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Optimal Energy Management in Combined Heat and Power System via A Decentralized Consensus-Based ADMM ⋆

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Abstract: This paper proposes a comprehensive optimization model that considers not only the economic dispatch (ED) of the combined heat and power (CHP) units, but also the demand response (DR) of consuming units in the energy management system, where each individual unit can exchange the information with its neighbours. In this optimization problem, there are energy balance constraints and individual local constraints. Particularly, the progresses of the power dispatch and the heat dispatch of each CHP unit are coupled through a feasible polygon region constraint, and the power demands of each consumer among different periods are also coupled due to the requirement of the total power consumption. To achieve the optimal energy coordination of the underlying system, we propose a decentralized alternating direction method of multipliers (ADMM), under connected communication network of individuals, such that each CHP unit and consumer can simultaneously implement their own optimal strategies based on an agreed energy price derived by a consensus protocol. The convergence and optimality of the proposed method are guaranteed under certain conditions. Simulation results are shown to demonstrate the developed results.

Keywords: Combined heat and power (CHP), demand response (DR), economic dispatch (ED), decentralized optimization, ADMM, consensus protocol.

1. INTRODUCTION

The energy network system has been largely impacted due to the increasing of energy demands, such as electric vehicles (EVs), air conditioners and heating pipes. Therefore, extensive research has been dedicated to how to mitigate these negative impacts by methods like the demand side management (DSM) Esther and Kumar (2016). Due to the privacy and autonomy of the individuals, as well as the heavy communication signals and high computational complexity of centralized methods, it is more practical to solve the DSM problems by the decentralized methods.

In Ma et al. (2016), the authors adopted the real-time price model, which has been widely applied for demand response (DR) as one of the main DSM activities, to shift the power consumption of deferrable loads in a decentralized way. DR can shift the loads in peak times and fill the valley, which not only reduces the users' energy bill but also increases the energy efficiency and quality.

DR problems mainly happen at the demand side, while at the supply side, economic dispatch (ED) approaches are implemented to obtain the optimal allocation of the generation units to satisfy the user demand which is usually assumed to be constant Nemati et al. (2018).

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Consequently, we are motivated to develop an energy service framework which integrates the ED and DR problems to maximize the total profit in the energy network including both generation and demand units. The combined heat and power (CHP) system as one of the co-generation units can produce electricity and thermal energy simultaneously with high energy efficiency, which has attracted the worldwide interest from academia and industry. In a CHP system, usually the power capacity limits are related to the heat productions of CHPs and vice versa, which brings difficulties to handle the power dispatch and the heat dispatch of CHPs Guo and Henwood (1996).

The proposed profit maximization problem for power and heat coordination is an optimization problem subject to the energy balance constraint and individual constraints. Furthermore, certain coupling constraints are introduced among different periods of the power demand of each individual consumer. The progresses of the heat dispatch and the power dispatch are also coupled by the heat-power feasible polygon region constraints of CHPs. We will implement the CHP economic dispatch and the consumer demand response with these constraints in an integrated way.

We further suppose in this paper that each individual is connected with each other via a communication network, and they can exchange information with their neighbors. In Nguyen et al. (2018), a decentralized approach based on consensus theory was derived to obtain the optimal energy updates, but it did not consider the coupling constraints in the energy system. While in Yi et al. (2020), a distributed neurodynamic-based approach considering coupled inequality constraints was proposed in a multi-energy system with the information exchanged among neighboring units, but the demand response of consumers was not considered.

For achieving the optimal solution to the underlying constrained problem, we introduce a decentralized Alternating Direction Method of Multipliers (ADMM) Boyd et al. (2010). In the proposed energy system, the communications graph topology is assumed to be connected. The decentralized coordination without any central entities is often formulated as a consensus problem. Hence, in the proposed ADMM method, we further adopt a typical consensus protocol to derive a pair of agreed parameters which are later proved to be the power and heat energy prices. It is verified that under certain conditions, the energy system can converge to the optimal solution by applying the proposed decentralized method.

The contributions of this paper are summarized as follows:

- We propose a novel multi-energy coordination problem which considers not only the economic dispatch of CHPs, but also the demand response of consumers. The satisfaction model of each individual and the related system profit maximization model are introduced with certain coupling constraints.
- A fully decentralized method is developed to effectively handle the constrained optimization problem with the optimal solution guaranteed.
- The proposed method based on the consensus protocol only utilizes the local communications among the neighboring individuals to get the agreed energy price.

The rest of the paper is organized as follows. Section 2 introduces the energy system model including both the CHPs and consumers, and formulates the energy coordination problems. In Section 3, a decentralized ADMM method based on the consensus protocol is developed to obtain the optimal solution to the underlying optimization problem. The convergence and optimality are guaranteed under certain conditions. Numerical illustrations are shown in Section 4. We conclude the paper in Section 5.

2. FORMULATION OF ENERGY COORDINATION

2.1 Energy System Model

We study the coordination problem in a specific energy system composed of CHP populations $\mathcal{N} \equiv \{1, ..., N\}$, and multiple consumers $\mathcal{M} \equiv \{1, ..., M\}$ over a multi-time horizon $\mathcal{T} \equiv \{1, ..., T\}$. For demonstration, we assume that there's no other power generators and heat boilers. The CHPs generate electricity and heat at the same time to satisfy the demands of consumers.

