2.1 Passification lemma

Definition 1. (Andrievskii and Fradkov (2006)). Given $g \in \mathbb{R}^l$, a scalar transfer function $g^T W(s) = g^T C(sI - A)^{-1} B$ is called *hyper-minimum-phase* (HMP) if it is minimum-phase and its leading coefficient $g^T CB$ is positive.

Lemma 1. (Passification lemma, Fradkov (1976, 2003)). A rational function $g^T W(s) = g^T C(sI - A)^{-1} B$ is HMP if and only if there exist a matrix P, a vector θ_*, and a scalar $\varepsilon > 0$ such that

$$P > 0, \quad PA_* + A_*^T P < -\varepsilon P, \quad PB = C^T g, \tag{2}$$

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where $A_* = A - B\theta_*^T C$.

Remark 1. If $g^T W(s) = g^T C(sI - A)^{-1}B$ is HMP, then there exists θ such that $u = -\theta^T y + v$ makes the system

$$\dot{x} = Ax + Bu, \quad y = Cx$$

strictly passive from a new input v to the output $g^T y$, i.e., there exist functions $V(x) = x^T Px$, with $P > 0$, and $\varphi(x) \geq 0$, such that $\varphi(x) > 0$ for $x \neq 0$, satisfying

$$V(x(t)) \leq V(x(0)) + \int_0^t \left[y^T(s)gv(s) - \varphi(x(s))\right] ds.$$

Lemma 2. (Fradkov (1994)). Let $g_p^T W_p(s) = g_p^T C_p(sI - A_p)^{-1}B_p$ be a minimum-phase transfer function with a relative degree $r > 1$ and a leading coefficient $g_p^T C_p A_p^{r-1} B_p > 0$. Let $P(s)$ and $Q(s)$ be Hurwitz polynomials of degrees $r-2$ and $r-1$ with positive coefficients. Then there exist a number $\kappa_0 > 0$ and a function $\lambda_0(\kappa) > 0$ such that $g_p^T W_p(s) + \kappa\lambda P(\lambda s)/Q(s)$ is HMP for any $\kappa > \kappa_0$, $0 < \lambda < \lambda_0(\kappa)$.

2.2 Quantizer model

Following Liberzon (2009), we introduce a *quantizer with a quantization range M and a quantization error bound* Δ_e as a mapping $q\colon y \mapsto q(y)$ from $\mathbb{R}^l$ to a finite subset of $\mathbb{R}^l$ such that

$$\|y\| \leq M \quad \Rightarrow \quad \|q(y) - y\| \leq \Delta_e. \tag{3}$$

We will refer to the quantity $e = q(y) - y$ as the *quantization error*. The concrete codomain of q is not important for our further analysis, therefore, can be chosen arbitrary. The value of M is usually dictated by the effective range of a sensor.

By *dynamic quantizer* we will mean the mapping

$$q_\mu(y) = \mu q\left(\frac{y}{\mu}\right), \tag{4}$$

where $\mu > 0$. For each positive μ, one obtains a quantizer with the quantization range μM and the quantization error bound $\mu\Delta_e$. We say that M and Δ_e are the nominal quantization range and quantization error bound. We can think of μ as the "zoom" variable: increasing μ corresponds to zooming out and essentially obtaining a new quantizer with larger quantization range and quantization error bound, whereas decreasing μ corresponds to zooming in and obtaining a quantizer with a smaller quantization range but also a smaller quantization error bound.

3. ADAPTIVE CONTROL WITH DYNAMIC QUANTIZATION

Consider the system (1) subject to Assumptions 1 and 2 with $r > 1$ being the relative degree of $g_p^T W_p(s)$. Let us fix some Hurwitz polynomials $P(s)$ and $Q(s)$ of degrees $r-2$ and $r-1$ with positive coefficients. Due to Lemma 2, there exist λ and κ such that $g_p^T W_p(s) + W_s(s)$ with $W_s(s) = \kappa\lambda P(\lambda s)/Q(s)$ is HMP. We consider a controller that consists of a shunt system

$$\begin{aligned} \dot{x}_s(t) &= A_s x_s(t) + B_s u(t), \\ y_s(t) &= C_s x_s(t) \end{aligned} \tag{5}$$