The generated power and heat of CHP $n \in \mathcal{N}$ are denoted by p_{nt} and h_{nt} (with units of MW) respectively, which are assumed to be constant at each time slot $t \in \mathcal{T}$. We consider the extraction-condensing CHP units whose feasible region can be described as a polygon region. The region of CHP n could be formulated as Wu et al. (2018):

$$\begin{cases} h_{n,min} \le h_{nt} \le h_{n,max} \\ p_{nt} \le w_n^1 - w_n^2 \cdot h_{nt} \\ p_{nt} \ge \max\{w_n^3 - w_n^4 \cdot h_{nt},\ w_n^5 \cdot h_{nt} + w_n^6\}, \end{cases} \tag{1}$$

where $h_{n,min}, h_{n,max}$ are the lower and upper bounds on the generated heat, and parameters $w_n^1, w_n^2, w_n^3, w_n^4, w_n^5, w_n^6$ reflect the ratios of generated power to heat under different operation mode of CHP n.

We say the operation strategy $\boldsymbol{u}_n \triangleq (u_{nt}; t \in \mathcal{T})$ with $u_{nt} = [p_{nt}, h_{nt}]^\top$ is admissible, if it satisfies the above inequality constrains for any time $t \in \mathcal{T}$. The set of admissible operation strategies of CHP n is denoted by $\mathcal{U}_n$, and the set of admissible strategies for CHP populations is denoted by $\mathcal{U}$ such that

$$\mathcal{U} \triangleq \{\boldsymbol{u} \equiv (\boldsymbol{u}_n; n \in \mathcal{N});\ \text{s.t. } \boldsymbol{u}_n \in \mathcal{U}_n\}. \tag{2}$$

Let $\bar{p}_{mt}$ and $\bar{h}_{mt}$ denote the power and heat demand of consumer $m \in \mathcal{M}$ at time t, and define the control strategy as $\bar{\boldsymbol{u}}_m \triangleq (\bar{u}_{mt}; t \in \mathcal{T})$ with $\bar{u}_{mt} = [\bar{p}_{mt}, \bar{h}_{mt}]^\top$. The strategy is admissible, if

$$\bar{p}_{mt} \begin{cases} \in [0, \gamma_{mt}], & \text{in case } t \in \mathcal{T}_m \\ = 0, & \text{otherwise} \end{cases}, \quad \text{and} \quad \sum_{t=1}^{T} \bar{p}_{mt} = \Gamma_m, \tag{3a}$$

$$\bar{h}_{mt} \begin{cases} \in [0, \varpi_{mt}], & \text{in case } t \in \mathcal{T}_m \\ = 0, & \text{otherwise} \end{cases}, \tag{3b}$$

where $\mathcal{T}_m$ with $\mathcal{T}_m \subset \mathcal{T}$, γ_{mt} and ϖ_{mt} respectively represent the consuming interval, upper bounds on the rate of consumed power and heat at t, and Γ_m denotes the total power demand over $\mathcal{T}$. The set of admissible control strategies for consumer m is denoted by $\bar{\mathcal{U}}_m$, and the set of admissible strategies for the consumer populations is denoted by $\bar{\mathcal{U}}$ such that

$$\bar{\mathcal{U}} \triangleq \{\bar{\boldsymbol{u}} \equiv (\bar{\boldsymbol{u}}_m; m \in \mathcal{M});\ \text{s.t. } \bar{\boldsymbol{u}}_m \in \bar{\mathcal{U}}_m\}. \tag{4}$$

For guaranteeing the power and heat balance in the energy system, the strategies of all individuals are required to satisfy the following equalities:

$$\sum_{n=1}^{N} (1 - \eta_n) p_{nt} = \sum_{m=1}^{M} (1 + \bar{\eta}_m) \bar{p}_{mt}, \tag{5a}$$

$$\sum_{n=1}^{N} (1 - \varrho_n) h_{nt} = \sum_{m=1}^{M} (1 + \bar{\varrho}_m) \bar{h}_{mt} \tag{5b}$$

for all $t \in \mathcal{T}, n \in \mathcal{N}, m \in \mathcal{M}$, where $\eta_n, \bar{\eta}_m, \varrho_n, \bar{\varrho}_m \in (0, 1)$ denote the power and heat loss coefficients. The generated losses are due to the energy transmitted through the transmission lines or pipes and low quality equipments.

2.2 Topology Description of Individual Interaction

We assume that each individual unit can exchange the local information with its neighbours to satisfy the supply-demand balance constraint in (5). The communication among individuals could be described by an undirected graph which is commonly used in the multi-agent system.The undirected graph of the system can be denoted by $\mathcal{G} \triangleq \langle \mathcal{O}, \mathcal{E} \rangle$ where $\mathcal{O} \equiv \{1, \cdots, N, N+1, \cdots, N+M\}$ and $\mathcal{E}$ denote the vertex set and edge set, respectively.

Let $e = (i, j)$ denote each edge of graph $\mathcal{G}$ with $i, j \in \mathcal{O}$, which means that individual i can interact with individual

j via the local information transmission. The neighboring set of individual i and its cardinality are denoted by $\mathcal{V}_i \triangleq \{j \in \mathcal{O}, (i,j) \in \mathcal{E}\}$ and $D_i \triangleq ||\mathcal{V}_i||$, respectively.

Denote by y_{ij} the element of the adjacency matrix $\mathcal{Y}$ of $\mathcal{G}$, such that $y_{ij} = 1$ in case $(i,j) \in \mathcal{E}$ and $y_{ij} = 0$ otherwise. Let $\mathcal{D} \triangleq \text{diag}\{D_i\}_{i=1,\cdots,N+M}$. Thus the Laplacian matrix of graph $\mathcal{G}$ is defined by $\mathcal{L} \triangleq \mathcal{D} - \mathcal{Y}$.

2.3 Problem Formulation

In this paper, we aim to minimize the total system cost consisting the operation cost of CHPs and the consumer's local cost during the whole coordination periods.

More specifically, the operation cost of CHP $n \in \mathcal{N}$ at t can be represented as below Fang et al. (2018):

$$\begin{aligned} C_n(u_{nt}) = c_{0,n} + c_{1,n}p_{nt} + c_{2,n}p_{nt}^2 \\ + c_{3,n}h_{nt} + c_{4,n}h_{nt}^2 + c_{5,n}p_{nt}h_{nt}, \end{aligned} \tag{6}$$

where $c_{0,n}, c_{1,n}, c_{2,n}, c_{3,n}, c_{4,n}, c_{5,n}$ are cost coefficients of CHP n. The last term $c_{5,n}p_{nt}h_{nt}$ reflects the coupling relationship of the heat and the electricity.

Consequently, the profit function of CHP n is

$$W_n(u_{nt}) \triangleq \pi_{pt}p_{nt}(1-\eta_n) + \pi_{ht}h_{nt}(1-\varrho_n) - C_n(u_{nt}), \tag{7}$$

where π_{pt} and π_{ht} are the prices of the electricity and heat at time t.

Meanwhile, the profit function of consumer m at t can be given:

$$\begin{aligned} \bar{W}_m(\bar{u}_{mt}) \triangleq & F_m(\bar{p}_{mt}) - \pi_{pt}\bar{p}_{mt}(1+\bar{\eta}_m) \\ & + H_m(\bar{h}_{mt}) - \pi_{ht}\bar{h}_{mt}(1+\bar{\varrho}_m), \end{aligned} \tag{8}$$

where $F_m(\bar{p}_{mt})$ and $H_m(\bar{h}_{mt})$ denote the satisfaction functions of consumer m at t with respect to its consumed electricity and heat. We will use the following form to represent $F_m(\bar{p}_{mt})$ which is a continuous differentiable concave function and has been widely adopted to model the behavior of power users Nguyen et al. (2018):

$$F_m(\bar{p}_{mt}) = \begin{cases} \beta_m\bar{p}_{mt} - \alpha_m\bar{p}_{mt}^2, & \text{if } \bar{p}_{mt} \le \dfrac{\beta_m}{2\alpha_m} \\ \dfrac{\beta_m^2}{4\alpha_m}, & \text{if } \bar{p}_{mt} > \dfrac{\beta_m}{2\alpha_m} \end{cases} \tag{9}$$

with prescribed parameters α_m and β_m which reflect the individual conditions.

The function $H_m(\bar{h}_{mt})$ is adopted with respect to the difference between the actual demand and the nominal demand (reference demand) $\bar{\boldsymbol{h}}_m^r \triangleq (\bar{h}_{mt}^r; t \in \mathcal{T})$ as below Wang et al. (2015):

$$H_m(\bar{h}_{mt}) = -\bar{\alpha}_{mt}(\bar{h}_{mt} - \bar{h}_{mt}^r)^2, \tag{10}$$

where $\bar{\alpha}_{mt} > 0$ denotes the deviation penalty parameter which reflects the relative importance or satisfaction of reaching the expected heat demand at different periods. $\bar{\boldsymbol{h}}_m^r$ can be predicted and anticipated based on the historical demand data. Actually, the user heat demand could be captured from the ambient and desired indoor temperature related to the user's comfort Wang et al. (2018).

The total system profit function is given as the summation of all individual profits over the whole periods $\mathcal{T}$. The maximization problem of the system profit can be transformed into a minimization problem of the system cost by changing the sign of the profit function. Thus, the objective function of the coordination problem is expressed as follows:

$$\begin{aligned} J(\boldsymbol{u}, \bar{\boldsymbol{u}}) &= -\sum_{t\in\mathcal{T}} \Big\{ \sum_{n\in\mathcal{N}} W_n(u_{nt}) + \sum_{m\in\mathcal{M}} \bar{W}_m(\bar{u}_{mt}) \Big\} \\ &= \sum_{t\in\mathcal{T}} \Big\{ \sum_{n\in\mathcal{N}} C_n(u_{nt}) - \sum_{m\in\mathcal{M}} Q_m(\bar{u}_{mt}) \Big\}, \end{aligned} \tag{11}$$

where $Q_m(\bar{u}_{mt}) = F_m(\bar{p}_{mt}) + H_m(\bar{h}_{mt})$ and the last equality holds by (5).

By considering the constraints proposed in Section 2.1, the centralized coordination problem is formulated as the following optimization problem:

Problem 1.

$$\begin{aligned} \min_{\boldsymbol{u}\in\mathcal{U}, \bar{\boldsymbol{u}}\in\bar{\mathcal{U}}} \; & \sum_{t\in\mathcal{T}} \Big\{ \sum_{n\in\mathcal{N}} C_n(u_{nt}) - \sum_{m\in\mathcal{M}} Q_m(\bar{u}_{mt}) \Big\} \\ \text{s.t. } & (5), \; \forall t \in \mathcal{T}, \end{aligned} \tag{12}$$

where $\mathcal{U}$ and $\bar{\mathcal{U}}$ are defined above. ■

The centralized methods in the literature can be applied to solve Problem 1. However, it may create privacy issues, heavy communication and computation burdens. Therefore, we will develop a decentralized consensus-based ADMM for the underlying constrained optimization problem.