with the transfer function $W_s(s)$ and switching adaptive control law

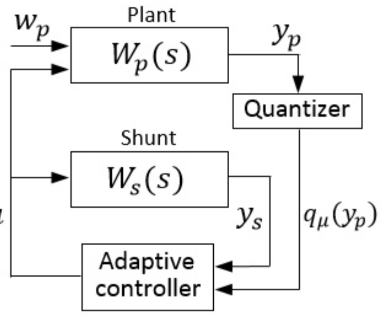

Fig. 1. A minimum-phase system with a shunt

$$\begin{aligned} u &= -\theta_p^T q_i(y_p) - \theta_s y_s, \\ \dot{\theta}_p &= \gamma q_i(y_p)[g_p^T q_i(y_p) + y_s] - \sigma_i \theta_p, \\ \dot{\theta}_s &= \gamma y_s[g_p^T q_i(y_p) + y_s] - \sigma_i \theta_s, \end{aligned} \tag{6}$$

for $t \in [iT, iT + T)$, $i \in \mathbb{N}_0$, where $\theta_p \in \mathbb{R}^l$, $\theta_s \in \mathbb{R}$ are adaptive coefficients, $q_i(y_p) = q_{\mu_i}(y_p)$ is the dynamic quantizer defined in (4), $T > 0$ is a switching period, $\gamma > 0$, and μ_i, σ_i are switching parameters to be defined later.

By introducing (5), we obtain an HMP augmented system. Without quantization, such a system can be stabilized using a regularized passification-based adaptive controller proposed in Narendra et al. (1971). The presence of quantization requires an additional analysis provided in the proof of Theorem 1 below. The switching procedure is introduced to take advantage of the dynamic quantization: when the system state converges to a smaller set, we can "zoom in" and reduce the quantization error, which allows one to adjust σ_i and ensure convergence to a smaller set.

Denote $x = \mathrm{col}\{x_p, x_s\}$, $y = \mathrm{col}\{y_p, y_s\}$, $w = \mathrm{col}\{w_p, 0\}$,

$$A = \begin{bmatrix} A_p & 0 \\ 0 & A_s \end{bmatrix}, \quad B = \begin{bmatrix} B_p \\ B_s \end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0 \\ 0 & C_s \end{bmatrix} \tag{7}$$

and consider the augmented system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t), \\ y(t) &= Cx(t), \end{aligned} \tag{8}$$

whose transfer function is

$$W(s) = \begin{bmatrix} W_p(s) \\ W_s(s) \end{bmatrix}.$$

For $g = \mathrm{col}\{g_p, 1\}$, we obtain that $g^T W(s) = g_p^T W_p(s) + W_s(s)$ is HMP. Without loss of generality, we assume that $\|g\| = 1$ (since for $\tilde{g} = g/\|g\|$ the function $\tilde{g}^T W(s)$ remains HMP). Then Lemma 1 guarantees that there exist a matrix P, a vector $\theta_* \in \mathbb{R}^{l+1}$, and a scalar $\varepsilon > 0$ such that (2) are satisfied (with A, B, and C given in (7)). For $\mu_0 > 0$, $V_0 > 0$ define the following quantities

$$\begin{aligned} \rho &= \frac{\|C\|}{\mu_0 \Delta_e \|\theta_*\|} \sqrt{V_0 \lambda_{\min}^{-1}(P)}, \\ \nu &= \frac{\varepsilon}{2} - \|\theta_*\| \mu_0^2 \Delta_e^2 V_0^{-1} - 2\frac{\mu_0 \Delta_e \|\theta_*\| \|C\|}{\sqrt{\lambda_{\min}(P) V_0}}, \\ \alpha &= \varepsilon - \nu - 2\rho^{-1} \lambda_{\min}^{-1}(P) \|C\|^2, \\ c_\gamma &= \gamma^{-1} \|\theta_*\|^2, \\ c_w &= \alpha^{-1} \nu^{-1} \lambda_{\max}(P), \\ c_e &= 2\alpha^{-1}(\|\theta_*\| + \|\theta_*\|^2 \rho). \end{aligned} \tag{9}$$