3. DECENTRALIZED IMPLEMENTATION

3.1 ADMM Method for Coordination Problem

In this section, Problem 1 will be dealt with by applying the ADMM method which is commonly adopted in network systems for the decentralized operation Boyd et al. (2010). For simplicity, we define a matrix $U \equiv \big[\{U_n\}_{n=1,\cdots,N}; \{\bar{U}_m\}_{m=1,\cdots,M}\big] = [\boldsymbol{u}; \bar{\boldsymbol{u}}]$, i.e., the matrix U is the combination of the vectors $\boldsymbol{u}$ and $\bar{\boldsymbol{u}}$.

To solve the underlying problem, we first introduce an auxiliary variable $Z = \big[\{Z_n\}_{n=1,\cdots,N}; \{\bar{Z}_m\}_{m=1,\cdots,M}\big] \equiv U$ with $Z_{nt} = [Z_{p,nt}, Z_{h,nt}]^\top$ and $\bar{Z}_{mt} = [\bar{Z}_{p,mt}, \bar{Z}_{h,mt}]^\top$, and define the closed convex sets: $\Pi_1 = \big\{U \in \mathbb{R}^{(2N+2M)T} : \sum_{n=1}^N (1-\eta_n)p_{nt} = \sum_{m=1}^M (1+\bar{\eta}_m)\bar{p}_{mt}, \sum_{n=1}^N (1-\varrho_n)h_{nt} = \sum_{m=1}^M (1+\bar{\varrho}_m)\bar{h}_{mt}, \; \forall t \in \mathcal{T}\big\}$ and $\Pi_2 = \big\{Z \in \mathbb{R}^{(2N+2M)T} : Z_n \in \mathcal{U}_n, n = 1, \cdots, N, \; \bar{Z}_m \in \bar{\mathcal{U}}_m, m = 1, \cdots, M\big\}$. The indicator functions of Π_1 and Π_2 are given as follows:

$$\mathcal{I}_1(U) = \begin{cases} 0, & \text{if } U \in \Pi_1 \\ +\infty, & \text{otherwise} \end{cases}, \; \mathcal{I}_2(Z) = \begin{cases} 0, & \text{if } Z \in \Pi_2 \\ +\infty, & \text{otherwise.} \end{cases}$$

Consequently, Problem 1 can be reformulated as below:

$$\begin{aligned} \min \; & \sum_{t\in\mathcal{T}} \Big\{ \sum_{n\in\mathcal{N}} C_n(u_{nt}) - \sum_{m\in\mathcal{M}} Q_m(\bar{u}_{mt}) \Big\} \\ & \qquad + \mathcal{I}_1(U) + \mathcal{I}_2(Z) \\ \text{s.t. } & U = Z. \end{aligned} \tag{13}$$

Remark: Since the coupling constraints illustrated in (3) are introduced, the above optimization problem can not be decomposed into sub-problems with respect to the individual time slots, while in Nguyen et al. (2018) the optimal solution is implemented sequentially and forwardly over the whole periods without considering these coupling constraints.

The indicator functions $\mathcal{I}_1(U)$ and $\mathcal{I}_2(Z)$ are closed, proper and convex since the sets of constraints in the optimization problem (13) are all closed nonempty convex sets. Consequently, the objective function in (13) is proper, closed, and convex, which establishes the sufficient conditions for the optimal solution by applying ADMM method Boyd et al. (2010).

The augmented Lagrangian of (13) is:

$$L_\rho(U,Z,V) = \sum_{t\in\mathcal{T}}\Big\{\sum_{n\in\mathcal{N}} C_n(u_{nt}) - \sum_{m\in\mathcal{M}} Q_m(\bar{u}_{mt})\Big\} + \mathcal{I}_1(U) + \mathcal{I}_2(Z) + \frac{\rho}{2}\|U - Z + V\|_2^2,$$

where $\rho > 0$ is a penalty factor and V is the scaled dual variable. The ADMM iterations are as follows with additional proximal terms:

$$\begin{aligned} U^{k+1} &:= \arg\min_U L_\rho(U, Z^k, V^k) + \frac{1}{2}(U-U^k)^\top \Phi (U - U^k) \\ Z^{k+1} &:= \arg\min_Z L_\rho(U^k, Z, V^k) + \frac{1}{2}(Z-Z^k)^\top \Psi (Z - Z^k) \\ V^{k+1} &:= V^k - \kappa\rho(U^k - Z^k), \end{aligned} \tag{14}$$

where $\Phi, \Psi, \kappa > 0$ satisfy:

$$\Phi > \rho(\frac{1}{\mu_1} - 1)I,\ \Psi > \rho(\frac{1}{\mu_2} - 1)I,\ \mu_1 + \mu_2 < 2 - \kappa$$

for $\mu_1 > 0$, $\mu_2 > 0$.

The variables U, Z, V are updated in parallel with respect to the values of previous iteration. The ADMM method in (14) is called Jacobian ADMM approach in the literature, whose convergence is verified in Deng et al. (2017).

For simplicity, let $\Phi = \phi I$ and $\Psi = \psi I$, such that

$$\phi > \rho(\frac{1}{\mu_1} - 1),\ \psi > \rho(\frac{1}{\mu_2} - 1).$$

Actually, Problem 1 can be directly solved in a decentralized way by the proposed Jacobian ADMM approach, in which each individual needs to share the local information with all other individuals. However, in our energy system, each individual can only interact with its neighbours.

3.2 The Consensus Update of U^{k+1}

This section illustrates an update method to implement U^{k+1} in a decentralized way based on the consensus protocol.