Theorem 1. Consider the system (1) subject to Assumptions 1 and 2. Let its output be dynamically quantized by

(4) with a nominal quantization range M and maximum quantization error Δ_e such that

$$\frac{c_e\|C\|^2}{\lambda_{\min}(P)} < \frac{M^2}{\Delta_e^2}. \tag{10}$$

For an arbitrary bounded set of initial conditions, let us choose

$$\mu_0^2 > \frac{c_w\|C\|^2}{M^2\lambda_{\min}(P) - c_e\Delta_e^2\|C\|^2}\Delta_w^2, \quad V_0 = \frac{M^2\lambda_{\min}(P)}{\|C\|^2}\mu_0^2, \tag{11}$$

such that

$$x^T(0)Px(0) + \gamma^{-1}\|\theta(0) - \theta_*\|^2 < V_0, \tag{12}$$

where $\theta = \mathrm{col}\{\theta_p, \theta_s\}$. Then the adaptive controller (5), (6) with

$$V_{i+1} = V_i e^{-\alpha T} + (1 - e^{-\alpha T})(c_\gamma + c_w\Delta_w^2 + c_e\mu_i^2\Delta_e^2), \tag{13}$$

$$\mu_i = \mu_0\sqrt{V_i V_0^{-1}}, \tag{14}$$

$$\sigma_i = \alpha + \gamma\mu_i^2\Delta_e^2(\rho + \|\theta_*\|^{-1}), \tag{15}$$

$$\gamma > \frac{\|\theta_*\|^2\|C\|^2}{\mu_0 M^2\lambda_{\min}(P) - c_w\Delta_w^2\|C\|^2 - c_e\mu_0^2\Delta_e^2}, \tag{16}$$

guarantees that

$$x^T(t)Px(t) < V_i, \quad t \in [iT, iT + T), \quad i \in \mathbb{N}_0. \tag{17}$$

Moreover, $\|\theta(t)\|$ is a bounded function and V_i monotonically decreases with

$$\lim V_i = \frac{c_\gamma + c_w\Delta_w^2}{1 - c_e\mu_0^2\Delta_e^2 V_0^{-1}}.$$

Remark 2. Condition (10) establishes the minimum precision level $\frac{M}{\Delta_e}$ of the nominal quantizer (with $\mu = 1$) such that the adaptive controller is capable of reducing the state norm. Namely, it guarantees that V_i, given by (13), decreases. The values of ν and ρ in (9) are chosen to minimize the limit of V_i.

Proof. First, we show that

$$c_\gamma + c_w\Delta_w^2 + c_e\mu_i^2\Delta_e^2 < V_i < V_{i-1}, \quad i \in \mathbb{N}. \tag{18}$$

Relations (10), (11), (16) guarantee that

$$c_\gamma + c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 < V_0. \tag{19}$$

Then (13) with $e^{-\alpha T} \in (0, 1)$ implies

$$c_\gamma + c_w\Delta_w^2 + c_e\mu_0^2\Delta_e^2 < V_1 < V_0.$$

Since $\mu_1^2 \overset{(14)}{=} \mu_0^2\left(\frac{V_1}{V_0}\right) < \mu_0^2$, the latter guarantees (18) for $i = 1$. If (18) holds for i, then (13) with $e^{-\alpha T} \in (0, 1)$ implies

$$c_\gamma + c_w\Delta_w^2 + c_e\mu_i^2\Delta_e^2 < V_{i+1} < V_i.$$

Since $\mu_{i+1}^2 \overset{(14)}{=} \mu_0^2\frac{V_{i+1}}{V_i}\frac{V_i}{V_0} < \mu_0^2\frac{V_i}{V_0} = \mu_i^2$, the latter guarantees (18) for $i + 1$. By induction, we obtain (18) for $i \in \mathbb{N}$. Thus, V_i monotonically decreases with the limit value obtained by solving (13), (14).

Consider the augmented system (8) and

$$V(x, \theta) = x^T Px + \gamma^{-1}\|\theta - \theta_*\|^2$$

with $\theta = \mathrm{col}\{\theta_p, \theta_s\}$ and P, θ_* satisfying (2). Below we show that

$$V(t) < V_i, \quad t \in [iT, iT + T), \quad i \in \mathbb{N}_0, \tag{20}$$

which implies (17) and the boundedness of $\|\theta(t)\|$. The control law (6) can be written in the form

$$\begin{aligned} u &= -\theta^T q_i(y), \\ \dot\theta &= \gamma q_i(y)q_i^T(y)g - \sigma_i\theta, \end{aligned} \qquad t \in [iT, iT + T), \tag{21}$$

where $q_i(y) = \mathrm{col}\{q_i(y_p), y_s\}$. Note that $q_i(y)$ formally is not a quantizer, since its codomain is not a finite set, but it satisfies the relation (3) with the same range and maximum error as $q_i(y_p)$.

Denote $e_i = q_i(y) - y$. Using $PB \overset{(2)}{=} C^T g$, for $t \in [iT, iT + T)$ we obtain

$$\begin{aligned} \dot V &= 2x^T P[Ax - B\theta^T q_i(y)] + 2x^T Pw \\ &\quad + 2(\theta - \theta_*)^T q_i(y)q_i^T(y)g - 2\sigma_i\gamma^{-1}(\theta - \theta_*)^T\theta \\ &= 2x^T P[Ax - B\theta_*^T Cx] + 2q_i^T(y)g(\theta_* - \theta)^T q_i(y) \\ &\quad - 2e_i^T g(\theta_* - \theta)^T q_i(y) - 2y^T g\theta_*^T e_i + 2x^T Pw \\ &\quad + 2(\theta - \theta_*)^T q_i(y)q_i^T(y)g - 2\sigma_i\gamma^{-1}(\theta - \theta_*)^T\theta. \end{aligned}$$

Let us assume that

$$|e_i(t)| \le \mu_i\Delta_e, \quad t \in [iT, iT + T), \; i \in \mathbb{N}_0. \tag{22}$$

Then, using Young's inequality, we obtain

$$\begin{aligned} &-2e_i^T(y)g(\theta_* - \theta)^T q_i(y) \le \\ &\qquad (\rho + \|\theta_*\|^{-1})\mu_i^2\Delta_e^2\|\theta_* - \theta\|^2 + \rho^{-1}\|y\|^2 + \|\theta_*\|\mu_i^2\Delta_e^2, \\ &-2y^T g\theta_*^T e_i \le \rho^{-1}x^T C^T gg^T Cx + \rho\mu_i^2\Delta_e^2\|\theta_*\|^2, \\ &2x^T Pw \le \nu x^T Px + \nu^{-1}\lambda_{\max}(P)\Delta_w^2, \\ &-2\sigma_i\gamma^{-1}(\theta - \theta_*)^T\theta \le -\sigma_i\gamma^{-1}\|\theta - \theta_*\|^2 + \sigma_i\gamma^{-1}\|\theta_*\|^2. \end{aligned}$$

Therefore, for $t \in [iT, iT + T)$,

$$\begin{aligned} &\dot V + \alpha V - \beta_i \le -(\varepsilon - \nu - 2\rho^{-1}\lambda_{\min}^{-1}(P)\|C\|^2 - \alpha)x^T Px \\ &\quad - (\sigma_i - \gamma\rho\mu_i^2\Delta_e^2 - \gamma\|\theta_*\|^{-1}\mu_i^2\Delta_e^2 - \alpha)\gamma^{-1}\|\theta_* - \theta\|^2 \\ &\quad + \nu^{-1}\lambda_{\max}(P)\Delta_w^2 + \sigma_i\gamma^{-1}\|\theta_*\|^2 + \rho\mu_i^2\Delta_e^2\|\theta_*\|^2 \\ &\quad + \|\theta_*\|\mu_i^2\Delta_e^2 - \beta_i. \end{aligned}$$

Taking $\beta_i = \nu^{-1}\lambda_{\max}(P)\Delta_w^2 + \sigma_i\gamma^{-1}\|\theta_*\|^2 + (\rho\|\theta_*\|^2 + \|\theta_*\|)\mu_i^2\Delta_e^2$ and substituting α from (9) and σ_i from (15), we find that $\dot V \le -\alpha V + \beta_i$. Thus,

$$V(t) \le \left(V(iT) - \frac{\beta_i}{\alpha}\right)e^{-\alpha(t - iT)} + \frac{\beta_i}{\alpha}, \qquad t \in [iT, iT + T). \tag{23}$$