It is easily known from the definition of the indication function $\mathcal{I}_1(U)$ that U^{k+1} in (14) can be obtained by solving the following optimization problem:

$$\begin{aligned} &\min_U\ F(U) \\ &s.t.\ \sum_{n\in\mathcal{N}}(1-\eta_n)p_{nt} = \sum_{m\in\mathcal{M}}(1+\bar{\eta}_m)\bar{p}_{mt},\ \forall t\in\mathcal{T} \\ &\quad\ \ \sum_{n\in\mathcal{N}}(1-\varrho_n)h_{nt} = \sum_{m\in\mathcal{M}}(1+\bar{\varrho}_m)\bar{h}_{mt},\ \forall t\in\mathcal{T} \end{aligned} \tag{15}$$

with $F(U) = \sum_{t\in\mathcal{T}}\Big\{\sum_{n\in\mathcal{N}} C_n(u_{nt}) - \sum_{m\in\mathcal{M}} Q_m(\bar{u}_{mt})\Big\} + \frac{\rho}{2}\|U - Z^k + V^k\|_2^2 + \frac{\phi}{2}\|U - U^k\|_2^2$.

It gives that the strong duality holds since (15) is a convex optimization problem. Hence, we can get the optimal solution U^{k+1} of (15) by applying the Karush-Kuhn-Tucker conditions Bertsekas (1999), such that:

$$\frac{\partial F(U)}{\partial u_{nt}}\Big|_{u_{nt}=u_{nt}^{k+1}} = \Big\{\hat{\lambda}_t^{k+1}\frac{\partial E_p}{\partial u_{nt}} + \hat{\gamma}_t^{k+1}\frac{\partial E_h}{\partial u_{nt}}\Big\}\Big|_{u_{nt}=u_{nt}^{k+1}} \tag{16a}$$

$$\frac{\partial F(U)}{\partial \bar{u}_{mt}}\Big|_{\bar{u}_{mt}=\bar{u}_{mt}^{k+1}} = \Big\{\hat{\lambda}_t^{k+1}\frac{\partial E_p}{\partial \bar{u}_{mt}} + \hat{\gamma}_t^{k+1}\frac{\partial E_h}{\partial \bar{u}_{mt}}\Big\}\Big|_{\bar{u}_{mt}=\bar{u}_{mt}^{k+1}} \tag{16b}$$

where $E_p = \sum_{n\in\mathcal{N}}(1-\eta_n)p_{nt} - \sum_{m\in\mathcal{M}}(1+\bar{\eta}_m)\bar{p}_{mt}$, $E_h = \sum_{n\in\mathcal{N}}(1-\varrho_n)h_{nt} - \sum_{m\in\mathcal{M}}(1+\bar{\varrho}_m)\bar{h}_{mt}$, $\hat{\lambda}_t^{k+1}$ and $\hat{\gamma}_t^{k+1}$ are the optimal Lagrange multipliers at iteration $k+1$, for all $t\in\mathcal{T}$, $n\in\mathcal{N}$ and $m\in\mathcal{M}$.

Consequently, based on (16a) and (6) we have the following update of p_{nt} and h_{nt}, $\forall n\in\mathcal{N}$

$$\begin{aligned} p_{nt}^{k+1} = \big\{&\hat{\lambda}_t^{k+1}(1-\eta_n) + \phi p_{nt}^k + \rho(Z_{p,nt}^k - V_{p,nt}^k) \\ &- c_{1,n} - c_{5,n}h_{nt}^{k+1}\big\}/\big\{2c_{2,n} + \rho + \phi\big\} \end{aligned} \tag{17a}$$

$$\begin{aligned} h_{nt}^{k+1} = \big\{&\hat{\gamma}_t^{k+1}(1-\varrho_n) + \phi h_{nt}^k + \rho(Z_{h,nt}^k - V_{h,nt}^k) \\ &- c_{3,n} - c_{5,n}p_{nt}^{k+1}\big\}/\big\{2c_{4,n} + \rho + \phi\big\} \end{aligned} \tag{17b}$$

which can be rewritten in short by

$$p_{nt}^{k+1} = a_{nt}^k\hat{\lambda}_t^{k+1} + b_{nt}^k\hat{\gamma}_t^{k+1} - c_{nt}^k \tag{18a}$$

$$h_{nt}^{k+1} = \tilde{a}_{nt}^k\hat{\lambda}_t^{k+1} + \tilde{b}_{nt}^k\hat{\gamma}_t^{k+1} - \tilde{c}_{nt}^k \tag{18b}$$

where the coefficients $a_{nt}^k, b_{nt}^k, c_{nt}^k$ and $\tilde{a}_{nt}^k, \tilde{b}_{nt}^k, \tilde{c}_{nt}^k$ are derived from (17).

And for consumer $m\in\mathcal{M}$, based on (16b), (9) and (10) we have the update of $\bar{p}_{mt}$ and $\bar{h}_{mt}$ at $k+1$, $t\in\mathcal{T}$ as follows:

$$\bar{p}_{mt}^{k+1} = \begin{cases} \dfrac{-\hat{\lambda}_t^{k+1}(1+\bar{\eta}_m) + \phi\bar{p}_{mt}^k + \rho(\bar{Z}_{p,mt}^k - \bar{V}_{p,mt}^k) + \beta_m}{2\alpha_m + \rho + \phi}, \\ \qquad\qquad\qquad \text{if } \bar{p}_{mt}^k \le \dfrac{\beta_m}{2\alpha_m}; \\ \dfrac{-\hat{\lambda}_t^{k+1}(1+\bar{\eta}_m) + \phi\bar{p}_{mt}^k + \rho(\bar{Z}_{p,mt}^k - \bar{V}_{p,mt}^k)}{\rho + \phi}, \\ \qquad\qquad\qquad \text{if } \bar{p}_{mt}^k > \dfrac{\beta_m}{2\alpha_m}; \end{cases} \tag{19a}$$