Note that

$$\frac{\beta_i}{\alpha} = c_\gamma + c_w\Delta_w^2 + c_e\mu_i^2\Delta_e^2.$$

Since $V(0) \overset{(12)}{<} V_0$ and $\beta_0/\alpha \overset{(19)}{<} V_0$, (23) implies (20) with $i = 0$. If (20) holds for some i, then $V(iT) < V_i$ and, by continuity, (23) yields

$$V(iT + T) \le \left(V(iT) - \frac{\beta_i}{\alpha}\right)e^{-\alpha T} + \frac{\beta_i}{\alpha} < \left(V_i - \frac{\beta_i}{\alpha}\right)e^{-\alpha T} + \frac{\beta_i}{\alpha} = V_{i+1}.$$

Then, using (23) on $[(i+1)T, (i+1)T + T]$ with $\frac{\beta_{i+1}}{\alpha} \overset{(18)}{<} V_{i+1}$, we obtain (20) for $i+1$. By induction, (20) holds for all $i \in \mathbb{N}_0$.

Now we show that (22) holds for all $i \in \mathbb{N}_0$. Since $\|y(0)\| \overset{(12)}{<} \mu_0 M$ and $y(t)$ is continuous, if $\|y(t)\| \ge \mu_0 M$ for some t, then there exists the smallest t_* such that $\|y(t_*)\| = \mu_0 M$. Consequently, $\|e_0(t)\| \overset{(3)}{\le} \mu_0\Delta_e$ holds for $t \in [0, t_*]$ implying $V(t) \overset{(20)}{<} V_0$ for $t \in [0, t_*]$. The latter

i	$V_i \times 10^{-2}$	μ_i	σ_i
0	1171.53	1	250.25
1	243.1	0.46	52.13
2	50.5	0.2	11.03
3	10.6	0.09	2.5
4	2.26	0.04	0.73
5	0.55	0.02	0.37

Fig. 2. Switching parameters: V_i — upper bounds for V on $[iT, iT+T)$, μ_i — zooming parameter, σ_i — regularizing parameter.

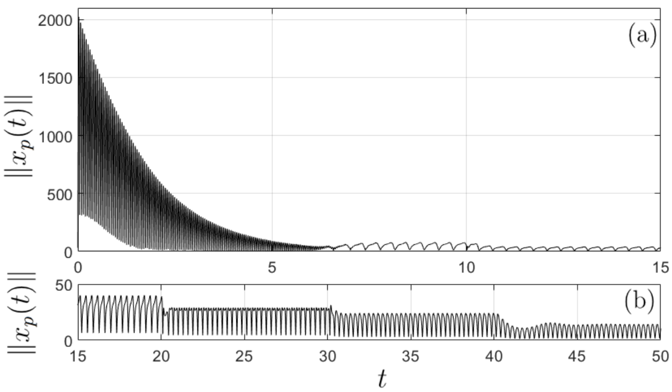

Fig. 3. Plant norm $\|x_p(t)\|$: (a) for $t \in [0, 15]$, (b) for $t \in [15, 50]$

yields $\|y(t_*)\| < \mu_0 M$, what contradicts the definition of t_*. Thus, $\|y(t)\| < \mu_0 M$ for $t \geq 0$. Then $V(T) \overset{(23)}{<} V_1 \overset{(14)}{=} \mu_1^2 V_0/\mu_0^2$ and, therefore, $\|y(T)\| < \mu_1 M$. Using the arguments similar to the above, we obtain (22) for $i = 1$. By induction, (22) holds for $i \in \mathbb{N}_0$. □

Remark 3. (Polytopic-type uncertainties). Our results are applicable to the system (1) with uncertain A_p that resides in a polytope. In this case, the matrix of the augmented system belongs to some polytope

$$A_\xi \in \left\{ \sum_{i=1}^{N} \xi_i A_i \,\middle|\, 0 \leq \xi_i, \sum_{i=1}^{N} \xi_i = 1 \right\}. \tag{24}$$

If $g^T W_\xi(s) = g^T C(sI - A_\xi)^{-1} B$ is HMP for all ξ from (24), then (2) are feasible for each ξ with some θ_ξ and P_ξ. To apply Theorem 1, one should take

$$\varepsilon = \min_{\xi \in \Xi} \varepsilon_\xi, \quad \theta_* = \operatorname*{argmax}_{\theta_\xi, \xi \in \Xi} \|\theta_\xi\| \tag{25}$$

and instead of quantities $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ substitute $\min_{\xi \in \Xi} \lambda_{\min}(P_\xi)$ and $\max_{\xi \in \Xi} \lambda_{\max}(P_\xi)$, respectively. The existence of these quantities follows from Lemma 1, compactness of the set of ξ, and continuity of A_ξ in ξ.

Relations (2) are feasible for $\theta_\xi = k_* g$ with a large enough k_* (Andrievskii and Fradkov (2006)). Since (2) are affine in A, to obtain the values from (25), one can solve linear matrix inequalities

$$P > 0,\ P(A_i - Bk_* g^T C) + (A_i - Bk_* g^T C)^T P < -\varepsilon P,$$
$$PB = C^T g, \quad i = 1, \dots, N$$

with a decision variable P and tuning parameters ε, k_*.

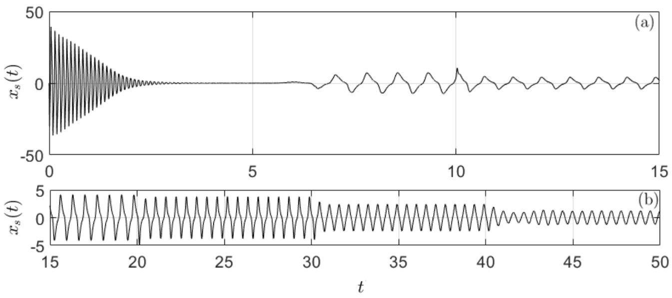

Fig. 4. Shunt state $x_s(t)$: (a) for $t \in [0, 15]$, (b) for $t \in [15, 50]$

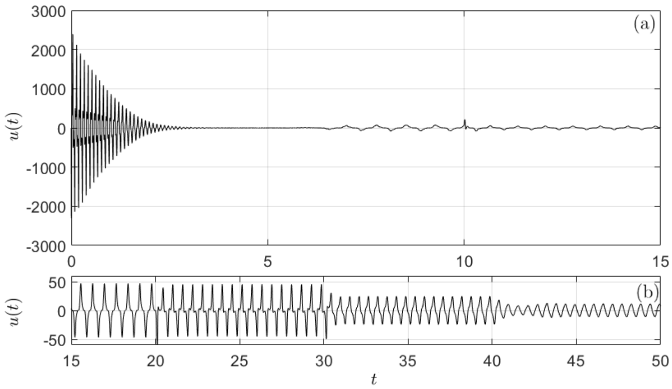

Fig. 5. Control input: (a) for $t \in [0, 15]$, (b) for $t \in [15, 50]$

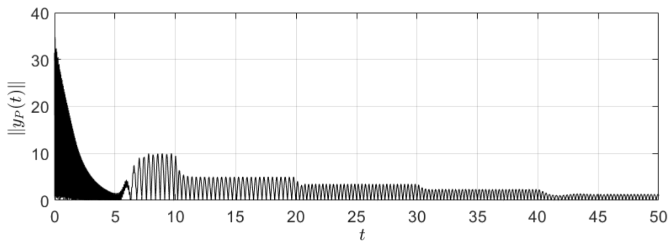

Fig. 6. The value of $\|y_p(t)\|$ for $t \in [0, 50]$

4. EXAMPLE: FLIGHT CONTROL

The lateral motion of an aircraft considered as a rigid body can be described by (Fradkov and Andrievsky (2011))

$$\begin{aligned} \dot{\beta}(t) &= r(t) + a_1 \beta(t) + b_1 \delta(t), \\ \dot{r}(t) &= a_2 \beta(t) + a_3 r(t) + b_2 \delta(t), \\ \dot{\psi}(t) &= r(t), \end{aligned} \tag{26}$$