$$\begin{aligned} \bar{h}_{mt}^{k+1} = \big\{&-\hat{\gamma}_t^{k+1}(1+\bar{\varrho}_m) + \rho(\bar{Z}_{h,mt}^k - \bar{V}_{h,mt}^k) \\ &+ \phi\bar{h}_{mt}^k + 2\bar{\alpha}_{mt}\bar{h}_{mt}^r\big\}/\big\{2\bar{\alpha}_{mt} + \rho + \phi\big\} \end{aligned} \tag{19b}$$

which can be rewritten in short by

$$\bar{p}_{mt}^{k+1} = -\theta_{mt}^k\hat{\lambda}_t^{k+1} + \vartheta_{mt}^k,\ \bar{h}_{mt}^{k+1} = -\tilde{\theta}_{mt}^k\hat{\gamma}_t^{k+1} + \tilde{\vartheta}_{mt}^k, \tag{20}$$

with $\theta_{mt}^k, \vartheta_{mt}^k$ and $\tilde{\theta}_{mt}^k, \tilde{\vartheta}_{mt}^k$ determined from (19).

Furthermore, the optimal solution U^{k+1} must satisfy the equality constraints in (15). Substituting (18) and (20) into these constraints leads to

$$\hat{\lambda}_t^{k+1} = \frac{\sum_{n=1}^N I_{c,nt}^k + \sum_{m=1}^M I_{\vartheta,mt}^k - \hat{\gamma}_t^{k+1}\sum_{n=1}^N I_{b,nt}^k}{\sum_{n=1}^N I_{a,nt}^k + \sum_{m=1}^M I_{\theta,mt}^k}$$

$$\hat{\gamma}_t^{k+1} = \frac{\sum_{n=1}^N I_{\tilde{c},nt}^k + \sum_{m=1}^M I_{\tilde{\vartheta},mt}^k - \hat{\lambda}_t^{k+1}\sum_{n=1}^N I_{\tilde{a},nt}^k}{\sum_{n=1}^N I_{\tilde{b},nt}^k + \sum_{m=1}^M I_{\tilde{\theta},mt}^k}$$

where $I_{c,nt}^k = (1-\eta_n)c_{nt}^k$, $I_{\vartheta,mt}^k = (1+\bar{\eta}_m)\vartheta_{mt}^k$, $I_{b,nt}^k = (1-\eta_n)b_{nt}^k$, $I_{a,nt}^k = (1-\eta_n)a_{nt}^k$, $I_{\theta,mt}^k = (1+\bar{\eta}_m)\theta_{mt}^k$, $I_{\tilde{c},nt}^k = (1-$

$\varrho_n)\tilde{c}^k_{nt}, I^k_{\tilde{\vartheta},mt} = (1+\bar{\varrho}_m)\tilde{\vartheta}^k_{mt}, I^k_{\tilde{a},nt} = (1-\varrho_n)\tilde{a}^k_{nt}, I^k_{\tilde{b},nt} = (1-\varrho_n)\tilde{b}^k_{nt}, I^k_{\tilde{\theta},mt} = (1+\bar{\varrho}_m)\tilde{\theta}^k_{mt}$.

It is known that the multipliers $\hat{\lambda}^{k+1}_t$ and $\hat{\gamma}^{k+1}_t$ are respect to the information of all individuals. However, the information of each individual can only be transmitted to its neighbours. Therefore, we introduce a decentralized manner illustrated in the following theorem to compute $\hat{\lambda}^{k+1}_t$ and $\hat{\gamma}^{k+1}_t$.

Definition 1. A graph $\mathcal{G}$ can be called connected if for every pair (i, j) of vertices, i is reachable from j.

Theorem 3.1. Assume that the graph $\mathcal{G}$ is connected, and the initial values of all individuals that need to be exchanged with their neighbours are set to be:

$$\begin{aligned} x^0_{nt} &= [I^k_{a,nt}, I^k_{b,nt}, I^k_{c,nt}, I^k_{\tilde{a},nt}, I^k_{\tilde{b},nt}, I^k_{\tilde{c},nt}]^\top, \\ \bar{x}^0_{mt} &= [I^k_{\theta,mt}, 0, I^k_{\vartheta,mt}, 0, I^k_{\tilde{\theta},mt}, I^k_{\tilde{\vartheta},mt}]^\top, \end{aligned} \tag{21}$$

for all $n \in \mathcal{N}, m \in \mathcal{M}$ and $t \in \mathcal{T}$.

Then we apply the following consensus protocol:

$$x^{\tau+1}_{it} = x^\tau_{it} - \zeta \sum_{j\in\mathcal{V}_i} y_{ij}(x^\tau_{it} - x^\tau_{jt}), \ \forall t \in \mathcal{T} \tag{22}$$

where τ represents the iteration step of the process and y_{ij} is given in Section 2.2, and $\zeta > 0$ satisfies

$$\zeta \max_{(i,j)\in\mathcal{E}} \{D_i - D_j\} < 2. \tag{23}$$

Consequently, all individuals can reach the consensus vector $x^*_t = [w^*_{1,t}, w^*_{2,t}, w^*_{3,t}, w^*_{4,t}, w^*_{5,t}, w^*_{6,t}]^\top$ as $\tau \to +\infty$, where