where $\psi(t)$ and $r(t)$ are the yaw angle and the yaw rate, $\beta(t)$ is the sideslip angle, $\delta(t)$ is the rudder angle (the control signal), a_i and b_i are the aircraft model parameters that depend on the flight conditions. Following Fradkov and Andrievsky (2011), we take $a_2 = 33$, $a_3 = -1.3$, $b_1 = 19/15$, $b_2 = 19$ and assume that $a_1 \in [-1.5, -0.7]$ is an uncertain parameter.

The first mode of the aircraft bending is modeled as

$$W_{bend}(s) = \frac{\Delta\psi(s)}{\delta(s)} = \frac{k_{bend}}{T_{bend}^2 s^2 + 2\xi_{bend} T_{bend} s + 1}, \tag{27}$$

where $k_{bend} = -1.5 \times 10^{-3}$ is the bending mode transition factor; $T_{bend} = \omega_{bend}^{-1}$ is the response time factor with $\omega_{bend} = 65\ \text{s}^{-1}$ being the first bending mode natural frequency; and $\xi_{bend} = 0.01$ is the damping ratio. The measured signal is given by

$$y(t) = \psi(t) + \Delta\psi(t). \tag{28}$$

The system (26)–(28) can be presented as (1) with

$$\left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & 0 \end{array}\right] = \left[\begin{array}{ccccc|c} a_1 & 1 & 0 & 0 & 0 & b_1 \\ a_2 & a_3 & 0 & 0 & 0 & b_2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-2\xi_{bend}}{T_{bend}} & 1 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_{bend}^2} & 0 & \frac{k_{bend}}{T_{bend}^2} \\ \hline 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right].$$

For $g_p = 1/\sqrt{2}$, the transfer function $g_p^T W_p(s) = g_p^T C_p(sI - A_p)^{-1} B_p$ is minimum-phase with relative degree $r = 2$. As a shunt transfer function, we take

$$W_s(s) = \frac{2}{s + 14}.$$

Then, for $g = \mathrm{col}\{1/\sqrt{2}, 1/\sqrt{2}\}$ (with $\|g\| = 1$), the function $g^T W(s)$ is HMP, where $W(s)$ is the transfer function of the augmented system (8). In a manner described in Remark 3, we find that (2) are feasible for $\varepsilon = 0.5$, $\theta_* = 20g$. Using Theorem 1 with

$$M = 10^4,\ \Delta_e = \Delta_w = 10^{-2},\ \mu_0 = 1,\ \gamma = 50,\ T = 10,$$

we obtain the switching parameters presented in Fig. 2. The decrease of μ_i corresponds to "zooming in" and obtaining more precise measurements. The set of initial conditions is given by $x^T(0)Px(0) \leq 1.17 \times 10^5$, while the limit set is $x^T(t)Px(t) \leq V_\infty = 34.68$.

The results of numerical simulations for $a_1 = -0.75$ are presented in Figs. 3–5. Note that at switching instants $T, 2T, \ldots$ the dynamics of the state significantly change. This happens due to the switching of the zooming parameter μ_i and regularizing parameter σ_i.

Though the system is stable, it possesses high-frequency oscillations. Such parasitic oscillations, reflecting unmodelled dynamics, are common for system with small parameters (see, e.g., Ioannou and Kokotovic (1983)). In our case, a shunt system (5) contains a small parameter λ in its transfer function W_s. It remains an open problem how to choose W_s to mitigate these oscillations.

5. CONCLUSIONS

We designed an adaptive controller for a linear minimum-phase systems of an arbitrary relative degree with bounded disturbances and quantized measurements. Our approach is based on the shunting method (parallel feedforward compensator) that extends the system to a hyper-minimum-phase one. The augmented system is further stabilized by a passification-based adaptive controller with dynamically quantized measurements. By constructing a switching procedure for the adaptive controller parameters and quantizer's zooming, we ensure convergence of the system state from an arbitrary set to an ellipsoid whose size depends on the disturbance bound.

Future work may be devoted to the optimization of the controller parameters.

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