$$w^*_{1,t} = \frac{\sum_{n=1}^N I^k_{a,nt} + \sum_{m=1}^M I^k_{\theta,mt}}{N+M}, \ w^*_{2,t} = \frac{\sum_{n=1}^N I^k_{b,nt}}{N+M},$$

$$w^*_{3,t} = \frac{\sum_{n=1}^N I^k_{c,nt} + \sum_{m=1}^M I^k_{\vartheta,mt}}{N+M}, \ w^*_{4,t} = \frac{\sum_{n=1}^N I^k_{\tilde{a},nt}}{N+M},$$

$$w^*_{5,t} = \frac{\sum_{n=1}^N I^k_{\tilde{b},nt} + \sum_{m=1}^M I^k_{\tilde{\theta},mt}}{N+M},$$

$$w^*_{6,t} = \frac{\sum_{n=1}^N I^k_{\tilde{c},nt} + \sum_{m=1}^M I^k_{\tilde{\vartheta},mt}}{N+M}.$$

Hence, we have

$$\hat{\lambda}^{k+1}_t = \frac{w^*_{3,t}w^*_{5,t} - w^*_{2,t}w^*_{6,t}}{w^*_{1,t}w^*_{5,t} - w^*_{2,t}w^*_{4,t}}, \ \hat{\gamma}^{k+1}_t = \frac{w^*_{1,t}w^*_{6,t} - w^*_{3,t}w^*_{4,t}}{w^*_{1,t}w^*_{5,t} - w^*_{2,t}w^*_{4,t}}. \tag{24}$$

Proof: For the sake of brevity, we omit the details of the proof Nguyen et al. (2018).

Consequently, $\hat{\lambda}^{k+1}_t$ and $\hat{\gamma}^{k+1}_t$ can be calculated by each individual in a decentralized way. Substituting (24) into (18) and (20), U^{k+1} in (14) is directly obtained.

3.3 The Summary of Proposed Decentralized ADMM

The optimal solution to Problem 1 can be implemented in parallel by applying the ADMM iterations in (14), which is summarized in Algorithm 1.

The update of Z is implemented in a decentralized way due to the decoupling property of the admissible set and the objective function in (14), such that each individual could get the optimal solution $Z^{k+1}_n, n \in \mathcal{N}$ or $\bar{Z}^{k+1}_m, m \in \mathcal{M}$ by solving the following local optimization problems:

$$\min_{Z_n\in\mathcal{U}_n} \ \frac{\rho}{2}\|U^k_n - Z_n + V^k_n\|^2_2 + \frac{\psi}{2}\|Z_n - Z^k_n\|^2_2, \tag{25a}$$

$$\min_{\bar{Z}_m\in\bar{\mathcal{U}}_m} \ \frac{\rho}{2}\|\bar{U}^k_m - \bar{Z}_m + \bar{V}^k_m\|^2_2 + \frac{\psi}{2}\|\bar{Z}_m - \bar{Z}^k_m\|^2_2. \tag{25b}$$

Similarly, the update V^{k+1} in (14) could be carried out in parallel. Each individual only needs to solve:

$$V^{k+1}_n := V^k_n - \kappa\rho(U^k_n - Z^k_n), \ n \in \mathcal{N}, \tag{26a}$$

$$\bar{V}^{k+1}_m := \bar{V}^k_m - \kappa\rho(\bar{U}^k_m - \bar{Z}^k_m), \ m \in \mathcal{M}. \tag{26b}$$

We will refer to $r^{k+1} = U^{k+1} - Z^{k+1}$ and $s^{k+1} = -\rho(Z^{k+1} - Z^k)$ as the primal residual and dual residual at iteration $k+1$, respectively. The update process in the proposed method is repeated until

$$\|r^{k+1}\| \le \epsilon^{pri} \text{ and } \|s^{k+1}\| \le \epsilon^{dual} \tag{27}$$

where $\epsilon^{pri} > 0$ and $\epsilon^{dual} > 0$ are feasibility tolerances.

Algorithm 1. (Implementation of Energy Coordination via the Decentralized Consensus-Based ADMM.)

- Set the initial state $U^0 = 0, Z^0 = 0, V^0 = 0$;
- Set the feasibility tolerances $\epsilon^{pri}, \epsilon^{dual}$;
- Set $k = 0$, $\|r\| > \epsilon^{pri}$ and $\|s\| > \epsilon^{dual}$;
- While $\|r\| > \epsilon^{pri}$ or $\|s\| > \epsilon^{dual}$
  - All individuals implement the consensus protocol (22) with the initial values in (21), then calculate $\hat{\lambda}^{k+1}_t$ and $\hat{\gamma}^{k+1}_t$ by (24).
  - Determine the update of $U^{k+1}_n, \bar{U}^{k+1}_m$ based on (17) and (19), the update $Z^{k+1}_n, \bar{Z}^{k+1}_m$ in (25), and $V^{k+1}_n, \bar{V}^{k+1}_m$ in (26) for $n \in \mathcal{N}, m \in \mathcal{M}$ independently.
  - Update $r = U^{k+1} - Z^{k+1}$; $s = -\rho(Z^{k+1} - Z^k)$;
  - Update $k := k+1$. ■

Remark: The convergence criterion in (27) could be computed by each individual in paralle based on the consensus protocol, which is introduced in Nguyen et al. (2018).

Furthermore, we consider the pricing scheme for the proposed energy system. At iteration step $k+1$ in Algorithm 1, denote the desired electricity price and heat energy price of each CHP unit and consumer respectively by $\pi^{k+1}_{p,nt}, \pi^{k+1}_{h,nt}$ and $\bar{\pi}^{k+1}_{p,mt}, \bar{\pi}^{k+1}_{h,mt}$ at t for all $n \in \mathcal{N}, m \in \mathcal{M}$.

When each CHP unit and consumer tend to maximize their own profits, based on (7) and (8) we have the following equalities:

$$\frac{\partial W_n(u_{nt})}{\partial p_{nt}} = \frac{\partial W_n(u_{nt})}{\partial h_{nt}} = \frac{\partial W_m(\bar{u}_{mt})}{\partial \bar{p}_{mt}} = \frac{\partial W_m(\bar{u}_{mt})}{\partial \bar{h}_{mt}} = 0,$$

which gives

$$\begin{aligned} \frac{\partial C_n}{\partial p_{nt}} &= (1-\eta_n)\pi^{k+1}_{p,nt}, \ \frac{\partial C_n}{\partial h_{nt}} = (1-\varrho_n)\pi^{k+1}_{h,nt}, \\ \frac{\partial F_m}{\partial \bar{p}_{mt}} &= (1+\bar{\eta}_m)\bar{\pi}^{k+1}_{p,mt}, \ \frac{\partial H_m}{\partial \bar{h}_{mt}} = (1+\bar{\varrho}_m)\bar{\pi}^{k+1}_{h,mt}. \end{aligned} \tag{28}$$

It means that the price profile is coincident with the marginal cost.

Let $\pi^*_{p,nt}, \pi^*_{h,nt}$ and $\bar{\pi}^*_{p,mt}, \bar{\pi}^*_{h,mt}$ denote the optimal (converged) prices of each CHP unit and consumer, λ^*_t and γ^*_t denote the optimal Lagrange multipliers at t, and V^*

represents the optimal value of scaled dual variable. It's verified that the optimal price profile can be obtained under certain conditions shown as theorem below.

Theorem 3.2. As the penalty parameter $\rho \to 0$, the optimal electricity prices $\pi^*_{p,nt}$ and $\bar{\pi}^*_{p,mt}$ approach the optimal Lagrange multiplier λ^*_t, and the optimal heat energy prices $\pi^*_{h,nt}$ and $\bar{\pi}^*_{h,mt}$ approach multiplier γ^*_t, $\forall n \in \mathcal{N}, m \in \mathcal{M}$.

Proof: It is derived based on Karush-Kuhn-Tucker conditions. For brevity, we omit the details of the proof. ■

4. NUMERICAL RESULTS

In this section, for illustrating the proposed results we will present the simulations on a network topology specified in Fig. 1, which composes of 2 CHP units and 3 aggregated demand units. The parameters of CHP units' cost functions and consumers' utility functions are taken from Guo and Henwood (1996); Nguyen et al. (2018). Assume all the users have a common power capacity size of 65 MWh. The stopping criterions ϵ^{pri} and ϵ^{dual} are all set as 10^{-3}, and the penalty parameter ρ is selected to be 0.01.

Fig. 2 shows the evolution of $\sum_{i\in\mathcal{N},\mathcal{M}} \|x_i^{\tau+1} - x_i^{\tau}\|_2^2$ given in Theorem 3.1 with $\zeta = 0.5$. We can know that the consensus protocol converges to the desired tolerance 10^{-3} only in about 12 iterations. Fig. 3 (left) displays the uncontrolled base demand, and the total demand which is the summation of base demand and all deferrable loads. It shows that the deferrable loads can be shifted to the valley by applying the proposed method. The heat demand is illustrated in Fig. 3 (right). There has a deviation between the reference heat demand and the actual demand, which is respect to the penalty parameter $\bar{\alpha}_{mt}$ given in (10).

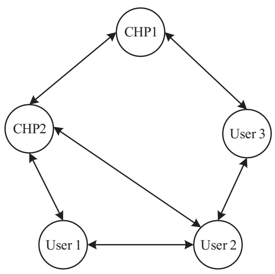

Fig. 1. Network topology of the energy system.

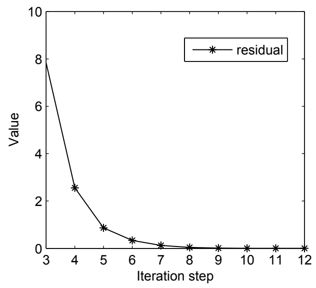

Fig. 2. Convergence of the consensus protocol.

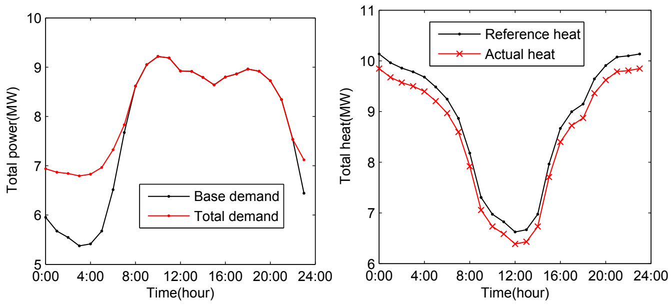

Fig. 3. Aggregate coordination strategies of power and heat.

5. CONCLUSION

The paper explores the coordination of the power and heat in the energy management system, which considers the ED problems of CHP units and DR problems of consumers. The objective is to maximize the total profit of the system subject to the energy balance constraint and local constraints of each individual. There is no central entity and the individual local information can only be exchanged with its neighbours. A decentralized consensus-based ADMM approach is developed to achieve the optimal solution, in which the agreed energy price is derived by applying the consensus protocol. The convergence and optimality are verified under certain conditions.

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